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Two-stage stochastic program for disassembly lot-sizing under random ordering lead time

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Abstract: This study addresses disassembly lot-sizing that determines the ordering and disassembly schedules of end-of-life (EOL) products and sub-assemblies to satisfy items demands. A stochastic version with uncertain ordering lead time (OLT) is considered for the first time. Here, OLT represents the time elapsed between placing an order and receiving it (only an EOL product can be ordered). Stochastic integer programming model is developed for the objective of minimizing the expected sum of setup, purchase, inventory and backlog costs. To test the performance of the proposed model, computational experiments were done on various test instances and are reported. The results show that the performance of the proposed model depends on the size of the ordering lead times, the number of periods and components.

Keywords: Disassembly lot-sizing, random ordering lead times, two-stage stochastic programming.

1. INTRODUCTION AND RELATED PUBLICATIONS

Sustainable development has become a philosophical concept for the future of the earth in the three main pillars of social equity, environmental protection and economic viability. Among the pillars, environmental protection is very urgent one due to significant environmental deteriorations such as excessive consumption of natural resources, global warming, etc. (Benaissa et al., 2018).

Treating end-of-life (EOL) products in a more economical and ecological way is one of the emerging challenges of sustainable development (Slama et al., 2020a). In general, EOL products can be recovered by collection, disassembly, inspection, recycling, repackaging, etc., which can be integrated as a reverse logistics framework (Ji et al., 2016).

Among the reverse logistics activities, this study focuses on the disassembly that separates the product into parts for the purpose of recovering materials, separating reusable parts/sub-assemblies, etc. (Lambert, 2007). Decision issues in disassembly systems can be categorized as design issues includes design for disassembly, disassembly line balancing, etc., and the operational problem includes process planning, lot-sizing, scheduling, etc. Note that disassembly is one of the key activities in reverse logistics, especially in product/material recovery such as refurbishing and recycling.

This study addresses the disassembly lot-sizing problem, one of disassembly production planning decisions. It aims to determine the quantities of disassembling an EOL product and its sub-assemblies to satisfy the demands of its parts (Slama et al., 2020a).

There are many variations of the disassembly lot-sizing problem, and they depending on the disassembly system (two or multi-level bill of materials) and on the planning horizon (one or multi period). The majority of the literature on disassembly lot-sizing system concerns the deterministic uncapacitated problem (e.g., Gupta and Lambert, 2016; Taleb et al., 1997; Kim et al., 2006a; Langella, 2007; Kim et al., 2009; Kang et al., 2016; Kim et al., 2018; Godichaud and Amodeo, 2018; Pour-Massahian-Tafti et al., 2019a,b, 2020), while a small number of studies focused on the deterministic capacitated problem (e.g., Lee et al., 2002; Kim et al., 2006b; Ji et al., 2016; Slama et al., 2020a). For the existing surveys, readers can refer to Slama et al. (2019a).

In industrial reality, the disassembly process is sensitive to various sources of uncertainty, such as uncertain demand for components, disassembly yield, disassembly lead times (Slama et al., 2020b). Indeed, these uncertainties create disruptions in the disassembly plan and lead to unmet demand from customers.

Many researchers have applied stochastic algorithms to the disassembly lot sizing problem. To position our research
in the existing literature, we review the researches on stochastic disassembly lot-sizing problems under uncertainty of demand, yield and disassembly lead time. Table 1 summarizes the literature on the considered problem, and indicates the resolution approach, the disassembly system, the planning horizon and the type of uncertainty.

As discussed in Guide (2000), the disassembly process is sensitive to the uncertainty of the ordering lead time of EOL product. This uncertainty can interrupt the disassembly process and the delivery of disassembled parts, and it increases backlog costs. However, to our knowledge, the uncertainty of the ordering lead time of EOL product (i.e. the time elapsed between placing a purchase order of EOL product and receiving it from the market) has not yet been studied. As the ordering lead time (OLT) of EOL product depends of the availability of the EOL product on the market, it cannot be estimated with precision. To fill this gap in the literature, this research studies the disassembly lot-sizing problem under the uncertainty of the OLT, for multi-period planning, single type of EOL product and multi-echelon disassembly bill of materials. The objective is to determine the ordering and disassembly schedules of EOL products while satisfying the demand of their components over a certain planning horizon.

This work is organised in the following way. Section 2 describes the studied problem. Section 3 presents the stochastic formulation for the optimisation problem considered. Section 4 deals with computational experiments. Section 5 provides conclusions as well as future research directions.

2. PROBLEM DESCRIPTION

A current industrial framework could be to consider a computer manufacturer, which voluntarily takes back old computers from end-of-life consumers. This manufacturer also has a disassembly and refurbishment facility, and responds to the demand for refurbished parts from its most environmentally conscious customers. This setting can be presented in Fig. 1.

In order to meet the demand for remanufactured parts, old computers (represented by boxes marked 1) must be ordered, collected and transported to the company’s remanufacturing facility. They must then be sorted and disassembled to obtain viable parts to use in the remanufacturing effort (the parts are represented by the circles denoted a, b, and c).

In practice, the time elapsed between placing a purchase order of an EOL product and receiving it, is random. This uncertainty can be mainly explained by the unavailability of EOL products on the market. Subsequently, in this paper, it is assumed that there is a shortage of the root item, that is, the end-of-life products cannot be obtained whenever they are ordered. To incorporate this uncertainty into the decision making process, a discrete set of scenarios is defined. The demand for parts and the time required to disassemble the EOL products are considered certain. Finally, the disassembly capacity is supposed unlimited.

We consider time-varying purchase costs depending on the market situation, which means that the purchase costs may vary according to different planning periods. When

![Fig. 1. Remanufacturing from collection to disassembly](image)

Each item is disassembled, the equipment required for that item has to be set up. This results in a setup cost. Also, inventory holding costs occur when items are held in order to satisfy future demand.

The risk of an uncertainty of ordering lead times occurs backlog and inventory holding costs. In fact, if the EOL products are not delivered at the desired time, the disassembly and refurbishing process are not starting as expected, and then a backlog cost is incurred. In the same manner, an inventory cost appears if the EOL product are received before the expected delivery date.

The case of multi-echelon disassembly system is considered in this research. As shown in Fig 2, a root item represents an EOL product to be ordered and disassembled, an intermediate items represent a sub-assemblies to be disassembled further (items 2-6) and a leaf items represent a components that cannot be disassembled (items a-d). The number in parentheses represents the number of components obtained by the disassembly operations.

As mentioned earlier, customers have a certain demand for disassembled items for all time periods. This demand is both external for leaf items and internal for the root and sub-assemblies items. Finally, the demand is satisfied only by disassembling the EOL products.

![Fig. 2. Multi-echelon disassembly lot-sizing system](image)

This study is supposing:

1. Once the EOL product is ordered for disassembly, it is available after a random ordering lead time:
2. The ordering lead time is a random discrete variable with a known probability distribution and a bounded over known interval $\text{OLT}_t^-$ and $\text{OLT}_t^+$;

The problem considered in this paper can be defined as the problem of determining the ordering schedules of the
Table 1. Summary of relevant literature under uncertainly

<table>
<thead>
<tr>
<th>Authors</th>
<th>Resolution</th>
<th>Uncertainty</th>
<th>Yield</th>
<th>Demand</th>
<th>DLT</th>
<th>OLT</th>
<th>#Periods</th>
<th>#Levels</th>
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<tbody>
<tr>
<td>Inderfurth and Langella (2006)</td>
<td>Heuristics</td>
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<td>1</td>
<td>✓</td>
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<td>2</td>
<td>Multi</td>
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<td>Kongar and Gupta (2006)</td>
<td>Fuzzy goal programming</td>
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<td>1</td>
<td>✓</td>
<td></td>
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<td>Kim and Xirochakis (2010)</td>
<td>MILP, Lagrangian heuristics</td>
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<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
<td>2</td>
<td>Multi</td>
</tr>
<tr>
<td>Wang and Huang (2015)</td>
<td>2S-MILP</td>
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<td>1</td>
<td>✓</td>
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<td>Multi</td>
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<tr>
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<td>1</td>
<td>✓</td>
<td></td>
<td>Multi</td>
<td>Multi</td>
</tr>
<tr>
<td>Fang et al. (2017)</td>
<td>MILP, Lagrangian heuristics</td>
<td>✓</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
<td>Multi</td>
<td>Multi</td>
</tr>
<tr>
<td>Liu and Zhang (2018)</td>
<td>MS-MILP, Outer-approximation</td>
<td>✓</td>
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<td>1</td>
<td>✓</td>
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<td>Multi</td>
</tr>
<tr>
<td>Quezada et al. (2020)</td>
<td>MS-MILP</td>
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<td>✓</td>
<td></td>
<td>Multi</td>
<td>Multi</td>
</tr>
<tr>
<td>Slama et al. (2019c)</td>
<td>S-LP</td>
<td>✓</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
<td>Multi</td>
<td>Multi</td>
</tr>
<tr>
<td>Slama et al. (2020a)</td>
<td>Analytical model, Newsboy formulaire</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>✓</td>
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<td>Multi</td>
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<tr>
<td>Slama et al. (2021a)</td>
<td>Analytical model, Newsboy formulaire</td>
<td>✓</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
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<td>Multi</td>
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<tr>
<td>Slama et al. (2021b)</td>
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<td>1</td>
<td>1</td>
<td>✓</td>
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<td>Multi</td>
<td>Multi</td>
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<tr>
<td>Slama et al. (2022)</td>
<td>Aggregate scenario</td>
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<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
<td>Multi</td>
<td>Multi</td>
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<tr>
<td>Current paper</td>
<td>2S-MILP</td>
<td>✓</td>
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<td>1</td>
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<td></td>
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</tr>
</tbody>
</table>

“MILP: Mixed Integer Linear Programming, SAA: Sample average approximation, GA: Genetic algorithm, 2S-MILP: Two stage-MILP, MS-MILP: Multi stage-MILP.”

EOL and the disassembly schedules of all parent items in the given disassembly system under random OLT, over a certain planning horizon. The objective is to minimise the sum of expected purchase, setup, inventory holding and backlog costs.

3. LINEAR STOCHASTIC MODEL

To formulate the problem considered in this research, the topological order is considered from left to right and from bottom to top for all items. Let \( i = 1 \) be the index of the root item and \( N \) the index of the last item obtained by disassembling it. Let the following three sets: (i) \( I \), the set of items \( i \) with \( i = 1, 2, \ldots, i_t-1, \ldots, N \), where \( i_t \) denote the index for the first leaf item, (ii) \( I_c \), the set of items \( i \) of the last level of product structure that can no longer be disassembled. These items verify the following equality: \( \forall i \in I, \forall j \in I \) \( a_{ji} = 0 \) and \( I_c = \{ i_t, \ldots, N \} \), and (iii) \( I_e \), the set of remaining items such as \( I_e = \{ 2, \ldots, i_t-1 \} \). The full list of notations used throughout this paper is given in Table 2.

In this research, we provide a scenario based stochastic optimization formulation. A scenario represents a possible realization of the ordering lead time for the EOL product in each period.

**Definition 3.1.** Let \( \Omega \) be the set of all possible scenarios. Each scenario \( \omega \) corresponds to a realization of the ordering lead times from periods 1 to \( t \). More precisely, if any purchase order of EOL product starts in period \( t \), the EOL product is available in period \( t + OLT^+ \) in scenario \( \omega \), and the probability of scenario \( \omega \) is \( P_\omega \). If the ordering lead times in each period are independent, \( P_\omega = \prod_{t \in T} P(OLT_t = OLT^+) \), \( \sum_{\omega \in \Omega} P_\omega = 1, \forall \omega \in \Omega \) and \( |\Omega| = \prod_{t \in T} (OLT^+ - OLT^- + 1) \).

The studied problem can be represented by the two-stage multi-period stochastic mixed-integer program (2S-MILP). The first stage variables correspond to the decisions made in period 0 before observing the OLT. These decisions are the setup, and disassembly quantity for item \( i \in I_c \), and procurement quantity of EOL product, in period \( t \). The second stage variables also called recourse decisions correspond to the inventory levels of item \( i \in I \) and backlog levels of item \( i \in I_c \) at the end of period \( t \).
These decisions are observed after the realization of the OLT for each scenario ω.

The stochastic formulation of the multi-level disassembly lot-sizing problem is based on the set Ω of all possible OLT scenarios and the probability Pω, of each scenario ω. The main objective is to minimize the expected objective value and to select the appropriate ordering and disassembly plan. Our model is a generalization of the classic work proposed by Lee and Xirochakis (2004) with deterministic ordering lead times and it can be expressed with the following 2S-MILP model:

\[
\mathbb{E}(TC) = \min \sum_{i \in T} \left( \sum_{\omega \in \Omega} P_{\omega} \left( \sum_{i \in I} b_i, I_{it}^\omega + \sum_{i \in I_c} b_i, B_{it}^\omega \right) + Z_t, c_t \sum_{i \in I_s} s_i, Y_{it}^\omega \right) + \sum_{i \in I_a} \sum_{\tau = 1}^{T} Q_{i\tau}^\omega \right) \quad \forall t \in T, \forall \omega \in \Omega
\]

\[
I_{it}^\omega = \sum_{\tau \in A_{it}^\omega} Z_{\tau} - \sum_{\tau = 1}^{T} Q_{1\tau}^\omega + I_{10} \quad \forall i \in I, \forall \omega \in \Omega
\]

\[
I_{it}^\omega = \sum_{j \in \phi(i)} a_{ji} Q_{j\tau}^\omega + I_{i0} - \sum_{\tau = 1}^{T} Q_{i\tau}^\omega \quad \forall i \in I_c, \forall t \in T
\]

\[
I_{it}^\omega - B_{it}^\omega = \sum_{j \in \phi(i)} a_{ji} Q_{j\tau}^\omega + I_{i0} - \sum_{\tau = 1}^{T} d_{i\tau} \quad \forall i \in I_c, \forall t \in T
\]

\[
Q_{it} \leq Y_{it} M \quad \forall i \in I_c, \forall t \in T
\]

\[
Q_{it} \geq 0 \quad \forall i \in I_c, \forall t \in T
\]

\[
Y_{it} \in \{0, 1\} \quad \forall i \in I_c, \forall t \in T
\]

\[
I_{it}^\omega \geq 0 \quad \forall i \in I, \forall t \in T, \forall \omega \in \Omega
\]

\[
B_{it}^\omega \geq 0 \quad \forall i \in I_c, \forall t \in T, \forall \omega \in \Omega
\]

\[
Z_{t} \geq 0 \quad \forall t \in T
\]

The objective function (Eq.1) minimizes the sum of the expected inventory holding, backlog and purchase costs as well as the setup cost over the planning horizon. Constraints (2), (3) and (4) define the inventory balance level for the EOL product, each sub-assembly item and each leaf item i at the end of each time period t for a scenario ω, respectively where \( A_{it}^\omega := \{ \tau \in T \mid \tau + OLT_{it}^\omega \leq t \} \) is the set of period whose disassembled parent items are available. Constraints (5) guarantee that a setup cost is generated in a period if any disassembly operation needs to be performed in that period. Constraints (6-10) provide the conditions framing the decision variables.

4. COMPUTATIONAL EXPERIMENTS

This section presents the results of experiments conducted to study the behavior of the proposed optimization approach. The proposed model is implemented in C with Concert Technology and solved with IBM CPLEX 12.9 on a PC with processor Intel (R) Core™ i7-5500 CPU @ 2.4 GHz and 8 Go RAM under Windows 10 Professional.

The numerical experiments are performed with two test beds. The first contains small size instance to illustrate the optimal 2S-MILP model solution (section 4.1), and the second contains large instances to show the limits of the proposed model (section 4.2).

4.1 Numerical example

First, in order to illustrate an optimal solution obtained by the optimization approach, this subsection presents a small numerical example of the 2S-MILP model. The effectiveness of our stochastic optimization approach was examined by solving the multi-level disassembly system shown in Fig. 3. We considered a finite planning horizon with 7 periods and a disassembly system with 5 items. Disassembly operation extracts one component of type 2 and 3, two components of type 4 and one component of type 5. The demands on components over the planning horizon \( d_{it} \) are listed in Table 3. The unit time inventory holding and backlog costs are \( h_i = 10 \forall i \in I \) and \( b_i = 100 \forall i \in I_c \), respectively. The setup cost \( s_i \) is equal to 500, \( \forall i \in I_c \). Finally, the purchase cost of the EOL product in each time is as follow: \( c_t = (10, 15, 70, 25, 36, 100, 50) \).

![Fig. 3. Example of multi-echelon disassembly system.](image)

In each time period, the EOL product can be delivered after a stochastic ordering lead time using a known probability distribution between 1 and 3 in each period \( t \) (see Fig. 4). Table 3 presents the exact optimal solution provided by the 2S-MILP model. More precisely, this table illustrates the optimal quantity to be ordered and disassembled in each period (first stage decisions) over the planning horizon. The second stage decisions, i.e, the expected inventory and stockout of component \( i \) at period \( t \) are also presented in the same table. The number of all possible scenarios is \( |\Omega| = 3^2 \). The optimal \( \mathbb{E}(TC) \) takes the value 23991.60 and is obtained in 26.79 seconds.

4.2 Performance analysis

To show the effectiveness of the 2S-MILP model more generally, we solved a large test problem, and the test results are given in this section. The large test problems consists of 10 problems for each combination of 3 levels of the number of components (10, 20 and 30) and 3 levels of the number of periods (5, 7 and 10). For each level of
Computation time is an important factor in the decision process. For this reason, we set a time limit of 3600 seconds for CPLEX to run. The performance measures used are:

- \( I^* \): Number of optimal (out of 10 instances) obtained by the CPLEX solver;
- \( CPU \): Computation times, in seconds, i.e. the time(s) needed to obtain optimal solutions.

Table 4. Characteristics of data sets

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>( D \sim U(10, 30) )</td>
</tr>
<tr>
<td>( e_i )</td>
<td>( D \sim U(40, 60) )</td>
</tr>
<tr>
<td>( s_i )</td>
<td>( D \sim U(500, 1000) )</td>
</tr>
<tr>
<td>( I_{it} )</td>
<td>( D \sim U(20, 100) )</td>
</tr>
<tr>
<td>( b_i )</td>
<td>( D \sim U(50, 100) )</td>
</tr>
<tr>
<td>( d_{it} )</td>
<td>( D \sim U(50, 160) )</td>
</tr>
<tr>
<td>( a_{ij} )</td>
<td>( D \sim U(1, 3) )</td>
</tr>
<tr>
<td>( OLT_{it} )</td>
<td>( D \sim U(OLT_{it}^- , OLT_{it}^+) )</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This study considered a multi-level disassembly lot-sizing problem for one type of EOL product. The time necessary to receive an EOL product after placing a purchase order is considered as a stochastic variable under the interval representation of uncertainty. The problem is to determine the ordering and the quantities of disassembling the EOL product and its subassemblies. An optimal two-stage multi period stochastic mixed-integer program is suggested for the objective of minimizing the expected total cost. This model describes the random parameter through the set of all possible scenarios. Then, it was illustrated with a numerical example. Our investigations into this area are still ongoing. First, it’s interesting to develop an heuristics to solve the large test instances. Also, with the classic tree representation scenario, the multi-stage model is an interesting direction for our future research.

REFERENCES


