



**HAL**  
open science

# Period-Doubling Bifurcation and Intermittency Route to Chaos in Semiconductor Lasers with Dual Feedback

Qin Zou, Kamel Merghem, Mwamsojo Nickson

## ► To cite this version:

Qin Zou, Kamel Merghem, Mwamsojo Nickson. Period-Doubling Bifurcation and Intermittency Route to Chaos in Semiconductor Lasers with Dual Feedback. Optique Nice 2022, Jul 2022, Nice, France. hal-03716166v2

**HAL Id: hal-03716166**

**<https://hal.science/hal-03716166v2>**

Submitted on 5 Dec 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Period-Doubling Bifurcation and Intermittency Route to Chaos in Semiconductor Lasers with Dual Feedback

Qin Zou, Kamel Merghem, Nickson Mwamsojo

Laboratoire Samovar, Télécom SudParis, Institut Polytechnique de Paris, Institut Mines-Télécom  
19 place Marguerite Perey, 91120 Palaiseau, France  
e-mail: [qin.zou@telecom-sudparis.eu](mailto:qin.zou@telecom-sudparis.eu)

**Abstract:** We investigate the bifurcations and the intermittency route to chaos in a semiconductor laser subject to a twofold external perturbation: a coherent optical feedback from a distant dielectrical mirror and an optoelectronic feedback originated from this external optical feedback. The characteristic behavior of the compound laser is described by means of the formalism that we developed. The transition to chaos via intermittency in this dual-feedback system is studied by using the phase of the dielectrical mirror as external parameter. First results are presented.

## 1. Description

The configuration of the considered system is schematized in Figure 1. The laser diode is driven by two electrical sources: a constant current  $J$  and a current proportional to a delayed electrical field from a distant dielectrical mirror with amplitude reflection coefficient  $r_2 \exp(-iD)$  ( $0 \leq r_2 \leq 1$ ). The alternating-current term, proportional to the intensity of the returned electrical field, is thus written as  $\rho\eta|E(t-\tau-t_0)|^2$ , where  $\tau$  is the external-cavity round-trip time,  $t_0$  is the delay time due to the optoelectronic loop,  $\eta$  (dimensionless) is the normalized feedback rate which is proportional to  $r_2$ , and  $\rho$  (in  $s^{-1}$ ) is a time-independent coefficient. In our study,  $\rho$  is assumed real. It can take positive as well as negative values, depending on the characteristics (amplification or attenuation) of the external electrical circuit associated with the photodiode PD.

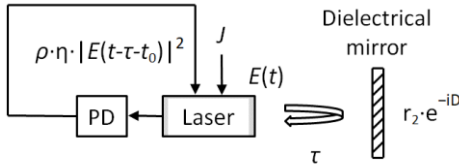


Figure 1. Principle scheme of the studied dual-feedback system. The external optical feedback is produced by a distant dielectrical mirror of reflection coefficient  $r_2$  and phase  $D$ . Compared with the commonly used configuration (as referred to in [1]), this structure involves two additional parameters: the phase factor  $\exp(-iD)$  of the dielectrical mirror and the gain (or loss) coefficient  $\rho$  of the optoelectronic loop.

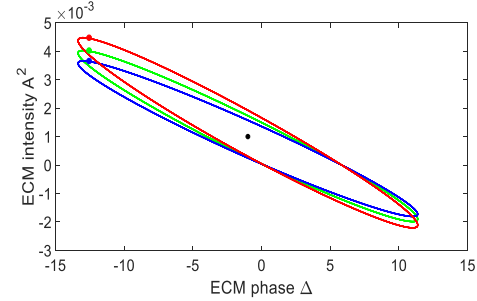


Figure 2. Dependence of the ellipticity of the so-called ECM ellipse on the sign of  $\beta$  (for  $\alpha=4$ ). Green curve:  $\beta=0$  (pure optical feedback); Blue curve:  $\beta=-0.1$  (dual-feedback with attenuating optoelectronic loop); Red curve:  $\beta=0.1$  (dual-feedback with amplifying optoelectronic loop). The color dots represent the corresponding maximum gain modes. The black dot refers to the solution for the solitary laser. The values of the parameters are  $P=10^{-3}$ ,  $\Delta_0=-1$ ,  $\theta=10^3$ , and  $\eta=0.003$  [2].

In this dual-feedback system, both the amplitude and the phase of the distant mirror can alter the properties of the returned electrical field. To model this system, we modified the equations of Lang-Kobayashi [1] by introducing the phase term  $D$  into the rate equation for the electrical field and the term  $\rho\eta|E(t-\tau-t_0)|^2$  into the rate equation for the carrier density. Assuming small perturbations from the external mirror ( $r_2 \ll 1$ ), we seek solutions for an external cavity mode (ECM) of the form  $Y = Ae^{i\phi}$ , as functions of  $\rho$  and  $D$ , to see how these two parameters affect the behavior of the compound laser. The dependence of the ECM ellipse on the parameter  $\beta$  ( $\beta = \rho\Gamma_0^{-1}$ , with  $\Gamma_0$ : inverse photon life time) is shown in Figure 2. We calculated the phase iterates and the corresponding Lyapunov exponent for three typical values of  $D$  (Figure 3). When  $D = 0$ , the phase delay between  $E(t)$  and  $E(t-\tau)$  is  $\Delta_0$ , which corresponds to the time delay  $\tau$ . In the case of  $D = -\Delta_0$ , the external mirror «compensates» the phase delay between  $E(t)$  and  $E(t-\tau)$ . If  $D = \Delta_0$ , the external mirror «doubles» the phase delay between  $E(t)$  and  $E(t-\tau)$ . We also investigated the so-called intermittency itinerary to chaos [3] of this system. The cobweb diagrams and the time-series plots for a given value of  $r_2$  with these three  $D$  values are illustrated in Figure 4.

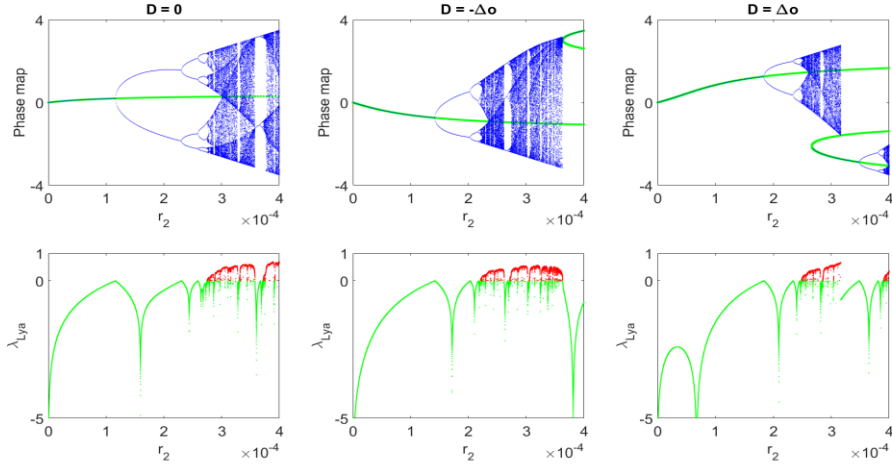


Figure 3. Normalized phase map (blue dots) of ECMs (We note that all possible stationary solutions, physical or not and dynamically stable or not, should be located on the green curves.) and corresponding Lyapunov exponent  $\lambda_{Lya}$  of the studied system with three typical values of  $D$ . Left column ( $D=0$ ): bifurcation route to chaos in usual configuration for external optical feedback. Middle column ( $D=-\Delta_0$ ): beginning of the second period-doubling sequence after a saddle-node bifurcation. Right column ( $D=\Delta_0$ ): a saddle-node bifurcation occurs at  $r_2 \approx 2.677 \cdot 10^{-4}$ , but chaotic behavior dominates in the range of  $r_2 \approx 2.677 \cdot 10^{-4}$  to  $r_2 \approx 3.158 \cdot 10^{-4}$  ( $\lambda_{Lya} > 0$ , as represented by red dots). This value ( $3.158 \cdot 10^{-4}$ ) is effectively the onset value of the second period-doubling sequence. We also observe a super-stable solution at  $r_2 \approx 0.69 \cdot 10^{-4}$  for the period-one ( $P_1$ ) modes. Finally, for these three cases, the onset values of  $r_2$  corresponding to the first  $P_1$ -to- $P_2$  bifurcation are respectively  $1.154 \cdot 10^{-4}$ ,  $1.419 \cdot 10^{-4}$ , and  $1.836 \cdot 10^{-4}$ . That means the compound laser constructed with  $D=\Delta_0$  would be more resistant to external optical feedback with respect to the former two cases.

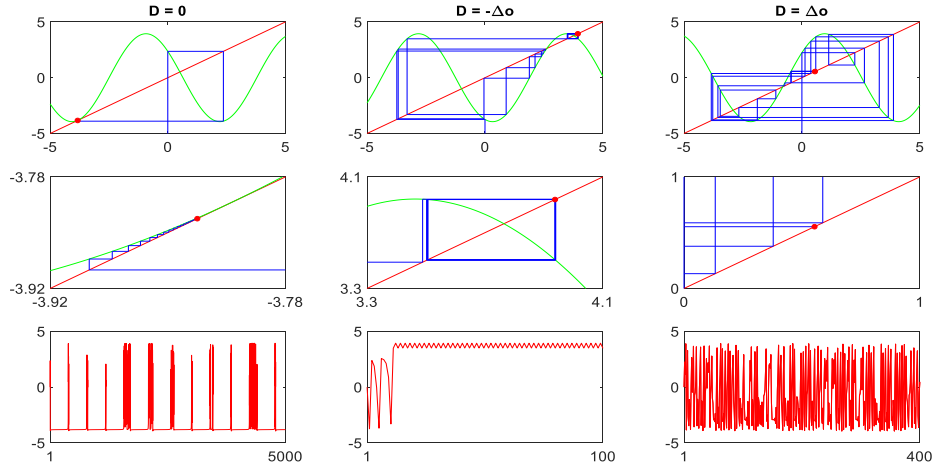


Figure 4. Transition to chaos via intermittency in the compound-laser system. Cobweb diagrams at  $r_2 = 1.7879 \cdot 10^{-3}$  [ $\alpha = 3$ ,  $\tan(\psi) = \alpha$ , and  $\Delta_0 = \pi + \psi$ ] for three typical values of  $D$ . The red dot represents the solution after 20 iterations (top). Zoomed windows (middle). Time-series plots (bottom). When  $D = -\Delta_0$ , the system trends to a stable-oscillation solution. In the case of  $D=0$ , the pulsating oscillations are interrupted by short irregular (chaotic) bursts. The system becomes truly chaotic if  $D = \Delta_0$ .

## 2. Conclusion

The dual-feedback system that we proposed and studied provides an alternative for better understanding the characteristic behavior of a laser diode in each of its operating regimes (periodic or quasi-periodic oscillation, bifurcation, or chaos). The chaotic dynamics of a coherent feedback system subject to a «forced perturbation» (optical injection) will be the object of our further investigation, both theoretically and experimentally.

## 3. References

- [1] R. Lang and K. Kobayashi, "External Optical Feedback Effects on Semiconductor Injection Laser Properties," IEEE Journal of Quantum Electronics, vol. 16, No. 3, 347-355 (1980).
- [2] T. Erneux and P. Glorieux, *Laser Dynamics*, (Cambridge University Press, 2010).
- [3] Heinz G. Schuster, «Deterministic Chaos: An Introduction», Weinheim: Physik-Verlag, 1984.