

Efficient Asymptotic Models for 3D Linear Ferromagnetic Materials in Eddy Current Problems

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Perturbation methods are often employed to generate accurate approximations to the solution of physical problems involving large or small parameters. These methods are accomplished through the introduction of asymptotic expansions in the limit of a vanishing small perturbation parameter. Indeed, when truncated at a certain order, we get a finite sum that approximates the exact solution with a given accuracy. Moreover, the coefficients of this sum are solutions to an ordered sequence of sub-problems that do not depend on the small parameter.

We aim to apply an asymptotic procedure to study time-harmonic eddy current problems in a material composed by a dielectric material of a constant permeability $\mu_0 > 0$ surrounding a linear ferromagnetic conductor of a constant permeability $\mu_r \mu_0$ where the relative permeability μ_r is assumed to be a large parameter. This method is a useful tool to model the skin effect, that is the fast decrease of the electromagnetic field with depth inside the metallic conductor.

Our work introduces an asymptotic expansion of the electromagnetic field when the surface of the conductor is smooth. First, we introduce two small parameters: $\delta L^{-1} \ll 1$, and $\varepsilon = (\mu_r \delta)^{-1} \ll 1$ where δ is the skin depth and L is the characteristic dimension of the object of study. Then, we exhibit the first terms of a singular expansion in power series of the skin depth δ in order to describe the skin effect in the conductor. In the dielectric domain, the electromagnetic field has a regular expansion in power series of ε . These asymptotic expansions are illustrated by numerical simulations.

Our approach leads to the classical Leontovich boundary condition on the surface of the conductor. Indeed, impedance boundary conditions methods (IBCs) lighten the computational efforts by avoiding to mesh the interior of the conductor domain. For this reason, we intend to compare our asymptotic solution up to the order ε with the Leontovich solution, and we will prove that the costs of our method are less than the resulting IBC regardless the number of physical parameters to be considered.

Since most electromagnetic devices require the modeling in three dimensions, we derive efficient asymptotic models of our eddy current problem in a three-dimensional setting. This study is an extension of our previous works that was restricted to a special class of two-dimensional eddy current problems for high conductive non-magnetic materials and linear ferromagnetic materials as well.