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S Subramanian

## ► To cite this version:

S Subramanian. Dividing a cake (or) Distributional values in the measurement of economic inequality: an expository note. *Journal of Philosophical Economics*, 2014, Volume VII Issue 2 (2), 10.46298/jpe.10661 . hal-03710422

**HAL Id: hal-03710422**

**<https://hal.science/hal-03710422v1>**

Submitted on 30 Jun 2022

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THE JOURNAL OF  
PHILOSOPHICAL ECONOMICS

Volume VII Issue 2 Spring 2014

ISSN 1843-2298

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*Dividing a cake (or)  
Distributional values in the  
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**S. Subramanian**



# Dividing a cake (or) Distributional values in the measurement of economic inequality: an expository note

*S. Subramanian*

**Abstract:** 'Distributional judgments'— judgments on the extent of inequality in the distribution of income and wealth— are routinely made by economists in exercises aimed at comparing inequalities in alternative situations. Yet the measurement of inequality is informed by certain nuances, which it would do well to be attentive to. In particular, the values underlying measurement protocols are not always made explicit, which tends to lend a somewhat misleading semblance of 'value-neutrality' to the activity of measurement. It is argued, with specific reference to the problem of inequality measurement, that such an orientation can compromise the possibility of accurate diagnosis and appropriate policy prescription. There is little that is original in this article, and much that is owed to the pioneering contributions of Serge-Christophe Kolm. The emphasis throughout is on explicating an important issue through a deliberate effort at achieving simplicity in both argument and expression.

**Keywords:** relative inequality measure, absolute measure, centrist measure, Kolm

*piece of cake*

*Meaning: A straightforward task that can easily be accomplished. (Phrases, sayings, idioms and expressions at The Phrase Finder. Available at:*

*<http://www.phrases.org.uk/meanings/piece-of-cake.html>)*

## Introduction

This expository piece is an elementary tract on the notions of 'relative', 'absolute' and 'intermediate' measures of inequality; on the role— often hidden— played by values in the distributional judgments underlying inequality comparisons; and on the importance of bringing these values out in the open so as to underline the possibility of plural approaches that can be adopted in the assessment and diagnosis of inequality. This is not so much an original piece of writing as a pedagogical essay aimed at explicating what appears to the author to be an important issue, and one, which was flagged, nearly forty years ago, by Serge-Christophe Kolm. (On the centrality of values in the enterprise of measurement, in general terms, the reader is referred to the many remarkable contributions to this subject that have been made by Amartya Sen and Anthony Atkinson, among others).

Economists are forever attempting to compare the extent of inequality in the distribution of income, or consumption expenditure, or wealth, or some other resource of interest, across alternative regimes. When one speaks of 'regimes', one could have in mind spatial regimes or temporal regimes or simply 'situational' or 'contextual' regimes. That is, one could mean two different regions (say countries, or provinces or states within a country) at some given point of time. Or one could mean the same region (say a country, or a province, or the entire globe) at two different points in time. These two types of comparison are, typically, referred to as 'cross-section' and 'time-series' comparisons respectively. Or one could simply have in mind two different 'situations' – say a pre-tax situation and a post-tax situation - in the same region and at the same time.

Whether the concern is with spatial or inter-temporal or inter-contextual comparisons of inequality in the distribution of a resource, it is often the case that (a) the average size of the resource is different in the two regimes; and (b) the numbers of people amongst whom the resource is divided are different in the two regimes. These two contingencies could be called, respectively, the 'Variable Size Problem' and the 'Variable Population Problem'. In this essay, only the first— or Variable Size— Problem will be dealt with, leaving the Variable Population Problem for another occasion: after all, sufficient unto the day...

## 'Scale Invariance'

To obtain an analytical fix on the problem of making comparative evaluative judgments on the equality or otherwise of any two given distributions— and by a distribution is meant a particular allocation of a resource amongst those contending for it— it is useful to simplify the problem greatly by considering only two-person societies. The resource in question can be thought of as 'cake', which is what economists typically think of as a 'good', something of which each person prefers more to less. The Problem of Variable Size alluded to earlier boils down to answering this question: when can it be said that a particular two-person division of a 'small' cake is exactly as unequal as the two-person division of a 'large' cake?

At first blush, one would imagine that this question is not exactly pregnant with momentous issues. A quick consultation with one's intuition would suggest something like the following: no matter whether the cake is small or large, if the poorer person in both cases gets a fourth, or a third, or whatever share of the cake, then the distribution in both cases is equally unequal. This judgment can be made a little more exact. To this end, a small investment in some very simple formalities will help.

Given a cake of finite size (by which is meant only that it is unprofitable to deal with the wildly unexciting problem of distributing nothing) in a world with two persons bearing the names of 1 and 2 respectively, let  $x_1$  be the amount of cake going to person 1, and  $x_2$  the amount of cake going to person 2. (It will be taken, of course, that neither  $x_1$  nor  $x_2$  is ever negative.) The convention will be observed of reserving the name '1' for the poorer of the two persons ('poorer', here, strictly meaning 'not richer than', so that the 'poorer' person is also allowed to have, at most, exactly as much cake as the second person; also, typically, the analysis will be confined to distributional changes in which the initially poorer person continues to remain the poorer of the two individuals). The total amount of cake to be divided, clearly, is  $x_1 + x_2$ . A typical 'distribution' will be written as a two-element list  $(x_1, x_2)$ , in which the poorer person's ownership of cake ( $x_1$ ) will always be written in the first place of the list, and the richer person's ownership ( $x_2$ ) will be written in the second place. Thus, if  $x_1$  is 4 and  $x_2$  is 6, then a distribution such as (4,6) conveys the information that person 1 receives 4 units and person 2 receives 6 units out of a total of 10 units of cake. A typical distribution such as  $(x_1, x_2)$  will be written, in brief, as simply  $x$ .

Now consider a distribution  $\mathbf{x} = (x_1, x_2)$ , and another distribution  $\mathbf{y} = (y_1, y_2)$ , which has the property that  $y_1 = kx_1$  and  $y_2 = kx_2$ , where  $k$  is some positive number. What

does this mean? For specificity, if  $x_1$  is 4,  $x_2$  is 6 and  $k$  is 2, then  $\mathbf{x} = (4,6)$  and  $\mathbf{y} = (8,12)$ : there is altogether twice as much cake (20 units) in  $\mathbf{y}$  as in  $\mathbf{x}$  (10 units), and each person in  $\mathbf{y}$  receives twice as much cake in  $\mathbf{y}$  as in  $\mathbf{x}$ . On the other hand, if  $k = 1/2$ , then  $\mathbf{x} = (4,6)$  and  $\mathbf{y} = (2,3)$ : there is altogether one-half as much cake (5 units) in  $\mathbf{y}$  as in  $\mathbf{x}$ , and each person receives one-half as much cake in  $\mathbf{y}$  as in  $\mathbf{x}$ . The distributions  $\mathbf{x}$  and  $\mathbf{y}$  have the property that the *proportions* of cake received by persons 1 and 2 are identically the same in  $\mathbf{x}$  and  $\mathbf{y}$ . That is,  $\mathbf{y}$  is simply a scaled-up, or scaled-down, version of  $\mathbf{x}$ . Whenever any distribution  $\mathbf{y}$  bears this relation to any distribution  $\mathbf{x}$ , it will be said that  $\mathbf{y}$  is a rescaled replica of  $\mathbf{x}$ . The earlier intuition about equal inequalities can now be stated more precisely, in terms of a property called the *Scale Invariance Property*:

*Scale Invariance*: For any two distributions  $\mathbf{x}$  and  $\mathbf{y}$ , if  $\mathbf{y}$  is a rescaled replica of  $\mathbf{x}$ , then the extent of inequality in the distribution  $\mathbf{y}$  is the same as the extent of inequality in the distribution  $\mathbf{x}$ .

Scale Invariance says, in effect, that an inequality measure is properly regarded as a *relative measure* as long as the relative shares of cake going to each person are the same, then no matter what the size of the cake is, measured inequality should be regarded as remaining unchanged.

Surely, Scale Invariance disposes of the Variable Size Problem? That would indeed appear to be the case, if one were to go by the conventions and practices of professional economists. Almost all known indices of inequality are scale-invariant or relative measures: common examples would include the coefficient of variation, the Gini coefficient, and the Theil Index. But before embracing Cole Porter's belief in the infallibility of consensus ('Fifty Million Frenchmen [or at least Scores Of Dismal Scientists] Can't Be Wrong'), it is worth examining the issue a little more closely.

It is possible to advance another property of an inequality index which it could be hard to disagree with. This is a property that one may call *Positive Responsiveness to the Income of the Rich* (or simply, *Positive Responsiveness*, in short), which says that given any two-person distribution, if the amount of cake received by the poorer person remains constant while the amount of cake received by the richer person increases, then measured inequality should be seen to increase, that is, inequality should rise monotonically with the richer person's ownership of cake, other things remaining the same. This is no more than the elementary requirement that when

the rich get richer while the poor stay where they are, inequality should be seen to increase:

*Positive Responsiveness to the Income of the Rich:* For any two distributions  $\mathbf{x} = (x_1, x_2)$  and  $\mathbf{y} = (y_1, y_2)$ , if  $y_1 = x_1$  and  $y_2 > x_2$ , then inequality in the distribution  $\mathbf{y}$  is greater than inequality in the distribution  $\mathbf{x}$ .

Positive Responsiveness, considered on its own, seems to be an eminently reasonable property to demand of an inequality index. However, and unfortunately, there is a specific sense in which one cannot be eminently, or even ordinarily, reasonable if one insists on the unexceptionableness of *both* Scale Invariance and Positive Responsiveness. To see what is involved, consider the two two-person distributions  $\mathbf{x} = (0,1)$  and  $\mathbf{y} = (0,100)$ . Notice that  $\mathbf{y}$  is just a 100-fold rescaled replica of  $\mathbf{x}$ : 1's receipt of cake in  $\mathbf{y}$  is 100 times her receipt of cake in  $\mathbf{x}$ , and likewise 2's receipt of cake in  $\mathbf{y}$  is 100 times her receipt of cake in  $\mathbf{x}$ . By the Scale Invariance property, the following judgment must be pronounced:

*Statement A.* The extent of inequality in  $\mathbf{x}$  is the same as the extent of inequality in  $\mathbf{y}$ .

However, notice also that in moving from distribution  $\mathbf{x}$  to distribution  $\mathbf{y}$ , the poorer person 1's receipt of cake has remained unchanged while the richer person 2's receipt has increased. By the Positive Responsiveness property, the following judgment must also be pronounced:

*Statement B.* The extent of inequality in  $\mathbf{y}$  is greater than the extent of inequality in  $\mathbf{x}$ .

Unhappily, Statements A and B are mutually incompatible. It may be objected that the problem just discussed arises only when the poorer person has no cake. But all situations in which  $x_1 = 0$  are part of the 'domain' of permissible income distributions on which an inequality measure must pronounce a judgment. Also, the case of perfect concentration, in which one person has no cake and the other all of the cake is such a special and distinguished polar case of the cake-division problem that it would be hard to find a reason for leaving it out of the reckoning. Finally, and as far as can be ascertained, all known relative inequality measures are presumed to subsume the case of perfect concentration just mentioned.

A second sort of objection might be that incompatibility is 'inevitable', because Scale Invariance upholds a relative view of inequality, while Positive Responsiveness upholds an absolute view. One's response would have to be that the appeal of specific axioms must be judged on the strength of their own respective merits, that is to say

on independent grounds, and not in anticipation of their joint compatibility or otherwise. Otherwise, and at some level, no 'impossibility theorem' would survive this sort of objection!

Briefly, it would appear that the economist's standard resolution of the Variable Size Problem, by resort to the Scale Invariance postulate, is subject to a rather elementary difficulty, as has just been demonstrated. Some other way of addressing the Variable Size Problem may have to be explored. Some economists have advanced one such alternative path, in terms of a property called Translation Invariance.

## 'Translation Invariance'

The Scale Invariance property, as discussed above, is concerned with the *ratio* of the poorer person's receipt to the richer person's receipt, and not with the *difference* in the two persons' respective receipts. Is this, perhaps, the source of the difficulty with Scale Invariance? If it is, then an alternative invariance property that naturally suggests itself is what economists call Translation Invariance, which simply requires that equal additions of cake to (or subtractions of cake from) what each person presently owns ought to leave the extent of measured inequality unchanged: the criterion for equal inequalities then shifts from equal *proportionate* changes to equal *absolute* changes:

*Translation Invariance:* For any two distributions  $\mathbf{x} = (x_1, x_2)$  and  $\mathbf{y} = (y_1, y_2)$ , if  $y_1 = x_1 + c$  and  $y_2 = x_2 + c$ , where  $c$  is any number such that  $x_1 + c \geq 0$ , then the extent of inequality in the distribution  $\mathbf{y}$  is the same as the extent of inequality in the distribution  $\mathbf{x}$ .

While Scale Invariance, as noted earlier, takes a *relative* view of an inequality measure, Translation Invariance takes an *absolute* view of an inequality measure: equal *absolute* changes, rather than equal *proportionate* changes, are what are required— under Translation Invariance— to preserve the extent of inequality in a distribution. A very well-known absolute measure of inequality or dispersion, frequently resorted to by statisticians and economists, is the index called the standard deviation.

Does Translation Invariance satisfactorily resolve the Problem of Variable Size? To answer this question, it is helpful to advance one additional property of an inequality index whose appeal, it can be maintained, would be hard to deny. This is the property of *Continuity*. In a general way, one might say that Nature abhors



a discontinuity, much as it is supposed to abhor a vacuum. By a discontinuity is meant, in ordinarily apprehended language, a situation in which a minor or marginal change in a cause results in a major or abruptly large change in its effect. (A rule of punishment, which stipulates that if a person is caught stealing ten dollars he will be, administered a mild cuff on his ear, but that if he is caught stealing ten dollars and a cent he will have his ear cut off, is a discontinuous rule of punishment. Such discontinuities are not, in general, attractive.) In the context of inequality measurement, the property of Continuity would simply demand that minor changes in a distribution should not produce major changes in distributional judgments:

*Continuity.* For any two distributions  $\mathbf{x}$  and  $\mathbf{y}$ , if  $\mathbf{x}$  and  $\mathbf{y}$  are 'similar' distributions, then inequality judgments about  $\mathbf{x}$  and  $\mathbf{y}$  should also be 'similar'.

It can be shown that Translation Invariance and Continuity are properties, which are mutually incompatible, if it is also insisted upon that an inequality index should be able, at the least, to differentiate between an outcome of perfect equality and an outcome of perfect concentration. To see this, consider the three two-person distributions  $\mathbf{x} = (0,1)$ ,  $\mathbf{u} = (99,100)$ , and  $\mathbf{v} = (100,100)$ . Since the distribution  $\mathbf{u}$  has been derived from the distribution  $\mathbf{x}$  by the addition of an equal amount of cake (99 units) to each person's ownership, Translation Invariance obliges one to subscribe to the following judgment:

*Statement C.* The extent of inequality in  $\mathbf{x}$  is the same as the extent of inequality in  $\mathbf{u}$ .

It is apparent, from mere inspection, that the distributions  $\mathbf{u}$  and  $\mathbf{v}$  are very similar to each other, whence the property of Continuity dictates endorsement of Statement D below:

*Statement D.* The extent of inequality in  $\mathbf{u}$  is *virtually* the same as the extent of inequality in  $\mathbf{v}$ .

Notice now that  $\mathbf{x}$  is a wholly concentrated distribution (2 has all the cake and 1 nothing), while  $\mathbf{v}$  is a perfectly equal distribution (each of 1 and 2 receives 100 units of cake). This fact, in conjunction with Statements C and D above, entails Statement E below:

*Statement E.* Distribution  $\mathbf{u}$  is, at one and the same time, both a perfectly concentrated and a virtually perfectly equal distribution.

Statement E is a plainly absurd judgment, and if one subscribes to Continuity, one must conclude that Translation Invariance, like its rival Scale Invariance, is not a problem-free invariance condition.

Again, it may be objected that this impossibility result is a perfectly 'obvious' one, one which is a straightforwardly foreseeable consequence of combining an absolute principle of inequality comparison (translation invariance), with a relative principle (continuity). Again, in response, it must be pointed out that the objective is not to claim any large (or even small!) 'surprise-value' for the impossibility result. Moreover, a person who subscribes to translation invariance is no doubt free to dismiss continuity as a desirable property, but if s/he were to do this only as a consequence of biting the bullet in the cause of unswerving loyalty to translation invariance, or as a response to the inevitable tension of combining absolute and relative principles with no regard for the stand-alone appeal of these principles, then we are ultimately dealing with a stance which hardly transcends a stonewalling insistence on brute consistency. Indeed, and as mentioned earlier, the essence of 'impossibility theorems' is that certain combinations of axioms result in a discovery of non-existence when one is unable to perceive de-merit in the axioms identified and assessed individually. The somewhat absurdly swift and direct results discussed in this note share the 'essence' just discussed with more complex impossibility results, and from this author's point of view, it is hard to see it as a compulsive criticism to suggest – *after the event*, so to speak – that, for instance, the outcome of the Arrow theorem is, after all, inevitable!

## The Normative Values Underlying Scale and Translation Invariance

It is, perhaps, just as well that neither Scale Invariance nor Translation Invariance furnishes a logically coherent resolution of the Problem of Variable Size. This is because a closer examination of the two properties uncovers their respective value-orientations to inequality, orientations, which many would pronounce to be 'extreme' and morally not very attractive. To see what is involved, consider first the distributions  $\mathbf{a} = (1,100)$  and  $\mathbf{b} = (2,105)$ . In moving from  $\mathbf{a}$  to  $\mathbf{b}$ , person 1's ownership of cake has increased by 100 per cent, and person 2's by only 5 per cent. For one who subscribes to Scale Invariance, the movement from  $\mathbf{a}$  to  $\mathbf{b}$  must be seen in the light of a vast improvement in distributional equity. However, it is also true that the large rate of increase in 1's receipt of cake has been achieved on a very

small base, while the relatively small rate of increase in 2's receipt of cake has been achieved on a very large base, with the result that of the total additional 6 units of cake generated in moving from **a** to **b**, the poorer person's share has been just 1/6<sup>th</sup>, while the richer person's share has been 5/6<sup>th</sup>.

This has serious implications for inequality comparisons. For instance, by adopting a purely relative approach to inequality measurement, as is done in the bulk of the economics literature, the growth in global income is frequently viewed as being unaccompanied by any alarming increase in inter-country inequality. An absolute measure of inequality, would, contrarily, show that roughly equal country-specific rates of growth of average income are compatible with large increases in absolute inter-country differences in income. Absolute measures of inequality thus take an altogether less conservative view of inequality than relative measures. This does not necessarily make absolute measures wholly exempt from normative criticism. Consider the distributions **a** = (1 million, 2 million) and **c** = (0, 1 million): **c** has been derived from **a** by the subtraction of an identical amount of 1 million units from each person's initial allocation of cake, and Translation Invariance (the inspiration behind absolute measures of inequality) would certify that both **a** and **c** have the same extent of inequality. This *is* a morally odd judgment, for in **a** it is two millionaires who are being compared, while in **c** it is a complete destitute who is being compared with a millionaire: surely the two inequality comparisons are not morally commensurable. Indeed, it is a well-known problem with absolute inequality measures that they must treat equal *increments* of income symmetrically with equal *decrements* of income – a requirement that is not always normatively appealing.

Notwithstanding the earlier reference to 50 million Frenchmen, it was one particular Frenchman— the economist-philosopher Serge-Christophe Kolm— who was largely responsible for drawing other economists' attention to the difficulties occasioned by taking either a wholly relative or a wholly absolute view of inequality. He characterized relative inequality measures as 'rightist', and absolute inequality measures as 'leftist' (Kolm, 1976a, b), and although his political inclination was perhaps more pronouncedly oriented to the left than to the right, he was seized of both the logical and normative problems associated with either extreme type of inequality measure. He advanced, in this connection, the possible virtues of a 'centrist' inequality measure, namely a measure which has the property that it will register an increase in value whenever any distribution is uniformly

scaled up by any factor, and a decline in value whenever any distribution is changed by adding a fixed sum to each person's ownership of cake.

## On a Particular Centrist Measure of Inequality

For a two-person distribution, the mathematician Manfred Krtscha (1994) has derived an ingenious 'centrist' measure of inequality with a number of appealing properties, drawing on rigorous normative and mathematical reasoning. This is not the place to get into any of the details of Krtscha's work, but it is useful to note that the Krtscha measure of inequality for a two-person distribution  $\mathbf{x} = (x_1, x_2)$  is given (at the cost of some simplification) by:

$$K(\mathbf{x}) = \frac{(x_2 - x_1)^2}{2\bar{x}},$$

where  $\bar{x} = (x_1 + x_2) / 2$  is the arithmetic mean of the distribution  $\mathbf{x}$ .

The Krtscha Index will now be aside for a moment. Appealing to the reader's raw intuition, it could be suggested that a rather simple, straightforward, and 'natural' relative measure of inequality for a two-person distribution of incomes would be obtained by expressing the difference in the two persons' incomes as a proportion of the total income: if  $R(\mathbf{x})$  is employed as a short-hand for the value of this 'rightist' or 'relative' inequality measure for the distribution  $\mathbf{x} = (x_1, x_2)$ , then one can write:

$$R(\mathbf{x}) = \frac{(x_2 - x_1)}{(x_1 + x_2)},$$

or, noting that the arithmetic average of the distribution is given by  $\bar{x} = (x_1 + x_2) / 2$ , one has:

$$R(\mathbf{x}) = \frac{(x_2 - x_1)}{2\bar{x}}.$$

One can see that the measure  $R$  lies between 0 and 1: when both persons receive an equal share of the cake ( $x_1 = x_2$ ),  $R(\mathbf{x}) = 0$ ; and when the poorer person receives no part of the cake ( $x_1 = 0$ ),  $R(\mathbf{x}) = 1$ .

Similarly, a 'natural' absolute or 'leftist' measure of inequality for a two-person distribution— call it  $L$ — would be given simply by the difference between the richer person's income and the poorer person's income:

$$L(\mathbf{x}) = (x_2 - x_1).$$

If the desire is to avoid the 'extreme' values inherent in both 'rightist' and 'leftist' inequality measures, then there would be an inclination to settle for a 'centrist' measure. A centrist measure, to recall, is one whose value goes up when there is a uniform scaling up of a distribution, and whose value goes down when the same absolute amount is added to each person's income. Given the rightist and leftist measures  $R$  and  $L$  respectively, a trivially simple way of obtaining a 'centrist' measure—call it  $C$ —is to just express it as a product of the rightist and leftist measures (a uniform scaling up of a distribution would leave the rightist measure unchanged while raising the value of the leftist measure and therefore of the product of the two measures; and a uniform addition of the same amount to both incomes would leave the leftist measure unchanged while reducing the value of the rightist measure and therefore of the product of the two measures):

$$C(\mathbf{x}) = R(\mathbf{x})L(\mathbf{x}) = \frac{(x_2 - x_1)^2}{2\bar{x}}.$$

A remarkable fact can now be noted. A fresh look again at the expressions for  $C$  and  $K$  indicates that they are one and the same: the centrist measure of inequality  $C$  is just the Krtsha Index  $K$ , which thus has a rather simple, intuitive rationalization! (Indeed, for a general,  $n$ -person distribution, it turns out that the Krtsha measure is a product of two well-known measures of inequality—one of which, the coefficient of variation, is a relative measure, and the other, the standard deviation, is an absolute measure.) Regrettably, centrist measures of inequality are rarely employed by economists in the empirical literature on inequality comparisons.

## Practical Implications

While the arguments presented above may be conceded to have some validity, as far as they go, the question arises: how far *do* they go, from a practical point of view? The distinction between different approaches to the conceptualization and measurement of inequality would be of no more than abstract logical interest and arcane value, if it were the case that in the generality of situations, alternative assessments of inequality actually agreed in their respective diagnoses. This is a matter for empirical verification. While a certain amount of concentrated theoretical work [1] has been done on the broad issues discussed in this note, it

remains true that the overwhelmingly favored approach to inequality measurement in applied work has been the *relative* approach [2], entailing the use of relative measures of inequality such as the (relative) Gini coefficient, or the coefficient of variation, or the Theil index, or the Atkinson index. In the comparatively restricted range of applied studies available which have advanced the merits of a plural approach to assessment— covering relative, absolute and centrist or intermediate conceptualizations of inequality— the evidence points strongly to the possibility that the outcome of inequality comparisons is a pronouncedly variable function of the particular value-orientation that is espoused by the practitioner. In what follows, and while taking care to avoid an emphasis on details of a wholly technical nature, a few studies, which uphold this proposition, are cited.

Consider the time-series evidence on inter-national income-inequality as reviewed by Bosmans, Decaneq and Decoster (2011). In a broad way, the authors find (ignoring *intra*-national inequality and focusing only on *inter*-country inequality), that the relative approach to inequality assessment suggests an over-time (1980 to 2009) decline in inequality, a trend which is, however, negated and reversed by both the absolute and intermediate approaches. To quote the authors (Bosmans et al., 2011: p.16):

Several popular relative [measures] ... indicate that inequality has decreased over time. However, we showed that one does not need to rely on genuine absolute measures to reverse the conclusion. For the changes in the world income distribution over the period 1980-2009, some intermediate level of relativeness ... suffices to produce unanimous agreement on increased inequality within the corresponding subset of inequality measures ... We have shown that the specific choice of invariance concept does indeed have a major impact on the answer one gives to the simple question of whether world inequality increased or not ...

A similar conclusion is arrived at by Atkinson and Brandolini (2004) in their work on the assessment of over-time trends in world income-inequality. In the matter of inter-national inequality over the period 1970 to 2000, the authors find— broadly speaking— that specific relative measures yield a declining trend while specific absolute and intermediate measures reverse this trend. In the matter of global inequality (where account is taken of both *intra*- and *inter*-national inequality), they discover that all approaches to inequality assessment suggest a rising trend— which is however steeper for absolute and intermediate measures than for relative measures. In their own words (Atkinson and Brandolini, 2004: pp.14, 16-17):

To sum up, when we adopt a relative view of inequality, the world distribution of real per capita GDP appears to have noticeably narrowed from 1970 to 2000. However, this conclusion does not survive a move towards non-relative conceptions of inequality. We find evidence of a substantial increase of international inequality, whether we adopt an absolute or intermediate conception, regardless of the measure chosen ... [T]he secular movement of the world income distribution [i.e. one which takes account of both intra- and inter-national distributions] does not change whether we look at relative or non-relative measures— inequality has been rising. The story is somewhat different, however, after the Second World War: the modest positive slope of relative inequality is matched by a steep ascent of absolute and intermediate inequality.

Specific country-experiences of inequality tend to replicate the evidence on the global front. In a study of changes in the distribution of household consumption expenditure in Spain between the years 1980-81 and 1990-91, Del Rio and Ruiz-Castillo (2001) find that a relative approach to inequality measurement suggests a decline and an absolute approach an increase; resort to intermediate approaches suggests that these— except when they lean heavily on the side of a right-of-centre orientation— signal an over-time increase in inequality. As the authors (Del Rio and Ruiz-Castillo, 2001; p. 221) put it:

The 1990-91 household expenditure distribution in Spain dominates, in the relative ('rightist') Lorenz sense, the 1980-81 distribution, but the latter dominates the former in the absolute ('leftist') Lorenz sense. This situation constitutes a textbook case for intermediate or 'centrist' notions of inequality and social welfare ... The data reveal that there is a decrease in expenditures inequality [only] for a small set of centrist attitudes.

Much the same sort of inference has been drawn by Subramanian and Jayaraj (2013a,b) about inequality in the inter-personal distribution of consumption spending in rural India. While a number of commentators (Ahluwalia 2011, Bhalla 2011, Bhagwati 2011) have maintained that rising inequality in rural India has not been a feature of the growth process, it turns out that this judgment is largely a matter of the particular approach to inequality measurement one adopts. Subramanian and Jayaraj, in the papers cited earlier, point out, on the basis of official surveys conducted in selected years between 1970-71 and 2009-10, that inequality in the distribution of consumption expenditure in India is indeed non-increasing when it is measured in terms of the relative Gini coefficient, but that this trend is reversed when inequality is measured in terms of the absolute and intermediate Gini coefficients. Similarly, in both rural and urban India, the

data surveyed at decadal intervals from 1960-61 to 2002-03, and from 1980-81 to 2002-03 respectively, suggest that the relative Gini measure of inequality in the inter-household distribution of *assets* is either roughly stationery or declining, while the absolute and intermediate Gini measures register a systematic over-time increase. Employing micro (household-level) data at five-yearly intervals from 1983 to 2009-10, and taking appropriate account of the uniform reliability of these data, the authors also find that the (relative) coefficient of variation in the interpersonal distribution of consumption expenditure displays no rising over-time trend in rural India, whereas this pattern is reversed by the (absolute) standard deviation and the (intermediate) Krtscha index, which latter is simply a product of the coefficient of variation and the standard deviation.

For egalitarians, inequality— other things equal— is an intrinsically unacceptable ethical failing, and one which is arguably particularly egregious in the presence of considerable amounts of poverty (as in the scheme of Harry Frankfurt's (1987) notion of 'sufficientarianism'). In large parts of the Third World, one finds evidence of the coexistence of poverty with rising levels of economic inequality. Apart from such intrinsic considerations of fairness, inequality is also regarded by many as a 'bad' from instrumental considerations relating, *inter alia*, to the deleterious effects of inequality on social cohesion, on efficiency, on the health status of a society, and on aggregate demand in an economy. As has been pointed out in Subramanian and Jayaraj (2013b) these are positions that can and should be debated: only, the debate is frequently short-circuited by denials of growth being accompanied by rising inequality— a product of findings based on an exclusive reliance on wholly relative approaches to the assessment of inequality. This section has been concerned to argue that such evidence on the matter as is available suggests that absolute and intermediate orientations to inequality do *not*, as a general rule, endorse the evaluative outcomes of purely relative approaches to the problem.

## Concluding Observations

In the social sciences (as perhaps in other spheres of life), 'experts' tend to stick to consensual modes of thinking, for reasons which Anthony Shorrocks (2005) has called 'network' and 'inertia' effects, without bothering overmuch to examine the normative and logical underpinnings of their conceptualizations and formulations of important phenomena such as disparity and deprivation. This has (obvious) unhappy implications for description, diagnosis, and remediation. This paper has



been concerned to highlight one particular example of this general issue— that of measuring inequality. The paper, which is largely bereft of nuance and detail, is primarily an effort to bring a technical problem with important logical and social implications to the attention of the philosophically- inclined general reader. It may be added that the criticism in this essay of certain aspects of 'mainstream' economic theorizing is not intended as a nihilistic dismissal of the enterprise of social and economic measurement. Far from it: that would only open the door to chaos, license, and the nonsensical excesses of 'relativism' gone overboard which often pass for reasoned discontentment with the Establishment. Rather, the objective here is to advocate the virtues of continual examination of even the seemingly trivial and the self-evident, and to emphasize the importance of carefulness and patience even at the cost of plodding, inconvenience, and possibly eventual admission of uncertainty or ignorance.

Finally, and in order to avoid any possibility of misunderstanding, it might be clarified that it is no part of this essay's intention to suggest that economists, in any exhaustive sense, are unaware of what they are measuring or that they do not appreciate important issues such as the distinctions between absolute and relative measures and the implications of scale and translation invariance. Unless *some* economists were deeply sensible of these issues, one should scarcely have been in a position to write a paper such as the present one! Indeed, there has been some care displayed in disowning much in the way of originality in the paper, and a principal motivation has been to *re-visit* certain important issues in the measurement of inequality. Having said this, it is nevertheless valid to maintain that there is a difference between the existential fact of knowledge of an issue, on the one hand, and the extent of its acknowledgement, on the other. It can scarcely be claimed, for example, that the limitations of the market mechanism are not known within the neo-classical economics profession, though it would perhaps also be fair to suggest that, in a general way, neo-classical economists are insufficiently sensitive to the limitations of the market mechanism. The situation, if anything, is worse when it comes to practitioners (especially at the empirical level) of the economics of inequality, with specific reference to the distinction (and implications thereof) between relative and absolute approaches to assessment. Hence, in large part, the expository-cum-critical orientation of the present essay.

## Acknowledgement

The author would like to thank Peter Lambert for valuable comments, as also an anonymous referee and the editor of this journal for extensive criticisms which he has sought to address to the best of his ability.

## Endnotes

[1] Some salient papers on non-relative measures of inequality would include Atkinson and Brandolini (2004), Azpitarte and Alonso-Villar (2011), Bosmans, Decancq and Decoster (2011), Bossert and Pfingsten (1990), Chakravarty and Tyagarupananda (1998, 2009), Del Rio and Alonso-Villar (2008, 2011), Del Rio and Ruiz-Castillo (2000, 2001), Jenkins and Jantti (2005), Kolm (1976a, 1976b), Krtscha (1994), Moyes (1987), Yoshida (2005), Zheng (2007), and Zoli (2012).

[2] It is instructive to take note of the view of Atkinson and Brandolini (2004; pp. 1-2) in this regard: 'At the empirical level, the acceptance of the relative criterion is almost unconditional. We have never seen official publications reporting estimates of absolute inequality, and even academic studies are rare ...'. In a similar vein, Bosmans, Decancq and Decoster (2011; p.2) say: '... [A]s far as the measurement apparatus used to assess inequality is concerned, there seems to be a remarkable tacit agreement to focus exclusively on the relative view of inequality, thereby ignoring the a priori equally relevant absolute and intermediate views.' For notable empirical studies of non-relative approaches to inequality assessment, which are also distinguished by their careful attention to the underlying theoretical issues involved, the reader is referred to Atkinson and Brandolini (2004), Bosmans, Decancq and Decoster (2011), and Del Rio and Ruiz-Castillo (2000, 2001). Of applied interest also are the papers by Subramanian and Jayaraj (2013a,b).

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S. Subramanian is Professor at the Madras Institute of Development Studies (Chennai, India) ([subbu@mids.ac.in](mailto:subbu@mids.ac.in))