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Multi-Criteria Information Fusion for Storm Prediction Based on Belief Functions

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***Abstract*—The objective of this paper is to present a general methodology for storm risk assessment and prediction based on several physical criteria thanks to the belief functions framework to deal with conflicting meteorological information. For this, we adapt the Soft ELECTRE TRI (SET) approach to this storm context and we show how to use it on outputs of atmospheric forecast model, given an estimate of the state of the atmosphere in a future time. This work could also serve as a benchmark for other methods dealing with multi-criteria decision-making (MCDM) support and conflicting information fusion.**

Keywords: storm risk assessment, information fusion, belief functions, decision-making, Soft ELECTRE TRI.

I. Introduction

In the context of storm prediction, many sources of observations of the atmosphere may be used. The aim of storm risk assessment is to exploit as best as possible some of these available data to evaluate the risk of thunderstorm at a given location in the surveillance area under concern in a close future. Each type of data is associated to a given source of information called a criterion in our context. In the present paper, the data used are coming from a numerical weather prediction model. These kinds of models allow to simulate the evolution of the state of the atmosphere by solving dynamical and thermodynamical equations, by including data assimilation of observations of the atmosphere (from satellite, rawinsonde or buoys, for instance) and by adding physical parametrization for unresolved processes as convection. The outputs of the Global Forecast System (GFS), developed by the Centers for Environmental Prediction (NCEP) have been used for our study [1]. The estimation of storm risk level is of prime importance for many applications (aeronautical safety, air traffic management, ...). In this work we present a general methodology showing how to use belief functions [2] coupled with the Soft ELECTRE TRI (SET) outranking method [3] to manage efficiently the conflicting sources of information in a multi-criteria decision-making context.

This paper is organized as follows. After a short presentation of the Soft ELECTRE TRI outranking method in Section II for Multi-Criteria Decision-Making (MCDM) support we introduce the storm risk assessment problematic in Section III, and we show how it can fit well with the Soft ELECTRE TRI framework. We also provide an example of our storm assessment methodology based on data set coming from the atmospheric forecast supplied by GFS, and we show the performances of SET approach with respect to "ground truth" obtained by the World Wide Lightning Location Network (WWLLN) [4]. Conclusions are given in section IV with some perspectives.

II. Soft ELECTRE TRI for MCDM

A. A short presentation of SET

The Soft ELECTRE TRI method (SET) proposed in [3] is an evolution of the ELECTRE TRI (ET)[1] method proposed by Roy in [5] for making the outranking of alternatives with respect to profiles of categories. The SET method is based on belief functions calculus [2] (see appendix) and improves the classical ET method because it does not require an arbitrary choice of λ-cut strategy for making the outranking of alternatives with respect to profiles of categories, nor an ad-hoc choice of decisional attitude for making the final assignment. Actually, the SET method solves the assignment problem in a soft manner. The Fig. 1 shows the general MCDM problem that can be addressed by the SET method. More precisely, SET solves an assignment problem in complex situations where a (or several) given alternative has to be assigned to predetermined categories based on multiple criteria values. Each criterion G_j $(j = 1, \ldots, n_G)$ is evaluated quantitatively. Each profile is defined by the green points limiting the bounds of each category with respect to each criterion. The red chain represents a "multi-criteria value" (i.e. an alternative $\mathbf{a}$) that one wants to assign to a predefined category.

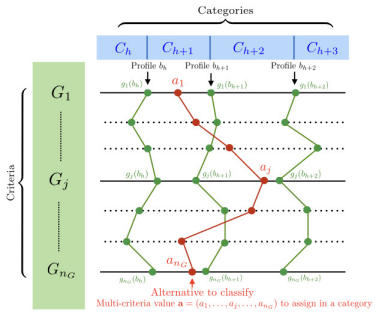

Figure 1: How to assign a category to an alternative?

[1]The acronym ELECTRE stands for "ÉLimination Et Choix Traduisant la RÉalité" (Elimination and Choice Expressing the Reality) [7], [8].

In the context of our storm risk assessment application each category C_h corresponds to a level of risk (low, moderate, strong, very strong, or extreme), and it is defined by its profile lower and upper bounds denoted respectively by $\mathbf{b}_{h-1}$ and $\mathbf{b}_h$ (see vertical green plots in Fig. 1). The profiles bounds define ad-hoc categories for the values of each criterion G_j corresponding to a meteorological parameter which is either a direct measure of a meteorological parameter, or an estimation of the parameter resulting from a sophisticate meteorological forecast model. These meteorological parameters (i.e. criteria) will be described in details in section III. Any alternative corresponds to a multi-criteria value $\mathbf{a} = (a_1, \ldots, a_j, \ldots a_{n_G})$ whose component a_j is nothing but the instance of the parameter (i.e. criterion) G_j for this alternative $\mathbf{a}$. Each alternative $\mathbf{a}$ is associated with a (2D) cell of the area of interest on the surface of the Earth.

The SET method allows to take into account the weight of importance of each criterion entering this assignment problem and to give a soft (i.e. probabilistic) assignment solution to commit any multi-criteria value $\mathbf{a}$ to a category. More precisely, SET calculates the probability that a chosen alternative $\mathbf{a}$ belongs to a predetermined category C_h based on all information available (the criteria values, the importances of the criteria, and the bounds of each predetermined category).

B. SET principle

We present briefly the principle and the steps of the Soft ELECTRE TRI (SET) outranking method developed originally in [3]. SET makes a soft assignment of $n_a \geq 1$ alternatives $\mathbf{a}_i$ in predefined ordered categories C_h $(h = 1, \ldots, n_h)$ according to criteria[2] measure $g_j(.)$, $j \in \mathbf{J} = \{1, \ldots, n_G\}$. Each category C_h is delimited by the set of its lower and upper limits b^j_{h-1} and b^j_h with respect to each criterion G_j measured by $g_j(\cdot)$. By convention, $b^j_0 \leq b^j_1 \ldots \leq b^j_{n_h}$. $\mathbf{b}_0 = (b^1_0, \ldots, b^j_0, \ldots, b^{n_G}_0)$ is the lower (minimal) profile bound and $\mathbf{b}_{n_h} = (b^1_{n_h}, \ldots, b^j_{n_h}, \ldots, b^{n_G}_{n_h})$ is the upper (maximal) profile bound. The overall profile $\mathbf{b}_h$ is defined by $(g_1(b_h), g_2(b_h), \ldots, g_{n_G}(b_h))$, and it is represented by the vertical plot joining the green dots in Fig. 1.

The outranking relations used in SET are based on the calculation of partial concordance and discordance indices from which global concordance and credibility indices are derived based on Basic Belief Assignment (BBA) modeling [2], and on an advanced fusion technique based on Proportional Conflict Redistribution rule no. 6 (PCR6) [9]–[11]. A soft assignment of each alternative $\mathbf{a}_i$ in a predetermined category is obtained by the calculation of the probabilized outranking relations, from which a final hard assignment can be drawn (if needed) for some action. In the storm risk assessment context, an action for instance could be the broadcast of an alert message to the air traffic management organisms or airports.

The Soft ELECTRE TRI method requires the following four steps:

[2]In our context, a criteria is a meteorological parameter.

SET-Step 1: Calculation of partial (local) concordance indices $c_j(\mathbf{a}_i, \mathbf{b}_h)$, partial discordances indices $d_j(\mathbf{a}_i, \mathbf{b}_h)$, and also partial uncertainty indices $u_j(\mathbf{a}_i, \mathbf{b}_h)$ between an alternative $\mathbf{a}_i$ and a profile $\mathbf{b}_h$ thanks to a smooth sigmoidal model [12]. The partial indices are encapsulated in BBAs $m^j_{ih}(.)$ for alternative $\mathbf{a}_i$ versus profile $\mathbf{b}_h$ (i.e. $\mathbf{a}_i$ vs. $\mathbf{b}_h$) as follows:

$$\begin{cases} c_j(\mathbf{a}_i, \mathbf{b}_h) \triangleq m^j_{ih}(c) & \text{(local concordance)} \\ d_j(\mathbf{a}_i, \mathbf{b}_h) \triangleq m^j_{ih}(\bar{c}) & \text{(local discordance)} \\ u_j(\mathbf{a}_i, \mathbf{b}_h) \triangleq m^j_{ih}(c \cup \bar{c}) & \text{(local uncertainty).} \end{cases} \quad (1)$$

where $m^j_{ih}(\cdot)$ is a Basic Belief Assignment (BBA) defined on the frame of discernement $\Theta \triangleq \{c, \bar{c}\}$, where c means that the alternative $\mathbf{a}_i$ is concordant (i.e. it agrees) with the assertion '$\mathbf{a}_i$ is at least as good as profile $\mathbf{b}_h$", and $\bar{c}$ means that the alternative $\mathbf{a}_i$ is opposed to this assertion (i.e. it is discordant, or it disagrees with this assertion). For each criterion G_j, a BBA m^j_{ih} defined on the power-set of Θ is obtained by the fusion of two simple BBAs $m^j_1(.)$ and $m^j_2(.)$ based on the following sigmoid models, see [12] for justification and details. Similarly, we compute also partial (local) concordance indices $c_j(\mathbf{b}_h, \mathbf{a}_i)$, partial discordances indices $d_j(\mathbf{b}_h, \mathbf{a}_i)$, and partial uncertainty indices $u_j(\mathbf{b}_h, \mathbf{a}_i)$ between a profile $\mathbf{b}_h$ and an alternative $\mathbf{a}_i$. This partial indices are encapsulated in BBAs $m^j_{hi}(.)$ for profile $\mathbf{b}_h$ versus alternative $\mathbf{a}_i$ (i.e. $\mathbf{b}_h$ vs. $\mathbf{a}_i$).

SET-Step 2: Calculation of the global (overall) concordance indices $c(\mathbf{a}_i, \mathbf{b}_h)$, $c(\mathbf{b}_h, \mathbf{a}_i)$, discordance indices $d(\mathbf{a}_i, \mathbf{b}_h)$, $d(\mathbf{b}_h, \mathbf{a}_i)$, and uncertainty indices $u(\mathbf{a}_i, \mathbf{b}_h)$, $u(\mathbf{b}_h, \mathbf{a}_i)$ by the fusion of local indices. More precisely, one must calculate

$$\begin{cases} m_{ih}(.) = [m^1_{ih} \oplus m^2_{ih} \oplus \ldots \oplus m^{n_g}_{ih}](.) \\ m_{hi}(.) = [m^1_{hi} \oplus m^2_{hi} \oplus \ldots \oplus m^{n_g}_{hi}](.) \end{cases} \quad (2)$$

where $\oplus$ denotes symbolically a chosen fusion operator.

To take into account the weight of importance $w_j \in [0, 1]$ of each criterion G_j, we propose two fusion methods:

1) Fusion method 1: we use the weighting averaging (WA) fusion rule because it is a very simple rule, and it can be processed very quickly. This is of prime importance in our storm risk assessment context because one can have millions of cells (depending the resolution cell we want to work with) in a wide surveillance areas.
2) Fusion method 2: we use a more sophisticate PCR6 fusion rule adapted with importance discounting presented in details in [13] if more computational power is available[3].

Once the BBAs $m_{ih}(.)$ and $m_{hi}(.)$ are obtained, the global indices are defined by

$$\begin{cases} c(\mathbf{a}_i, \mathbf{b}_h) \triangleq m_{ih}(c)\alpha(\mathbf{a}_i, \mathbf{b}_h) \\ d(\mathbf{a}_i, \mathbf{b}_h) \triangleq m_{ih}(\bar{c})\beta(\mathbf{a}_i, \mathbf{b}_h) \\ u(\mathbf{a}_i, \mathbf{b}_h) \triangleq 1 - c(\mathbf{a}_i, \mathbf{b}_h) - d(\mathbf{a}_i, \mathbf{b}_h). \end{cases} \quad (3)$$

[3]Due to the complexity of this fusion rule and computational burden, only problems of relatively small dimensions, say for $n_G \leq 6$, can be addressed by this second method.

The discounting factors $\alpha(\mathbf{a}_i, \mathbf{b}_h)$ and $\beta(\mathbf{a}_i, \mathbf{b}_h)$ in (3) are defined in [3]. They are not given here due to space limitation restraints. $c(\mathbf{b}_h, \mathbf{a}_i)$, $d(\mathbf{b}_h, \mathbf{a}_i)$ and $u(\mathbf{b}_h, \mathbf{a}_i)$ are similarly computed using the dual formula of (3).

The belief and plausibility of the outranking propositions $X =$ "$\mathbf{a}_i > \mathbf{b}_h$" ($\mathbf{a}_i$ outranks $\mathbf{b}_h$), and $Y =$ "$\mathbf{b}_h > \mathbf{a}_i$" ($\mathbf{b}_h$ outranks $\mathbf{a}_i$) are then given by

$$\text{Bel}(X) = c(\mathbf{a}_i, \mathbf{b}_h) \text{ and } \text{Bel}(Y) = c(\mathbf{b}_h, \mathbf{a}_i) \quad (4)$$

$$\text{Pl}(X) = 1 - d(\mathbf{a}_i, \mathbf{b}_h) \text{ and } \text{Pl}(Y) = 1 - d(\mathbf{b}_h, \mathbf{a}_i) \quad (5)$$

SET-Step 3: Calculation of the probabilized outranking relations. In SET-Step 2 we have characterized the outrankings $X =$ "$\mathbf{a}_i > \mathbf{b}_h$" and $Y =$ "$\mathbf{b}_h > \mathbf{a}_i$" by their imprecise probabilities $P(X) \in [\text{Bel}(X); \text{Pl}(X)]$ and $P(Y) \in [\text{Bel}(Y); \text{Pl}(Y)]$. Solving the outranking problem consists in choosing (deciding) if finally X dominates Y (in such case we must decide X as being the valid outranking), or if Y dominates X (in such case we decide Y as being the valid outranking). This hard assignment problem is difficult in general because $P(X)$ in $[\text{Bel}(X); \text{Pl}(X)]$ and $P(Y)$ in $[\text{Bel}(Y); \text{Pl}(Y)]$ and these belief intervals can partially overlap. Fortunately, a soft (probabilized) outranking solution is possible by computing the probability that X dominates Y (or that Y dominates X) by assuming uniform distribution of unknown probabilities between their lower and upper bounds. To get the probabilized outrankings, we have to calculate $P_{X>Y} \triangleq P(P(X) > P(Y))$ and $P_{Y>X} \triangleq P(P(Y) > P(X))$ which are given by the ratio of two polygonal areas, or can be estimated using sampling techniques, as explained and illustrated in [3]. The probabilities of outrankings are denoted $P_{ih} \triangleq P_{X>Y}$ where $X \triangleq$ "$\mathbf{a}_i > \mathbf{b}_h$" and $Y \triangleq$ "$\mathbf{b}_h > \mathbf{a}_i$". Reciprocally, we denote $P_{hi} \triangleq P_{Y>X} = 1 - P_{ih}$. This probabilization of outrankings is directly obtained by this Step 3 of SET, and thus eliminates the arbitrary λ-cut strategy used in classical ELECTRE TRI method.

SET-Step 4: Final soft assignment of $\mathbf{a}_i$ into a category C_h. From the probabilized outrankings obtained in SET-Step 3, we can make directly the soft assignment of alternatives $\mathbf{a}_i$ to categories C_h defined by their profiles $\mathbf{b}_h$. This is easily obtained by the combinatorics of all possible sequences of outrankings taking into account their probabilities P_{ih} to calculate all the assignment probabilities $P(\mathbf{a}_i \to C_h)$. Moreover, this soft assignment mechanism provides also the probability $\delta_i \triangleq P(\mathbf{a}_i \to \emptyset)$ reflecting the impossibility to make a coherent outranking. This soft assignment procedure of the SET method does not require an arbitrary choice of decisional attitude unlike to what is proposed in the classical ET method. A simple detailed example of this SET-Step 4 is given in [3] for convenience.

III. Application of SET to storm risk assessment

A. Surveillance zone and data set

We apply the SET method briefly presented in the previous section to storm risk assessment problematic. For this, we consider in this study five meteorological parameters (i.e. criteria) drawn from GFS (Global Forecast System) open data available on the web [1]. We have used GFS data for the 9th May 2016 at 3h UTC. The GFS data used in this study are available in [22]. The wide surveillance area covers Atlantic ocean from $[-1, 70.5]$ degrees in latitude, and $[-100, 10.5]$ degrees in longitude. We have 21592 cells of size $0.5 \times 0.5\,\text{deg}^2$ to evaluate. Each cell corresponds to an alternative $\mathbf{a}_i$ that must be assigned to a storm risk category C_h by the SET method. The table I shows the five ($n_G = 5$) meteorological parameters (i.e. criteria) used in our study, their units, their preference ordering, and their qualitative importance chosen for this problematic.

Criteria	Units	Preference ordering	Importance weight
$G_1 =$ PConv	kg/m^2	increasing	very high
$G_2 =$ LI	$^\circ K$	decreasing	very high
$G_3 =$ CAPE	J/kg	increasing	high
$G_4 =$ DivB	s^{-1}	decreasing	low
$G_5 =$ DivS	s^{-1}	increasing	low

Table I: Criteria used for SET method.

where

- PConv is the 3-h accumulated precipitation induced by convective process (in kg/m^2) [24];
- LI is the lifted index which characterizes the instability of the atmosphere (in $^\circ K$). This parameter, developed by Galway [25], is the gap between the environmental temperature and the temperature of a parcel lifted dry-adiabatically to saturation then moist-adiabatically to 500 hPa;
- CAPE is the convective available potential energy (in J/kg). This parameter is the potential energy available to the parcel to lift up beyond the level of free convection. As the lifted index, this parameter relies on the difference between the environmental temperature and the temperature of a parcel lifts adiabatically [26];
- DivB is the low-level wind divergence if there is convective clouds in the cell (in s^{-1}). This parameter is derived from horizontal wind component and the pressure level of the bottom of convective cloud [27];
- DivS is the divergence of the wind above the top of the convective clouds (in s^{-1}). Indeed, isolated storm cloud is associated with low-level wind convergence and divergence near the top of the cloud [27];

PConv has an increasing preference order which means that bigger the PConv value is, higher is the storm risk. LI has a decreasing preference order meaning that lower the LI value is, higher is the storm risk. To work with quantitative importance weights, we need to transform qualitative labels (low, high, very high) into numerical values. For this we use the following mapping: Low importance $\mapsto 1$, Moderate importance $\mapsto 2$, High importance $\mapsto 3$ and Very high importance $\mapsto 4$. This mapping is quite ad-hoc, and could be changed/adapted for reflecting a better subjective interpretation of the importance level expressed by the expert who provides these qualitative importance factors. This mapping is specific of the fusion

system design. After the normalization of the numerical importance factors, we get the following normalized weights of importance of each criterion[4]: $w_1 = 4/12$, $w_2 = 4/12$, $w_3 = 2/12$, $w_4 = 1/12$, and $w_5 = 1/12$.

In our study, we consider five ($n_h = 5$) levels of risk defined qualitatively as: Low, Moderate, Strong, Very strong and Extreme which respectively correspond quantitatively to the levels 1, 2, 3, 4 and 5 that will be shown in on the followings figures. The profile bounds for each category of risk and each criterion are given in Table II.

Criteria\Bounds of categories	b_1	b_2	b_3	b_4
$G_1 =$ PConv	0	1.5	7	10
$G_2 =$ LI	0	-2	-6	-10
$G_3 =$ CAPE	0	1000	2000	4000
$G_4 =$ DivB	0	-0.1	-0.5	-1
$G_5 =$ DivS	0	0.1	0.5	1

Table II: Bounds of categories of risk.

For convenience, the figure 2 presents the flow chart of the proposed method related to our storm application.

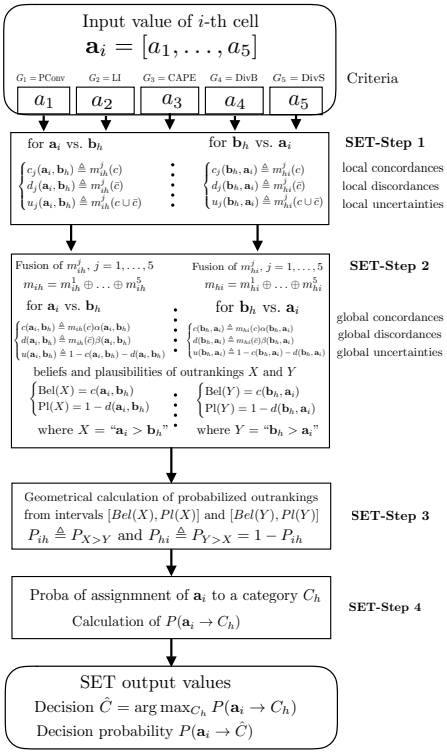

Figure 2: Flow chart of the method related to the application.

The bounds for LI and CAPE criteria are those defined by Wesoleck in [23]. The bounds for PConv criteria have been deduced from the different thresholds used to distinguish light, moderate and heavy rainfall [28] and adapted to our geographical area. Heavy rain, correspondings to accumulated precipitation above 10-30 mm/h, is associated to severe storm risk [29]. The bounds for low-level convergence wind and high-level divergence wind have been chosen larger than usual threshold [30], because the most important information is the sign of the divergence merge with the presence of convective cloud in the cell. Hence, if the PConv value for the cell under analysis is greater than $b_4^{j=1} = 20$, then the storm risk for this cell is considered as extreme (risk=5). If the PConv value belongs to $(b_3^{j=1}, b_4^{j=1}] = (10, 20]$ then the storm risk for this cell is considered as very high (risk=4), etc. If the LI value is lower than $b_4^{j=3} = -10$ then the storm risk for this cell is considered as extreme (risk=5), but if the LI value is between $b_4^{j=3} = -10$ and $b_3^{j=3} = -6$ the storm risk for this cell is considered only as very high (risk=4).

The figures 3–7 show the risk levels (1, 2, 3, 4 and 5) corresponding to each criterion for the 21592 cells covering the Atlantic ocean surveillance area for each criterion considered separately. The Dark blue cells with values -1 correspond to ground cells which are not taken into account in this study. One clearly sees the difference of risks drawn from the five meteorological parameters and the conflicting information between these five maps of risks of storm that illustrate the input data we have to process by the SET method to get the global risk assessment.

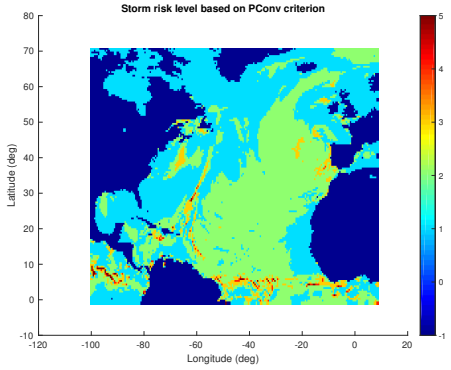

Figure 3: Storm risk levels based on PConv criterion. 0 means no risk; 1, low level of risk; 2, moderate level of risk; 3, strong level of risk; 4, very strong level of risk and 5, extreme risk. Risk are not calculated over earth, fixed to -1 value (dark blue).

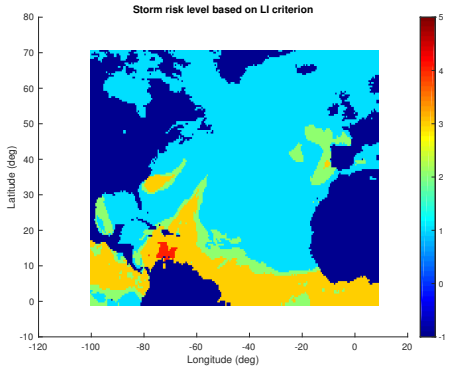

Figure 4: Storm risk levels based on LI criterion.

[4]In this work and for simplicity, the importance factor w_j of each criteria G_j used to make the importance discounting of BBAs in the SET method is chosen independently of the profile bound values.

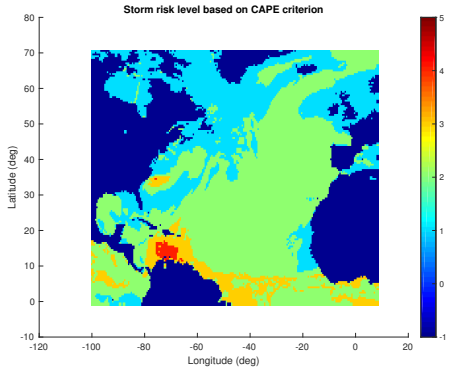

Figure 5: Storm risk levels based on CAPE criterion.

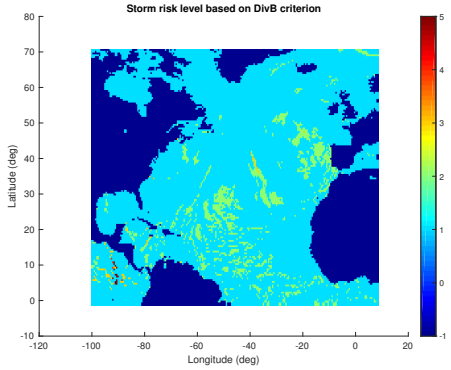

Figure 6: Storm risk levels based on DivB criterion.

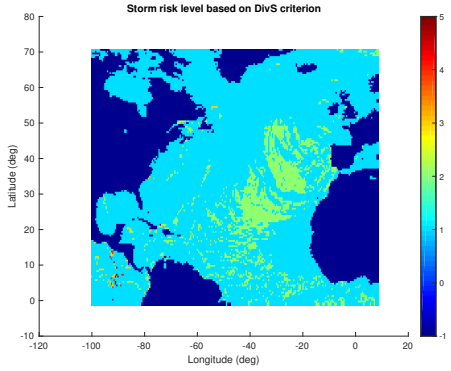

Figure 7: Storm risk levels based on DivS criterion.

To estimate the variability (randomness) of GFS data in each cell, we estimate for each category C_h of risk the probability $P(C_h)$ by counting the number of criteria associated with C_h divided by $n_G = 5$. This level of randomness is characterized by Shannon's entropy. Hence, for the cell #i, if we have the probability measure $\mathbf{p}_i = (p_1, p_2, \ldots, p_{n_h})$, the normalized Shannon entropy[5] is given by

$$H(\mathbf{p}_i) = -\frac{1}{\log_2 n_h} \cdot \sum_{h=1}^{n_h} p_h \log_2 p_h \qquad (6)$$

[5]One takes $p_h \log_2 p_h = 0$ if $p_h = 0$.

$H(\mathbf{p}_i) = 0$ when all meteorological parameters agree with the same storm risk category, and $H(\mathbf{p}_i)$ is maximum if $\mathbf{p}_i$ is the uniform pmf. One defines the mean entropy $\bar{H}$ of the GFS data by averaging the entropy values of the $N = 21592$ cells of the surveillance area by

$$\bar{H} = \frac{1}{N} \sum_{i=1}^{N} H(\mathbf{p}_i) \qquad (7)$$

Figure 8 shows the normalized entropies of the meteorological GFS data we have used in this study. The mean entropy of these GFS data is $\bar{H} = 0.2989$, and only 32% of the data are totally in agreement on the same risk level (shown in green color on Fig 8). As we observe on this figure most of the data are conflicting because the entropies values are much bigger than zero. The high level of randomness of these data justifies a sophisticate MCDM method able to deal efficiently with conflicting sources of information. This motivates the use of SET approach proposed in this work.

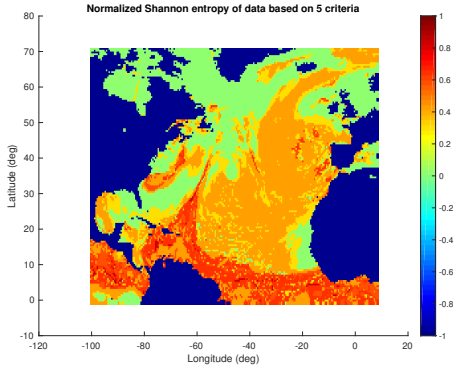

Figure 8: Normalized Shannon entropy of GFS data.

B. Results based on the weighted averaging rule

The figure 9 shows the storm risk levels based on the weighted average[6] of risk levels shown figures 3–7, with the weights of importance $w_1 = 4/12$, $w_2 = 4/12$, $w_3 = 2/12$, $w_4 = 1/12$, and $w_5 = 1/12$.

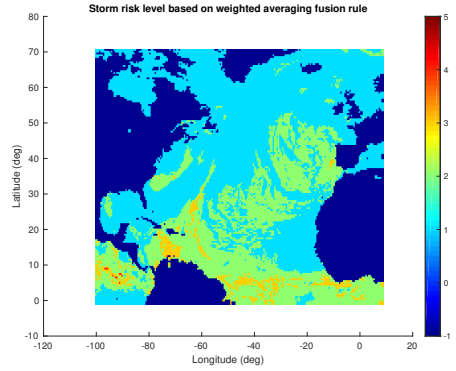

Figure 9: Storm risk levels based on weighted average of risks.

[6]For representation convenience and comparison with SET results, the risk values of Fig. 9 have been rounded to their closest integer value.

Based on this simple fusion rule one observes that the strong (and higher) risks of thunderstorms are located mainly in the intertropical convergence zone (around the equatorial line), on the Caribbean Sea, and aside the Portugal coast. However the method of fusion does not provide a measure of the trustfulness (confidence) of this result, and it does not manage precisely the level of conflict between the different sets of data.

C. Results based on the SET approach

The figure 10 shows the map of storm risk levels based on weighted averaging fusion rule used in SET-step 2, whereas the figure 11 shows the result when the PCR6 fusion rule[7] is used in SET-step 2. These two resulting maps of risk levels can be interpreted as the *SET-combination* of maps shown in figures taking into account the importance and contradiction of the five meteorological criteria.

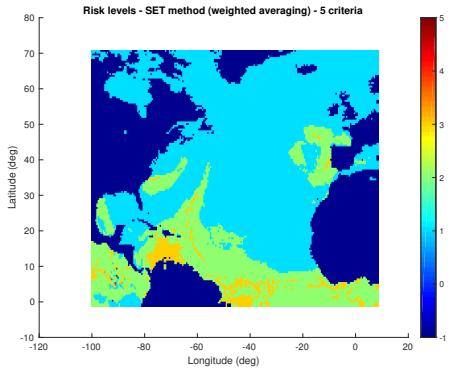

Figure 10: Storm risk levels based on SET (averaging rule).

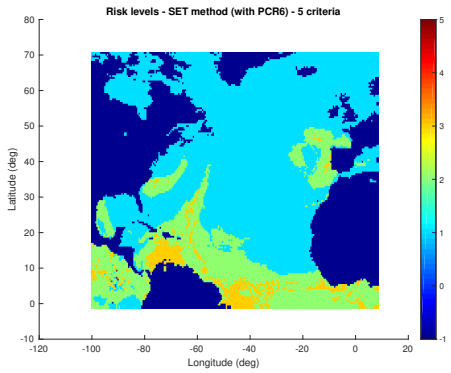

Figure 11: Storm risk levels based on SET (PCR6 rule).

The confidence in the resulting storm risk maps of figures 10 and 11 are shown in figures 12 and 13 respectively.

D. Performances analysis

To measure the performance of our method of storm risk assessment we need to compare our SET results with some ground truth. For this, we consider as ground truth the

[7]with importance discounting of BBAs, as explained in [13].

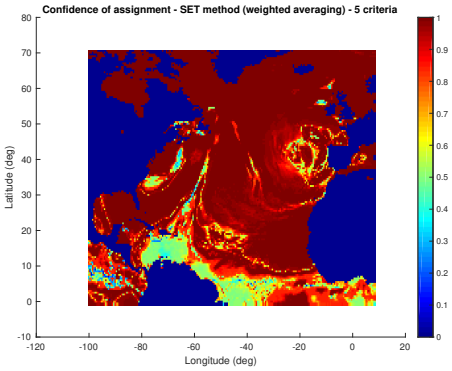

Figure 12: Confidence in decision (SET with averaging rule).

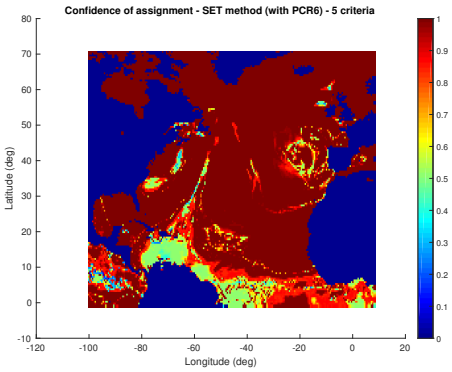

Figure 13: Confidence in decision (SET with PCR6 rule).

information of location of strokes supplied by the World Wide Lightning Location Network (WWLLN) [4]. WWLLN archival data are copyrighted by the University of Washington and are available to the public at nominal cost. For a given date, at a time T, all cells where strokes impacts have been detected by WWLLN network, in the time interval [T +/- 1h30], have been tagged. These data consist of the $N_L = 223$ locations of all the detected lightnings on May 9th, 2016 in the time interval [1h30 - 4h30 UTC] which are shown as red dots in Fig 14.

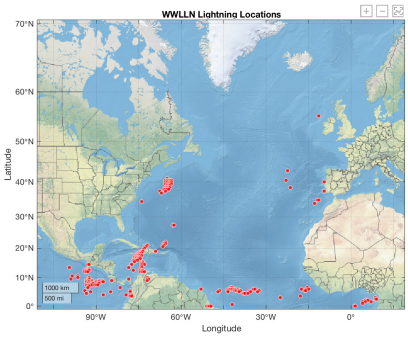

Figure 14: WWLLN lightning detections.

The performance of the method are evaluated by the estimation of the detection probability $\hat{P}_d = P(\hat{C} > C_1 | d_W = 1)$

of lightnings, and the false alarm probability $\hat{P}_{fa} = P(\hat{C} > C_1|d_W = 0)$, where $\hat{C}$ denotes the decision (i.e. the category of the risk) taken for a chosen confidence threshold, $d_W = 1$ indicates that a lightning flash has been detected by the WWLLN for the cell under analysis, and $d_W = 0$ indicates no detection. These probabilities are empirically estimated by

$$\hat{P}_d = P(\hat{C} > C_1|d_W = 1) \approx \frac{n(\hat{C} > C_1, d_W = 1)}{n(d_W = 1)} \quad (8)$$

$$\hat{P}_{fa} = P(\hat{C} > C_1|d_W = 0) \approx \frac{n(\hat{C} > C_1, d_W = 0)}{n(d_W = 0)} \quad (9)$$

where $n(\hat{C} > C_1, d_W = 1)$ is the number of cells for which the joint event $\hat{C} > C_1$ and $d_W = 1$ has occurred, and $n(d_W = 1)$ is the number of cells for which one has got a WWLLN detection $d_W = 1$. Similarly, $n(\hat{C} > C_1, d_W = 0)$ is the number of cells where events $\hat{C} > C_1$ and $d_W = 0$ have occurred, and $n(d_W = 0)$ is the number of cells having no WWLLN detection (i.e. $d_W = 0$).

The tables III indicates the estimations of the detection probability $\hat{P}_d$ of lightnings, and of the false alarm probability $\hat{P}_{fa}$ obtained by the three methods tested based on WWLLN set of detections.

Methods	$\hat{P}_d$	$\hat{P}_{fa}$
Weighted Averaging Rule	0.9775	0.3601
SET with averaging rule	0.9462	0.2945
SET with PCR6 rule	0.9507	0.2954

Table III: $\hat{P}_d$ and $\hat{P}_{fa}$ performances.

Our results show that SET approach (with averaging rule, or with PCR6 rule) provides interesting results because it allows to identify and predict the areas with high risk of storm that are coherent with the real location of lightnings detected by the WWLLN. One sees that the direct weighted averaging fusion of the risk maps of the five criteria shown in Fig. 9 produces notably more false alarms than with SET method, and only a little increase detection probability. In this work there is no clear advantage of using the PCR6 rule with respect the weighted averaging rule in step 2 of SET method because the performances in term of probability of detection and false alarms are very close. In terms of computational time, the direct weighted averaging fusion of the risk maps is the fastest method which takes few seconds[8] with MatLab (R2108a version) running with a MacBookPro laptop computer (2.8 GHz Intel Core i7), then the second fastest method is the SET method using weighted averaging rule taking 1mn12sec, and the slowest (and most complicate) method is the SET method based on PCR6 rule which takes approximately 26mn to produce the results. One important avantage of the SET method (aside its aforementioned performances) is its ability to provide the confidence map of the solutions obtained by SET (i.e. the predicted risk levels) as shown in Fig. 12 and 13. These confidence maps are useful to identify areas of risks where the confidences are low and thus very uncertain if some important decision but be taken based on these solutions (for instance the diverting of the flight of an aircraft, etc). Such type of useful confidence map cannot be drawn form the direct weighted averaging fusion of the risk maps. Due to space restraint, we did not include the decision inconsistency maps[9] (i.e. the map of the probabilities) $P(\mathbf{a}_i \rightarrow \emptyset)$ reflecting the impossibility to make a coherent outranking (see SET step 4), but these maps obtained by SET (with weighted averaging or with PCR6 rule) reveal actually only very few cells located mainly at the west of Panama. This very small number of cells yielding decision inconsistency indicates that SET has provided solutions in good decision-making conditions in general.

[8]Once the five maps of risks have been computed.

IV. Conclusions

In this paper we have presented an application of belief functions for storm prediction based on multi-criteria analysis and the Soft Electre Tri (SET) methodology. We have shown that SET allows to reduce notably the false alarms rate with respect to a simple weighted averaging fusion method without sacrificing much the detection of lightnings, and to provide the confidence map of the solutions obtained. The SET method based on PCR6 rule of combination performs well but it has a high computational burden which prevent it to use it for quasi-real time applications, and for working with multi-criteria problems involving many more criteria. This work, we hope, could serve as a benchmark problem for testing many MCDM methods in future. More investigations are currently done to apply this type of new methodology using more meteorological criteria on other type of real data sets, and more refined parameter settings that will be reported if possible in a forthcoming publication.

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Appendix

Basic definitions of belief functions

In this appendix we provide basics of belief functions (BF) introduced by Shafer [2] to model epistemic uncertainty to reason about uncertainty. We assume that the answer of the problem under concern belongs to a known finite discrete frame of discernement (FoD) $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, with $n > 1$, and where all elements of Θ are exhaustive and exclusive. The set of all subsets of Θ (including empty set $\emptyset$, and Θ) is the power-set of Θ denoted by 2^Θ. The number of elements (i.e. the cardinality) of 2^Θ is $2^{|\Theta|}$. A (normal) basic

[9]They have been included with the data set files in [22] for convenience.

belief assignment (BBA) associated with a given source of evidence is a mapping $m(\cdot): 2^\Theta \to [0,1]$ satisfying $m(\emptyset)=0$ and $\sum_{A\in 2^\Theta} m(A)=1$. The number $m(A)$ is called the mass of A committed by the source of evidence. The subset $A\in 2^\Theta$ is called a focal element (FE) of the BBA $m(\cdot)$ if and only if $m(A)>0$. The set of all the focal elements of the BBA $m(\cdot)$ is noted by $\mathcal{F}_\Theta(m)=\{X\in 2^\Theta | m(X)>0\}$. The belief of A denoted $Bel(A)$ and the plausibility of A denoted $Pl(A)$ are usually interpreted respectively as lower and upper bounds of an unknown (subjective) probability measure $P(A)$. They are respectively defined for any $A\in 2^\Theta$ from the BBA $m(\cdot)$ by

$$Bel(A)=\sum_{X\in 2^\Theta | X\subseteq A} m(X) \tag{10}$$

and

$$Pl(A)=\sum_{X\in 2^\Theta | A\cap X\neq\emptyset} m(X)=1-\text{Bel}(\bar{A}). \tag{11}$$

where $\bar{A}$ represents the complement of A in Θ, that is $\bar{A}\triangleq\Theta-\{A\}=\{X|X\in\Theta \text{ and } X\notin A\}$. The symbol $\triangleq$ means *equal by definition*, and the minus symbol denotes the set difference operator. The vacuous BBA (VBBA for short) representing a totally ignorant source is defined as $m_v(\Theta)=1$.

PCR6 rule of combination

The PCR6 rule proposed in [10], [11] is an interesting alternative of original PCR rule of combination no. 5 (PCR5) proposed in [9], [19]. PCR6 and PCR5 rules coincide if we combine only two BBAs defined on the same FoD. The PCR6 fusion of $S>2$ BBAs is obtained by $m^{\text{PCR6}}_{1,2,\ldots,S}(\emptyset)=0$, and for all $A\in 2^\Theta\setminus\{\emptyset\}$ by

$$\begin{aligned} m^{\text{PCR6}}_{1,2,\ldots,S}(A) = {} & m^{\text{Conj}}_{1,2,\ldots,S}(A) \\ & + \sum_{j\in\{1,\ldots,\mathcal{F}\} | A\in\mathbf{X}_j\wedge\pi_j(\emptyset)} \\ & \Big[\Big(\sum_{i\in\{1,\ldots,S\}|X_{j_i}=A} m_i(X_{j_i})\Big) \\ & \cdot \frac{\pi_j(\emptyset)}{\sum_{X\in\mathbf{X}_j}\Big(\sum_{i\in\{1,\ldots,S\}|X_{j_i}=X} m_i(X_{j_i})\Big)}\Big] \end{aligned} \tag{12}$$

where $m^{\text{Conj}}_{1,2,\ldots,S}(A)=\sum_{\substack{\mathbf{X}_j\in\mathcal{F}(m_1,\ldots,m_S)\\ X_{j_1}\cap\ldots\cap X_{j_S}=A}}\prod_{i=1}^S m_i(X_{j_i})$ is the conjunctive fusion rule, and where $\pi_j(X_{j_1}\cap X_{j_2}\cap\ldots\cap X_{j_S})\triangleq\prod_{i=1}^S m_i(X_{j_i})$, and $\pi_j(\emptyset)$ in (12) is the concise notation of $\pi_j(X_{j_1}\cap X_{j_2}\cap\ldots\cap X_{j_S})$ when $X_{j_1}\cap X_{j_2}\cap\ldots\cap X_{j_S}=\emptyset$. The $x\wedge y$ is the logical conjunction operator meaning that conditions x and y must be satisfied. PCR6 rule is quasi-associative and it offers a more precise conflict redistribution than DS rule but it requires a higher computational burden. PCR6 does not preserve the neutrality of the vacuous BBA however. PCR6 is simpler to implement than PCR5. Very basic Matlab$^{\text{TM}}$ codes of PCR5 and PCR6 rules can be found in [13], [20], and also from the BFAS (Belief Functions and Applications Society) repository [21].

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