Design Techniques of Spatially Coupled Low-Density Parity-Check Codes: A Review and Tutorial on 5G New Radio

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Abstract

As active as the research is on the various possible uses of 5G and B5G (beyond 5G), we herein make a tutorial and review on the existing spatial coupling techniques that are used in the protograph-based design of Spatially Coupled Low-Density Parity-Check (SC-LDPC) codes. We unroll useful details for the computing of these techniques, implement them in the context of the 5G standard and draw up their performances. As a main result in terms of lesson learnt, a guide is provided to select the most appropriate spatially coupled technique for the three main 5G services and for three of its numerous use cases. Three research tracks are pointed out.

Keywords

5G; LDPC codes; SC-LDPC codes; tail-biting SC-LDPC codes; braided SC-LDPC codes; tail-biting braided SC-LDPC codes.

I INTRODUCTION

The 5G, 5th generation of mobile communication standards, is since 2019 to nowadays a reality in several cities around the world. Research on 5G is increasingly active to propose, in connection with its three types of uses (enhanced Mobile Broad Band (eMBB), Ultra-Reliable Low-Latency Communications (URLLC), massive Machine Type Communications (mMTC)), a diversity and multitude of use cases (e.g.: in health, smart societies, education, etc.). At the same time, the B5G (beyond 5G) and the evolution toward the 6G standard are underway. In this dynamic, many works have focused on improving the performance of the Low-Density Parity-Check (LDPC) codes. In the 5G New Radio (5G NR), the air interface of the 5G standard, block LDPC codes have been specified [1]. Indeed block LDPC codes have shown to achieve performance very close to the Shannon limit [2]. A new family of LDPC codes, originally referred to as LDPC convolutional (LDPC-CC) codes [3] and more recently as spatially coupled LDPC (SC-LDPC) codes [4], are built from block LDPC codes by introducing a memory effect [3]. SC-LDPC codes have excellent performance promise over a wide variety of channel conditions [4]. Many papers are written in the published literature on the conception of SC-LDPC codes [5]-[9]. However, of the different ways of achieving spatial coupling, there are only five: one is based on an algebraic structure and four are protograph-based.

The basic idea of the first construction technique of SC-LDPC codes was introduced in 1981 through a patent [10]. Rediscovered years later [11], it is based on an algebraic structure. The second technique is based on a construction called unwrapping procedure. It uses protograph to generate large graphs [3]. The third technique is called tail-biting. It uses protographs to design SC-LDPC with rate-loss mitigation [5]-[8]. The fourth technique is related to braided convolutional codes. It is based on protographs and founded on the braided concatenation of convolutional codes [9]. We denoted by braided SC-LDPC codes, the resulting code from this technique. The fifth technique is the tail-biting version of braided SC-LDPC codes [12] [13]. It allows to avoid the rate-loss due to termination saturation. Works involving all these techniques demonstrated good Bit Error Rate (BER) performance over theoretical block LDPC codes [14]-[23]. This performance having been obtained on theoretical LDPC codes, we are interested here in how these same techniques would behave on codes used in practice, in this case the LDPC codes used in the 5G NR. We are also interested in knowing what pratical lessons we could draw from the observed results.

Our objective is to focus on the spatial coupling techniques used in the construction of protograph-based SC-LDPC codes in order, for each of these techniques, to make a state of the art, and explain step by step how they can be implemented on 5G LDPC codes. It is also for us to analyze these techniques together with the results that they can yield when applied to 5G codes in order to rank them according to their suitability to each of the three 5G uses.

Our literature search involved the keywords "LDPC-CC", "SC-LDPC codes" and "protographs". Scientific databases of IEEE and Elsevier were used together with other resources like channel coding books and doctoral dissertations and the Google scholar search engine. In the remainder of this paper, Section II deals with the review and tutorial on the different techniques of spatial coupling, Section III is about a comparative analysis together with main conclusions and future research directions, Section IV concludes the paper.

II SC-LDPC CODES DESIGN: A REVIEW AND A TUTORIAL

In this Section, we firstly present the different techniques used to achieve spatial coupling in the design of SC-LDPC codes. Secondly, related works are presented for each of these techniques. Thirdly, we explain step by step how to implement each technique for the construction of a 5G SC-LDPC code. For background on LDPC and SC-LDPC codes, one could refereed to the works in [3]-[4] and [24]-[32]. Details about the specification of the LDPC code for the 5G standard are also given in Annex 1.

2.1 Unwrapping technique

The unwrapping technique used to design SC-LDPC codes was presented by Jiménez-Feltström and Zigangirov in 1999 [3]. In [6], the notions of unterminated and terminated regular LDPC-CC are evoked. Even though the Tanner graph representation of an LDPC-CC code proceeds to infinity in time, practically speaking, it always has a finite start and end time, i.e., the Tanner graph terminates both at the beginning and at the ending. We speak of a completed Tanner graph, and of a terminated LDPC-CC. It is shown in [24] [25] that for a set of terminated LDPC-CC, the iterative decoding thresholds are better than for the corresponding regular and irregular block LDPC codes. In the papers [21] [34], the authors present the fundamental concepts of coding for lightwave systems, namely the use of spatially coupled codes. In [35] and [36], the IEEE 1901 standard for power line communications is studied, where an LDPC-CC with unwrapping technique is specified.

The unwrapping technique is based on a matrix unpacking scheme to get the parity-check matrix of a periodically time varying convolutional code from the block code parity-check matrix. The process performs copy and paste as well as diagonal matrix expansion operations on a block parity-check matrix to generate a bi-infinite convolutional parity-check matrix [3]. Details on how to design code using the unwrapping technique can be found in Annex 2.

2.2 Tail-Biting technique

LDPC-CC have some advantages over block LDPC codes, especially for the transmission of streaming data [7] or data in packets of various lengths, as the same encoder can be employed to encode data blocks of various lengths. An interesting aspect of LDPC-CC is that the same encoder may be used to generate code sequences of different lengths with reasonably high performance by choosing several termination lengths. However, the insertion of a termination produces a rate-loss, which is particularly noticeable for short frame lengths. The introduction of the tail-biting technique avoids this loss. Tail-biting was originally presented by Solomon and Van Tilborg [6] and, separately, by Ma and Wolf [5] as a technique for terminating a convolutional code with none of the rate-loss due to standard termination. The tail-biting codes that result from have the minimum distance of both convolutional and block codes.

A spatially coupled tail-biting LDPC code can be produced from an SC-LDPC code by combining the control nodes at times $t = L, L + 1, ..., L + m_s - 1$ with the matching control nodes of the same design at times $t = 0, 1, ..., m_s - 1$, respectively. Therefore, the base matrix corresponding to a tail-biting SC-LDPC code defined in this way has a size equal to $b_c.L \times b_v.L$. By comparing the two base matrices (terminated SC-LDPC and tail-biting SC-LDPC), one can easily see that the base matrix of tail-biting SC-LDPC can be obtained from the base matrix of terminated SC-LDPC by adding its last $b_c \times m_s$ rows to the first $b_v \times m_s$ rows. As such, tail-biting SC-LDPC codes have the same code rate and degree distribution as the matching protograph codes.

2.3 Braided technique

Braided block codes were introduced for the first time in [37], [38]. This method of constructing LDPC codes by spatial coupling is analogous to the design of LDPC codes using protographs and constitutes a novel approach to get code sets having robust distance properties, robust iterative thresholds, and low-complexity encoding/decoding from basic codes [12]. The codes resulting from this technique fall into the category of LDPC-CC [9] [13], called braided LDPC-CC. However, compared to braided block codes (BBC) [37]-[39], braided convolutional codes (BCC) [9] use low-stress length convolutional codes as candidate codes. BBC and BCC are both similar in their coding procedures. They are built using a bidimensional infinite array composed of a horizontal and a vertical encoder. Both encoders are connected by a return of parity. In this way, parity and systematic symbols are braided at the same time.

2.4 Tail-Biting braided technique

As in [5] and [12], it is possible to derive tail-biting versions of braided LDPC-CC codes to eliminate the problem of rate-loss due to termination saturation. The tail-biting periodic LDPC-CC codes are quasi-cyclic codes. Therefore, it is possible to encode them in linear time, by use of circuits relying on shift registers [40]. The methods described in [41] can also be used and very successful coding structures can be generated. The implementation of this technique is a combination of the two previous methods, that is, the tail-biting braided SC-LDPC codes are constructed using an array composed of a vertical and a horizontal encoder. Both encoders are connected by a parity return to braid together the parity and systematic symbols. Indeed, introducing a termination always results in a rate-loss. A new code is generated from the braided code by combining the control nodes with corresponding control nodes of the same nature, respectively.

III COMPARATIVE ANALYSIS, MAIN RESULTS AND RESEARCH TRACKS

Starting from the 5G standard block LDPC code, we have developed a redesigned version to evaluate an SC-LDPC code with $m_s = 1$ and $L = 10$. Let's denote by LDPC-5G this 5G block LDPC code. Let's also denote by : SC-LDPC-5G the modified version of LDPC-5G using the unwrapping technique, SC-LDPC-Tail-Biting-5G the modified version of LDPC-5G using the tail-biting technique, SC-LDPC-Braided-5G the modified version of LDPC-5G using the braided technique and SC-LDPC-Tail-Biting-Braided-5G the modified version of LDPC-5G using tail-biting braided technique. The resulting BER performance of the five codes (LDPC-5G, SC-LDPC-5G, SC-LDPC-Tail-Biting-5G, SC-LDPC-Braided-5G and SC-LDPC-Tail-Biting-Braided-5G) is shown in Figure 1. In Annex 3, useful information about the simulation is provided.

The unwrapping technique code achieves 0.8 dB additional coding gain at a BER of 10^{-6}. Good performance is so obtained over the standard 5G code. However, as an insufficiency the introduction of a termination introduces a rate-loss. As about the code designed using the tail-biting technique, 0.9 dB of additional coding gain is obtained for a BER of 10^{-6}. The performance is little better than the unwrapping technique with at plus no rate-loss. The evaluation of the code built on the braided technique yields an additional coding gain of 1.1 dB for a BER of 10^{-6}. So further better performance is achieved and this third technique outperforms the first two ones. This is in line with the literature based on theoretical codes, as it has been illustrated that the iterative decoding performance is significantly increased with block-braided codes. However, the limitation of this method remains the presence of rate-loss. As about the last technique consisting of a hybridation of the tail-biting technique and the braided technique, a 1.5 dB of additional coding gain at a BER of 10^{-6} is achieved. The best performance is so achieved compared to all the previous techniques with the advantages of both the tail-biting and the braiding : a code without rate-loss nor performance degradation. As a conclusion, spatial coupling gives a significant improvement in overall BER performance regardless of the coupling method that is used.

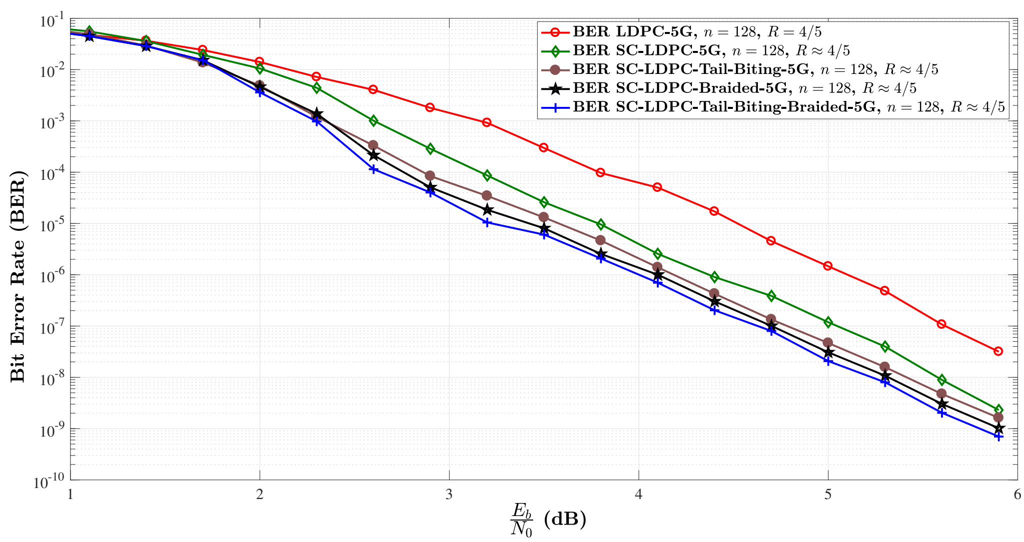

Figure 1: BER of various techniques of SC-LDPC 5G codes with $N = 128$ and $R = 4/5$.

From the simulation results discussed above, we derived some main results as lessons learnt in Table 1 and Table 2. Indeed, Table 1 is provided as a guide to better choose the spatial coupling technique according to the three main categories of 5G uses. Table 2 is more focused on three 5G use cases, selected as challenging for many African countries, to show how each of the four spatial coupled techniques can suit to each of these three use cases. The marker "+" means that the technique is appropriate, and the more it is repeated, the more the corresponding technique is appropriate. The choice of one of the spatial coupling techniques for a given service is fundamentally related to reliability, throughput, latency, complexity and number of the connected objects.

Spatial coupling technique / 5G type of use	Benchmark	Unwrapping	Tail-Biting	Braided	Tail-Biting Braided
URLLC	+	++	+++	+++	+++
eMBB	+	++	++	++	++
mMTC	++	+++	++++	+++++	++++++

Table 1: Spatial coupling techniques and their level of suitability to 5G type of uses.

Spatial coupling technique / 5G use cases	Benchmark	Unwrapping	Tail-Biting	Braided	Tail-Biting Braided
Enhanced agricultural productivity	+	+++	++++	+++	++++
Improved remote education	+	++	+++	++	+++
Advanced healthcare	+	++	++	++	++

Table 2: Spatial coupling techniques and their level of suitability to 5G use cases.

URLLC transmissions require, for certain uses, high reliability with very constrained latency times [47]. The eMBB aims to support the ever-increasing end-user data rate and capacity of the system [48]. Also the mMTC communications aim for robust and cost-effective connection of billions of devices without overloading the network [48]. For the improved remote education, high speed, higher capacity and high reliability is required [50]. Reliability, mobility and capacity is important for the case of advanced healthcare [49]. Cost reduction, resource consumption reduction and high capacity connectivity is imposed for the case of agricultural productivity improvement [50].

From this study, we have drawn above practical lessons for the three 5G basic types of uses and for three of its many use cases. As about its implications for research, we can note three tracks for further research. The first track relies on the fact that our study was limited to a spatial coupling memory equal to unity. It seems to us important to investigate how larger values of the coupling memory act on the performance. The second track is to analyze the performance of these techniques in terms of Block Error Rate (BLER). Indeed, for pratical interest, BLER performance analysis is more relevant. The third track is to study the competitiveness of these different techniques with respect to the decoder blocking problem in a sliding window decoding scheme.

IV CONCLUSION

In this paper, we made a review of the different techniques for performing spatial coupling of LDPC codes for 5G. For each technique, we presented the related works. We then made a tutorial exhibition of how to implement each of them. We finally proceeded with a comparative analysis using simulation results, and provided implications for industry and research. In a nutshell, this study could be used as a guide to make a judicious choice of the SC-LDPC code design technique that suits for a target 5G use case and opens a path toward the next wireless communication standard.

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A ANNEX 1 : 5G STANDARD LDPC CODES

We technically present the 5G code and its construction. Quasi-cyclic LDPC (QC-LDPC) codes have been accepted by 3GPP (3rd Generation Partnership Project) as a channel coding scheme for data channels [1]. These codes are developed to support various expansion factors. To correctly adapt to various code rates and block sizes, they also have rate-compatibility properties. Support for multiple expansion factors is handled by these codes and they have rate compatibility properties. The block sizes of the codes and the coding rates required by 3GPP are specified in papers [29][49]. The base matrix is subdivided into sub-matrices (SM1, SM2, SM3, SM4 and SM5) [28], for an efficient implementation of the codec (Figure 2). LCE are extension checks and LCN are core checks.

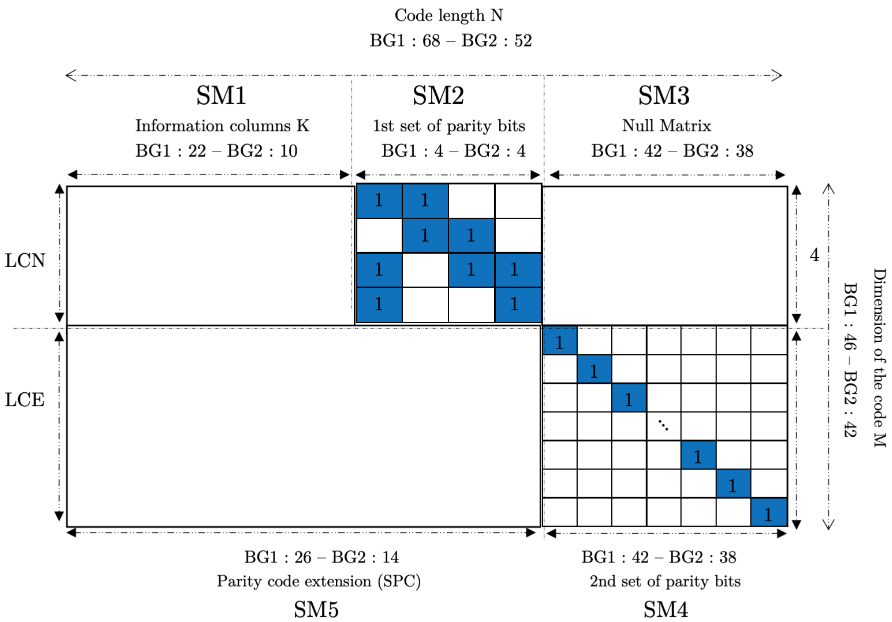

Figure 2: Structure of 5G LDPC Codes Base Matrix.

Index Set	Values of A	Values of j	Set of expansion factors Z
0	2	$\{0,1,2,3,4,5,6,7\}$	$\{2,4,8,16,32,64,128,256\}$
1	3	$\{0,1,2,3,4,5,6,7\}$	$\{3,6,12,24,48,96,192,384\}$
2	5	$\{0,1,2,3,4,5,6\}$	$\{5,10,20,40,80,160,320\}$
3	7	$\{0,1,2,3,4,5\}$	$\{7,14,28,56,112,224\}$
4	9	$\{0,1,2,3,4,5\}$	$\{9,18,36,72,144,288\}$
5	11	$\{0,1,2,3,4,5\}$	$\{11,22,44,88,176,352\}$
6	13	$\{0,1,2,3,4\}$	$\{13,26,52,104,208\}$
7	15	$\{0,1,2,3,4\}$	$\{15,30,60,120,240\}$

Table 3: Supported Expansion Factors in the 5G New Radio.

The set of expansion factors supported for the graph are all values of the form $Z = A2^j$ for $A \in \{2,3,5,7,9,11,13,15\}$ and $j \in \{0,1,2,3,4,5,6,7\}$. The expansion factors supported for the 5G NR QC-LDPC code are listed in Table 3 and are organized into 8 sets, one for each value of A. Two base graphs (BG1 and BG2) are defined to cover the wide range of data rates and block sizes that must be supported in 5G. The details of the BG1 and BG2 matrices are summarized in Table 4 and [49].

B ANNEX 2 : CONSTRUCTION OF UNWRAPPED SC-LDPC CODE

Let $(b,c) \in N \times N$. A protograph is a bipartite graph that has $c-b$ constraint nodes and c variable nodes with b representing the size of the coded block [38]. A protograph is used to represent the parity-check matrix of a block code of rate $R = b/c$. The corresponding base matrix is of dimension $(c-b) \times c$ with c also representing the block length. To design a base matrix of a spatially coupled code from a base matrix B of an block LDPC code,

Parameters	BG1	BG2
Minimum coding rate	1/3	1/5
Maximum coding rate	8/9	2/3
Size of the base matrix	46 × 68	42 × 52
Number of systematic columns	22	{6, 8, 9, 10}
Number of non-zero elements	316	197
Number of parity check matrices	51	51
Maximum expansion factor	384	384
Minimum expansion factor	2	2
Maximum information block size	8448	3840

Table 4: BG1 and BG2 parameters of 5G LDPC code.

some memory is introduced into the coding process. An unwrapping of the protograph branches of B is performed to obtain a convolutional protograph (CC-P). For this purpose, an unwrapping constraint is first determined, it is defined by :

$$\sum_{i=0}^{m_s} B_i = B \tag{1}$$

where B_i, $0 \leq i \leq m_s$, are submatrices of B and m_s is the memory size of the CC-P. The unwrapping mechanism is equivalent to the subdivision of B such that (1) is satisfied. The terminated CC-P corresponds to a convolutional base matrix (CC-BM) in the case of terminated SC-LDPC codes. The CC-BM matrix is defined by:

$$B_{[0,L-1]} = \begin{bmatrix} B_0 & & \\ \vdots & \ddots & \\ B_{m_s} & & B_0 \\ & \ddots & \vdots \\ & & B_{m_s} \end{bmatrix}$$

where L is called the termination factor. The resulting base matrix $B_{[0,L-1]}$ is of dimension $(L+m_s).b_c \times L.b_v$ and each submatrix B_i is of dimension $b_c \times b_v$. The coding rate of SC-LDPC is then $R_{SC} = [L.b_v - (L+m_s).b_c]L.b_v$. We get a terminated regular SC-LDPC ensemble, if the graph expansion process (also called graph covering by the Z-factor [39]) is applied to the CC-P.

We explain now how we begin from the design of standard codes to produce a novel spatially coupled code. The base matrix B is decomposed into two sub-matrices B_0 and B_1 to generate a memory effect with depth m_s. This decomposition is illustrated on an example with a clear explanation of the approach that we followed to obtain the new codes.

To illustrate, we consider a block code protograph with 2 variable nodes and 1 constraint node. The base matrix $B = [3, 3]$ defines the parity-check matrix of dimension 1×2. Let be a convolutional protograph coupled with the introduction of a memory equal to 2. The unwrapping is given by the relation $B = B_0 + B_1 + B_2$ and B_{cc} represents the completed convolutional base matrix. The block code is of dimension 1×2 with a coding rate $1/2$. The convolutional base matrix B_{SC} is of dimension $(L + 2) \times 2.L$ with code rate $(L - 2)/2.L$ and constraint length $6Z$. Now, the graph expansion results in an LDPC convolutional code set with a regular termination $(3, 6)$.

$$\mathbf{B}_{cc} = \begin{bmatrix} 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & & & \\ 1 & 1 & 1 & 1 & \ddots & & \\ & & 1 & 1 & \ddots & 1 & 1 \\ & & & & \ddots & 1 & 1 \\ & & & & & 1 & 1 \end{bmatrix} \qquad \mathrm{B}_{[0,3]} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

C ANNEX 3 : USEFUL INFORMATION FOR SIMULATIONS

We simulated the performance of the newly designed codes in order to analyze the increase in reliability that can be achieved by introducing spatial coupling according to the different techniques reviewed in Section II. We considered a BPSK (Binary Phase Shift Keying) transmission on an additive white Gaussian noise (AWGN) channel. The belief propagation (BP) decoding approach was employed to do decoding of 5G standard LDPC codes that are based on the parity-check matrices in [45]. The maximum number of decoding iterations was set to 200 to achieve the best performance. We consider that channel codes with short block length and low code rate are recommended for highly reliable data transmissions [46]. We present BER performance results for 5G codes and make a comparison with the performance of their spatially coupled counterparts that are obtained by the newly developed conceptions. The performance of different LDPC-5G codes with rates $4/5$ are considered in our simulations, representing a favorable condition for high-speed transmission. We constructed the SC-LDPC-5G, SC-LDPC-Tail-Biting-5G, SC-LDPC-Braided-5G and SC-LDPC-Tail-Biting-Braided-5G codes with 128 bit block lengths from different SC-LDPC codes. We have constructed modified versions to obtain the SC-LDPC codes with coupling length $L = 10$ and memory $m_s = 1$.