Computation for the Summation of Binomial Expansions and Geometric Series of Multiples of Powers of Two
Chinnaraji Annamalai

To cite this version:
Chinnaraji Annamalai. Computation for the Summation of Binomial Expansions and Geometric Series of Multiples of Powers of Two. 2022. hal-03710654

HAL Id: hal-03710654
https://hal.science/hal-03710654
Preprint submitted on 30 Jun 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution 4.0 International License
Computation for the Summation of Binomial Expansions and Geometric Series of Multiples of Powers of Two

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: This paper presents computing technique for the summation of binomial expansions and geometric series of multiples of powers of two. This computing technique is a methodological advance which is useful for researchers who are working in science, economics, engineering, computation, and management.

MSC Classification codes: 05A10, 40A05 (65B10)

Keywords: computation, combinatorics, geometric series, binomial coefficient

1. Introduction
The author of this article introduces an innovative binomial coefficient [1-3] along with a traditional binomial coefficient. The innovative binomial coefficient is shown below:

\[ V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!}, \quad (n \geq 1, r \geq 0 \text{ and } n,r \in \mathbb{N} = \{0,1,2,3,\ldots\}). \]

Traditional binomial coefficient: \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \), where \( n,r \in \mathbb{N} \).

Let us compare the innovative binomial coefficient \( V_x^y \) with the traditional binomial coefficient as follows:

Let \( z = x + y \). Then, \( zC_x = \frac{z!}{x!y!} \). Here, \( V_x^y = V_y^x \implies zC_x = zC_y, \quad (x,y,z \in \mathbb{N}). \)

For example,

\[ V_3^5 = V_5^3 = (5+3)C_3 = (5+3)C_5 = 56. \]

Also, \( V_n^0 = V_0^n = nC_0 = nC_n = \frac{n!}{n!0!} = 1 \) and \( V_0^0 = 0C_0 = \frac{0!}{0!} = 1 \)(\( \because 0! = 1 \)).

2. Computation of Binomial Expansions and Geometric Series
In this section, the author of this article provides a theorem based on binomial expansions and geometric series [4-13].

Theorem:

\[ \sum_{i=1}^{1} i \times V_i^{1-i} + \sum_{i=1}^{2} i \times V_i^{1-i} + \sum_{i=1}^{3} i \times V_i^{1-i} + \cdots + \sum_{i=1}^{n} i \times V_i^{1-i} = (n-1)2^n + 1. \]

Proof. Let us find the value of each binomial expansion in the binomial theorem step by step.

Step 1: \( 1 \times V_1^0 = 1 \implies \sum_{i=1}^{1} i \times V_i^{1-i} = 1 = 1 \times 2^0, \quad (\because V_n^0 = V_0^n = 1 \text{ and } n = 0,1,2,3,\ldots). \)
Step 2: \[ \sum_{i=1}^{2} i \times V_{i}^{1-i} = 1 \times V_{1}^{1} + 2 \times V_{2}^{0} = 2 + 2 = 4 = 2 \times 2^{1}. \]

Step 3: \[ \sum_{i=1}^{4} i \times V_{i}^{1-i} = 1 \times V_{1}^{2} + 2 \times V_{2}^{1} + 3 \times V_{3}^{0} = 3 + 6 + 3 = 12 = 3 \times 2^{2}. \]

Step 4: \[ \sum_{i=1}^{4} i \times V_{i}^{1-i} = 1 \times V_{1}^{3} + 2 \times V_{2}^{2} + 3 \times V_{3}^{1} + 4 \times V_{4}^{0} = 4 + 12 + 12 + 4 = 4 \times 2^{3}. \]

Similarly, we can continue the expressions up to "step n" such that \[ \sum_{i=0}^{n} i \times V_{i}^{n-i} = n2^{n-1}. \]

Now, by adding these expressions on both sides, it appears as follows:
\[ \sum_{i=1}^{1} i \times V_{i}^{1-i} + \sum_{i=1}^{2} i \times V_{i}^{1-i} + \sum_{i=1}^{3} i \times V_{i}^{1-i} + \sum_{i=1}^{4} i \times V_{i}^{1-i} \ldots + \sum_{i=1}^{n} i \times V_{i}^{1-i} = \sum_{i=1}^{n} i \times 2^{i-1}. \]

where \[ \sum_{i=1}^{n} i \times 2^{i} = (n - 1)2^{n} + 1 \]

is the geometric series of multiples of powers of two[3].

\[ \therefore \sum_{i=1}^{1} i \times V_{i}^{1-i} + \sum_{i=1}^{2} i \times V_{i}^{1-i} + \sum_{i=1}^{3} i \times V_{i}^{1-i} + \ldots + \sum_{i=1}^{n} i \times V_{i}^{1-i} = (n - 1)2^{n} + 1. \]

Hence, theorem is proved.

3. Conclusion
This article presented a numerical computational technique for the summation of binomial expansions and geometric series of multiples of powers of two. This technique and its results can be useful for researchers who are working in science, economics, engineering, management, and medicine [14].

References


