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Abstract

This paper studies, in a unified and dynamic framework, the impact of fund managers compensation (symmetric and asymmetric fees including a penalty component) as well as their investment in the fund when managers exhibit a loss aversion utility function. Contrary to the vast majority of the existing literature, the benchmark portfolio, relative to which a fund's performance is measured, is risky. The optimal portfolio value comprises a call option and a term resembling the optimal value when the benchmark is riskless. The proportion invested in the risky security is a speculative position, while the fraction invested in the benchmark contains both a hedging addend and a speculative element. Our model and simulations show that (i) a risky benchmark substantially modifies the manager's allocation compared to a riskless benchmark; (ii) optimal positions are less risky when the manager is compensated by symmetric fees or faces a penalty; (iii) a relatively large manager's stake (30%) in the fund considerably reduces her risk-taking behaviour and results in an almost identical terminal portfolio value for the different fees schemes; (iv) optimal weights significantly react to different parameter values; (v) these results may have important implications on regulation.

Résumé

L'objectif de cet article est d'étudier, dans un cadre unifié et dynamique, l'impact de la rémunération des gestionnaires de fonds (commission de performance symétrique ou non-symétrique avec potentiellement un malus), ainsi que leur propre investissement dans le fond lorsqu'ils sont caractérisés par une fonction d'utilité de type « aversion aux pertes ». Contrairement à la littérature existante, la performance du fond est appréciée par rapport à une référence (un indice, par exemple) risquée. Les principales conclusions de notre modèle et de nos simulations sont les suivantes : (i) une référence risquée modifie sensiblement l'allocation d'actifs ; (ii) les proportions optimales sont moins risquées dans le cas d'une commission de performance symétrique ; (iii) la détention par le gestionnaire d'une part relativement importante (30%) du fonds altère son comportement risqué et permet d'obtenir une valeur terminale du portefeuille quasi-identique quel que soit le type de rémunération ; (iv) les proportions optimales sont sensibles aux variations des valeurs des paramètres ; (v) ces résultats peuvent avoir des conséquences sur la régulation.

In most developed countries, a substantial part of portfolios invested in financial markets are managed by institutional investors, typically mutual funds, insurance companies, pension funds and hedge funds. Recent decades have witnessed a tremendous increase in the assets under management by institutional investors - in 2019, these assets amounted to USD 52 trillion representing 58% of the global market (see Boston Consulting Group 2020 Report). In view of the size of this industry and its implications for investors, managers and regulators, one of the most important aspects of delegated portfolio management consists in examining the implications of the fee structures and the fund managers' attitude toward the risk on asset allocation decisions. Moreover, since portfolio managers are rewarded for performance relative to a benchmark, delegation and benchmarking are closely related (see, for example, Leippold and Rohner, 2011). The objective of active portfolio managers is to overperform a relevant benchmark. Ma et al. (2019) documented that for 79% of active US. open-end mutual funds, managers compensation is directly related to fund performance and 78% "disclose the benchmark used to evaluate performance".

The goal of this paper is to study the impact of different performance-based fee structures as well as manager's personal stake in the fund on portfolio management in the setting of prospect theory, in an unified and dynamic framework in relation to regulation in force. In stark contrast to the vast majority of relevant literature, but in accordance with market practice, the benchmark portfolio evolves randomly over time. Indeed, the benchmark plays an important role in portfolio management since managers' performance is compared to the performance of a benchmark, which evolves randomly over time. Basak et al. (2006) demonstrated that the over or underperformance of a portfolio depends on the manager's risk aversion and the choice of the benchmark. Moreover, for instance, Servaes and Sigurdsson (2020) reported that "seventy percent of the (European) funds have a stochastic benchmark against which performance is measured, generally a stock index" and showed that performance fees funds perform poorly when the benchmark is not stochastic or when it does not reflect funds' assets nature. In addition, the design of performance-based fees depends on the benchmark (see, among others, Admati and Pfleiderer, 1997; Dybvig et al., 2009; Lioui and Poncet, 2013; Cvitanic and Xing, 2017). It would be therefore of great interest in the portfolio choice problem

to take not only the distribution of the portfolio into account, but also that of the benchmark as well as their relationship.

The compensation of portfolio managers often includes incentive fees, which depend on their performance relative to some relevant benchmark portfolio representing the nature of the assets held by the manager. In a so-called asymmetric fee contract, managers receive a performance bonus, a call option, for exceeding a benchmark (the strike price) without being penalized for poor performance. To this bonus an analogous underperformance penalty, with a fee rate smaller than that of the bonus, may be added (see Golec and Starks, 2002; Cuoco and Kaniel, 2011; Buraschi et al., 2014; Barucci et al., 2021), which is equivalent to a put option. In the United States, the 1970 Amendment to the Investment Advisers Act of 1940 prohibited the use of asymmetric fees in the mutual fund industry[1]. In contrast, in the European Union both symmetric (or fulcrum) and asymmetric fees were permitted under the UCITS (Undertakings for Collective Investment in Transferable Securities) directive. Following a controversial and intense debate[2], more recently in November 2020, the European Securities and Markets Authority (ESMA) published the official translations of the guidelines on performance fees in UCITS and certain types of AIFs (Alternative Investment Funds) imposing symmetric fees to fund managers. However, asymmetric fees are widely used in the hedge fund industry (see Ackermann et al., 1999; Elton et al., 2003; Goetzmann et al., 2003, Agarwal et al., 2004; Ben-David et al., 2020), by pension funds (Cuoco and Kaniel, 2011) and by European mutual funds as well (Ma et al., 2019; Servaes and Sigurdsson, 2020). Another practice characterizing funds management is that managers often invest a fraction of their personal wealth in the fund they manage. Since 2004, the SEC (Securities and Exchange Commission) has made it compulsory for fund managers to disclose their ownership in a fund. These regulations have a common objective, to align managers and investors interests, and are supposed to influence managers' risk-taking.

The widespread conviction that such a convex fees schedule makes a manager more willing to bear risks is only partially founded. Ross (2004) questioned this common wisdom and showed that

[1] According to this amendment only symmetric fees are permitted: The fund manager is equally rewarded or penalized for overperforming or underperforming the benchmark respectively.
[2] See Starks, 1987; Grinblatt and Titman, 1989; Golec and Starks, 2002; Ou-Yang, 2003; Agarwal et al., 2007; Zou, 2017; Cvitanic and Xing, 2018.

further conditions on the utility function are required before concluding about the effect of asymmetric compensation schemes on manager's attitude toward the risk. It follows that it is of key importance to understand how incentive fees impact managers' investment strategy by paying particular attention to managers' utility function. To take into account the results of empirical studies having shown the failures of the neoclassical theory concerning the decision of rational individuals having to choose among risky gambles, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) elaborated an alternative theory of choice under risk, known as the prospect theory. In the framework of this theory, experimental studies have shown, in particular, that individuals maximize their objective function in terms of gains and losses both measured relative to some reference point as opposed to final wealth in the traditional theory. Moreover, they are risk averse when they face gains, while risk-seeking when they face losses (asymmetric risk preferences) resulting in a concave-convex value function (the so-called S-shaped function). Recent empirical studies have found strong evidence that professional traders and investors are loss-averse (Coval and Shumway, 2005; Haigh and List, 2005; Hwang and Satchell, 2010; Lee and Veld-Merkoulova, 2016) and prospect theory implies an optimal portfolio composition different to that of the classical utility function (Berkelaar et al., 2004; Levy and Levy, 2004; Zarrow and Zhao, 2006; Di Giorgi and Legg, 2012; Fulga, 2016; Bilsen et al., 2020; Choi et al., 2021).

More specifically, the main objective of this study is to develop a partial equilibrium model in order to address, in a continuous-time context, the issue of deriving optimal asset allocation for a manager, exhibiting loss aversion, (i) compensated by asymmetric and symmetric fees when the funds' performance may be compared to both a risky and a riskless benchmark and (ii) owning a part of the fund under management. Thus, with regard to the relevant literature, we integrate in our study the main types of fee structures employed by portfolio managers, whose performance may also be assessed relative to a risky benchmark. Then we examine how these features impact fund managers' portfolio composition when they exhibit risk-aversion and risk-seeking.

A few papers examine incentive fees in a portfolio choice dynamic setting. In the traditional expected utility setting, Carpenter (2000) studied the optimal investment choice of a fund manager who is compensated with a call-option contract. According to Ross's (2004) analysis, she found that

when the portfolio value is below the benchmark, the manager will try to increase the risk of the fund's value. If, in contrast, the value of the portfolio lies above the benchmark, the manager moderates portfolio's volatility in order to lock in her (his) gains, owing to her (his) risk aversion. In the same spirit, Panageas and Westerfield (2009) showed that the risk-seeking managers behaviour relies on both option-like compensation and finite investment horizons. Higher risk increases current option prices embedded in manager's remuneration but may reduce anticipated future option values since it also increases the probability that fund asset values will decrease in the future while the strike price will remain unchanged. In line with the above mentioned paper, Guasoni and Obloj (2011) found that incentive schemes are ineffective in increasing risk-taking for risk-neutral managers, but they lead very risk-averse managers to bear more risk. By investigating the impact of fund flows on fees and portfolio decisions (see, for instance, Hugonnier and Kaniel, 2010), Barucci and Marazzina (2016) concluded that performance-based fees result in a more risky manager behaviour[3]. Cuoco and Kaniel (2011) pursued another goal by analyzing the implications that performance-based fees may have on asset prices and volatilities from a general equilibrium standpoint.

In contrast to these papers, Kouwenberg and Ziemba (2007), opted for the prospect theory and obtained two main results for asymmetric fees including a performance bonus: (i) Loss-averse managers become more risk-seeking with higher incentive fees; (ii) this behaviour is, however, tempered when the manager invests a large part of her (his) wealth in the fund. Zou (2017) reached the same conclusions[4]. He and Kou (2018) considered a fund manager who invests her (his) own capital in the fund and receives a specific remuneration known as the first-loss scheme. Under this scheme, fund managers invest typically 10% of their own capital in the fund to cover first-loss. In return, they receive higher performance fees, typically 40%. They found that the first-loss scheme (10% first-loss capital and 30% performance fee) provides better results in terms of utility improvement for both managers and investors, and fund risk reduction than the traditional scheme (10% internal capital and 20% performance fee).

[3] Nicolosi et al. (2018) extended this framework by incorporating mean-reverting prices of risk and volatility.
[4] For an application to defined contribution pension plans, see Dong et al. (2021).

The above mentioned papers did not investigate different compensation schemes simultaneously and assumed that the benchmark is constant or deterministic (a riskless asset, for instance)[5] with two notable exceptions. Cuoco and Kaniel (2011) who considered, however, a benchmark consisting of the same assets as the portfolio under management (see also Berkelaar et al., 2004). Basak et al. (2007) studied optimal asset allocation of fund receiving a convex flow scheme relative to a risky benchmark and found a relationship between portfolio's volatility and the manager's attitude towards the risk.

Our model and simulations show that performance-based fees and the nature of the benchmark have a considerable impact on the manager's portfolio composition not only from a quantitative point of view, but also with relation to her (his) attitude towards the risk. In the case of a risky benchmark, the optimal portfolio value at any date comprises a term resembling the optimal value when the benchmark is riskless and a call option on the benchmark depending on the probability that this latter may be greater or lower than the fund value. It follows that the manager takes a speculative position in the risky security and a position in the risky benchmark, which contains both a hedging addend and a speculative element. The two positions behave in an opposite manner. In line with the existing literature, in the region of losses, asymmetric fees encourage managers to bear more risk than symmetric fees. The presence of a penalty dampens the asymmetric fees effect. The manager's own investment in the fund plays a crucial role. Based on our simulations, a stake as high as 30% not only mitigates manager risk-taking behaviour, but, more importantly, results in an optimal wealth almost identical for symmetric and asymmetric fees (penalty included). To meet regulators' considerations, for funds having a low value, a high manager stake could achieve the same objectives as different fee structures, while for large funds a combination of fees and a low stake could be an alternative. We also investigate the reaction of the optimal proportions to the value of the parameters in the case of a risky benchmark for different compensation schemes. Thus, simulations show that these proportions significantly respond to different values of the correlation coefficient, the volatilities and prices of risk of the risky asset and the risky benchmark.

[5] Note that Carpenter (2000) also examines the case of a particular stochastic benchmark, the growth optimal portfolio.

The remainder of the paper is organized as follows. In Section 1, the economic framework is described and the fund manager's optimization problem is formulated. Section 2 is devoted to the derivation of the optimal final wealth. The optimal weights invested in the risky security and the risky benchmark are derived in Section 3. Section 4 offers some concluding remarks and suggests some potential future extensions.

1. The Economy

The uncertainty in a frictionless continuous-time economy is represented by a complete probability space (Ω, F, P) with a standard filtration $F = \{F(t) : t \in [0,T]\}$, a finite time period [0, T] and two independent standard Brownian motions, $z_B(t)$ and $z_\varepsilon(t)$, defined on (Ω, F).

The investment opportunities are represented by a riskless asset, $\beta(t) = \beta(0)e^{rt}$ - with $\beta(0) > 0$ and a constant interest rate, r - and two risky assets, *B(t)* and *S(t)*, which satisfy the following stochastic differential equations (SDEs hereafter):

$$\frac{dS(t)}{S(t)} = (r + \sigma_S \theta_S)dt + \sigma_S \left[\rho dz_B(t) + \sqrt{1-\rho^2}\, dz_\varepsilon(t)\right] \tag{1}$$

$$\frac{dB(t)}{B(t)} = (r + \sigma_B \theta_B)dt + \sigma_B dz_B(t) \tag{2}$$

with initial conditions *S(0) > 0* and *B(0) > 0*. θ_i and σ_i, for i = *S, B*, represent the constant market price of risk and the constant, strictly positive, instantaneous volatility of the risky assets respectively. ρ stands for the correlation coefficient and $\sigma_{SB} = \rho\sigma_S\sigma_B$ represents the covariance between the instantaneous returns of two risky assets $dS(t)/S(t)$ and $dB(t)/B(t)$.

Because of high information and transaction costs, for example, an investor delegates investment decisions to a fund manager who acts on the investor's behalf. As we shall show shortly, the manager is compensated on the basis of her (his) portfolio performance relative to that of a prespecified benchmark, say risky asset *B(t)*, which is supposed to be a sufficiently diversified portfolio so that it carries only the market risk. (S)he invests in the riskless asset, in the risky security *S(t)* and in the benchmark. For tractability, we assume that withdrawals or injections of funds by the

investor are not permitted. $\pi_S(t)$ and $\pi_B(t)$ denote the proportion of the portfolio invested in the benchmark and in the risky asset *S(t)* respectively. $\{\pi_S(t), \pi_B(t) : 0 \le t \le T\}$ is an admissible, self-financing trading strategy[6] such that wealth under management, at any date *t*, evolves according to:

$$\frac{dW(t)}{W(t)} = [r + \pi(t)'\sigma\theta]dt + \pi(t)'\sigma dz(t) \tag{3}$$

with the initial condition *W(0)>0*. $\pi(t)' = [\pi_S(t) \quad \pi_B(t)]$, $\theta = \begin{pmatrix} \frac{\theta_S - \rho\theta_B}{\sqrt{1-\rho^2}} \\ \theta_B \end{pmatrix}$, $\sigma = \begin{pmatrix} \rho\sigma_S & \sqrt{1-\rho^2}\sigma_S \\ \sigma_B & 0 \end{pmatrix}$

and $dz(t) = \begin{pmatrix} dz_B(t) \\ dz_\varepsilon(t) \end{pmatrix}$.

A manager receives a compensation for her (his) management activities from the investor. The fees charged by the manager depend on the terminal value of the wealth under management, *W(T)*, and on the terminal value of the benchmark, *B(T)*. The latter may, in general, be constant, a riskless asset or a risky portfolio (an index). The fees, denoted by $F(W(T), B(T))$, are received at the end of the investment period and are of the form:

$$F(W(T), B(T)) = bW(T) + c_1[W(T) - B(T)]^+ - c_2[B(T) - W(T)]^+ \tag{4}$$

where b and c_i, for $i = 1, 2,$ are constants specified in advance, and $y^+ = \max[0, y]$. If $c_1 > c_2 > 0$, fees are asymmetric containing an underperformance penalty component. If $c_2 = 0$, the latter vanishes. Finally, if $c_1 = c_2$, fees are symmetric. $0 \le b < 1, 0 \le c_i \le 1$, $0 < b + c_i < 1$ so that the fees are an increasing function of *W(T)* and a decreasing function of *B(T)*. Since the manager can invest in the benchmark portfolio, fees are strictly positive and the manager's compensation cannot exceed the terminal wealth. When $W(T) < B(T)$, in order to have strictly positive fees, $W(T) > \frac{c_2}{b + c_2} B(T) \equiv \underline{W}(T)$ for $0 < b + c_2 < 1$.

Incentive fees contain three components: a term proportional to the terminal value of the fund, a performance adjustment component, which depends on the managed portfolio's performance relative to that of a benchmark portfolio and an underperformance element. The performance term can then be

[6] See Cox and Huang (1989) for the conditions that an admissible trading strategy must satisfy.

viewed as a European call option that gives a manager the right to exchange a fraction of her (his) portfolio for the benchmark portfolio, while the penalty addend is a European put option.

The fund manager may invest a part of her (his) own wealth in the fund that (s)he manages (see Hodder and Jackwerth, 2007; Khorana et al., 2007; Kouwenberg and Ziemba, 2007). A proportion *a*, with $0 \le a < 1$, of the fund (resp. *1-a*) is owned by the manager (resp. the investor). The manager's portfolio terminal value, $W^m(T)$, is:

$$\begin{aligned} W^m(T) &= aW(T) + (1-a)F(T) + \beta(T)(W^m(0) - aW(0)) = \\ &aW(T) + (1-a)bW(T) + (1-a)c_1[W(T) - B(T)]^+ - (1-a)c_2[B(T) - W(T)]^+ + Y(T) \end{aligned} \quad (5)$$

where $W^m(0)$ is the initial value of the manager's wealth. $Y(T) = \beta(T)(W^m(0) - aW(0))$ represents the wealth that the manager invests on her (his) own. If $a = W^m(0) = 0$ then the manager is assumed not to have personal wealth and therefore maximizes her (his) expected utility, which is a function of her (his) management fees.

Traditionally, in the asset allocation problem rational investors are supposed to be risk-averse and their utility is a monotone strictly increasing concave function of their lifetime consumption and/or terminal wealth. Based on experiments, Kahneman and Tversky (1979) found that the economic agents' behaviour does not conform to this risk assumption and proposed the prospect theory. In the framework of this theory, the utility function differs from the traditional one by three properties. First, wealth is measured relative to a given reference point. Second, the utility function is concave for gains and convex for losses reflecting the fact that individuals are risk-averse in the domain of gains, while risk-seeking in the domain of losses. Third, as investors are more sensitive to losses than to gains, the utility function is steeper for losses than for gains. These properties result in an S-shaped utility function.

This utility function is more involved than the traditional one since it is both concave and convex and has a kink at the reference point, $\Theta(T)$. Moreover, the specification of the latter depends on the situation faced by an economic agent. The reference point is a neutral outcome below which an agent will perceive all outcomes as losses and above it as gains. In the context of incentive fees, if the terminal value of the wealth under management is greater than the benchmark value, the manager

earns performance fees, which is considered as a gain. In contrast, a value of the wealth lower than the benchmark value is experienced as a loss since the proportional component is reduced by the penalty term. According to Kouwenberg and Ziemba (2007), we assume that the reference point is the terminal fund value when the manager just performs the benchmark ($W(T) = B(T)$): $\Theta(T) = (a + (1-a)b)B(T) + Y(T)$. It is easy to show that $W^m(T) \geq \Theta(T)$ is equivalent to $W(T) \geq B(T)$.

Following De Giorgi et al. (2004) and Köberling and Wakker (2005), the manager's value function is a CARA (Constant Absolute Risk Aversion) utility function of the form:

$$U(W(T), B(T)) = \begin{cases} 1 - \exp\{-\alpha_1 \gamma_1 (W(T) - B(T))\} & \text{if } W(T) > B(T) \\ \lambda[\exp\{\alpha_i \gamma_2 (W(T) - B(T))\} - 1] & \text{if } W(T) \leq B(T) \end{cases} \quad (6)$$

for *i = 1,2*, where $0 < \gamma_i < 1$ and $\gamma_1 > \gamma_2$ are the curvature parameters, $\lambda > 1$ is the loss aversion coefficient, $\alpha_1 = a + (1-a)(b + c_1)$ and $\alpha_i = a + (1-a)(b + c_i)$. If $c_1 \neq c_2 \neq 0$, then $\alpha_2 = a + (1-a)(b + c_2)$. If $c_2 = 0$ $(\alpha_2 = a + (1-a)b)$, the underperformance penalty component disappears. If $c_1 = c_2$ $(\alpha_1 = \alpha_2)$, fees are symmetric.

To determine asset allocation, the fund manager maximizes the expected utility of her (his) terminal value of fees. The market described above is dynamically complete since the number of sources of risk is equal to that of the traded risky securities. Under the martingale approach pioneered by Karatzas et al. (1987) and Cox and Huang (1989; 1991), the static problem is written as:

$$\max_{\{W(T)\}} \mathrm{E}[U(W(T), B(T)) | F_t] \quad (7)$$

$$\text{s.t. } W(0) = G(0)\mathrm{E}\left[\frac{W(T)}{G(T)}\right]$$

$$W(T) \geq 0$$

where $\mathrm{E}[\cdot | F_t]$ denotes the expectation conditional on the information, F_t, available at time *t*, and

$$G(t) = \frac{\beta(t)}{\xi(t)} = G(0)\exp\left\{\left(r + \frac{1}{2}\sigma_G^2\right)t + \theta_B z_B(t) + \frac{(-\rho\theta_B + \theta_S)}{\sqrt{1-\rho^2}} z_\varepsilon(t)\right\}, \quad \text{with} \quad G(0) = \beta(0) \quad \text{and}$$

$\sigma_G^2 = \frac{\theta_B^2 - 2\rho\theta_S\theta_B + \theta_S^2}{1-\rho^2}$. It represents the numéraire or optimal growth portfolio such that the value of any admissible portfolio relative to this numéraire is a martingale under *P* (see Long, 1990; Merton, 1990; Bajeux-Besnainou and Portait, 1997). $\xi(t)$ is the Radon-Nikodym derivative of the so-called, unique, risk-neutral probability measure *Q* equivalent to the historical probability *P* , such that the relative price (with respect to the savings account chosen as numéraire), of any risky security is a *Q*-martingale (see Harrison and Pliska, 1981). The first constraint is the usual budget constraint binding at the optimum, while the second is a solvency constraint imposed on the manager in order to follow a trading strategy, which avoids a negative terminal value of the portfolio under management (see Nguyen and Portait, 2002).

2. The terminal value of the optimal portfolio

In this section, we determine the optimal terminal wealth by distinguishing asymmetric fees (with and without a penalty) and symmetric fees. We also provide an illustration to show how the terminal wealth behaves in favourable and in unfavourable states of the world.

As indicated above, the optimal portfolio policy may be determined by using the martingale approach. However, unlike standard concave maximization problems, the objective function is neither a concave function nor continuously differentiable at $W(T) = B(T)$, not only because of the optional feature of the fee schedule but also due to the shape of the manager's objective function. The martingale approach cannot therefore be directly applied to solve this optimization problem. Berkelaar et al. (2004) used a method, first proposed by Basak and Shapiro (2001), to take into account non-concave and non-differentiable utility functions, to a loss aversion value function. In this paper, we privilege this technique to tackle our optimization problem. Proposition 3.1 establishes the optimal solution.

PROPOSITION 2.1. Given the economic framework described above, the optimal terminal wealth of a manager with a CARA prospect utility function (6) is:

$$W^*(T) = \begin{cases} \left[B(T) - \frac{1}{\alpha_1 \gamma_1} Ln\left(\frac{\eta}{\alpha_1 \gamma_1 G(T)} \right) \right] & \text{if } G(T) > \overline{G}(T) \\ 0 & \text{if } G(T) \leq \overline{G}(T) \end{cases} \quad (8)$$

where $\overline{G}(T) \equiv \overline{G}(B(T))$ is a solution to the equation $f(\overline{G}(T), B(T)) = 0$ such that, for *i = 1, 2*:

$$f(x, B(T)) = 1 + \lambda \left[1 - e^{-\alpha_i \gamma_2 B(T)} \right] - \left[B(T) + \frac{1}{\alpha_1 \gamma_1} \left(1 - Ln\left(\frac{\eta}{\alpha_1 \gamma_1 x} \right) \right) \right] \frac{\eta}{x} = 0 \quad (9)$$

i) Asymmetric fees: If, in the exponential in *f(x)*, $c_1 = 0, c_2 > 0$ ($c_1 = c_2 = 0$), the underperformance penalty is positive (null) ;

ii) Symmetric fees: In the exponential in *f(x)*, $c_1 = c_2$.

$\eta \geq 0$ is the Lagrangian associated with the static program solving: $W(0) = \beta(0) E\left[\frac{W^*(T)}{G(T)} \right]$

Proof. See Appendix A.

When the benchmark is a riskless asset, *B(T)* is replaced by $\beta(T)$. The manager's terminal wealth is either equal to the positive part of an expression involving the benchmark and the optimal growth portfolio at date *T* in some states of the nature or equal to zero otherwise. It is worth pointing out that, in contrast to other models, the final wealth evolves randomly over time as a function not only of the optimal growth portfolio but also of the risky benchmark. $G(T) > \overline{G}(T)$ represents the states in which the numéraire portfolio is above a critical value. In other words, this inequality represents the states in which each manager receives a compensation for performance that exceeds the benchmark or, put differently, the states in which the call option included in her (his) fees can be exercised. The inclusion in the fees of an underperformance penalty does not have an impact on the portfolio's terminal value, but rather on the threshold $\overline{G}(T)$, which is a solution to the equation $f(G(T), B(T))$.

To get more insights into the behavior of optimal wealth and optimal demands (see next Section), we provide various simulations represented in Figures 1 to 14. Table 1 summarizes the values of the parameters used in our simulations. There are three kinds of parameters. First, for the loss aversion utility function, Tversky and Kahneman (1992) estimated the parameters as λ = 2.25 and

$\gamma_1 = \gamma_2 = 0.88$. Köberling and Wakker (2005) demonstrated, however, that for CARA utility functions to satisfy some desirable properties, $\gamma_1 > \gamma_2$. This is why we opt for $\gamma_1 = 0.9$ and $\gamma_2 = 0.8$. Secondly, the choice of fee parameter values is in accordance with market practices. In a typical asymmetric compensation scheme $b = 2\%$, $c_1 = 20\%$[7] and as the penalty component is lower than the performance element, we choose $c_2 = 4\%$. When fees are symmetric, the funds charge different fees for over/underperformance relative to the benchmark. A typical fulcrum scheme is $c_1 = c_2 = 10\%$. Finally, the risk-return characteristics over a one year investment horizon of the Amundi European Equity Value Fund having as benchmark the MSCI (Morgan Stalney Capital International) Europe Value Equity serves as a reference to determine their return, volatility correlation coefficient and risk-prices (Sharpe ratios).

[Insert Table 1 about here]

Figure 1 depicts the evolution of the optimal terminal wealth, *W(T)*, as a function of the optimal growth portfolio, *G(T)*, of a loss-averse manager compensated with asymmetric fees (with and without a penalty) and with symmetric fees when the benchmark is risky. In good states of the world, *W(T)* is, as expected, an increasing function of *G(T)*. In bad states, however, *W(T)* jumps to zero. This pattern is similar to that found in other papers (see, for instance, Berkelaar et al., 2004).

[Insert Figure 1 about here]

Including in the fees an underperformance penalty results in a lower critical value $\overline{G}(T)$. This latter is at the lowest when fees are symmetric. As a consequence, this increases the spectrum of states in which the terminal wealth becomes positive. Similar results are obtained for a riskless benchmark.

3. The fund manager's optimal asset allocation

In order to determine the manager's intermediate optimal wealth and proportions invested in the risky asset, two benchmarks serve as examples to illustrate the manager's portfolio decisions: a riskless and a risky benchmark. We derive expressions for the manager's optimal wealth and portfolio strategies at any time $t < T$ for asymmetric and symmetric fees.

[7] Agarwal et al. (2000), for hedge funds, and Servaes and Sigurdsson (2020), for mutual funds, found a performance fee percentage with a median of 20%.

For a loss-aversion utility function, given the expression of $f(G(\mathrm{T}),\mathrm{B}(\mathrm{T}))$, $\overline{G}(\mathrm{t}) \equiv \overline{G}(\mathrm{B}(\mathrm{t}))$ is a function of the benchmark. To derive tractable solutions, we assume that $\overline{G}(\mathrm{t})$ obeys the following equation:

$$\frac{d\overline{G}(t)}{\overline{G}(t)} = \mu_{\overline{G}}(t) + \sigma_{\overline{G}}(t) dz_B(t)$$

The expectation and volatility of the instantaneous return of $\overline{G}(\mathrm{t})$ are also functions of *B(t)*: $\mu_{\overline{G}}(t) \equiv \mu_{\overline{G}}(B(t))$ and $\sigma_{\overline{G}}(t) \equiv \sigma_{\overline{G}}(B(t))$. This equation serves for notations only in the rest of the paper since there is not an analytical solution to the equation $f(G(\mathrm{T}),\mathrm{B}(\mathrm{T}))$.

PROPOSITION. 3.1. For a manager characterized by a utility function (6), at any date *t*, $0 \le t \le T$:

a) The optimal wealth is given by

$$W^*(t) = B(t)N(d_1) - e^{-r(T-t)}(e^{rT}B(0))N(d_2) + e^{-r(T-t)}\left\{(e^{rT}B(0))N(d_2) + \frac{1}{\alpha_1\gamma_1}\left[\left(LnG(t) + \left(r - \frac{1}{2}\sigma_G^2\right)(T-t) - Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right)\right)N(d_2) + \sigma_G\sqrt{T-t}\,n(d_2)\right]\right\} \quad (10)$$

b) The optimal proportions invested in the risky security and the risky benchmark can be written respectively:

$$\pi_S^*(t) = \frac{\theta_S - \rho\theta_B}{W^*(t)(1-\rho^2)\sigma_S\sigma_{G-\overline{G}}(t,T)}\left\{B(t)n(d_1) - e^{-r(T-t)}(e^{rT}B(0))n(d_2) + e^{-r(T-t)}\left[(e^{rT}B(0))n(d_2) + \frac{1}{\alpha_1\gamma_1}\left[\sigma_{G-\overline{G}}(t,T)N(d_2) + \left(LnG(t) + \left(r - \frac{1}{2}\sigma_G^2\right)(T-t) - Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right) - \sigma_G\sqrt{T-t}\,d_2\right)n(d_2)\right]\right]\right\} \quad (11)$$

$$\pi_B^*(t) = \frac{1}{W^*(t)}\left\{\left[N(d_1) + \frac{\theta_B - \rho\theta_S - (1-\rho^2)\sigma_{\overline{G}}(t)}{(1-\rho^2)\sigma_B\sigma_{G-\overline{G}}(t,T)}n(d_1)\right]B(t) + \frac{e^{-r(T-t)}}{\alpha_1\gamma_1(1-\rho^2)\sigma_B\sigma_{G-\overline{G}}(t,T)}\left[(\theta_B - \rho\theta_S)\sigma_{G-\overline{G}}(t,T)N(d_2) + (\theta_B - \rho\theta_S - (1-\rho^2)\sigma_{\overline{G}}(t))\left(LnG(t) + \left(r - \frac{1}{2}\sigma_G^2\right)(T-t) - Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right) - \sigma_G\sqrt{T-t}\,d_2\right)n(d_2)\right]\right\} \quad (12)$$

where $d_1 = \dfrac{Ln\left(\dfrac{G(t)}{\overline{G}(t)}\right) + \left(r - \dfrac{1}{2}\sigma_G^2\right)(T-t) - \delta(t,T) + \sigma_B\left(\theta_B(T-t) - \overline{\sigma}_{\overline{G}}(t,T)\right)}{\sigma_{G-\overline{G}}(t,T)}$,

$d_2 = d_1 - \dfrac{\sigma_B\left(\theta_B(T-t) - \overline{\sigma}_{\overline{G}}(t,T)\right)}{\sigma_{G-\overline{G}}(t,T)}$, $\delta(t,T) = \int_t^T \left(\mu_{\overline{G}}(u) - \theta_B\sigma_{\overline{G}}(u) - \frac{1}{2}\sigma_{\overline{G}}^2(u)\right)du$, $\overline{\sigma}_{\overline{G}}(t,T) = \int_t^T \sigma_{\overline{G}}(u)du$,

$\sigma_{G-\overline{G}}^2(t,T) = \int_t^T \left(\sigma_G^2 + \sigma_{\overline{G}}^2(u) - 2\theta_B\sigma_{\overline{G}}(u)\right)du$,

N(.) and n(.) denote the standard normal cumulative distribution and density functions respectively.

Proof. See Appendix B.

COROLLARY. 3.1. In the case of a riskless benchmark, $B(t) = \beta(t)$, the optimal wealth and proportion become respectively:

$$W_R^*(t) = e^{-r(T-t)}\left\{e^{rT}\beta(0)N(d_{R}) + \frac{1}{\alpha_1\gamma_1}\left[\left(LnG_R(t) + \left(r - \frac{1}{2}\theta_S^2\right)(T-t) - Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right)\right)N(d_{R}) + \theta_S\sqrt{T-t}\,n(d_{R})\right]\right\} \tag{13}$$

$$\pi_{S_R}^*(t) = \frac{e^{-r(T-t)}}{W_R^*(t)\sigma_S\sqrt{T-t}}\left\{e^{rT}\beta(0)n(d_{R}) + \frac{1}{\alpha_1\gamma_1}\left[\theta_S\sqrt{T-t}\,N(d_{R}) + \left(LnG_R(t) + \left(r - \frac{1}{2}\theta_S^2\right)(T-t) - Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right) - \theta_S\sqrt{T-t}\,d_{R}\right)n(d_{R})\right]\right\} \tag{14}$$

where $d_{R} = \dfrac{Ln\left(\dfrac{G_R(t)}{\overline{G}_R}\right) + \left(r - \dfrac{1}{2}\theta_S^2\right)(T-t)}{\theta_S\sqrt{T-t}}$, $\overline{G}_R$ is a solution to the equation:

$$f(x) = 1 + \lambda\left[1 - \exp\left\{-\alpha_i\gamma_2\beta(T)\right\}\right] - \left[\beta(0)e^{rT} + \frac{1}{\alpha_1\gamma_1}\left(1 - Ln\left(\frac{\eta_R}{\alpha_1\gamma_1 x}\right)\right)\right]\frac{\eta_R}{x} = 0$$

i) Asymmetric fees: If, in the first exponential in *f(x)*, $c_1 = 0, c_2 > 0$ ($c_1 = c_2 = 0$), the underperformance penalty is positive (null) ,

ii) Symmetric fees: In the first exponential *f(x)*, $c_1 = c_2$,

and $G_R(t) = \beta(0)\exp\left\{\left(r + \frac{1}{2}\theta_S^2\right)t + \theta_S z_S(t)\right\}$. $\eta_R \geq 0$ is the Lagrangian associated with the static program solving: $W_R(0) = \beta(0)E\left[\frac{W_R^*(T)}{G_R(T)}\right]$.

Proof. Equations (13) and (14) can directly be derived from Proposition 4.1.

The manager's optimal wealth at time *t* (Eq. 10), in the case of a risky benchmark, may be decomposed into two components. The first one is the price of a call option, having a maturity *T*, written on the benchmark with an exercise price equal to $e^{rT}B(0)$. This term appears due to the stochastic character of the benchmark. It should be noted, however, that the price of the option is derived in terms of the probability that $G(T) > \overline{G}(T)$ ($G(T) \leq \overline{G}(T)$) in favourable (unfavourable) states, that is, when the portfolio's final value overperforms (underperforms) the benchmark. By comparing Equations (10) and (13), it can be seen that the second component has a similar, although more involved, expression to that of the optimal wealth when the benchmark is the riskless asset. This term contains two addends. The first one is a cash-or-nothing call option, which pays a cash amount $e^{rT}B(0)$ (or $e^{rT}\beta(0)$) in favourable states, and nothing otherwise. The second addend is an adjustment term depending on the predetermined fees parameters *b* and c_i, i =1, 2, and associated with the optimal growth portfolio, which is a function of the prices of risk reflecting risk preferences.

Equation (11) shows that the optimal proportion invested in the risky security comprises a part invested in the benchmark and an addend resembling the proportion when the riskless asset serves as the benchmark (Eq. 14). As a function of the excess returns of the risky security and the risky benchmark, this position is a speculative position. It depends on the difference between Sharpe ratios associated with the risky security and the risky benchmark multiplied by the correlation coefficient between the two assets, which is reasonably supposed to be positive. For a given correlation coefficient, the higher the former ratio and the lower the latter, ceteris paribus, the higher the weight invested in the risky security, and vice versa. A lower correlation coefficient amplifies this difference. Indeed, since the manager's objective is to beat the benchmark, the greater this difference, the higher the probability that the terminal portfolio value will exceed the benchmark value. The contrary

prevails for the benchmark. Unlike $\pi_S^*(t)$, the fraction invested in the benchmark contains a hedging and a speculative term. The fund manager not only forms her (his) anticipations about the future evolution of the benchmark but also, to achieve her (his) objective, (s)he engages in a hedging procedure against adverse fluctuations in the benchmark.

[Insert Figure 2 about here]

When the option in the fees is deeply in the money (*G(t)* tends to infinity), the optimal wealth tends to infinity. In contrast, in bad states of the world (*G(t)* goes to zero), the optimal wealth tends to zero to satisfy the solvency constraint. As shown in Figure 2, the behavior of the wealth at any date *t* is in accordance with Propositions 3.1. The same conclusions are derived for a riskless benchmark (Corollary 3.1).

[Insert Figure 3 about here] [Insert Figure 4 about here]

In the case of a risky benchmark, the optimal weight of the risky security, $\pi_S^*(t)$, exhibits a humped shape (see Figure 3) and $\pi_{S_R}^*(t)$ has a similar pattern (Figure not reproduced) (see also Berkelaar et al., 2004; Kouwenberg and Ziemba, 2007; Buraschi et al., 2014). In contrast, the proportion invested in the benchmark, $\pi_B^*(t)$, evolves in an opposite way (see Figure 4). They reach a peak or a trough (maximum or minimum) respectively when the fund value is close to the benchmark value because the uncertainty of being in the region of losses or in the region of gains is the highest. Moreover, as a function of *G(t)*, they evolve distinctly for different fee structures. The dividing point, $\overline{G}(\mathrm{T})$, separating those two regions, shifts toward the right hand side from symmetric to asymmetric fees as do optimal proportions making it more risky to do better than the benchmark. In bad states, when the benchmark exceeds wealth value, asymmetric fees induce high variations, as a function of *G(t)*, in the proportions invested in the risky asset and the benchmark. If managers are, however, totally (symmetric fees) or partially (asymmetric fees with a penalty) penalized for underperformance, they enter in less fluctuating positions expecting to increase the value of the portfolio and to avoid such a painful situation. In good states, weights are decreasing functions of *G(T)* and for exceptionally advantageous situations, optimal weights converge to a steady state close to zero. Indeed, for high

values of wealth, in the region of gains, fund managers decrease their investment in the two risky assets to lock in a profit.

[Insert Figure 5 about here] [Insert Figure 6 about here]

Another important feature of our model is related to manager's ownership in the fund. To study its impact, we consider four different values: a = 0% (pictured in Figure 5), 10%, 20% and 30% (pictured in Figure 6). When the manager does not invest in the fund, symmetric fees offer a better "insurance" against unfavourable states than asymmetric fees with or without a penalty. In good states, however, the portfolio's terminal value with symmetric fees is lower than the value of the other schemes. An a = 10% drive the manager to bear less risk and as it increases and reaches the value of 30%, fund value is almost the same whatever the scheme and the states of the nature. To moderate a risk-taking behavior, a relatively high stake in the fund (30% in our case) produces a better effect than different compensation schemes. Kouwenberg and Ziemba (2007) and Zou (2017) reached a similar conclusion.

[Insert Figure 7 about here] [Insert Figure 8 about here]

We also examine the impact of different parameter values on the optimal weights $\pi_S^*(t)$ and $\pi_B^*(t)$ in the case of a risky benchmark for both asymmetric fees (with and without a penalty) and for symmetric fees. To save space, we reproduce the figures when this impact is significant and for representative cases. For the correlation coefficient, we choose three values: ρ = -0.5, ρ = 0 and ρ = 0.82. As an example, Figures 7 and 8 picture this impact for asymmetric fees with a penalty. For the other compensation schemes, the optimal proportions exhibit similar shapes. For positive values of ρ, when ρ decreases to 0, the optimal proportions invested in the risky asset also decrease. However, for negative values of ρ, $\pi_S^*(t)$ for ρ = -0.5 lie between those for ρ = 0 and ρ = 0.82. Indeed, $\pi_S^*(t)$ crucially depends on the term $(\theta_S - \rho\theta_B)/(1-\rho^2)$, which deceases for $\rho > 0$ and increases for $\rho < 0$. In contrast, as expected, when ρ declines and passes through 0 to become negative the sign of $\pi_B^*(t)$ reverses and become positive.

[Insert Figure 9 about here] [Insert Figure 10 about here]

[Insert Figure 11 about here] [Insert Figure 12 about here]

[Insert Figure 13 about here] [Insert Figure 14 about here]

To study the role of the volatilities of the risky asset and the risky benchmark, they take the following values: $\sigma_S = 0.16, \sigma_S = 0.262$ and $\sigma_S = 0.35$, and $\sigma_B = 0.13, \sigma_B = 0.235$ and $\sigma_B = 0.3$. σ_S and σ_B heavily influence $\pi_S^*(t)$ and $\pi_B^*(t)$ respectively. Since there is an inverse relation between volatilities and optimal proportions, the lower the volatility the higher in absolute values $\pi_S^*(t)$ and $\pi_B^*(t)$ (see Figures 9 and 10 for asymmetric fees as an example). Prices of risk, θ_S and θ_B, strongly affect optimal weights, and their effect is more pronounced for symmetric fees. This is why we reproduce the figures for those fees. An increasing θ_S results in higher, in absolute values, $\pi_S^*(t)$ and $\pi_B^*(t)$ (see Figures 11 and 12). Unlike θ_S, θ_B has the opposite effect (see Figures 13 and 14).

4. Conclusion

We examined the impact of performance-based fees on asset allocation by a manager investing a fraction of her (his) own wealth in the fund and exhibiting a loss aversion utility function. The performance of the portfolio under management is assessed relative to a risky benchmark and the fees scheme comprises symmetric and asymmetric fees with a performance bonus as well as an underperformance penalty. These features turn out to be very important when determining optimal demands and studying managers' risk-taking behavior. A riskless benchmark (rarely used in practice) implies an artificially higher optimal intermediate portfolio value compared to a risky benchmark, which is effectively used. In addition, portfolio choice is substantially modified, since the manager takes a position in the risky security and the benchmark. Those positions are very volatile when the manager underperforms the benchmark and is compensated by asymmetric fees. However, this risk-taking behavior is considerably reduced by symmetric fees and, especially, when the manager's stake in the fund is relatively high (for example, 30%). Those results may be insightful to regulators and contribute to the debate on managers' compensation.

One important question related to delegated portfolio management deals with the design of incentive contracts and the agency problems arising between investors and managers (see Admati and

Pfleiderer, 1997; Ou-Yang, 2003; Dybvig et al., 2009; Lioui and Poncet, 2013; Cvitanić and Xing, 2018). This question may be examined within the framework of the prospect theory by focusing the analysis on the fee schedule and the nature of the benchmark. Cuoco and Daniel (2011) have studied, in a general equilibrium setting, the influence of managers' fees structure on asset prices. Their analysis may be extended to the managers' behavior vis-à-vis the risk and different fee compensations. Finally, liquidity concerns impose restrictions on risky assets trading limiting manager's portfolio choice when, in particular, (s)he faces losses. How can illiquidity modify manager's asset allocation when maximizing incentive fees?

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Appendix A: Proof of Proposition 2.1.

The utility function is a S-Shaped function having a concave and a convex part. We denote $U_i(W_i^*(T)), i=1,2$ the corresponding utility functions, where $W_i^*(T)$ is the optimal terminal wealth. $U_1(W_1^*(T))$ represents the concave part and the Lagrange technique can be applied:

$$L(W(T),\eta) = E\left[1-\exp\left\{-\gamma_1\left((b+c)\left(W_1(T)-B(T)\right)\right)\right\}\right]+\eta\left[W(0)-E\left[\frac{W_1(T)}{G(T)}\right]\right]$$

$\frac{\partial L(W(T),\eta)}{\partial W_1(T)} = 0$ implies that $W_1(T)^* = B(T) - \frac{1}{(b+c)\gamma_1} Ln\left(\frac{\eta}{(b+c)\gamma_1 G(T)}\right)$

where η denotes the Langrange multiplier associated with the optimization problem.

$U_2(W_2^*(T))$ is the convex part of the utility function. It follows that $W_2^*(T)$ is located at one of the two boundaries $W_2^*(T)=0$ or to $W_2^*(T)=B(T)$. To determine the global maximum, we compare the local maxima $W_1^*(T)$ and $W_2^*(T)$. Consider the function:

$$f(G(T),B(T))=U_1(W_1^*(T))-\eta\frac{W_1^*(T)}{G(T)}-\left[U_2(W_2^*(T))-\eta\frac{W_2^*(T)}{G(T)}\right]$$

If $f(G(T),B(T))\geq 0$, then $W_1^*(T)$ is the optimal solution.

Let us first compare $W_1^*(T)$ and $W_2^*(T)=B(T)$.

$$f_1(G(T),H(T))=1-\frac{\eta}{\alpha_1\gamma_1 G(T)}\left[1-Ln\left(\frac{\eta}{\alpha_1\gamma_1 G(T)}\right)\right]$$

It can be shown that $f_1(G(T),H(T))\geq 0,\ \forall \mathrm{G(T)}$, and B(T) is not an optimal solution.

Compare $W_1^*(T)$ and $W_2^*(T)=0$.

$$f_2(G(T),H(T))=1-\frac{\eta}{\alpha_1\gamma_1 G(T)}\left[1-Ln\left(\frac{\eta}{\alpha_1\gamma_1 G(T)}\right)\right]-\eta\frac{B(T)}{G(T)}+\lambda\left[1-\exp\left(-\alpha_i\gamma_2 B(T)\right)\right]$$

It can be shown that $f_1(G(T),H(T))\leq 0$ if $\mathrm{G(T)}\leq\overline{\mathrm{G}}\mathrm{(T)}$. Thus 0 is an optimal solution.

Appendix B: Proof of Proposition 3.1

a) To calculate the optimal wealth at any date t, $0\leq \mathrm{t}\leq \mathrm{T}$, from expression (8), we have:

$$W^*(t)=E\left[\frac{G(t)}{G(T)}W^*(T)\right]=E\left[\frac{G(t)}{G(T)}W_1^*(T)1_{\{G(T)\geq\overline{G}(T)\}}\right]$$
$$=E\left[\frac{G(t)}{G(T)}\left[B(T)-\frac{1}{(b+c)\gamma_1}Ln\left(\frac{\eta}{(b+c)\gamma_1 G(T)}\right)\right]1_{\{G(T)\geq\overline{G}(T)\}}\right]$$

$$W^*(t)=E\left[\frac{G(t)}{G(T)}B(T)1_{\{G(T)\geq\overline{G}(T)\}}\right]-\frac{1}{(b+c)\gamma_1}Ln\left(\frac{\eta}{(b+c)\gamma_1}\right)E\left[\frac{G(t)}{G(T)}1_{\{G(T)\geq\overline{G}(T)\}}\right]$$
$$+\frac{1}{(b+c)\gamma_1}E\left[\frac{G(t)}{G(T)}LnG(T)1_{\{G(T)\geq\overline{G}(T)\}}\right] \qquad \text{(B.1)}$$

From the expression of G(t), we get:

$$\frac{G(t)}{G(T)} = \exp\left\{-\left(r + \frac{1}{2}\sigma_G^2\right)(T-t) - \theta_B z_B(T-t) - \frac{(\theta_S - \rho\theta_B)}{\sqrt{1-\rho^2}} z_\varepsilon(T-t)\right\}$$

where $\sigma_G^2 = \frac{(\theta_S^2 - 2\rho\theta_S\theta_B + \theta_B^2)}{(1-\rho^2)}$ and $z_B(T-t)$ and $z_\varepsilon(T-t)$ are two Brownian motions such that: $z_j(T-t) = z_j(T) - z_j(t),\ j = B, \varepsilon$. From $\overline{G}(T)$ dynamics, we have:

$$\overline{G}(T) = \overline{G}(t)\exp\left\{\int_t^T \left(\mu_{\overline{G}}(s) - \frac{1}{2}\sigma_{\overline{G}}^2(s)\right)ds + \int_t^T \sigma_{\overline{G}}(s)dz_B(T-s)\right\}$$

We can now compute the expectations involved in equation (C.1). From the second expectation we have:

$$E\left[\frac{G(t)}{G(T)}1_{\{G(T)\geq\overline{G}(T)\}}\right] = e^{-r(T-t)}E\left[\exp\left\{-\frac{1}{2}\sigma_G^2(T-t) - \theta_B z_B(T-t) - \frac{(\theta_S - \rho\theta_B)}{\sqrt{1-\rho^2}} z_\varepsilon(T-t)\right\}1_{\{G(T)\geq\overline{G}(T)\}}\right]$$

Let $\overline{P}$ be a probability measure equivalent to *P* such that:

$$\frac{d\overline{P}}{dP}\Big|F(t) = \exp\left\{-\frac{1}{2}\sigma_G^2(t) - \theta_B z_B(t) - \frac{(\theta_S - \rho\theta_B)}{\sqrt{1-\rho^2}} z_\varepsilon(t)\right\}$$

By virtue of Girsanov's theorem:

$$dz_S(t) = d\overline{z}_B(t) - \theta_B dt$$
$$dz_\varepsilon(t) = d\overline{z}_\varepsilon(t) - \frac{(\theta_S - \rho\theta_B)}{\sqrt{1-\rho^2}}dt$$

Under $\overline{P}$:

$$G(T) = G(t)\exp\left\{\left(r - \frac{1}{2}\sigma_G^2\right)(T-t) + \theta_B\overline{z}_B(T-t) + \frac{(\theta_S - \rho\theta_B)}{\sqrt{1-\rho^2}}\overline{z}_\varepsilon(T-t)\right\}$$

$$\overline{G}(T) = \overline{G}(t)\exp\left\{\int_t^T \left(\mu_{\overline{G}}(s) - \theta_B\sigma_{\overline{G}}(s) - \frac{1}{2}\sigma_{\overline{G}}^2(s)\right)ds + \int_t^T \sigma_{\overline{G}}(s)d\overline{z}_B(T-s)\right\}$$

Bayes formula implies that:

$$E\left[\frac{G(t)}{G(T)}1_{\{G(T)\geq\overline{G}(T)\}}\right] = e^{-r(T-t)}E^{\overline{P}}\left[1_{\{G(T)\geq\overline{G}(T)\}}\right] =$$

$$e^{-r(T-t)}E^{\overline{P}}\left[1_{\left\{G(t)e^{r(T-t)}\exp\left\{-\frac{1}{2}\sigma_G^2(T-t)+\theta_B\overline{z}_B(T-t)+\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}}\overline{z}_\varepsilon(T-t)\right\}\geq\overline{G}(t)e^{\int_t^T\left(\mu_{\overline{G}}(s)-\theta_B\sigma_{\overline{G}}(s)\right)ds}\exp\left\{-\frac{1}{2}\int_t^T\sigma_{\overline{G}}^2(s)ds+\int_t^T\sigma_{\overline{G}}(s)\overline{z}_B(T-s)\right\}\right\}}\right]$$

Calculating the above expectation and rearranging terms gives:

$$e^{-r(T-t)}E^{\bar{P}}\left[1_{\{G(T)\geq\bar{G}(T)\}}\right]=e^{-r(T-t)}N(d_2)$$

$$\text{where } d_2=\frac{Ln\left(\frac{G(t)}{\bar{G}(t)}\right)+\left(r-\frac{1}{2}\sigma_G^2\right)(T-t)-\int_t^T\left(\mu_{\bar{G}}(s)-\theta_B\sigma_{\bar{G}}(s)-\frac{1}{2}\sigma_{\bar{G}}^2(s)\right)ds}{\sigma_{G-\bar{G}}(t,T)}$$

$$\sigma_{G-\bar{G}}^2(t,T)=\sigma_G^2(T-t)+\int_t^T\sigma_{\bar{G}}^2(s)ds-2\theta_B\int_t^T\sigma_{\bar{G}}(s)ds$$

The first expectation in C.1 gives:

$$E\left[\frac{G(t)}{G(T)}B(T)1_{\{G(T)\geq\bar{G}(T)\}}\right]=B(t)E\left[\exp\left\{-\frac{1}{2}\sigma_G^2(T-t)-\theta_B z_B(T-t)-\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}}z_\varepsilon(T-t)\right\}\right.$$
$$\left.\exp\left\{\left(\theta_B\sigma_B-\frac{1}{2}\sigma_B^2\right)(T-t)+\sigma_B z_B(T-t)\right\}1_{\{G(T)\geq\bar{G}(T)\}}\right]$$
$$=B(t)E\left[\exp\left\{-\frac{1}{2}\left(\sigma_G^2+\sigma_B^2-2\sigma_B\theta_B\right)(T-t)-\left(\theta_B-\sigma_B\right)z_B(T-t)\right.\right.$$
$$\left.\left.-\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}}z_\varepsilon(T-t)\right\}1_{\{G(T)\geq\bar{G}(T)\}}\right]$$

The probability measure $\bar{\bar{P}}$ is defined by its Radon-Nikodym derivative with respect to P:

$$\frac{d\bar{\bar{P}}}{P}\Big|F(t)=\exp\left\{-\frac{1}{2}\left(\sigma_G^2+\sigma_B^2-2\sigma_B\theta_B\right)(T-t)-\left(\theta_B-\sigma_B\right)z_B(T-t)-\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}}z_\varepsilon(T-t)\right\}$$

and the processes by Girsanov's theorem:

$$d\bar{\bar{z}}_B(t)=dz_S(t)+(\theta_B-\sigma_B)dt$$
$$d\bar{\bar{z}}_\varepsilon(t)=d\bar{z}_\varepsilon(t)$$

are, under $\bar{\bar{P}}$, standard Brownian motions, and:

$$G(T)=G(t)\exp\left\{\left(r-\frac{1}{2}\sigma_G^2+\theta_B\sigma_B\right)(T-t)+\theta_B\bar{\bar{z}}_B(T-t)+\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}}\bar{\bar{z}}_\varepsilon(T-t)\right\}$$

$$\bar{G}(T)=\bar{G}(t)\exp\left\{\int_t^T\left(\mu_{\bar{G}}(s)-\frac{1}{2}\sigma_{\bar{G}}^2(s)-\left(\theta_B-\sigma_B\right)\sigma_{\bar{G}}(s)\right)ds+\int_t^T\sigma_{\bar{G}}(s)d\bar{\bar{z}}_B(T-s)\right\}$$

Thanks to Bayes formula:

$$E\left[\frac{G(t)}{G(T)}B(T)1_{\{G(T)\geq\bar{G}(T)\}}\right]=B(t)E^{\bar{\bar{P}}}\left[1_{\{G(T)\geq\bar{G}(T)\}}\right]=B(t)N(d_1)$$

$$\text{where } d_1 = \frac{Ln\left(\frac{G(t)}{\overline{G}(t)}\right) + \left(r - \frac{1}{2}\sigma_G^2\right)(T-t) + \sigma_B \int_t^T \left(\theta_B - \sigma_{\overline{G}}(s)\right) ds - \int_t^T \left(\mu_{\overline{G}}(s) - \theta_B \sigma_{\overline{G}}(s) - \frac{1}{2}\sigma_{\overline{G}}^2(s)\right) ds}{\sigma_{G-\overline{G}}(t,T)}$$

$$d_2 = d_1 - \frac{\sigma_B \int_t^T \left(\theta_B - \sigma_{\overline{G}}(s)\right) ds}{\sigma_{G-\overline{G}}(t,T)}$$

It remains to calculate the following expectation:

$$E\left[\frac{G(t)}{G(T)} LnG(T) 1_{\{G(T) \geq \overline{G}(T)\}}\right] = e^{-r(T-t)} E^{\overline{P}}\left[LnG(T) 1_{\{G(T) \geq \overline{G}(T)\}}\right]$$

Result. Let x and y two random variables normally distributed, $x \sim N(\mu_x, \sigma_x^2)$ and $y \sim N(\mu_y, \sigma_y^2)$, with covariance $\text{cov}(x, y) = \rho \sigma_x \sigma_y$. Then for arbitrary positive numbers *k* and *l* we have:

$$E\left[Lnke^x 1_{\{ke^x > le^y\}}\right] = (Lnk + \mu_x) N(d) + \sigma_x n(d)$$

where $1_{\{\ \}}$ is the indicator function and $\sigma_{x-y}^2 = Var(x-y)$.

This result gives:

$$e^{-r(T-t)} E^{\overline{P}}\left[LnG(T) 1_{\{G(T) \geq \overline{G}(T)\}}\right] = e^{-r(T-t)}\left[\left(LnG(t) + \left(r - \frac{1}{2}\sigma_G^2\right)(T-t)\right) N(d_2) + \sigma_G \sqrt{T-t}\, n(d_2)\right]$$

Substituting the expressions of the expectations into B.1 leads to equation (10) in the main text.

b) The following derivatives with respect to *G(t)* and $\overline{G}(t)$ are given, for *i = 1, 2*, by:

$$N_G'(d_i) = \frac{n(d_i)}{G(t)\sigma_{G-\overline{G}}(t,T)}, \qquad N_{\overline{G}}'(d_i) = -\frac{n(d_i)}{\overline{G}(t)\sigma_{G-\overline{G}}(t,T)}, \qquad n_G'(d_i) = \frac{d_i n(d_i)}{G(t)\sigma_{G-\overline{G}}(t,T)},$$

$$n_{\overline{G}}'(d_i) = -\frac{d_i n(d_i)}{\overline{G}(t)\sigma_{G-\overline{G}}(t,T)}.$$

Applying Itô's lemma to the optimal wealth (equation 10) and rearranging terms gives:

$$\frac{dW^*(t)}{W^*(t)}=\mu_{W^*}(t)dt+\frac{1}{W^*(t)}\left\{\left[\sigma_B N(d_1)+\frac{(\theta_B-\sigma_{\bar{G}}(t))}{\sigma_{G-\bar{G}}(t,T)}n(d_1)\right]B(t)+\frac{e^{-r(T-t)}(\theta_B-\sigma_{\bar{G}}(t))}{\alpha_1\gamma_1\sigma_{G-\bar{G}}(t,T)}\right.$$
$$\left.\left[\left(\frac{\theta_B\sigma_{G-\bar{G}}(t,T)}{(\theta_B-\sigma_{\bar{G}}(t))}N(d_2)+\left(LnG(t)+\left(r-\frac{1}{2}\sigma_G^2\right)(T-t)-Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right)-\sigma_G\sqrt{T-t}d_2\right)n(d_2)\right)\right]\right\}dZ_B(t)$$
$$+\frac{(\theta_S-\rho\theta_B)}{\sqrt{1-\rho^2}\sigma_{G-\bar{G}}(t,T)W^*(t)}\left\{n(d_1)B(t)+\frac{e^{-r(T-t)}}{\alpha_1\gamma_1}\left[\left(\sigma_{G-\bar{G}}(t,T)N(d_2)\right.\right.\right.$$
$$\left.\left.\left.+\left(LnG(t)+\left(r-\frac{1}{2}\sigma_G^2\right)(T-t)-Ln\left(\frac{\eta}{\alpha_1\gamma_1}\right)-\sigma_G\sqrt{T-t}d_2\right)n(d_2)\right)\right]\right\}dZ_S(t)$$

The admissible wealth writes:

$$\frac{dW(t)}{W(t)}=\left[r+\pi(t)'\sigma\theta\right]dt+\pi(t)'\sigma dz(t) \tag{3}$$

Identifying the diffusion terms of the admissible wealth and the optimal wealth yields equations (11) and (12).

r	σ_S	σ_B	θ_S	θ_B	ρ	$W(0)$	$B(0)$
0.005	0.262	0.235	0.32	0.19	0.82	1	1
$\beta(0)$	b	c_1	c_2	λ	γ_1	γ_2	T
1	0.02	0.2	0.04	2.25	0.9	0.8	1

Table 1. Numerical values of the parameters. This table gathers the parameter values of the loss-aversion utility function (see Tversky and Kahneman, 1992 and Köberling and Wakker, 2005), the fee scheme in accordance with market practices and the risk-return characteristics of the fund (Amundi European Equity Value Fund) and the benchmark (MSCI Europe Value Equity).

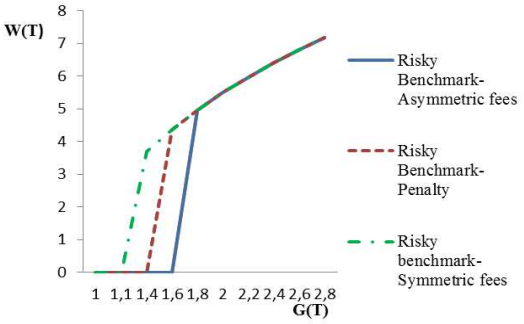

Figure 1. Optimal portfolio terminal value-Risky benchmark. This figure plots the terminal wealth as a function of *G(T)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees (solid line), asymmetric fees with a penalty (dashed line) and with symmetric fees (dashed-dotted line). The parameter values are given in Table 1.

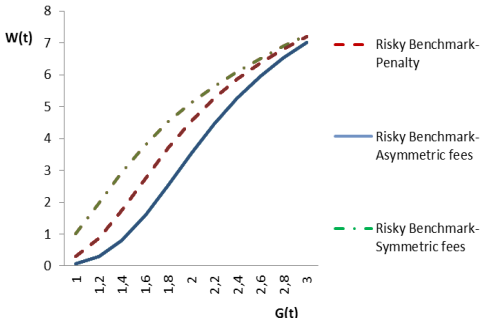

Figure 2. Optimal portfolio value at date *t*- Risky benchmark. This figure plots the intermediate wealth at time *t* as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees (solid line), asymmetric fees with a penalty (dashed line) and with symmetric fees (dashed-dotted line). The parameter values are given in Table 1.

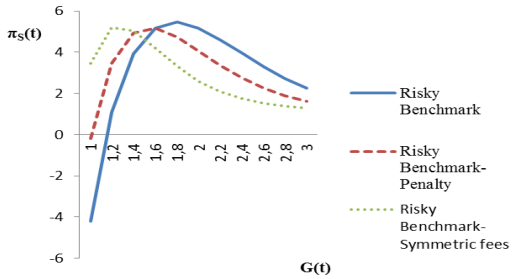

Figure 3. Optimal proportion invested in the risky asset - Risky benchmark. This figure plots $\pi_S(t)$ at time *t* as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees (solid line), asymmetric fees with a penalty (dashed line) and symmetric fees (dotted line). The parameter values are given in Table 1.

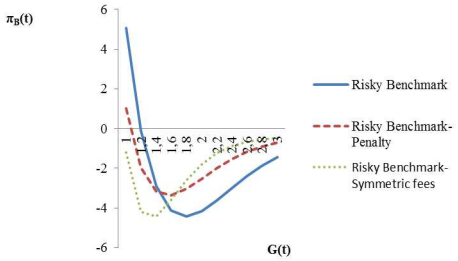

Figure 4. Optimal proportion invested in the risky risky benchmark - Risky benchmark. This figure plots $\pi_B(t)$ at time *t* as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees (solid line), asymmetric fees with a penalty (dashed line) and with symmetric fees. The parameter values are given in Table 1.

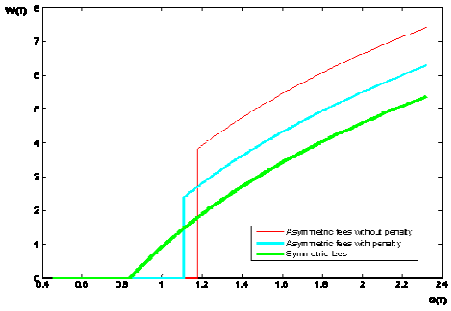

Figure 5. Optimal portfolio's terminal value. This figure plots the terminal portfolio's value as a function of *G(T)* when fund manager's investment in the fund is a = 0% for symmetric fees (bold line), asymmetric fees with a penalty (medium line) and asymmetric fees (thin line). The parameter values are given in Table 1.

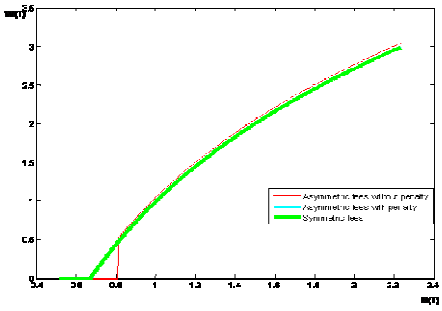

Figure 6. Optimal portfolio's terminal value. This figure plots the terminal portfolio's value as a function of *G(T)* when fund manager's investment in the fund is a = 30% for symmetric fees (bold line), asymmetric fees with a penalty (medium line) and asymmetric fees (thin line). The parameter values are given in Table 1.

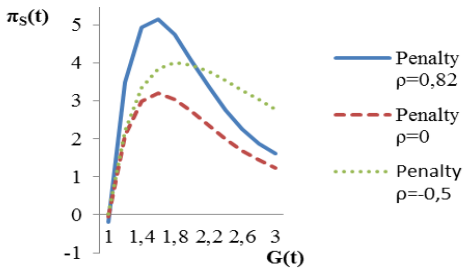

Figure 7. Optimal proportion invested in the risky asset as a fnction of ρ - Risky benchmark. This figure plots $\pi_S(t)$ at time t as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees with penalty when ρ = 0,82 (solid line), ρ = 0 (dashed line) and ρ = -0,5 (dotted line). The other parameter values remain unchanged and are given in Table 1.

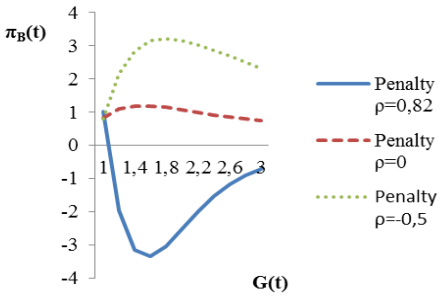

Figure 8. Optimal proportion invested in the risky benchmark as a function of ρ - Risky benchmark. This figure plots $\pi_B(t)$ at time t as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees with a penalty when ρ = 0,82 (solid line), ρ = 0 (dashed line) and ρ = -0,5 (dotted line). The other parameter values remain unchanged and are given in Table 1.

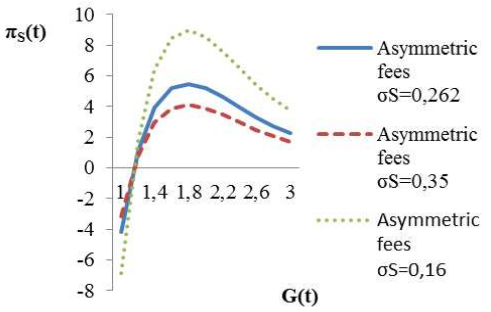

Figure 9. Optimal proportion invested in the risky asset as a function of σ_S - Risky benchmark. This figure plots $\pi_S(t)$ at time t as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees when σ_S = 0,16 (dotted line), σ_S = 0,262 (solid line) and σ_S = 0,35 (dashed line). The other parameter values remain unchanged and are given in Table 1.

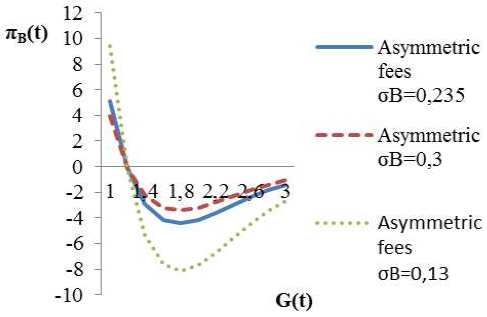

Figure 10. Optimal proportion invested in the risky benchmark as a function of σ_B - Risky benchmark. This figure plots $\pi_B(t)$ at time t as a function of *G(t)* of a fund manager with loss aversion for a risky benchmark with asymmetric fees when σ_B = 0,13 (dotted line), σ_B = 0,235 (solid line) and σ_B = 0,3 (dashed line). The other parameter values remain unchanged and are given in Table 1.

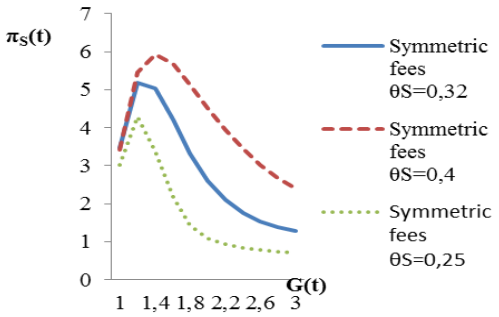

Figure 11. Optimal proportion invested in the risky asset as a function of θ_S - Risky benchmark. This figure plots $\pi_S(t)$ at time t as a function of $G(t)$ of a fund manager with loss aversion for a risky benchmark with symmetric fees when θ_S = 0,25 (dotted line), θ_S = 0,32 (solid line) and θ_S = 0,40 (dashed line). The other parameter values remain unchanged and are given in Table 1.

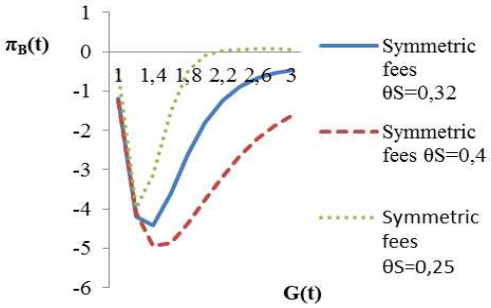

Figure 12. Optimal proportion invested in the risky benchmark as a function of θ_S - Risky benchmark. This figure plots $\pi_B(t)$ at time t as a function of $G(t)$ of a fund manager with loss aversion for a risky benchmark with symmetric fees when θ_S = 0,25 (dotted line), θ_S = 0,32 (solid line) and θ_S = 0,40 (dashed line). The other parameter values remain unchanged and are given in Table 1.

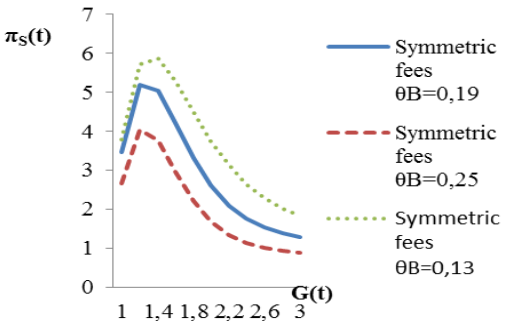

Figure 13. Optimal proportion invested in the risky asset as a function of θ_B - Risky benchmark. This figure plots $\pi_S(t)$ at time t as a function of $G(t)$ of a fund manager with loss aversion for a risky benchmark with symmetric fees when θ_B = 0,13 (dotted line), θ_B = 0,19 (solid line) and θ_B = 0,25 (dashed line). The other parameter values remain unchanged and are given in Table 1.

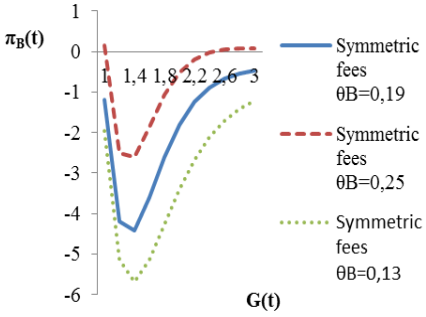

Figure 14. Optimal proportion invested in the risky benchmark as a function of θ_B - Risky benchmark. This figure plots $\pi_B(t)$ at time t as a function of $G(t)$ of a fund manager with loss aversion for a risky benchmark with symmetric fees when θ_B = 0,13 (dotted line), θ_B = 0,19 (solid line) and θ_B = 0,25 (dashed line). The other parameter values remain unchanged and are given in Table 1.