A Study of the Impact of Predictive Maintenance Parameters on the Improvment of System Monitoring

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A Study of the Impact of Predictive Maintenance Parameters on the Improvment of System Monitoring

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Abstract: Predictive maintenance can be efficiently improved by studying the sensitivity of the maintenance decisions with respect to changes in the proposed model parameters (costs, duration of reparation, etc.). To address this issue, we first propose an original approach that includes both maintenance costs and maintenance risks in the same objective function to minimize. This approach uses the RUL as an indicator of the health state of the system and supposes that the system is under regular inspections and can only be replaced by a new system in case of serious deterioration or failure. Then, we present a process of human decision making under uncertainty based on several criteria. Finally, we study and analyze the influence of the model parameters and their implications on the obtained maintenance policies. The study will lead to some recommendations that can improve the predictive maintenance decisions and help experts better handle maintenance costs.

Keywords: predictive maintenance; sensitivity analysis; remaining useful life; cost optimization; human decision; mechanical bearing system

MSC: 68U35

Citation: Louhichi, R.; Sallak, M.; Pelletan, J. A Study of the Impact of Predictive Maintenance Parameters on the Improvment of System Monitoring. *Mathematics* **2022**, *10*, 2153. https://doi.org/10.3390/math10132153

Academic Editor: Tadashi Dohi

Received: 29 April 2022
Accepted: 16 June 2022
Published: 21 June 2022

1. Introduction

Industry 4.0, initially emerged in Germany, is now attracting more and more scientists and is starting to be widely adopted in modern societies [1–3]. According to [1], a fundamental concept of industry 4.0 is "cyberphysical systems" where the physical and the digital level merge. An example of "cyberphysical systems" is the use of process parameters (such as stress, temperature, etc...) of a mechanical system undergoing a physical degradation [1]. These parameters are recorded digitally (mainly through sensors) and the real conditions of the system are defined from the physical object and the parameters of its digital process [1]. This is generally used in the area of predictive maintenance.

Predictive maintenance is a widespread maintenance practice that is based on regular monitoring of the condition of the system in order to determine the right time to maintain the system [1,4–7].Predictive maintenance is *realized "following a forecast derived from repeated analysis or known characteristics and evaluation of the significant parameters of the degradation of thesystem"* [6]. System condition monitoring, fault diagnosis, and fault prognosis are the pillars of predictive maintenance. Several approaches for fault diagnosis and prognosis are described in the literature [8–10]. These approaches can be data-driven approaches or physical-based model approaches. Data-driven approaches use real data on the health state of the system to predict the failure of the system while model-based approaches use physical models in the form of physical equations when enough knowledge on the real physical dynamics of a system are available [8,9]. In the existing literature, the health state of the system can be evaluated using different measures [5,11–15]. We cite the "Remaining

Useful Life" (*RUL*) as one of the most widely known measures [16–19]. The *RUL* is defined as the expected duration left for the system before it fails [5,11–13].

Even if predictive maintenance provides more advantages than other types of maintenance (such as corrective maintenance or preventive maintenance) [11,20–22], it remains a challenging task for industry to find the optimal time for predictive maintenance. The existing literature provides a large panel of methods to optimize the predictive maintenance strategy [23]. A cost model considering finite repair, durations of maintenance, and costs due to testing, repair, maintenance, and lost production or accidents was developed in [24]. The objective of the maintenance optimization is to minimize a total cost rate thanks to an appropriate selection of two intervals: one for inspections and one for replacements [24]. The case of predictive maintenance for systems exhibiting two-phase behavior: new condition and worn condition were analyzed and cost-minimizing policies were developed in [25] in order to determine when the system should be monitored. A sequential imperfect preventive maintenance policy was developed in [14,26], and the optimal schedule for maintenance, which minimizes the cost rate during the system's life cycle, was identified in [14,26] using the reliability as a measure of the health state of the system. More recently, a dynamic predictive maintenance policy for complex multicomponent systems was developed in order to minimize the long-term mean maintenance cost per unit time [27], and a decision-making method based on cyber manufacturing and mission reliability state was developed in order to identify the optimal way to maintain a production system [28]. Last but not least, an approach to jointly schedule missions and maintenance actions for a deteriorating vehicle was described in a recent work [29]. However, maintenance optimization may include risk minimization as described in [30,31]. According to the considered system, the available input data and the objective to be attained, risk minimization in maintenance can take two different types: optimization of an objective function where we aim to minimize a risk under constraints [11,31] and comparison of the evaluated risks with risk acceptance criteria [30].

In this work, we describe an approach for predictive maintenance optimization that includes both maintenance costs and maintenance risks in the same objective function to minimize. This approach uses the *RUL* as an indicator of the health state of the system and supposes that the system is under regular inspections and can only be replaced by a new system in case of serious deterioration or failure. In the context of industry 4.0, the process of inspection is automated as the system is equipped with connected sensors that collect instant data on the health state of the system. The process of transforming this data into a measurable value such as *RUL* or reliability level of the system is conducted by machines using techniques from artificial intelligence. Industry 4.0 has facilitated the decision making under time constraints [32,33]. The time between the detection of a potential failure and the time that the failure occurs can be seen as an opportunity for the decision makers to use adapted algorithms that allow them to minimize maintenance costs [32,33]. The process of decision making is complex, as it is dependent on the psychology of the decision maker and on whether the decision maker is risk averse or not. This complexity of decision making process under uncertainty has been tackled by Tversky and Kahneman in [34,35].

In this work, predictive maintenance is only performed once the *RUL* of the system reaches a certain threshold called RUL_{lim} under which the system is considered as deteriorated. The outputs of the proposed optimization approach are then the value of RUL_{lim} and the regular inspection step. This approach was briefly described in our previous work [11,36]. The novelty of the paper lies in the following points:

- A detailed analysis of the influence of input parameters of our approach on the optimization results. This study will help us understand the stakes of each input parameter and its role in the optimization results.
- Recommendations will be drawn from the influence analysis, which can lead to better results mainly through greater reduction of maintenance costs.

- A proposition of several criteria (Hurwicz, Wald, optimistic, etc.) to address the situations where the decision maker needs to make a decision on maintenance on the basis of an interval of RUL_{lim}.
- A case study concerning a rolling-element bearing system in order to illustrate the use of the three previous propositions.

2. Methodology Description

2.1. Assumptions

Our methodology is based on the following assumptions:

- The system under study is a unique component and it is integrated into a multicomponent system. This multicomponent system has a duration of exploitation D supposed to be known and constant.
- The system under study undergoes regular perfectly reliable inspections. An inspection informs the experts on the health state of the system. The inspection is performed thanks to connected sensors. The connected sensors transmit information on significant health parameters (such as, for example, temperature, pressure, etc.) to adapted software able to use these parameters to evaluate the RUL of the system using techniques inspired from artificial intelligence. An inspection gives a real estimation on the RUL of the system. After multiple simulations, the RUL is evaluated as the expected interval of time the system is likely to operate before it falls down. The RUL of the system can be evaluated thanks to Equation (1) [36]:

$$RUL(t) = E[T - t \backslash T > t] = \frac{\int_t^{\infty} (u - t) \cdot f(u) \cdot du}{S(t)} \quad (1)$$

  where T is the time of failure of the system, f is the failure density function of the system, and S is the survival function of the system.
- At $t = 0$, the system is new and there is no need to maintain it. However, we consider that an inspection at $t = 0$ is required. Once the system reaches the instant $t = D$, there is no need to perform inspection, and the system needs to be replaced by a new one (see Figure 1).

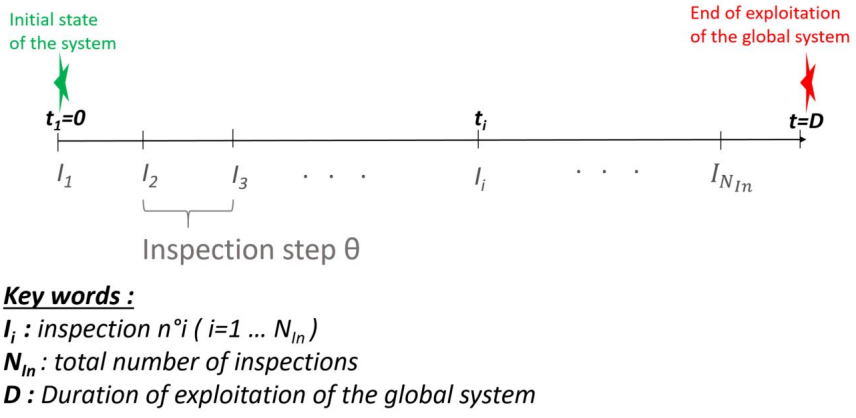

Figure 1. Inspection procedure.

- Between two consecutive inspections, one of these following scenarios may happen:
  - *Predictive maintenance scenario:* The RUL of the system attains some threshold value called RUL_{lim} under which the system is considered as deteriorated and should then be replaced by a new one.

- *Non-predictive maintenance scenario:* In this scenario, the system is not replaced by predictive maintenance. In this case, the system may fall down or continue to operate:
  - If the system fails before inspection $i+1$ knowing that he was operating at inspection i, a corrective maintenance should be performed on the system. The probability of occurrence of this scenario is equal to $\int_{t_i, T \geq t_i}^{t_{i+1}} f(t) \cdot dt = \frac{\int_{t_i}^{t_{i+1}} f(t) \cdot dt}{S(t_i)}$, where t_i is the time of the ith inspection.
  - If the system operates normally between inspections i and $i+1$, this scenario occurs with the complementary probability $1 - \int_{t_i, T \geq t_i}^{t_{i+1}} f(t) \cdot dt$.

- However, if the system reaches the duration of exploitation D without being replaced because the deterioration zone has not been reached, it is considered as bad as old and has to be replaced by a new one.
- Predictive and corrective replacements are assumed to have constant and known durations.
- Predictive and corrective replacements, as well as the inspection, are assumed to have constant and known costs.

2.2. Maintenance Costs

The maintenance costs include the cost of predictive maintenance, the cost of corrective maintenance, the cost of inspection, and the cost of operating loss due to maintenance. These types of costs are the widely used in literature [14,15,24,25]. Maintenance costs may include also the cost of maintenance risks. These risks can be of different types: human, environmental, or financial [11,37]. The reader may refer to our previous work in [11].

- *Predictive maintenance cost:* The cost of predictive maintenance C_p during the time cycle D can be evaluated using Equation (2):

$$C_p = \sum_{i=1}^{N_{in}-1} c_p \cdot N_i \tag{2}$$

  where c_p is the cost of a predictive replacement and N_i is a binary decision variable taking the value of 1 in case of a predictive maintenance between inspections i and $i+1$ and 0 elsewhere.
  Equation (2) takes into consideration the fact that predictive maintenance is not systematic as in some cases, corrective maintenance may be preferable to predictive maintenance.
- *Corrective maintenance cost:* The corrective cost for the ith inspection is paid only if there is no predictive replacement and if the system fails before the next inspection. Therefore, the cost of corrective maintenance during the time cycle D can be evaluated using Equation (3):

$$C_c = \sum_{i=1}^{N_{In}-1} c_c \cdot (1 - N_i) \cdot \int_{t_i, T > t_i}^{t_{i+1}} f(t)\, \mathrm{d}t + c_c \cdot (1 - N_{N_{in}}) \cdot \int_{t_{N_{in}}, T > t_{N_{in}}}^{D} f(t)\, \mathrm{d}t \tag{3}$$

  where c_c is the cost of a corrective replacement.
- *Inspection cost:* The total cost of inspection C_i during the time cycle D can be evaluated using Equation (4):

$$C_i = N_{in} \cdot c_i \tag{4}$$

  where c_i is the cost of an inspection.
  Note that as the inspection is performed regularly on the system starting from $t = 0$, the step of inspection θ is linked to the number of inspections N_{in}:

$$\theta = \frac{D}{N_{in}} \tag{5}$$

- *Operating loss cost:* The operating loss cost is due to the loss of the system's operation capacity due to a failure of the system or due to performing a maintenance activity on the system. The operating loss cost c_{ol} contains the operating loss cost due to predictive maintenance and the operating loss cost due to corrective maintenance (Equation (6)):

$$C_{ol} = \sum_{i=1}^{N_{in}-1} c_{dt} \cdot D_p \cdot N_i + \sum_{i=1}^{N_{in}-1} c_{dt} \cdot D_c \cdot (1 - N_i) \cdot \int_{t_i, T>t_i}^{t_{i+1}} f(t)\,\mathrm{d}t + c_{dt} \cdot D_c \cdot (1 - N_{N_{in}}) \cdot \int_{t_{N_{in}}, T>t_{N_{in}}}^{D} f(t)\,\mathrm{d}t \quad (6)$$

where D_p is the duration of a predictive replacement, D_c is the duration of a corrective replacement, and c_{dt} is the cost of system downtime per unit of time.

- *Indirect maintenance costs:* The indirect maintenance costs $C_{indirect}$ include the expected cost of human risks R_h, the expected cost of financial risks R_f, and the expected cost of ecological risks R_e due to maintenance. These three types of risks can be evaluated using the equations below [11]:

$$C_{indirect} = R_h + R_f + R_e \quad (7)$$

$$R_h = (VSL \cdot \sum_{j=1}^{n} p_j^d) \cdot \sum_{i=1}^{N_{in}} (1 - N_i) \cdot \int_{t_i, T>t_i}^{t_{i+1}} f(t) \cdot dt \quad (8)$$

$$R_f = M \cdot C \cdot \frac{x \cdot \sum_{i=1}^{N_{in}-1} N_i + y \cdot \sum_{i=1}^{N_{in}} (1 - N_i) \cdot \int_{t_i, T>t_i}^{t_{i+1}} f(t) \cdot dt}{100} \quad (9)$$

$$R_e = (\sum_{j=1}^{m} P_j \cdot V_j \cdot \rho_j \cdot D_{aj}) \cdot \sum_{i=1}^{N_{in}} (1 - N_i) \cdot \int_{t_i, T>t_i}^{t_{i+1}} f(t) \cdot dt \quad (10)$$

These risk equations are established under the following assumptions:

- n persons may be affected by a potential failure of the system. The probability of death of a person j is denoted by p_j^d, and the Value of Statistical Life *(VSL)* is used to evaluate the monetary loss of human life [38,39]. By way of similarities, Equation 8 can be applied to evaluate the risk of human injuries: we may consider different levels of injuries with their corresponding compensation costs.
- A business loses x% of customers in case of predictive maintenance and y% of customers in case of corrective maintenance.
- A failure of the system is eventually responsible for emitting m toxic pollutants with emission probabilities $(P_1, P_2, \ldots P_m)$ and emission volumes $(V_1, V_2, \ldots V_m)$. Each possibly emitted toxic pollutant j is characterized by its density ρ_j and its environmental damage cost D_{aj}.

The indirect costs of maintenance have not been considered in the rest of this paper. They will be treated in a future research work.

2.3. Process of Maintenance Cost Optimization

The objective function that we want to minimize is the total cost $C_{tot} = C_p + C_c + C_i + C_{ol}$. The decision variables of the optimization program are the number of inspections N_{in} and the binary variables N_i, $i \in \{1 \ldots N_{in}\}$. The constraints that need to be satisfied are:

- the decision variables N_i are binary
- the number of inspections N_{in} should be at least equal to 1: $N_{in} \geq 1$ as the system requires at least one inspection in its early life (see Figure 1).

Finally, the input data of the optimization program include the duration parameters: D, D_p, and D_c; the cost parameters: c_p, c_c, c_i, c_{dt} and the parameters of the Weibull distribution characterizing the failure evolution of the system.

Once the decision variables are evaluated, the decision maker faces one of these possible cases:

- $N_i = 1$ *for some* $i \in \{1 \dots N_{in}\}$*:* a predictive maintenance should be performed as the RUL of the system has reached some threshold RUL_{lim}. In this case, the system should be predictively replaced at inspection i, and the cost optimization process must be reset.
- $N_i = 0$*:* There is no predictive replacement of the system in this case. However, the system may fail or not before the next inspection $i + 1$:
  - If the system fails before inspection $i + 1$, a corrective replacement of the system should be performed and the cost optimization process should be reset.
  - If the system does not fail, we plan to perform the next inspection $i + 1$ on the system at $t_{i+1} = t_i + \theta$.

In Figure 2, we give the global approach followed in this paper and published in our previous work [11,36] in order to minimize the total cost of maintenance.

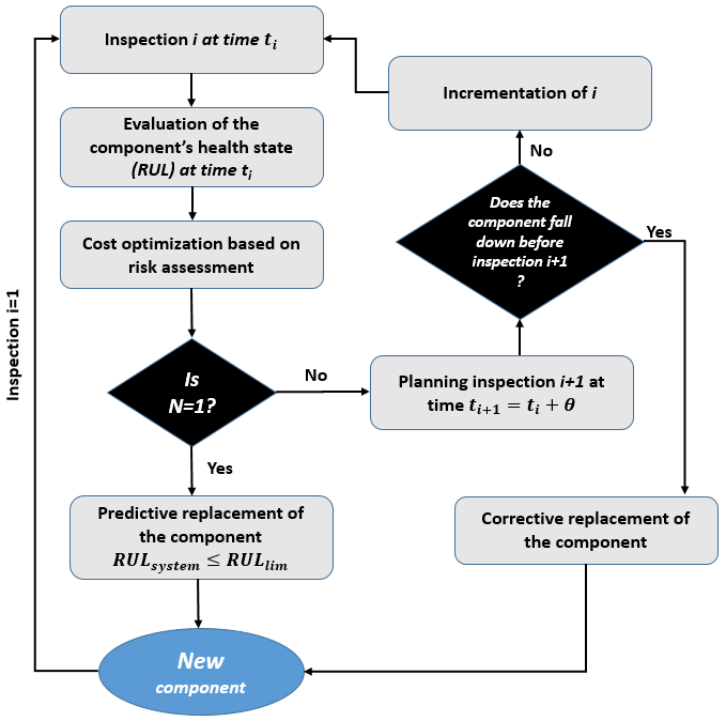

Figure 2. Flowchart of the optimization process for predictive maintenance planning.

3. Process of Human Decision Making under Uncertainty

Human, financial, and economic risks in maintenance as described above can be estimated using the known probabilities and consequences of the different failure scenarios and knowing that risks are defined in general as the product of events' probabilities and consequences. However, the situation where the decision maker needs to make a decision on maintenance on the basis of an interval of RUL_{lim} is uncertain, as at each inspection i, the expert needs to compare the real RUL of the system with the interval of RUL_{lim} evaluated by the optimization process described above, with no prior knowledge on the probability distribution of RUL_{lim} [40]. According to [40], the cognitive processes when making decisions under risk are different from when making decisions under uncertainty. In risky situations, value-based statistical thinking such as Bayesian theory is enough to make good decisions while in uncertain situations, statistical thinking is no longer enough and heuristic thinking is required [40]. Savage made it clear that *"applying Bayesian theory to decisions in uncertain situations would be utterly ridiculous because it is impossible to know all the alternatives, consequences and probabilities"* [40,41]. Therefore, the brain needs more than

just Bayes's rule to make optimal decisions in uncertain situations [40,41], and heuristic thinking becomes required [40].

If $RUL_{system} \leq RUL_{inf}$, the expert has to replace the deteriorated component with a new one. If $RUL_{system} \geq RUL_{sup}$, the component is not yet deteriorated and can continue to operate. The confusion comes when RUL_{system} is within the tolerance interval of RUL_{lim}. The expert needs to make a decision whether to replace the system before inspection $i+1$ or let it work until inspection $i+1$. In this situation, the decision D to make has two options O:

- O_1: *"The component is replaced at inspection i"*.
- O_2: *"The component is not replaced at inspection i"*.

For each decision option, there is a corresponding event E. In our case, two types of events can occur:

- E_1: "$RUL_{lim} = RUL_{inf}$".
- E_2: "$RUL_{lim} = RUL_{sup}$".

For each decision option O_i and event E_j, there is a corresponding conditional result $r_{i,j}$, where $(i,j) \in \{1,2\}$, which measures the expected monetary loss of each situation.

Let us note C_{t_i}, the expected monetary loss between inspection i and $i+1$. C_{t_i} can be expressed as follow:

$$C_{t_i} = \begin{cases} c_p + c_{dt} \cdot D_p + (c_c + c_{dt} \cdot D_c) \cdot \int_{t_1}^{t_2} f(t) \cdot dt & \text{if } r_{i,j} = r_{1,1} \\ c_p + c_{dt} \cdot D_p + (c_c + c_{dt} \cdot D_c) \cdot \int_{t_1}^{t_2} f(t) \cdot dt & \text{if } r_{i,j} = r_{1,2} \\ (c_c + D_c \cdot c_{dt}) \cdot \int_{t_i}^{t_{i+1}} f(t \backslash RUL_{lim} = RUL_{inf}) \cdot dt & \text{if } r_{i,j} = r_{2,1} \\ (c_c + D_c \cdot c_{dt}) \cdot \int_{t_i}^{t_{i+1}} f(t \backslash RUL_{lim} = RUL_{sup}) \cdot dt & \text{if } r_{i,j} = r_{2,2} \end{cases} \quad (11)$$

where $f(t \backslash RUL_{lim} = RUL_{inf})$ and $f(t \backslash RUL_{lim} = RUL_{sup})$ are the respective conditional probability densities of system failure knowing that $RUL_{lim} = RUL_{inf}$ and $RUL_{lim} = RUL_{sup}$.

Table 1 gives the conditional results of the different combinations O-E. Because we cannot obtain the probability values of events E_1 and E_2, we will only consider the monetary loss of each situation.

Table 1. Conditional results of the different combinations decision option—event.

	E_1	E_2
O_1	$c_p + c_{dt} \cdot D_p + (c_c + c_{dt}.D_c) \cdot \int_{t_1}^{t_2} f(t) \cdot dt + \sum_{j=2}^{j=N_{in}} C_{t_j}$	$c_p + c_{dt} \cdot D_p + (c_c + c_{dt} \cdot D_c) \cdot \int_{t_1}^{t_2} f(t) \cdot dt + \sum_{j=2}^{j=N_{in}} C_{t_j}$
O_2	$(c_c + D_c \cdot c_{dt}) \cdot \int_{t_i}^{t_{i+1}} f(t \backslash RUL_{lim} = RUL_{inf}) \cdot dt + \sum_{j=i+1}^{j=N_{in}} C_{t_j}$	$(c_c + D_c \cdot c_{dt}) \cdot \int_{t_i}^{t_{i+1}} f(t \backslash RUL_{lim} = RUL_{sup}) \cdot dt + \sum_{j=i+1}^{j=N_{in}} C_{t_j}$

3.1. Optimistic Criterion

Following Table 1, an optimistic expert would opt for the decision that maximizes the profit/minimizes the monetary loss. This means in our case study that an optimistic expert would prefer to believe E_1 rather than E_2 [42,43].

3.2. Wald Criterion

The decision-theoretic view of statistics introduced by Wald had an obvious interpretation in terms of decision making under complete absence of knowledge, in which the maximin strategy was shown to be the best response against natures' minimax strategy. Wald's criterion is extremely conservative, even in a context of complete absence of knowledge. However, this conservatism may sometimes make good sense [44]. The maximin criterion is a pessimistic approach. It suggests that the decision maker considers only the minimum payoffs of alternatives and chooses the alternative with the least bad outcome. This criterion applies to decision makers who are cautious and who seek insurance that

in the case of an unfavorable outcome, at least, there is a known minimum payoff. This approach may be justified because the minimum payoffs may have a higher probability of occurrence or the lowest payoff may lead to an extremely unfavorable outcome. A pessimistic expert would choose the decision that has the greatest minimal profit (the "less worse option"). This means in our case study that a pessimistic expert would prefer to believe E_2 rather than E_1 [42,43].

3.3. Criterion of Minimum Regret

If we note that m_j the best result associated to the event E_j: $m_j = min\{r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}\}$. It is obvious that $m_j = 0$ in our case study. The regret corresponding to the combination $O_i - E_j$ is equal to the difference $m_j - r_{i,j}$. The expected regret resulting from the option O_i is therefore calculated as:

$$E[Regret \backslash O_i] = \sum_j m_j - r_{i,j} \tag{12}$$

Therefore, the option O_i that maximizes the expected profit by minimizing the monetary loss is the one that minimizes the expected regret [42,43].

3.4. Hurwicz Criterion

This criterion is a combination between optimistic and Wald criteria. We note that α is the optimism coefficient, m is the minimum profit, and M is the maximum profit for each decision. For each decision, we evaluate: $M \cdot \alpha + m \cdot (1 - \alpha)$ and retain the option with the best result [42,43]. In our context, the profit corresponds to the monetary loss following the decision to replace or not the system. The optimism coefficient α reflects the feedback experience of the decision maker toward similar systems under similar conditions of operation. The lower the value of α is, the more the decision maker is considered to be risk averse and vice versa.

The value of α reflects the cognitive process that underpins the economic decision making. In fact, according to Kahneman and Tversky, people tend to place more importance on events that easily come to their mind and compare them relatively to a baseline situation [34,45]. For example, brokers tend to be risk averse if they won the day before but take more risk if they lost the day before [46]. According to [34], people often predict the alternatives that are most representative of the input. As for example, if the description of a company is favorable, a high profit is more representative of the situation even if the description is unreliable or does not allow accurate prediction [34]. This constitutes an anchoring bias, which leads to trusting the information received first in decision making. Furthermore, we mention the work of Paul Slovic that proves that emotional judgements in relation to a situation of uncertainty are able to predict the attitude and behavior of the decision maker to this situation of uncertainty [47,48]. In other terms, positive and negative perception of the same hazard can induce different choices in decision making.

The same can be applied here to our work; past experiences that come easily to the expert's mind will have an impact on his or her attitude to risk taking. The fact that we focus on monetary loss and not on monetary gain is another aspect that impact the attitude to risk taking. This obviously results in how the value of α is chosen to reflect the best the attitude of expert to risk.

The choice of the criteria depends on the context of decision making and the feedback experience of the decision maker. The most difficult choice remains the choice of the adapted human decision criteria and the choice of the value of α [42,43].

4. Case Study

4.1. System Description

We consider in this work a train system, seen as a complex system: the train is composed of several subsystems providing a set of basic functions, all contributing to realizing the main function of the train, which is transporting passengers from point A to point B [49] (Figure 3). The traction system contains the electric motor of the train,

which is also considered as a complex system. Some of the components of the electric motor are critical for the operation of the train [49] (Figure 4). For instance, we point out the mechanical bearing system as the most faulty component in the train motor. In fact, according to [49,50], faults arising in motors are often due to bearing faults.

A rolling-element bearing is a type of a bearing system that contains rolling elements (such as balls or rollers) placed between two races (inner race and outer race) [50–52] as shown in Figure 5.

The rolling-element bearing system allows rotational movement while reducing friction and stress. They resemble wheels, and they enable motor devices to roll on the motor shaft, which reduces the friction between the surface of the bearing and the surface that it is rolling over [50–52].

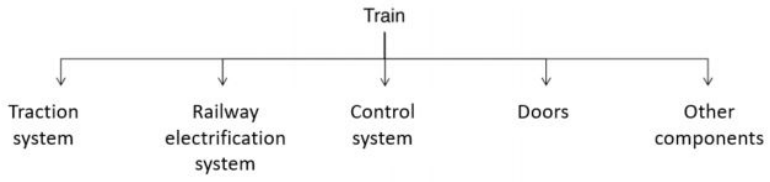

Figure 3. Train system [49].

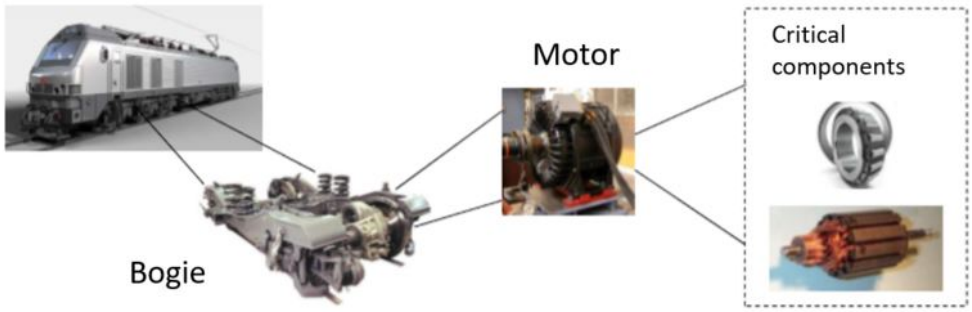

Figure 4. Critical components of the train motor [49].

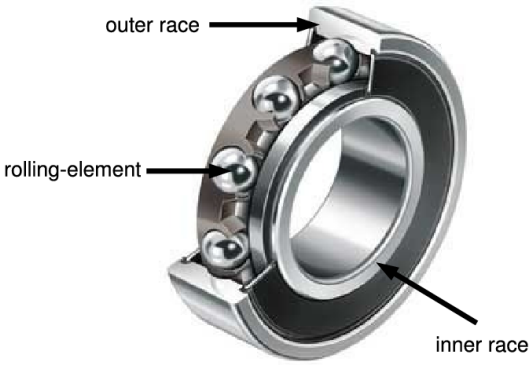

Figure 5. Mechanical rolling bearing system [49].

4.2. Main Characteristics of the System

The mechanical bearing system is the system under study. It is considered to be a critical component of the complex system: train motor. The train motor is considered in its turn a critical component of the complex system: train.

Table 2 summarizes the main characteristics of the mechanical bearing system. We refer to the nomenclature table at the end of this paper to help the reader understand the different cost parameters and the sources used to set them. The following data are realistic and are given for information (Table 2).

Table 2. Main characteristics of the mechanical bearing system.

Parameter	*Value*	*Unit*
c_p	200	
c_c	800	
c_i	150	
c_{dt}	1000	
D	25,000	hours
D_p	2	hours
D_c	10	hours

The failure evolution of the system follows the Weibull distribution with a constant scale parameter λ = 27,000 and a variable shape parameter $k \geq 1$ (the failure rate increases with time). The value of the shape parameter k is equal to 1.1 at t = 0 and should be updated at each inspection on the basis of real data coming from sensors. There are different methods to estimate the Weibull distribution parameters. We refer for example to the work of Pasquale Erto and Massimiliano Giorgio on how to estimate the Weibull distribution parameters on the basis of prior consideration of λ and k [53].

4.3. Application

As $N_{in} \geq 1$, we have run the optimization program for different values of N_{in} starting from N_{in} = 2. For a predefined value of N_{in}, if the optimization results show that N_i = 1 for some $i \in \{1...N_{in}\}$, it means that RUL_{lim} has been crossed by the system between inspections $i-1$ and i, and the system should be predictively replaced at inspection i. This provides an interval containing RUL_{lim}: in this case, RUL_{lim} should belong to the interval of time $[RUL_{system}(t_i), RUL_{system}(t_{i-1})]$. Let us call this interval of RUL_{lim}: $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$.

Table 3 summarizes the updated values of Weibull shape parameters k at each inspection i and the values of the decision variables obtained by our optimization program. The parameter λ is remained fixed while the parameter k has a linear dependency on $(i-1)\cdot\theta$. Therefore, the different values of k_i vary according to the number of inspection N_{in} and the order of inspection i. Table 3 gives only the value of N_i = 1 for some $i \in \{1 \ldots N_{in}\}$, as N_j = 0 for $j \in \{1 \ldots N_{in}\} \backslash \{i\}$. As the failure rate of the system increases ($k_i \geq 1$ for $i \in \{1 \ldots N_{in}\}$, it is almost expected to have predictive maintenance in the time interval $[t_{N_{in}}, D]$ where the system is most deteriorated.

Table 3. Values of the updated shape parameters k and of the decision variables (N_{in} and N).

N_{in}	k	N
2	$k_1 = 1.1, k_2 = 9.85$	$N_2 = 1$
3	$k_1 = 1.1, k_2 = 6.93, k_3 = 12.77$	$N_3 = 1$
4	$k_1 = 1.1, k_2 = 5.47, k_3 = 9.85, k_4 = 14.22$	$N_4 = 1$
5	$k_1 = 1.1, k_2 = 4.6, k_3 = 8.1, k_4 = 11.6, k_5 = 15.1$	$N_5 = 1$
6	$k_1 = 1.1, k_2 = 4.02, k_3 = 6.93, k_4 = 9.85, k_5 = 12.77, k_6 = 15.68$	$N_6 = 1$
7	$k_1 = 1.1, k_2 = 3.6, k_3 = 6.1, k_4 = 8.6, k_5 = 11.1$ $k_6 = 13.6, k_7 = 16.1$	$N_7 = 1$
8	$k_1 = 1.1, k_2 = 3.29, k_3 = 5.47, k_4 = 7.66, k_5 = 9.85$ $k_6 = 12.04, k_7 = 14.22, k_8 = 16.41$	$N_8 = 1$
9	$k_1 = 1.1, k_2 = 3.04, k_3 = 4.99, k_4 = 6.93, k_5 = 8.88,$ $k_6 = 10.82, k_7 = 12.77, k_8 = 14.71, k_9 = 16.65$	$N_9 = 1$

We have stopped the simulations at $N_{in} = 9$ because for a number of inspections superior to 9, N_i remains equal to 0, $i \in \{1..N_{in}\}$ for $N_{in} \geq 9$, meaning that predictive

maintenance becomes too expensive for all the cases, and it is then wiser to let the system operate until failure.

The variations of $C_p + C_i$ and C_c per N_{in} are illustrated in Figure 6, and the variations of C_{ol} and C_{tot} per N_{in} are illustrated in Figure 7.

Figure 8 illustrates how the interval of RUL_{lim} varies when varying N_{in}.

The more N_{in} increases, the better experts become able to predict the failure of the system and, therefore, the less we pay for corrective maintenance and for the loss of operation capacity (see Figures 6 and 7). However, predictive maintenance becomes expensive in return (see Figure 6). This cost compensation is reflected in the shape of the curve of C_{tot}: C_{tot} seems to decrease when N_{in} increases until reaching a minimum of 4988.71 euros at $N_{in} = 6$, beyond which $C_p + C_i$ become expensive and compensate the reduction of C_c and C_{ol} (see Figures 6 and 7).

Figure 8 shows as expected that the more inspections we perform on the system, the more we are able to estimate precisely the interval of RUL_{lim}. This is due to reduction of the length of the interval $[t_{i-1}, t_i]$.

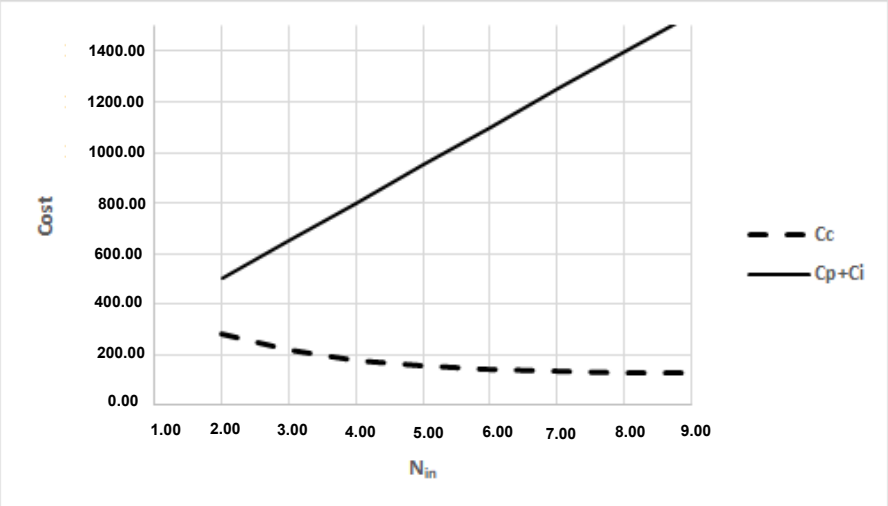

Figure 6. Variations of $C_p + C_i$ and C_c per N_{in}.

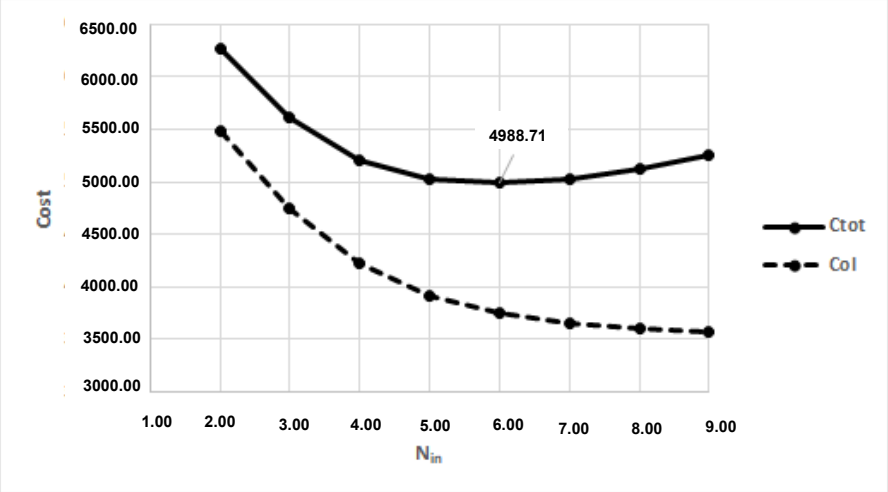

Figure 7. Variations of C_{ol} and C_{tot} per N_{in}.

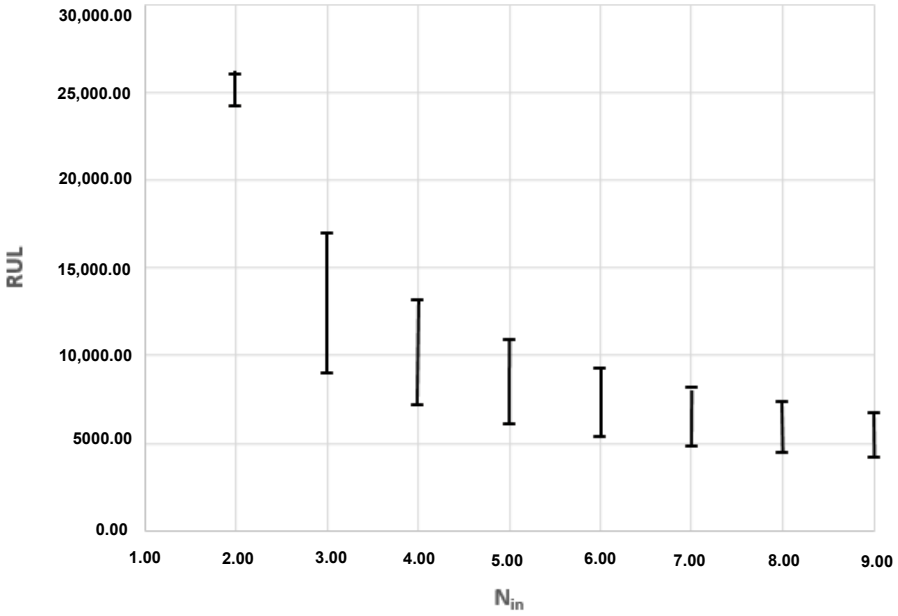

Figure 8. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per N_{in}.

5. Sensitivity Analysis under Variation of Cost and Time Parameters

In this section, we study the impact of variations of time and cost parameters on the optimization results. This study aims at understanding the influence of the different parameters on the optimization results, which cannot be done just by looking at the optimization program. Identifying the most/least influencing parameters on the maintenance strategy can be of great help for the decision maker, as it allows him to better manipulate the parameters in order to better reduce maintenance costs.

5.1. Study of the Impact of Cost Parameters on the Optimization Results

5.1.1. Variation of c_i

In this section, we have varied c_i from 10 to 200, while maintaining the other parameters fixed.

Figures 9–11 illustrate the variations of the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim} per c_i, respectively.

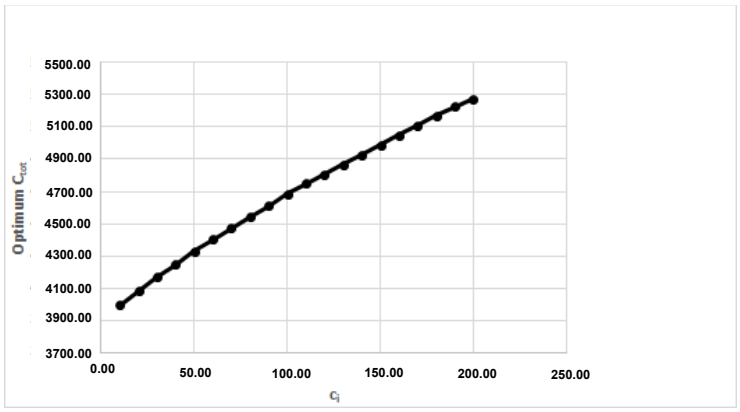

Figure 9. Variations of optimum C_{tot} per c_i.

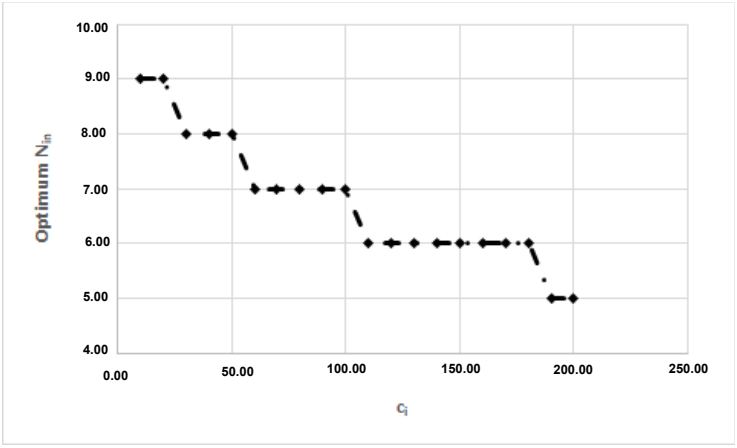

Figure 10. Variations of optimum N_{in} per c_i.

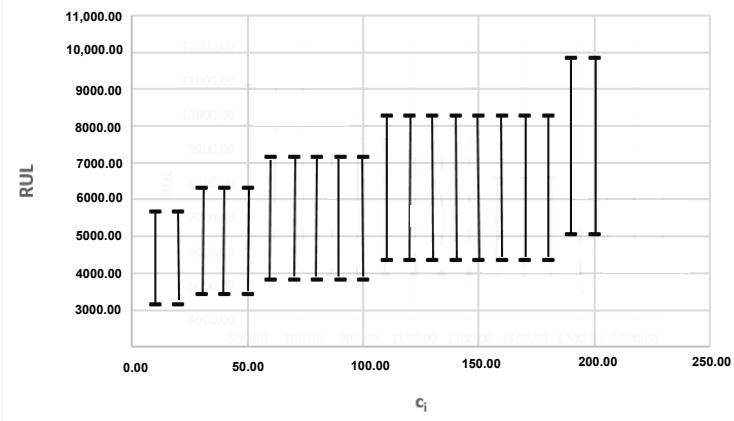

Figure 11. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per c_i.

It is obvious that an increase in c_i leads to an increase in the optimum of C_{tot} (see Figure 9). Predictive maintenance becomes expensive, which leads to a decrease in the optimal value of N_{in} (see Figure 10). In other words, the lower c_i is, the better experts are able to perform failure prognosis, and the system can be replaced predictively at its late life. This leads obviously to decreasing the optimum of C_{tot} as we optimize the exploitation of the system (see Figure 9).

The decrease in the value of N_{in} leads to more imprecision on the estimation of RUL_{lim}: the length of the interval of RUL_{lim} increases as c_i increases (see Figure 11).

5.1.2. Variation of c_p

In this section, we have varied c_p from 100 to 1500 while maintaining the other parameters fixed.

Figures 12–14 illustrate the variations of the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim} per c_p, respectively.

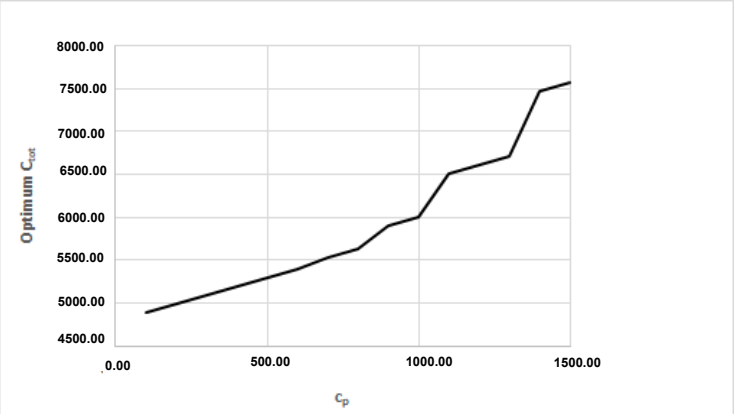

Figure 12. Variations of optimum C_{tot} per c_p.

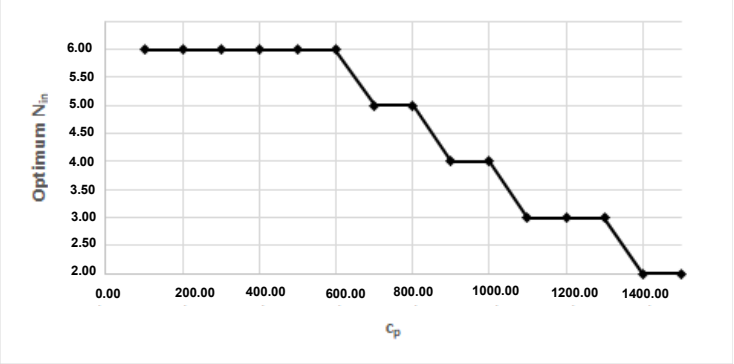

Figure 13. Variations of optimum N_{in} per c_p.

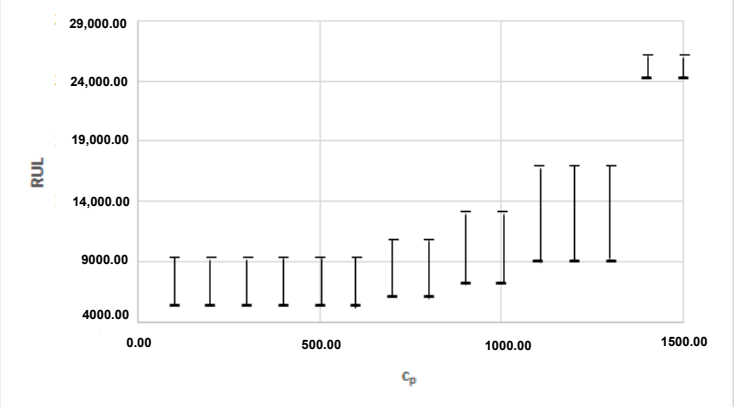

Figure 14. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per c_p.

For small values of c_p ($c_p \leq 600$), the optimum N_{in} is equal to 6. As soon as c_p gets higher values, the optimum N_{in} tumbles quickly to converge to the case where predictive maintenance becomes expensive in comparison with corrective maintenance (see Figure 13). This is reflected on the curve of C_{tot} where C_{tot} varies linearly as a function of c_p for $c_p \leq 600$, and then it increases rapidly for higher values of c_p (see Figures 12 and 13). The approximation of the RUL_{lim} follows N_{in}. The more c_p increases, the more N_{in} decreases and, therefore, the less precise becomes the estimation of RUL_{lim} (see Figure 14). We make an exception for the case where $N_{in} = 2$ for $c_p \geq 1400$, as this case study is not realistic.

5.1.3. Variations of c_c

In this section, we have varied c_c from 100 to 1500 while maintaining the other parameters fixed.

Figures 15–17 illustrate the variations of the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim} per c_c, respectively.

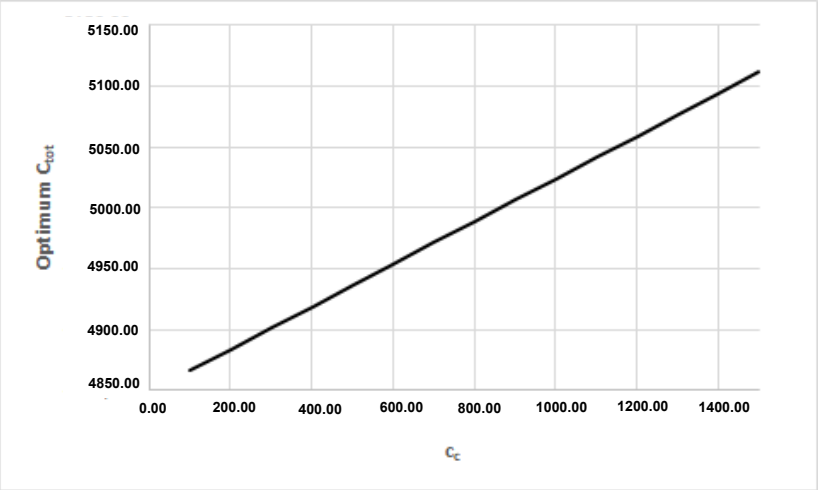

Figure 15. Variations of optimum C_{tot} per c_c.

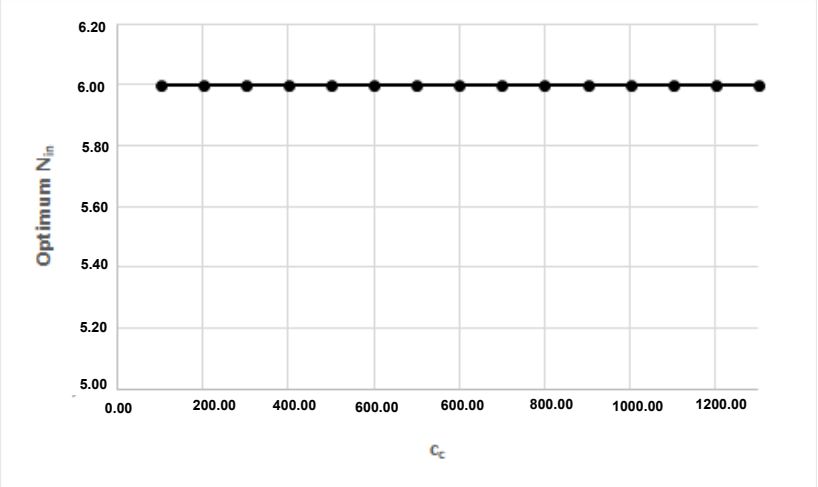

Figure 16. Variations of optimum N_{in} per c_c.

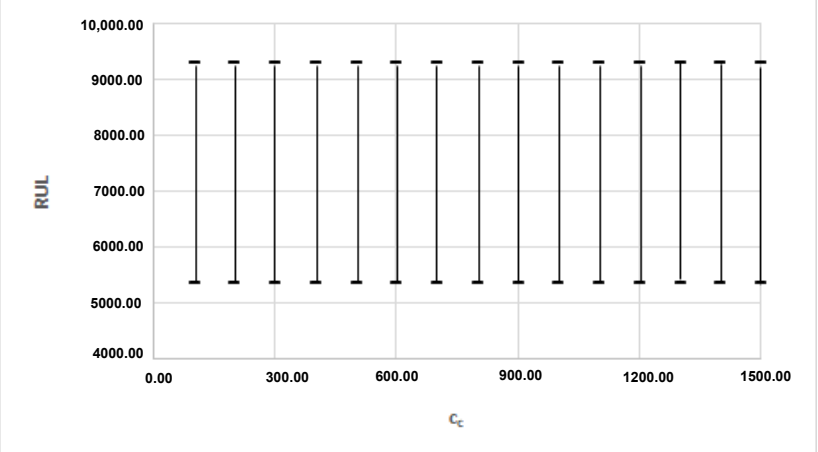

Figure 17. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per c_c.

The parameter c_c seems to have only an impact on C_{tot} through the cost C_c (see Figure 15). However, it seems to have no impact on the optimal number of inspections or

on the interval of RUL_{lim} (see Figures 16 and 17). This may be explained by the fact that the influence of corrective maintenance on the optimization results is mainly done through the loss of operation capacity due to corrective maintenance.

5.1.4. Variation of c_{dt}

In this section, we have varied the parameter c_{dt} from 500 to 2000 while maintaining the other parameters fixed.

Figures 18–20 illustrate the variations of the optimal C_{tot}, the optimal N_{in}, and the interval of RUL_{lim} per c_{dt}, respectively.

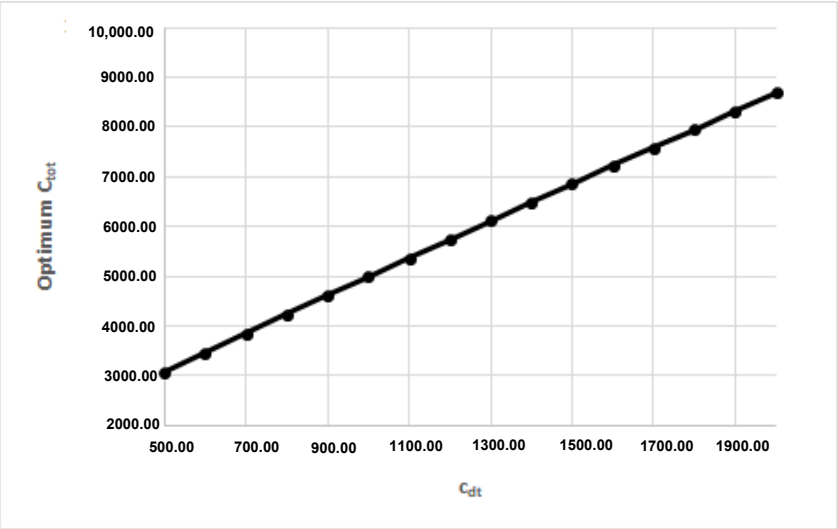

Figure 18. Variations of optimum C_{tot} per c_{dt}.

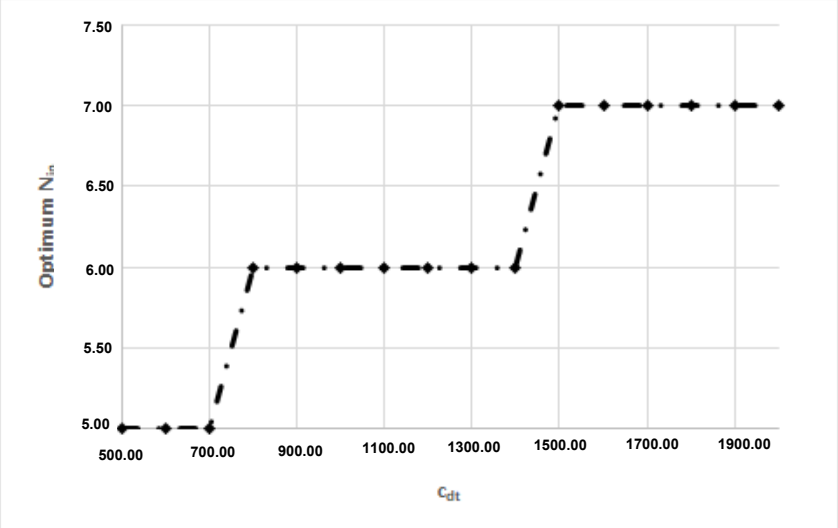

Figure 19. Variations of optimum N_{in} per c_{dt}.

The more c_{dt} is expensive, the more we should avoid the failure of the system as the corrective maintenance becomes expensive (through the term $c_{dt} \times D_c$, which remains too high comparing to the term $c_{dt} \times D_p$). Therefore, we should perform more inspections on the system. This leads to more precision on the approximation of RUL_{lim} (see Figures 19 and 20).

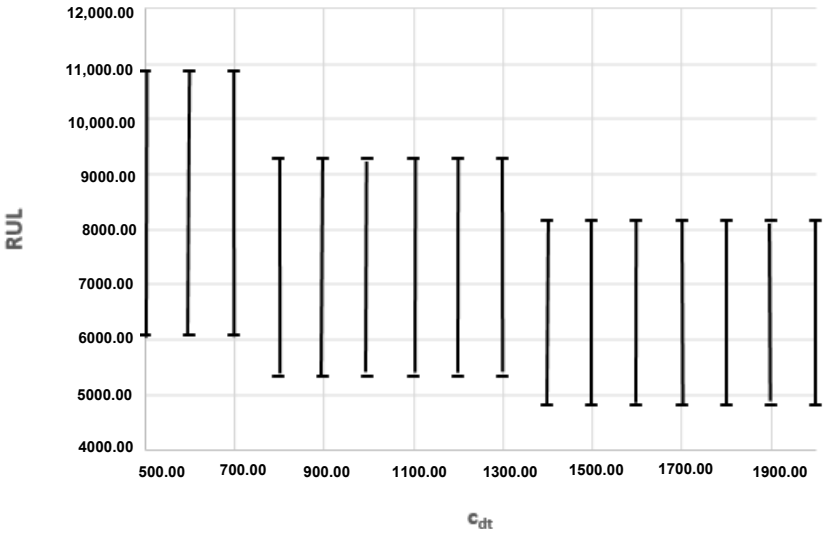

Figure 20. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per c_{dt}.

Preliminary conclusions can be drawn from the study of variations of cost parameters:

- The cost parameter c_{dt} is the most influencing parameter on the optimum C_{tot}.
- The parameter c_{cc} has almost no impact on the optimization results. The main impact of corrective maintenance on the optimization results is made through the cost of loss of operating capacity due to corrective maintenance.
- The parameters c_p and c_i have almost similar impact on the optimization results: these two parameters have an impact on the optimum of C_{tot}, but they have greater impact on the optimum N_{in} and the interval of RUL_{lim}.

5.2. Study of the Impact of Time Parameters on the Optimization Results

5.2.1. Variation of D_p

We have varied D_p from 0.5 to 5 h while keeping the other parameters fixed.

Figures 21–23 illustrate the variations of the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim} per D_p, respectively.

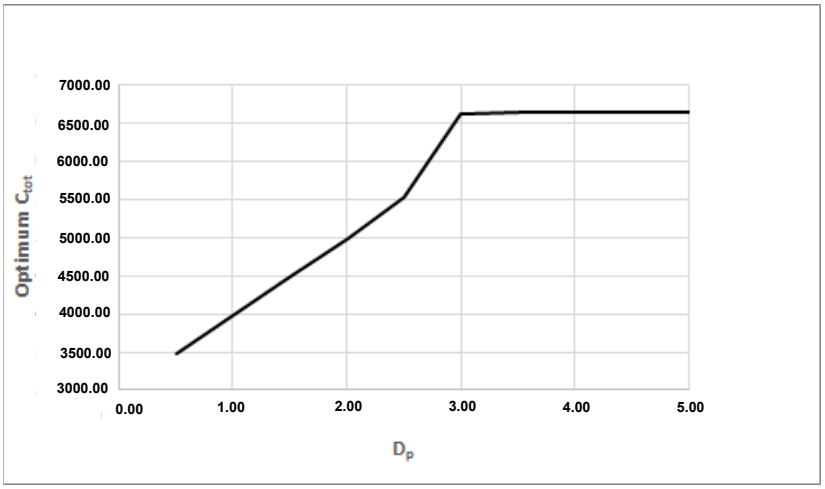

Figure 21. Variations of optimum C_{tot} per D_p.

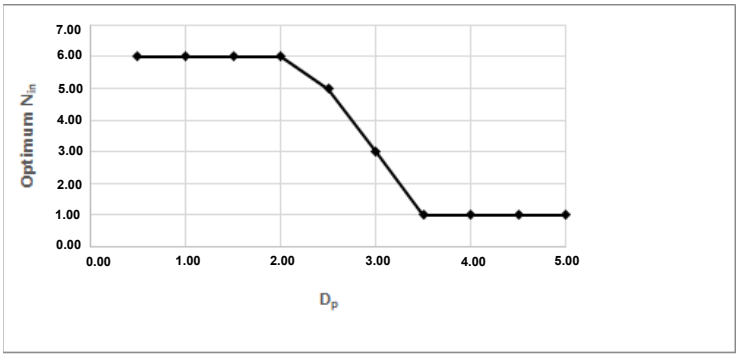

Figure 22. Variations of optimum N_{in} per D_p.

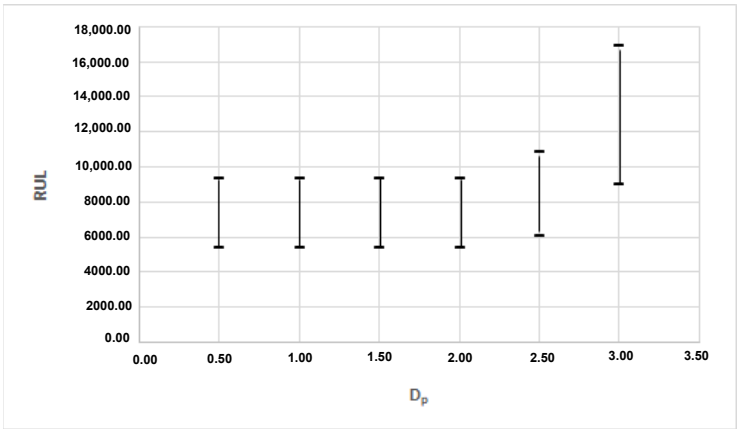

Figure 23. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per D_p.

The longer D_p is, the more expensive predictive maintenance becomes. This results, therefore, in a decrease in the optimum N_{in} (see Figure 22) and in more imprecision on RUL_{lim} estimation (see Figure 23). We can see from Figures 21 and 22 that the optimum C_{tot} remains constant and the optimum N_{in} remains equal to 1 for $D_p \geq 3.5$ h, meaning that in this case, it is useless to perform predictive maintenance, as it becomes more expensive than corrective maintenance.

5.2.2. Variation of D_c

We have varied in this section the parameter D_c from 1 to 10 h while keeping the other parameters fixed.

Figures 24–26 illustrate the variations of the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim} per D_c, respectively.

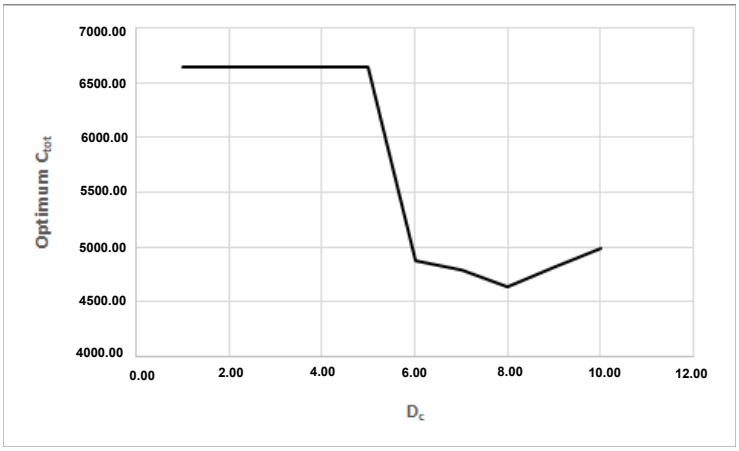

Figure 24. Variations of optimum C_{tot} per D_c.

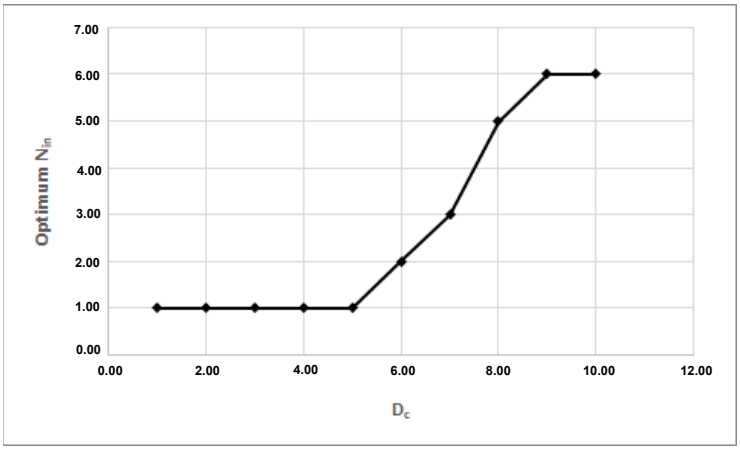

Figure 25. Variations of optimum N_{in} per D_c.

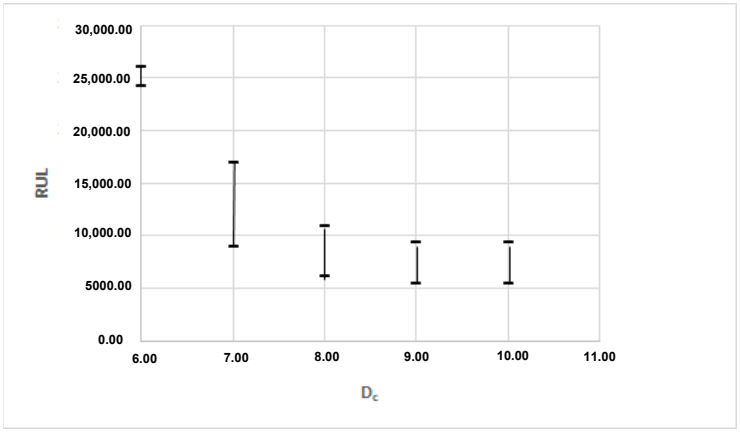

Figure 26. Variations of $[RUL_{inf}^{lim}, RUL_{sup}^{lim}]$ per D_c.

For low values of D_c ($D_c \leq 5$ h), it is more expensive to perform predictive maintenance than corrective maintenance as the cost of loss of operation capacity due to corrective maintenance becomes less expensive and comparable to the cost of loss of operation capacity due to predictive maintenance. Therefore, the optimum N_{in} is equal to 1 (corresponding to the fist inspection performed at $t = 0$) and the optimum C_{tot} corresponds to the total cost of the nonpredictive maintenance scenario (see Figure 24).

For higher values of D_c, predictive maintenance becomes interesting, as it becomes less expensive than corrective maintenance, which allows to reduce C_{tot}. This explains the shape of the curve of C_{tot} and N_{in} for $D_c \in [5, 8]$ h (see Figures 24 and 25).

For $D_c \geq 8$ h, even if predictive maintenance remains interesting, predictive maintenance can no longer compensate the cost increase in corrective maintenance and the optimal N_{in} then remains stable. This explains the shape of the curve of C_{tot} and N_{in} for $D_c \geq 8$ h (see Figures 24 and 25).

This result is reflected on the variations of RUL_{lim} interval: the more the optimal N_{in} increases (for $D_c \in [7, 8]$ h), the more precise our estimation of RUL_{lim} is through the reduction of inspection intervals. As the optimal N_{in} remains stable (for $D_c \geq 8$ h), the interval of RUL_{lim} does not change (see Figure 26).

A preliminary conclusion can be drawn from the study of variations of time parameters. If we want to minimize C_{tot} and have more precision on the estimation of RUL_{lim}, it is better to have low values of D_p. However, the influence of the parameter D_c on the optimization results seems to be less evident; this parameter needs to be considered with more precaution, as reducing D_c does not necessarily lead to reducing C_{tot} (see Figure 24).

6. Discussion

This work has allowed us to understand the implications of the different input parameters (time and cost parameters) of the proposed optimization methodology on the different output parameters, namely the optimum C_{tot}, the optimum N_{in}, and the interval of RUL_{lim}.

6.1. Cost Parameters

The major part of the optimum C_{tot} is borne by C_{ol}. Therefore, the impact of the parameter c_{dt} on the optimum C_{tot} is relatively important; a variation of c_{dt} from 500 to 2000 leads to a variation of C_{tot} from 3064.51 to 8685.15 (see Figure 19), meaning that the variation rate of the curve C_{tot} per c_{dt} is equal to 3.75. The parameter c_i also has an important impact on C_{tot}: a variation of c_i from 10 to 200 leads to a variation of the optimum C_{tot} from 3994.56 to 5275.30, meaning that the variation rate of the curve C_{tot} per c_i is equal to 6.74 (see Figure 9), followed by the parameter c_p, which has a little impact on the optimum C_{tot} comparing to c_{dt} and c_i (see Figure 12).

From Figures 10, 13 and 19, we can see that the optimal N_{in} is sensitive to c_i variations and, to a lesser extent, to c_p and c_{dt} variations.

The variations of the parameter c_{cc} has almost no impact on the optimization results (see Figures 16 and 17). Its only impact appears on the variations of the optimal C_{tot}, but it remains insignificant in comparison with the other cost parameters (see Figure 15).

6.2. Time Parameters

D_p and D_c have inverse impact on RUL_{lim} estimation: while the increase of D_p leads to more imprecision on the estimation of RUL_{lim} (see Figures 22 and 23), the increase of D_c, however, allows to estimate more precisely the RUL_{lim} (see Figures 25 and 26).

While the impact of D_p on the optimum C_{tot} seems intuitive (an increase in the parameter D_p leads to increasing C_{tot} until reaching a stable value of C_{tot} corresponding to the nonpredictive maintenance scenario; see Figure 21), the impact of D_c on the optimum C_{tot} is less evident (see Figure 24). The study of variations of D_c shows that there is a value of D_c where C_{tot} reaches its minimal value (see Figure 24).

6.3. Recommendations

It is a challenging task to reduce the optimum C_{tot} and the imprecision on RUL_{lim} estimation at the same time. The only possible way to do it is eventually through the parameters c_p and D_p. It is obvious that these two parameters are linked together as a reduction in D_p leads to a reduction in c_p. Some practical measures can be applied to reduce D_p, such as, for example, reducing supply times by providing a backup stock of new parts or by having more staff available for the task of maintenance.

The best way to increase the precision on the estimation of RUL_{lim} is through the parameter c_i. This will lead, however, to increasing the optimal value of C_{tot}.

The best way to decrease the optimum C_{tot} is through the parameters c_i and c_{dt}. We can imagine an automated inspection procedure that does not involve manpower and that is economically cheap.

To have better values of C_{tot}, experts need to identify the target value of D_c that allows C_{tot} to reach its minimum, knowing the values of the other parameters. Practical measures need to be implemented later in order to reach this target value of D_c. These measures may be similar to that used to reduce D_p.

7. Conclusions

In this paper, we describe a methodology for predictive maintenance optimization and provide an analysis of the influence of the input parameters of this methodology on the optimization results.

First, this analysis has allowed us to classify the input parameters from most influencing parameters to least influencing parameters. Second, this analysis has allowed us to

identify the way the input parameters influence the optimization results; some parameters have more influence on C_{tot} than others, and some other parameters have more influence on the interval of RUL_{lim} than others. Finally, thanks to this study, some reflections on how to handle the input parameters were drawn from the sensitivity analysis in order to help experts better improve the results of the proposed optimization methodology.

The optimization approach proposed in this paper may be improved by integrating other types of costs such as indirect costs related to maintenance risks [11] or by considering other decision variables such as reliability or deterioration index [14,15]. The sensitivity analysis may be improved by using the design of experiments to study the influence of two or more parameters on the cost optimization results. Finally, we mention that in applying the optimization methodology described in this paper, the input parameters may change due to inherent conditions such as experience and learning effect. This learning effect makes experts better familiar with some process (such as inspection process), which can lead to reduction of some cost and time parameters.

Author Contributions: Methodology, R.L.; validation, R.L.; formal analysis, R.L.; writing—original draft preparation, R.L.; writing—review and editing, J.P. and M.S.; supervision, J.P. and M.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research benefited from the support of ANR MAPSYD project, with partnership of Louis Bachelier Institute, Sector, and Synox.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: Many thanks go to the anonymous peer reviewers for their insightful comments.

Conflicts of Interest: The authors declare no conflict of interest.

Notations

The following notations are used in this manuscript:

c_c	Cost of a corrective replacement
c_{dt}	Cost per hour of the system down time
c_p	Cost of a predictive replacement
C	Cost of loss of one customer for a business
C_c	Expected cost of corrective maintenance during D
$C_{indirect}$	Expected cost of maintenance risks
C_{ol}	Expected cost of loss of operating capacity of the system during D
C_p	Expected cost of predictive maintenance during D
C_{t_i}	Expected monetary loss between inspection i and $i+1$
C_{tot}	Expected total cost of maintenance during D
D	Duration of exploitation of the whole global system containing our system under study
D_a	Vector of cost of damage per tonne emission of pollutants due to system failure
D_c	Duration of a corrective replacement
D_p	Duration of a predictive replacement
$f(t)$	Failure probability density function at time t
k	Shape parameter of Weibull distribution (to be updated at each inspection)
M	Number of potential customers at the beginning of the period D
N	Vector of binary decision variables: $N_i = 1$ in case of predictive maintenance between inspections i and $i+1$, and $N_i = 0$ elsewhere

N_{in}	Number of inspections during D
P	Vector of emission probabilities of toxic pollutants due to system failure
R_e	Expected cost of environmental maintenance risks
R_f	Expected cost of financial maintenance risks
R_h	Expected cost of human maintenance risks
$RUL(t)$	Remaining useful life of the system at t
RUL_{lim}	Threshold of RUL under which the system is considered as deteriorated and should be replaced before failure
RUL_{inf}^{lim}	The lower bound of the interval of RUL_{lim}
RUL_{sup}^{lim}	The upper bound of the interval of RUL_{lim}
$S(t)$	Survival function of the system at t
V	Vector of emission volumes of toxic pollutants due to system failure
α	Optimism coefficient
λ	Scale parameter of Weibull distribution (constant)
ρ	Density of possibly emitted toxic pollutant due to system failure
θ	Inspection step

Nomenclature

The following table is a nomenclature table of all the parameters of the different cost functions used in this manuscript:

Parameter	Cost function	Source
C	$C_{indirect}$	The evaluation of these parameters
c_c	C_c	takes in consideration
c_{dt}	C_{ol}	several aspects: cost
c_p	C_p	of a new component, labor
c_i	C_i	cost per hour, cost of logistics
D	C_c, C_i, C_{ol}	cost of bus/train ticket...
D_c	C_{ol}	In practice, the expertise and historical
D_p	C_{ol}	data on similar systems
M	$C_{indirect}$	help to set them
$(P_1, P_2, \dots P_m)$	$C_{indirect}$	
$(V_1, V_2, \dots V_m)$	$C_{indirect}$	
$(D_{a1}, D_{a2}, \dots D_{am})$	$C_{indirect}$	Official reports (CAFE Program [54])
VSL	$C_{indirect}$	Official data (3 millions euros in France [55])
$\lambda, k_i (i = 1 \dots N_{in})$	$C_c, C_{ol}, C_{indirect}$	Sensor data

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