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Risk Management Model for Simultaneous participation of a Distribution Company in Day-ahead and Real-time Markets

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Abstract

In Active Distribution Networks (ADNs), Distribution Company (Disco) follows two main strategies of dispatching of Distributed Energy Resources (DERs) and trading energy with wholesale energy markets, including Day-Ahead (DA) and Real-Time (RT) markets, to meet the demand. An attempt is made in this paper to model the strategic behavior of the Disco, in the wholesale DA and RT energy markets, through a bi-level optimization approach. While the objective of the upper-level problem is to minimize the expected cost of the Disco, the lower-level problem (with two optimization problems) formulates to simultaneously maximize the social-welfare of the DA market and minimize the cost of the RT market. Furthermore, uncertain behavior of renewable energy sources as well as demand is tackled into the problem formulation. To this end, Disco decision-making represents as a risk-based two-stage stochastic problem where the Disco's risk aversion is modeled using conditional value at risk (CVaR) method. Generally stated, the proposed model is a non-linear bi-level problem which may be transformed into a non-linear but single-level problem through Karush-Kuhn-Tucker (KKT) conditions and dual theory. Detailed numerical results on a 6-bus and RTS 24-bus power systems are used to demonstrate efficiency of the proposed model. Moreover, sensitivity analysis is carried out to investigate the effect of risk-aversion parameter on the decision making of the Disco and the offers/bids in both the DA and RT markets.

Keywords: Active distribution networks; Bi-level approach; Day-ahead and Real-time markets; Karush-Kuhn-Tucker; Risk management.

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Nomenclature

Acronyms		**Parameters:**	
		$E_{e,ini}^{ES}$	Initial energy stored in ES (MWh)
ADN	Active distribution network	$\overline{F}_{n-r}^{TN}$	Capacity limit of each TN line n-r (MW)
DA/RT	Day-ahead/Real-time market	$\overline{L}_{d,t}^{TN}$ /$\overline{LB}_{b,d,t}^{TN}$	Maximum demand/size of TNL energy block in the DA market (MW)
DER	Distributed energy resource	$\bar{P}_g$/ $\overline{PB}_{b,g}^{TN}$	Maximum production/size of Genco energy block (MW)
Disco	Distribution company	$P_{g,ini}^{TN}$	Initial production of Genco (MW)
DGA	Distributed generator aggregator	$P_t^{RTC\_UP}$/$P_t^{RTC\_DN}$	Maximum up/down regulation power capacity in the RT market (MW)
DN/DNL	Distribution network/Distribution network load	$\underline{P}^{Dis\_TN}$/ $\overline{P}^{Dis\_TN}$	Power trading limits of the Disco in the DA and RT markets (MW)
DRA	Demand response aggregator	$\overline{P}^{Dis\_Sub}$ / $\underline{P}^{Dis\_Sub}$	Disco's main substation capacity limit (MW)
ESA	Energy storage aggregator	$P_{l,t}^{DNL\_DA}$/ $P_{l,t,\omega}^{DNL\_RT}$	Deterministic/stochastic DNL (MW)
Genco	Generation company	$\underline{P}_k^{DG}$, $\bar{P}_k^{DG}$	Capacity limits of DGs (MW)
IL	Interruptible load	$P_{k,ini}^{DG}$	Initial output power of DGs (MW)
ISO	Independent system operator	$\bar{P}_e^{ch}$, $\bar{P}_e^{dch}$	Maximum charging/discharging power of ES (MW)
LL/UL	Lower/Upper level	$\bar{P}_f^{RES}$/$P_{f,t,\omega}^{RES\_RT}$	Capacity limits of RESs/ Maximum forecasted output power of RES (MW)
MILP	Mixed integer linear programming	$\overline{RG}_g^{UP}$, $\overline{RG}_g^{DN}$	Maximum up/down regulation reserve of Genco (MW)
MINLP	Mixed integer non-linear programming	$\overline{RL}_d^{DN}$	Maximum down regulation reserve of responsive TNL (MW)
MG	Micro-grid	RU_g/ RD_g	Ramp-up/down limits of Genco (MW/h)
		RU_k/RD_k	Ramp-up/down limits of DG (MW/h)
MCP	Market clearing price	α/β	Confidence level/Risk-aversion parameters
NLP	Non-linear programming	$\overline{P}^{IL}$	Maximum load interruption (MW)
RESA	Renewable energy source aggregator	η_{ch}/ η_{dch}	Charging/discharging efficiency of ES
TC/ETC	Total cost of Disco/Expected total cost of Disco ($)	λ_t^{RER}/ λ_t^{DG} /λ_t^{IL}	Offers of RESA/DGA/DRA ($/MWh)
TN/TNL	Transmission network/Transmission network load	λ_t^{ESdch}, λ_t^{ESch}	Offers/bids of ESA ($/MWh)
VaR/CVaR	Value at Risk/Conditional Value at Risk ($)	λ_t^{DNL}	Selling energy price to consumer by the Disco ($/MWh)
		τ_ω	Occurrence probability of each scenario

Indices and sets:		**Variables:**	
a, A	Index and set of IL		
b, B	Index and set of energy and offers/bids block of Genco/TNL	$c_t^{Dis\_DA}/c_{t\omega}^{Dis\_RT}$	Offers/bids of Disco to the DA/RT market ($/MWh)
d, D	Index and set of TNL	$en_{e,t}^{ES\_DA}/en_{e,t,\omega}^{ES\_RT}$	The amount of energy stored in ES (MWh)
e, E	Index and set of ES	$i_{i,j,t}^{DN\_DA}, i_{i,j,t,\omega}^{DN\_RT}$	The current amplitude of which flows from DN feeders in DA and RT markets (kA)
g, G	Index and set of Genco	$l_{d,t}^{TN\_DA}, lb_{b,d,t}^{TN}$	The amount of day-ahead TNL and its block (MW)
i, I/ j, J	Index and set of DN bus	$l_{d,t,\omega}^{TN\_RT}$	The amount of real-time TNL (MW)
f, F	Index and set of RES	$p_{g,t}^{TN}, pb_{b,g,t}^{TN}$	Output power of Genco and its block (MW)
k, K	Index and set of DG	$p_t^{Dis\_DA}/p_{t,\omega}^{Dis\_RT}$	Disco power exchange with DA/RT markets (MW)
l, L	Index and set of DNL	$p_{k,t}^{DG}$	Output power of DG (MW)
M_n^G/M_n^D	Set of Genco/TNL located at bus n	$p_{e,t}^{ESch\_DA}/p_{e,t}^{ESdch\_DA}$	DA Charging/Discharging power of ES (MW)
M_i^{DER}	Set of DERs located at bus i	$p_{e,t,\omega}^{ESch\_RT}/p_{e,t,\omega}^{ESdch\_RT}$	RT Charging/Discharging power of ES (MW)
n, N/ r, R	Index and set of TN bus	$p_{a,t}^{IL}$	The amount of load interruption (MW)
t, T	Index and set of time period	$p_{f,t}^{RES\_DA}$	Output power of the RESs (MW)
ω, W	Index and set of scenario	$rp_{g,t}^{UP\_DA}/rp_{g,t}^{DN\_DA}$	DA maximum up/down regulation power of Genco (MW)
Λ_n^{TN}	Set of buses directly connected to TN bus n	$rl_{d,t}^{DN\_DA}$	DA maximum down regulation power of responsive TNL (MW)
Parameters:		$rp_{g,t,\omega}^{UP\_RT}/rp_{g,t,\omega}^{DN\_RT}$	RT up/down regulation power of Genco (MW)
B_{n-r}	Susceptance of each TN line n-r	$rl_{d,t,\omega}^{DN\_RT}$	RT down regulation power of responsive TNL (MW)
$C_{b,g,t}^{TN}/C_{b,d,t}^{TN\_DA}$	Offers/bids block of Genco/TNL in the DA market ($/MWh)	$u_{k,t}^{DG}$	Binary variable (=1, if DG is on; =0 otherwise)
$C_{d,t}^{TN\_RT}$	Bids of TNL in the RT market ($/MWh)	$v_{i,t}^{DN\_DA}/v_{i,t,\omega}^{DN\_RT}$	The voltage amplitude of each DN buses (kV)
$C_{g,t}^{UP\_DA}/C_{g,t}^{DN\_DA}$	Up/Down regulation offer of Genco in the DA market ($/MWh)	$v_{e,t}^{ch}/v_{e,t}^{dch}$	Binary variable related to charging/ discharging of ES
$C_{d,t}^{DN\_DA}$	Down regulation offer of responsive TNL in the DA market ($/MWh)	$\theta_{n,t}^{TN\_DA}/\theta_{n,t,\omega}^{TN\_RT}$	Angle of TN bus after DA/RT market clearing (rad)
$C_{g,t}^{UP\_RT}/C_{g,t}^{DN\_RT}$	Up/Down regulation offer of Genco in the RT market ($/MWh)	$\lambda_{n,t}^{TN\_DA}/\lambda_{n,t,\omega}^{TN\_RT}$	DA/RT MCP at TN buses n
$C_{d,t}^{DN\_RT}$	Down regulation offer of responsive TNL in the RT market ($/MWh)	ξ, η_ω	Auxiliary variables used in CVaR calculation
D_t	Duration of time interval t (hour)		
$\underline{E}_e^{ES}, \overline{E}_e^{ES}$	Minimum/Maximum energy stored in ES (MWh)		

1. Introduction

A. Motivation and aim

Low energy efficiency of conventional generation and transmission systems besides environmental issues cause many power system industries to set an ambitious target of Distributed Energy Resources (DERs) [1]. DERs can be managed by aggregators or be operated by Distribution Companies (Discos). Therefore, decision making procedure, in Active Distribution Networks (ADNs), faces restructuring in such a way that the Disco may trade energy with DERs aggregators and participate in the wholesale markets [2]. In this way, Independent System Operator (ISO) clears the market based on the accrued energy transactions in the Day-Ahead (DA) market, where there are various decision-makers including Discos, generation companies (Gencos) [3], wind power producers (WPPs) [4], large consumers, and aggregators. On the other hand, the aforementioned decision-makers need to participate in real-time (RT) market to serve demand with minimum cost.

The players' operation problems involve uncertainties due to uncertain behavior of the power resources and demands. Therefore, some considerations should be made regarding energy balance in both the DA and RT markets.

Not only Disco needs to participate in the DA market, but also should participate in the RT market to deal with the uncertain behavior of Renewable Energy Sources (RESs) and demand. Therefore, development of a comprehensive framework of energy management process seems necessary to facilitate modelling of the Disco behavior in both the DA and RT markets. In such framework, uncertain behavior of the RESs and demand should be modelled using an appropriate risk management index. In this paper an attempt is made to model the decision-making problem of a Disco, considering uncertain behavior of RESs and demand, in both DA and RT energy markets.

B. Literature review and contributions

The decision-making framework of a Disco, as a price-taker player in wholesale market, has been modeled from different points of view. Interaction between the Disco and microgrids (MGs) is modeled using the system of system approach in [5]. The model is deterministic and thus does not take account of demand uncertainty. In [6], operation problem of a Disco in DA market is formulated to determine the decision variables of interest including, power exchange with the DA market, the output power of Distributed Generators (DGs), and the amount of interruptible loads (ILs). A bi-level approach, which considers the profit of a Disco and the operation cost of MGs as the Upper Level (UL) and Lower Level (LL) problems, is introduced in [7]. In [8], interactions between a Disco and DERs are modeled through a bi-level optimization approach in which the contract prices of the DERs are determined in the UL problem and the operation cost of the Disco is minimized in the LL problem. In [9], the operation problem of a Disco in presence of Electric Vehicles (EVs) aggregators is modeled as a risk-based bi-level optimization problem in which the profits of the Disco and EVs aggregators are maximized in the UL and LL problems, respectively. In [10], a metaheuristic-based approach is proposed to deal with the probabilistic operation of a Disco in presence of EVs aggregators and RESs. In [11], bidding strategy of an aggregator of prosumers in the DA and secondary reserve markets is modeled using a stochastic optimization approach. The aggregator tries to optimize the prosumers' flexibility with the aim of minimizing/maximizing the cost of purchased/sold energy. The authors of [12] introduce a model for operation of a Disco in DA and reserve markets. In [13], operation problem of a Disco, in cooperation with MGs, in local energy and reserve markets is modeled as a bi-level approach in which the Disco and MGs are considered as the UL and LL problems, respectively. A two-stage model for a Disco is presented in [14] in which the Disco participates in the energy and reserve markets. A bi-level optimization approach is developed in [15, 16] to achieve the optimal decisions of the

Disco in the presence of DER aggregators. A two-stage optimization approach is introduced in [17, 18] to formulate the behavior of a Disco, as a price-taker player, in the DA and RT markets. In [19], operation problem of a Disco in the DA and RT markets is formulated as a stochastic problem which in turn controls the uncertainties of RT energy prices and demand. A stochastic approach to derive optimal bidding strategies for a wind power producer and energy storage in the Spanish multi-stage market, consisting of DA, intraday, and RT markets, is proposed in [20]. Generally stated, in [6-20], Disco is considered as a price-taker player in the DA and RT markets.

In presence of DERs, Discos further contribute in markets and thus the markets' outcomes may change. Operation problem of a Disco, as a price-maker player in wholesale market, in presence of DERs has been investigated in few studies. The bidding strategy of the Disco in the DA market is formulated as a bi-level optimization model in [21]. In [22], a bi-level approach is introduced to model Disco's behavior in the DA wholesale market. The decision-making problem of a price-maker Disco in the DA energy and reserve markets is formulated using a bi-level approach in [23]. Accordingly, the operation cost of the Disco and the wholesale energy and reserve markets are modeled as the UL and LL problems, respectively. In [16], Disco's profit in the RT market is formulated as a bi-level optimization in which the Disco problem, including DR program, is considered as a one-stage deterministic model. The bidding strategy of the Disco in the DA and RT energy markets is modeled as a bi-level optimization in [15] in which the problem of the Disco and the clearing process of the markets are modeled in the upper- and lower-level problems. Table 1 reviews the decision-making frameworks of the so far researches in the field of operation problem formulation of the Disco. A fast review of the reported models reveals that the models consider Disco as price-taker player in the DA market or as the price-maker player in the both DA and RT markets. Although Ref. [25] mathematically formulated the participation of Disco in the DA and the RT markets,

it fails to model important issues in dealing with the uncertainties as well as the technical constraints of the decision-makers in the markets, including:

- The main advantage of the co-optimization of the Disco's operation problem in the DA and RT markets is to make optimal decisions in the DA, aiming to control the effect of uncertainties, including risk-level of the Disco, on the RT operation. Indeed, the risk-level of the Disco has effects on Disco decisions to participate in the markets and optimally schedule the DERs. However, what is missed is appropriate modeling of the risk-level of Disco through a risk management method.
- Although the Disco is responsible to meet the demand of network, the uncertainty of the demand is not modeled in operation problem of the Disco in the DA and RT markets.
- The ramp-up/down limitations of Gencos – refers as to a single generation sites connected to a single transmission network node - have major impact on the DA and RT market outcomes. However, these constraints are not modeled in the problem of the DA and RT markets. In this way, power generation of Gencos in the DA and the RT markets may be higher than their ramp-rate limitations which in turn leads to unreal market output results.

In response to the above crudities, a risk-based model is proposed to formulate simultaneous participation of a Disco in the DA and RT markets. The proposed model is a two-stage bi-level optimization problem in which the Disco and the markets are considered as the UL and LL problems, respectively. Moreover, the Conditional Value-at-Risk (CVaR) index is used to control the effect of uncertainties on the Disco’s decisions. On the other hand, technical constraints of the Gencos, including ramp-rate limitations, are modeled in the both DA and RT markets. The main contributions of this paper are twofold:

- Modeling the simultaneous participation of Disco in the DA and RT markets using a risk-based approach. This allows to formulate the uncertainties of RESs and demand as well as the risk-level of the Disco in the bidding strategies.
- Modeling ramp-rate limitations of Gencos in both the DA and RT market clearing problems.

Table 1
Comparison between the proposed model in this paper with the previous studies

Ref.	The role of Disco in wholesale markets	Type of markets	Model	UL model	Modeling MG or aggregator	Risk management	Approach	Important findings
[6]	Price-taker	DA wholesale market	Bi-level	Deterministic	MG: DG	No	NLP	Optimal decisions of the Disco to purchase power from the DGs change the traded power with the DA market. This decreases the operation cost.
[8]	Price-taker	DA wholesale and local energy markets	Bi-level	Deterministic	MG: DG, DR, and ES	No	MILP	Two clearing mechanisms, i.e., pay-as-bid and uniform, for local energy market are compared where the Disco and the MGs earn more profit in pay-as-bid and uniform mechanisms, respectively.
[12]	Price-taker	DA and reserve wholesale markets	Single-level	One-stage probabilistic	-	No	NLP	The Disco increases its profit in both the DA and reserve wholesale markets
[15]	Price-taker	RT wholesale and local energy markets	Bi-level	Two-stage stochastic	Aggregator: WT, PV, DG, and ES	No	NLP	Optimal cooperation of the Disco with the different aggregators decreases the operation cost of the Disco in RT market and increases the profit of the aggregators.
[16]	Price-taker	RT wholesale and local energy markets	Bi-level	Deterministic	Aggregator: DR	No	NLP	Optimal cooperation of the Disco with the DR aggregator decreases the operation cost of the Disco in RT market and increases the profit of the aggregator.
[17, 18]	Price-taker	DA and RT wholesale markets	Single-level	Deterministic	-	No	MILP and NLP	The Disco trades energy with the DERs which in turn leads to decreasing of the operation cost.
[19]	Price-taker	DA and RT wholesale markets	Single-level	Two-stage stochastic	-	Yes	MILP and NLP	The Disco manages the risk by purchasing energy from the DA market and optimal

								scheduling of DGs. This increases the total operation cost of the Disco.
[21]	Price-maker	DA wholesale market	Bi-level	Deterministic	-	No	NLP	The Disco decreases the operation cost through optimal scheduling of DGs and ILs
[22]	Price-maker	DA wholesale market	Bi-level	Two-stage stochastic	Aggregator: DG, DR, and ES	Yes	MILP	The Disco decreases the operation cost by participating in the DA market and trading energy with the DGs and ESs.
[23]	Price-maker	DA and reserve wholesale markets	Bi-level	One-stage probabilistic	Aggregator: IL and DG	No	MILP	The Disco participates in the both DA energy and reserve markets through optimal scheduling of DERs. This decrease the operation cost.
[24]	Price-maker	RT wholesale market	Bi-level	Deterministic	-	No	NLP	The Disco employs load shifting to decrease the operation cost.
[25]	Price-maker	DA and RT wholesale markets	Bi-level	Two-stage stochastic	Aggregator: WT and PV	No	NLP	The Disco earn more profit from participating in both the DA and RT markets, considering optimal interactions with RESA.
This paper	Price-maker	DA and RT wholesale markets	Bi-level	Two-stage stochastic	Aggregator: WT, PV, DG, DR, and ES	Yes	MILP	The risk-level of the Disco to manage the uncertainties has significant impact on the bidding/offering strategies in both the DA and RT markets.

C. Paper Organization

The rest of this paper is organized as follows. Section 2 presents the problem description. The problem mathematical formulation is explained in section 3. The numerical results are investigated in section 4 and finally, section 5 concludes the paper.

2. Problem Description

Figs. 1 and 2 show the proposed decision-making framework of the Disco in the DA and RT markets. Indeed, Disco in Fig. 1 interacts with different aggregators, including RES aggregator (RESA), DR aggregator (DRA), DG aggregator (DGA), and Energy Storage (ES) aggregator (ESA). The Disco receives bids/offers and technical data from the aggregators and decides on the optimal scheduling of resources. For this purpose, the Disco needs to map the

reaction of the market results onto the associated bidding/offering strategies. Therefore, the proposed decision-making framework is modeled as a bi-level optimization approach in which the Disco and both the DA and RT markets are modeled as the leader and the follower, respectively as shown in Fig. 2. Therefore, clearing process of the markets is modeled from the Disco point of view to simultaneously determine the strategies in compliance with the markets and the aggregators.

There are several uncertain parameters, including wind speed, solar radiation, and demand, in decision-making formulation of the Disco which could be dealt through several scenarios, as reported in Appendix A. Therefore, problem of Disco operation, as the upper-level problem, is formulated using a two-stage stochastic optimization approach as presented in Fig. 2. In this way, energy trading of the Disco with the DA market, RESA, DRA, ESA, and DGA considers as the first-stage or here-and-now decisions. Interaction of Disco with ESA and trading energy with the RT market, which depend on realization of scenarios, consider as the second-stage or wait-and-see decisions. The logic behind such modeling of aggregators in the problem of Disco operation is described as follows.

Generally, forecast value of RESs is formulated in the first-stage problem and deviation of the output power of RESs, in each scenario, from the scheduled output power of the first-stage is modelled in the second stage problem. In this paper, it is assumed that the DRA can sign contract with the Disco to curtail the loads. For this purpose, a bid-based mechanism for load curtailment is assumed wherein DRA submits the offers, in terms of maximum amount of load curtailment and the associated price, on an hourly basis. The proposed mechanism for the DRA is formulated in the first-stage problem. Disco should provide the maximum amount of the associated load in the DA horizon through purchasing power from the DA market (first-stage problem) and the DGA, considering high dispatchable capacity of the resources. Therefore, interactions between Disco and DGA are modeled in the first-stage problem. On the other hand,

there are two main reasons to model the ESA in the both stage problems: 1) to manage the deviation, caused by the uncertain behavior of RESs and demand, in the power balance constraint of the Disco in the real time, and 2) to allow Disco to participate in the RT market considering RT down and up regulation behavior of the players, i.e., Gencos and responsive TNLs.

In the real markets, the market players, including Gencos and Discos, submit their bids/offers to the DA market. Afterwards, ISO clears the market according to the received bids/offers and technical concepts, aiming to maximize the social welfare. Meanwhile, the market players need to participate in the RT market to control the uncertainties of the power resources/demand. In this paper, the clearing process of the DA and RT markets is modeled as the LL problem as shown in Fig. 2. Of note that, as the behavior of Gencos and TNLs in the markets are not strategic, their offers and bids are only considered in the LL problems.

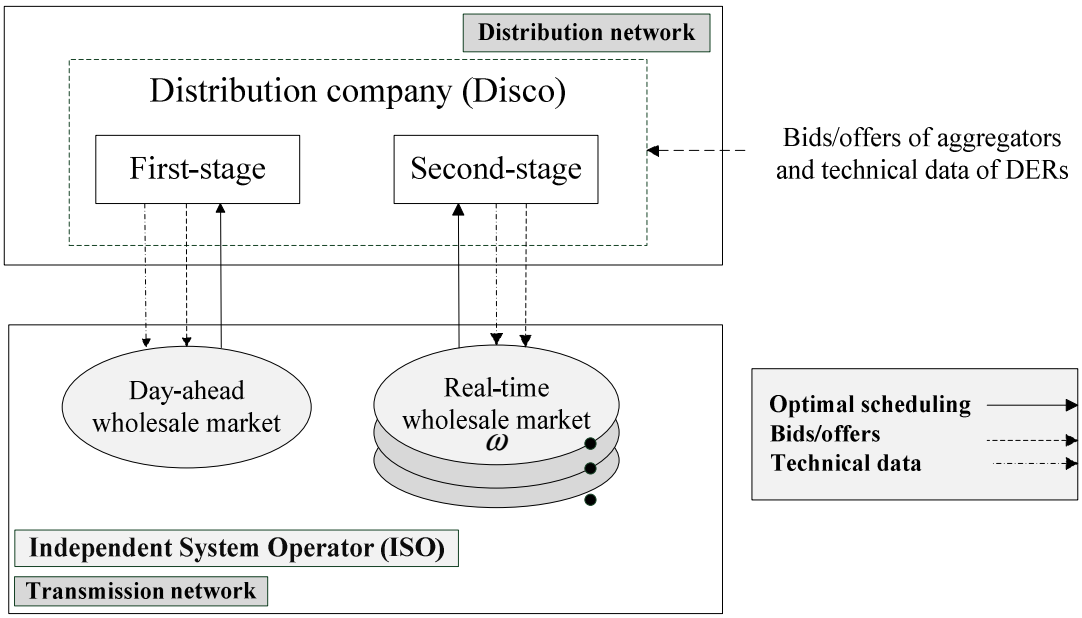

Fig. 1. Proposed decision-making framework of the Disco in the DA and RT markets

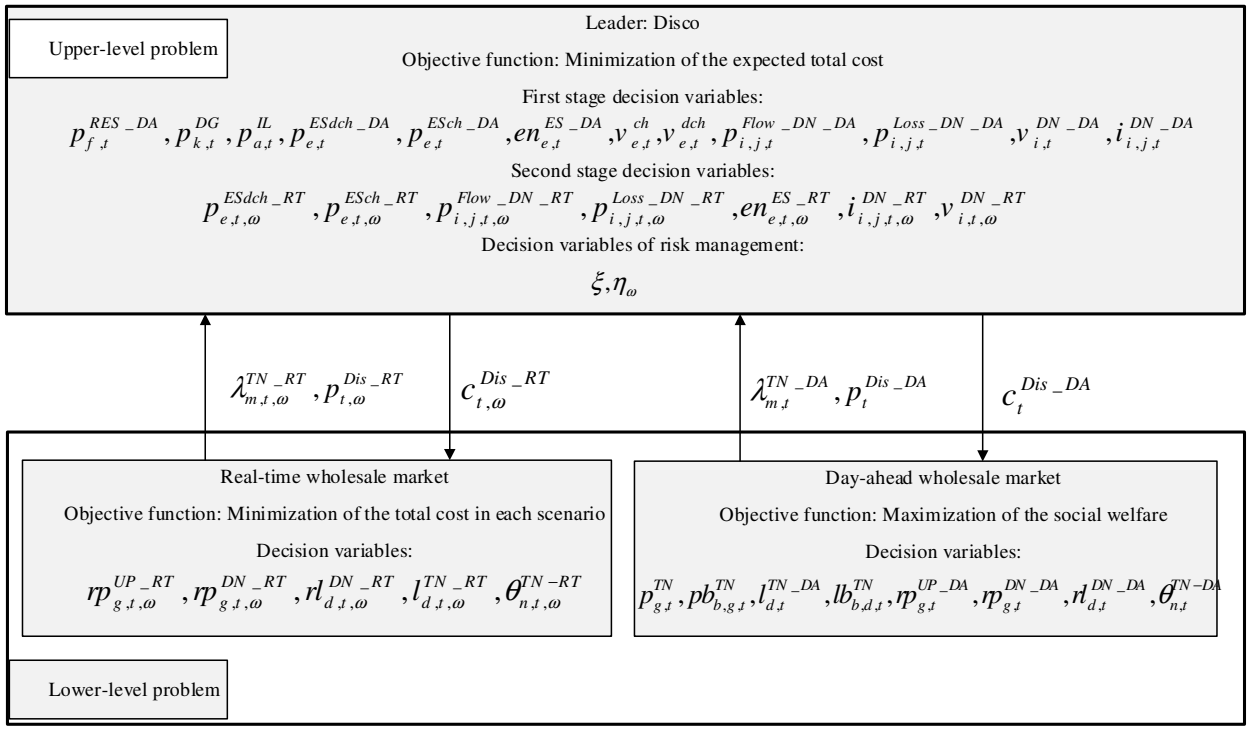

Fig. 2. Risk-based two-stage stochastic bi-level optimization problem

3. Mathematical Modeling

Operation problem of Disco, as a price-maker player in the markets, is formulated as a bi-level optimization problem in this section.

A. Operation problem of the Disco: UL problem

The Disco decision making process, as the UL problem, is formulated as:

B.1. Objective function

The expected total and total costs of the Disco, including DA and RT markets costs, are formulated using Eqs. (1) and (2), respectively.

$$\text{Minimize } ETC = \sum_{\omega=1}^{W} \tau_{\omega} TC_{\omega} \tag{1}$$

$$TC_{\omega} = TC^{DA} + TC_{\omega}^{RT} \tag{2}$$

B.2. First-stage decisions of the Disco:

1) Total cost of the Disco in DA market: The Disco operation cost in the first-stage decision making process (TC^{DA}) is modeled by Eq. (3). It consists of the costs of exchanged power with

DA market, purchased power from RESA, DGA, DRA, and ESA and revenue from power sold to consumers, respectively.

$$TC^{DA} = \sum_{t=1}^{T} \left[\begin{array}{l} \lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA} + \sum_{f=1}^{F} \lambda_t^{RES} p_{f,t}^{RES\_DA} + \sum_{k=1}^{K} \lambda_t^{DG} p_{k,t}^{DG} + \sum_{a=1}^{A} (\lambda_t^{IL}) p_{a,t}^{IL} \\ + \sum_{e=1}^{E} \lambda_t^{ESdch} p_{e,t}^{ESdch\_DA} - \sum_{e=1}^{E} \lambda_t^{ESch} p_{e,t}^{ESch\_DA} - \lambda_t^{DNL} (\sum_{l=1}^{L} P_{l,t}^{DNL\_DA} - \sum_{a=1}^{A} p_{a,t}^{IL}) \end{array} \right] \quad (3)$$

Subject to:

2) DA power balance constraint: The power balance constraints are modeled as:

$$\begin{aligned} &\sum_{f \in M_i^{RES}}^{F} p_{f,t}^{RES\_DA} + \sum_{k \in M_i^{DG}}^{K} p_{k,t}^{DG} + \sum_{a \in M_i^{IL}}^{A} p_{a,t}^{IL} + \sum_{e \in M_i^{ES}}^{E} p_{e,t}^{ESdch\_DA} - \sum_{e \in M_i^{ES}}^{E} p_{e,t}^{ESch\_DA} + \\ &p_t^{Dis\_DA} - \sum_{l \in M_i^{DNL}}^{L} P_{l,t}^{DNL\_DA} = \sum_{j \in Conec(i,j)}^{J} 0.5(p_{i,j,t}^{Flow\_DN\_DA} + p_{i,j,t}^{Loss\_DN\_DA}) \quad : \forall t, i = 1 \end{aligned} \quad (4.a)$$

$$\begin{aligned} &\sum_{f \in M_i^{RES}}^{F} p_{f,t}^{RES\_DA} + \sum_{k \in M_i^{DG}}^{K} p_{k,t}^{DG} + \sum_{a \in M_i^{IL}}^{A} p_{a,t}^{IL} + \sum_{e \in M_i^{ES}}^{E} p_{e,t}^{ESdch\_DA} - \sum_{e \in M_i^{ES}}^{E} p_{e,t}^{ESch\_DA} - \\ &\sum_{l \in M_i^{DNL}}^{L} P_{l,t}^{DNL\_DA} = \sum_{j \in Conec(i,j)}^{J} 0.5(p_{i,j,t}^{Flow\_DN\_DA} + p_{i,j,t}^{Loss\_DN\_DA}) \quad : \forall t, i \neq 1 \end{aligned} \quad (4.b)$$

3) RESA constraint: Eq. (5) stands to limit the purchased power from RESA ($p_{f,t}^{RES\_DA}$).

$$0 \le p_{f,t}^{RES\_DA} \le \overline{P}_f^{RES} \qquad : \forall f, t \quad (5)$$

4) DGA constraints: Eqs. (6) and (7) impose the lower and upper bounds and ramp-rate limitations (RU_k / RD_k) on the power generation of DGs ($p_{k,t}^{DG}$), respectively.

$$\underline{P}_k^{DG} u_{k,t}^{DG} \le p_{k,t}^{DG} \le \overline{P}_k^{DG} u_{k,t}^{DG} \quad : \forall k, t \quad (6)$$

$$RD_k \le p_{k,t}^{DG} - p_{k,t-1}^{DG} \le RU_k \quad : \forall k, t > 1 \;, \quad RD_k \le p_{k,t}^{DG} - p_{k,ini}^{DG} \le RU_k \quad : \forall k, t = 1 \quad (7)$$

5) DRA constraint: Eq. (8) is developed to tackle the upper bound of the load interruption ($p_{a,t}^{IL}$), provided by DRA, into problem formulation.

$$0 \le p_{a,t}^{IL} \le \overline{P}_{a,t}^{IL} \quad : \forall a, t \quad (8)$$

6) *ESA constraints:* Eqs. (9)-(12) are formulated to describe the technical constraints of ESA. While Eq. (9) limits the power charging/discharging ($p_{e,t}^{ESch\_DA} / p_{e,t}^{ESdch\_DA}$) of ESs, the associated binary variables ($v_{e,t}^{ch} / v_{e,t}^{dch}$) are used to control charge/discharge procedures. If $\lambda_t^{ESdch} > \lambda_t^{ESch}$ for all *t*, the optimization problem would automatically set either the charging or discharging

power to 0 for the battery. However, if $\lambda_t^{ESdch} < \lambda_t^{ESch}$ for a typical time-step, the Disco can charge the ES with a high price and, at the same time, discharge it with low price to earn the profit. Since occurring such case is not possible in the real operation, these binary variables are used in equation 9. Eq. (10) limits the stored energy ($en_{e,t}^{ES\_DA}$) in ESs. Eqs. (11) and (12) explain the dynamic behavior of stored energy in ESs.

$$0 \le p_{e,t}^{ESch\_DA} \le \overline{P}_e^{ch} v_{e,t}^{ch} \;\; , \;\; 0 \le p_{e,t}^{ESdch\_DA} \le \overline{P}_e^{dch} v_{e,t}^{dch} \;\; , \;\; v_{e,t}^{ch} + v_{e,t}^{dch} \le 1 \quad : \forall e,t \tag{9}$$

$$\underline{E}_e^{ES} \le en_{e,t}^{ES\_DA} \le \overline{E}_e^{ES} \qquad : \forall e,t, \tag{10}$$

$$en_{e,t}^{ES\_DA} = en_{e,t-1}^{ES\_DA} + \left(p_{e,t}^{ESch\_DA}\eta_{ch}\right) - \left(p_{e,t}^{ESdch\_DA} / \eta_{dch}\right) \qquad \forall e,t > 1 \tag{11}$$

$$en_{e,t}^{ES\_DA} = E_{e,ini}^{ES} + \left(p_{e,t}^{ESch\_DA}\eta_{ch}\right) - \left(p_{e,t}^{ESdch\_DA} / \eta_{dch}\right) \qquad \forall e,t = 1 \tag{12}$$

6) *DN technical constraints:* Technical constraints of distribution network are presented in Eqs. (13)-(16). Feeder's current and the associated upper and lower bounds are defined by (13) and (14), respectively. Moreover, the limitations of bus voltages are represented in (14). Eqs. (15) and (16) are used to model active power flow in DN feeders ($p_{i,j,t}^{Flow\_DN\_DA}$) and active power losses ($p_{i,j,t}^{Loss\_DN\_DA}$) of the network. In (15) and (16), $p_{i,j,t}^{Fm\_DA}$ refers to as the active power flows from bus *i* to bus *j* and $p_{i,j,t}^{To\_DA}$ refers to as the active power flows from bus *j* to bus *i*. Eq. (16) calculates the power losses in each feeder (if: $i_{i,j,t}^{DN\_DA}$ or $i_{j,i,t}^{DN\_DA} \geq 0$; otherwise returns 0). Of note that the non-linear terms of ($\left(v_{i,t}^{DN\_DA}\right)^2$ and $\left(i_{i,j,t}^{DN\_DA}\right)^2$) are linearized using the piecewise liner technique of [8].

$$i_{i,j,t}^{DN\_DA} = \frac{v_{i,t}^{DN\_DA} - v_{j,t}^{DN\_DA}}{Z_{i,j}^{DN}} \quad \forall i,j,t \tag{13}$$

$$-\overline{I}_{i,j}^{DN} \le i_{i,j,t}^{DN\_DA} \le \overline{I}_{i,j}^{DN} \quad \forall i,j,t \;\; , \;\; \underline{V}_i^{DN} \le v_{i,t}^{DN\_DA} \le \overline{V}_i^{DN} \quad \forall i,t \tag{14}$$

$$p_{i,j,t}^{Fm\_DA} - p_{i,j,t}^{To\_DA} = \left(\frac{R_{i,j}^{DN}}{\left(Z_{i,j}^{DN}\right)^2}\right)\left(\left(v_{i,t}^{DN\_DA}\right)^2 - \left(v_{j,t}^{DN\_DA}\right)^2\right) \quad \forall i,j,t \tag{15}$$

$$p_{i,j,t}^{Fm\_DA} + p_{i,j,t}^{To\_DA} = R_{i,j}^{DN}\left(i_{i,j,t}^{DN\_DA}\right)^2 \quad \forall i,j,t \tag{16}$$

B.3. Second-stage decisions of the Disco:

1) Total cost of the Disco in RT market: The Disco operation cost in the second-stage decision making process (TC_{ω}^{RT}) is modeled by Eq. (17). It consists of cost of exchanged power with the RT market, costs/revenues due to power deviation of the RESA ($P_{f,t,\omega}^{RES\_RT} - p_{f,t}^{RES\_DA}$) and consumers ($P_{t,\omega}^{DNL\_RT} - P_{t}^{DNL\_DA}$), and financial trading with ESA.

$$TC_{\omega}^{RT} = \sum_{t=1}^{T} \left[\begin{array}{l} \lambda_{m,t,\omega}^{TN\_RT} p_{t,\omega}^{Dis\_RT} + \sum_{f=1}^{F} \lambda_{t}^{RES} \left(P_{f,t,\omega}^{RES\_RT} - p_{f,t}^{RES\_DA} \right) + \sum_{e=1}^{E} \lambda_{t}^{ESdch} p_{e,t,\omega}^{ESdch\_RT} \\ - \sum_{e=1}^{E} \lambda_{t}^{ESch} p_{e,t,\omega}^{ESch\_RT} - \sum_{l=1}^{L} \lambda_{t}^{DNL} (P_{l,t,\omega}^{DNL\_RT} - P_{l,t}^{DNL\_DA}) \end{array} \right] \quad (17)$$

Subject to:

2) RT power balance constraint: Eq. (18) refers to as RT power balance constraint of DN buses.

$$\begin{aligned} &\sum_{f \in M_i^{RES}}^{F} \left(P_{f,t,\omega}^{RES\_RT} - p_{f,t}^{RES\_DA} \right) + \sum_{e \in M_i^{ES}}^{E} p_{e,t,\omega}^{ESdch\_RT} - \sum_{e \in M_i^{ES}}^{E} p_{e,t,\omega}^{ESch\_RT} + p_{t,\omega}^{Dis\_RT} - \\ &\sum_{l \in M_i^{L}}^{L} (P_{l,t,\omega}^{DNL\_RT} - P_{l,t}^{DNL\_DA}) = \sum_{j \in Conec(i,j)}^{J} 0.5 (p_{i,j,t,\omega}^{Flow\_DN\_RT} + (p_{i,j,t,\omega}^{Loss\_DN\_RT} - p_{i,j,t}^{Loss\_DN\_DA})) \quad : \forall t, \omega, i = 1 \end{aligned} \quad (18.a)$$

$$\begin{aligned} &\sum_{f \in M_i^{RES}}^{F} \left(P_{f,t,\omega}^{RES\_RT} - p_{f,t}^{RES\_DA} \right) + \sum_{e \in M_i^{ES}}^{E} p_{e,t,\omega}^{ESdch\_RT} - \sum_{e \in M_i^{ES}}^{E} p_{e,t,\omega}^{ESch\_RT} - \\ &\sum_{l \in M_i^{L}}^{L} (P_{l,t,\omega}^{DNL\_RT} - P_{l,t}^{DNL\_DA}) = \sum_{j \in Conec(i,j)}^{J} 0.5 (p_{i,j,t,\omega}^{Flow\_DN\_RT} + (p_{i,j,t,\omega}^{Loss\_DN\_RT} - p_{i,j,t}^{Loss\_DN\_DA})) \quad : \forall t, \omega, i \neq 1 \end{aligned} \quad (18.b)$$

3) Exchanged power of Disco with the markets: Eq. (19) imposes the upper and lower bounds on the summation of exchanged power between the Disco and the DA and RT markets.

$$\underline{P}^{Dis\_TN} \leq p_{t}^{Dis\_DA} + p_{t,\omega}^{Dis\_RT} \leq \overline{P}^{Dis-TN} \quad : \forall t, \omega \quad (19)$$

4) ESA constraints: Eqs. (20)-(23) are formulated to describe the technical constraints of ESA. Eq. (20) limits the charging/discharging ($p_{e,t,\omega}^{ESch\_RT}/p_{e,t,\omega}^{ESdch\_RT}$) of ESs. Eq. (21) limits the amount of stored energy ($en_{e,t,\omega}^{ES\_RT}$) in ESs. Eqs. (22) and (23) represent dynamic behavior of stored energy in ESs. Of note that, as the ES will be reset at the beginning of the next day, SOCs of each ES at hours 1 and 24 are same.

$$0 \leq p_{e,t}^{ESch\_DA} + p_{e,t,\omega}^{ESch\_RT} \leq \overline{P}_{e}^{ch} \quad , \quad 0 \leq p_{e,t}^{ESdch\_DA} + p_{e,t,\omega}^{ESdch\_RT} \leq \overline{P}_{e}^{dch} \quad : \forall e, t, \omega \quad (20)$$

$$\underline{E}_e^{ES} \le en_{e,t}^{ES\_DA} + en_{e,t,\omega}^{ES\_RT} \le \overline{E}_e^{ES} \quad : \forall e,t,\omega \tag{21}$$

$$en_{e,t,\omega}^{ES\_RT} = en_{e,t-1,\omega}^{ES\_RT} + \left(p_{e,t,\omega}^{ESch\_RT}\eta_{ch}\right) - \left(p_{e,t,\omega}^{ESdch\_RT}/\eta_{dch}\right) \quad \forall e,t>1,\omega \tag{22}$$

$$en_{e,t,\omega}^{ES\_RT} = \left(p_{e,t,\omega}^{ESch\_RT}\eta_{ch}\right) - \left(p_{e,t,\omega}^{ESdch\_RT}/\eta_{dch}\right) \quad \forall e,t=1,\omega \tag{23}$$

5) DN technical constraints: Eqs. (24)-(27) describe the power flow constraints in the second-stage decision-making process, according to the power flow results of the first-stage decision-making.

$$i_{i,j,t,\omega}^{DN\_RT} = \frac{v_{i,t,\omega}^{DN\_RT} - v_{j,t,\omega}^{DN\_RT}}{Z_{i,j}^{DN}} \quad \forall i,j,t,\omega \tag{24}$$

$$-\overline{I}_{i,j}^{DN} \le i_{i,j,t,\omega}^{DN\_RT} \le \overline{I}_{i,j}^{DN} \quad \forall i,j,t,\omega \ , \ \underline{V}_i^{DN} \le v_{i,t,\omega}^{DN\_RT} \le \overline{V}_i^{DN} \quad \forall i,t,\omega \tag{25}$$

$$p_{i,j,t,\omega}^{Fm\_RT} - p_{i,j,t,\omega}^{To\_RT} = \left(\frac{R_{i,j}^{DN}}{\left(Z_{i,j}^{DN}\right)^2}\right)\left(\left(v_{i,t,\omega}^{DN\_RT} - v_{i,t}^{DN\_DA}\right)^2 - \left(v_{j,t,\omega}^{DN\_RT} - v_{j,t}^{DN\_DA}\right)^2\right) \quad \forall i,j,t,\omega \tag{26}$$

$$p_{i,j,t,\omega}^{Fm\_RT} + p_{i,j,t,\omega}^{To\_RT} = R_{i,j}^{DN}\left(i_{i,j,t,\omega}^{DN\_RT}\right)^2 \quad \forall i,j,t,\omega \tag{27}$$

The decision variables of the UL problem is described as $\mathrm{X}_{\mathrm{UL}}^{\mathrm{Dis}} = \{c_t^{Dis\_DA}, c_{t,\omega}^{Dis\_RT}, p_{f,t}^{RES\_DA}, p_{k,t}^{DG}, p_t^{IL}, p_{e,t}^{ESch\_DA}, p_{e,t,\omega}^{ESch\_RT}, p_{e,t,\omega}^{ESdch\_RT}, p_{e,t}^{ESdch\_DA}, v_{e,t}^{ch}, v_{e,t}^{dch}, en_{e,t}^{ES\_DA}, en_{e,t,\omega}^{ES\_RT}\}$.

B.4. Risk management

CVaR approach of [19] is used to manage risk of Disco. The value of CVaR at the confidence level of α can be defined as the expected cost in the $(1-\alpha)\times 100$ percent of the worst scenarios, as [24]:

$$\text{Minimize}_{\xi,\eta_\omega} \ CVaR = \xi + \frac{1}{1-\alpha}\sum_{\omega=1}^{W}\tau_\omega \eta_\omega \tag{28}$$

$$TC_\omega - \xi - \eta_\omega \le 0 \tag{29}$$

$$\eta_\omega \ge 0 \tag{30}$$

Therefore, the final objective function of the UL problem is formulated as:

$$\text{Minimize} \quad ETC + \beta CVaR \tag{31}$$

where β represents the risk-aversion parameter. When β is zero, the Disco is a risk-neutral decision maker. The Disco becomes more risk-averse as β increases.

B. *DA market problem formulation: LL problem*

The DA market problem, as the LL problem, formulates as:

1) Objective function: DA market clearing problem formulates based on the offer of Gencos, the bid of the transmission network load (TNL), the bid/offer of the Disco, the regulation reserve offer of Gencos and responsive TNLs, as:

$$\text{Minimize } \sum_{t=1}^{T} D_t \left[\begin{array}{l} \sum_{g=1}^{G}\sum_{b=1}^{B}(C_{b,g,t}^{TN} pb_{b,g,t}^{TN}) - \sum_{d=1}^{D}\sum_{b=1}^{B}(C_{b,d,t}^{TN\_DA} lb_{b,d,t}^{TN}) - c_t^{Dis\_DA} p_t^{Dis\_DA} + \\ \sum_{g=1}^{G}\left(C_{g,t}^{UP\_DA} rp_{g,t}^{UP\_DA} + C_{g,t}^{DN\_DA} rp_{g,t}^{DN\_DA}\right) + \sum_{d=1}^{D} C_{d,t}^{DN\_DA} rl_{d,t}^{DN\_DA} \end{array} \right] \quad (32)$$

Subject to:

2) Power balance constraints: Eqs. (33) and (34) stand to satisfy the power balance constraint in DN and TN buses, respectively.

$$\sum_{g\in M_n^G} p_{g,t}^{TN} - p_t^{Dis\_DA} = \sum_{r\in\Lambda_n^{TN}} B_{n-r}\left(\theta_{n,t}^{TN\_DA} - \theta_{r,t}^{TN\_DA}\right) : \lambda_{n,t}^{TN\_DA} \qquad \forall n = m, t \quad (33)$$

$$\sum_{g\in M_n^G} p_{g,t}^{TN} - \sum_{d\in M_n^D} l_{d,t}^{TN\_DA} = \sum_{r\in\Lambda_n^{TN}} B_{n-r}\left(\theta_{n,t}^{TN\_DA} - \theta_{r,t}^{TN\_DA}\right) : \lambda_{n,t}^{TN\_DA} \qquad \forall n \neq m, t \quad (34)$$

3) Constraint of power trading with DA market: Eq. (35) limits the transferred power (p_t^{Dis-DA}) between the Disco and the market.

$$\underline{P}^{Dis\_TN} \le p_t^{Dis\_DA} \le \overline{P}^{Dis\_TN} : \underline{\mu}_t^{1\_DA}, \overline{\mu}_t^{1\_DA} \qquad \forall t \quad (35)$$

4) Gencos constraints: Eqs. (36) and (37) represent the limitations of the power generation ($p_{g,t}^{TN}$) by Gencos and the DA maximum up/down regulation power ($rp_{g,t}^{UP\_DA}/rp_{g,t}^{DN\_DA}$), respectively. Eqs. (38) and (39) model ramp-up (RU_g) and ramp-down (RD_g) limits for the Gencos. Eq. (40) establishes the maximum number of energy blocks ($pb_{b,g,t}^{TN}$). Eq. (41) indicates that summation of energy blocks, for a typical Genco, is equal to the total output power.

$$0 \le p_{g,t}^{TN} + rp_{g,t}^{UP\_DA} \le \overline{P_g} : \underline{\mu}_{g,t}^{2\_DA}, \overline{\mu}_{g,t}^{2\_DA} \quad , \quad 0 \le p_{g,t}^{TN} - rp_{g,t}^{DN\_DA} : \mu_{g,t}^{3\_DA} \qquad \forall g, t \quad (36)$$

$$0 \le rp_{g,t}^{UP\_DA} \le \overline{RG}_g^{UP} : \underline{\mu}_{g,t}^{4\_DA}, \overline{\mu}_{g,t}^{4\_DA} \quad , \quad 0 \le rp_{g,t}^{DN\_DA} \le \overline{RG}_g^{DN} : \underline{\mu}_{g,t}^{5\_DA}, \overline{\mu}_{g,t}^{5\_DA} \qquad \forall g, t \quad (37)$$

$$p_{g,t-1}^{TN} - p_{g,t}^{TN} \le RD_g : \mu_{g,t}^{9\_DA} \qquad \forall g, t > 1 \quad , \quad P_{g,ini}^{TN} - p_{g,t}^{TN} \le RD_g : \mu_{g,t}^{10\_DA} \qquad \forall g, t = 1 \quad (38)$$

$$p_{g,t}^{TN} - p_{g,t-1}^{TN} \le RU_g : \mu_{g,t}^{11\_DA} \qquad \forall g, t > 1 \quad , \quad p_{gt}^{TN} - P_{g,ini}^{TN} \le RU_g : \mu_{g,t}^{12\_DA} \qquad \forall g, t = 1 \quad (39)$$

$$0 \le pb_{b,g,t}^{TN} \le \overline{PB}_{b,g,t}^{TN} : \underline{\mu}_{b,g,t}^{13\_DA}, \overline{\mu}_{b,gt}^{13\_DA} \qquad \forall b, g, t \quad (40)$$

$$p_{g,t}^{TN} = \sum_{b=1}^{B} pb_{b,g,t}^{TN} \quad :\lambda_{g,t}^{1\_DA} \qquad \forall g \ ,t \tag{41}$$

5) TNLs constraints: Eq. (42) limits the TNLs consumption ($l_{d,t}^{TN\_DA}$) in the DA. Eq. (44) imposes the limitation on the DA maximum down regulation power ($rl_{d,t}^{DN\_DA}$) for each responsive TNL. Eq. (43) stands to link Eqs. (42) to (44). It reveals that if the purchased power by the TNLs becomes zero in response to price elasticity, DA maximum down regulation of the responsive TNLs should be zero. Eq. (45) imposes the upper bound on number of energy blocks related to TNLs consumption ($lb_{b,d,t}^{TN}$). Eq. (46) indicates that summation of energy blocks, related to the TNL, is equal to the total TNL consumption.

$$0 \le l_{d,t}^{TN\_DA} \le \overline{L}_{d,t}^{TN} \quad :\underline{\mu}_{d,t}^{6\_DA} \ ,\overline{\mu}_{d,t}^{6\_DA} \quad \forall d \ ,t \tag{42}$$

$$0 \le l_{d,t}^{TN\_DA} - rl_{d,t}^{DN} \quad :\mu_{d,t}^{7\_DA} \qquad \forall d \ ,t \tag{43}$$

$$0 \le rl_{d,t}^{DN\_DA} \le \overline{RL}_{d}^{DN} \ : \ \underline{\mu}_{d,t}^{8\_DA} \ ,\overline{\mu}_{d,t}^{8\_DA} \qquad \forall d \ ,t \tag{44}$$

$$0 \le lb_{b,d,t}^{TN} \le \overline{LB}_{b,d,t}^{TN} \quad :\underline{\mu}_{b,d,t}^{14\_DA} \ ,\overline{\mu}_{b,d,t}^{14\_DA} \qquad \forall b,d \ ,t \tag{45}$$

$$l_{d,t}^{TN\_DA} = \sum_{b=1}^{B} lb_{b,d,t}^{TN} \quad :\lambda_{d,t}^{2\_DA} \qquad \forall d \ \ ,t \tag{46}$$

6) TN power flow constraint: Eq. (47) shows the capacity limitation ($\overline{F}_{n-r}^{TN}$) of TN line, specifies with sending end of n and receiving end of r.

$$-\overline{F}_{n-r}^{TN} \le B_{n-r}\left(\theta_{n,t}^{TN\_DA} - \theta_{r,t}^{TN\_DA}\right) \le \overline{F}_{n-r}^{TN} \quad :\underline{\mu}_{n,r,t}^{15\_DA} \ ,\overline{\mu}_{n,r,t}^{15\_DA} \qquad \forall n \ ,r \in \Lambda_{n}^{TN} \ ,t \tag{47}$$

7) TN voltage angle constraints: While Eq. (49) defines TN bus as the reference, Eq. (48) imposes the bound on DA TN voltage angle ($\theta_{n,t}^{TN-DA}$).

$$-\pi / 2 \le \theta_{n,t}^{TN\_DA} \le \pi / 2 \ :\underline{\mu}_{n,t}^{16\_DA} \ ,\overline{\mu}_{n,t}^{16\_DA} \qquad \forall n \ \ ,t \tag{48}$$

$$\theta_{n,t|n=slack}^{TN\_DA} = 0 \quad :\lambda_{n,t|n=slack}^{5\_DA} \qquad \forall t \tag{49}$$

8) Maximum up/down regulation power constraints: Eqs. (50) and (51) show that summation of the DA maximum up/down regulation power, provided by each Genco and responsive TNL, is equal to the total up/down regulation power capacity ($P_t^{RTC\_UP}/P_t^{RTC\_DN}$).

$$\sum_{g=1}^{G} rp_{g,t}^{UP\_DA} + \sum_{d=1}^{D} rl_{d,t}^{DN\_DA} = P_t^{RTC\_UP} \ :\lambda_t^{3\_DA} \qquad \forall t \tag{50}$$

$$\sum_{g=1}^{G} rp_{g,t}^{DN\_DA} = P_t^{RTC\_DN} \quad :\lambda_t^{4\_DA} \qquad \forall t \tag{51}$$

C. RT market problem formulation: LL problem

The RT market problem formulation is described as:

1) Objective function of RT market problem: Objective function of the problem, i.e. Eq. (52), is to minimize the total operation cost in each scenario. The objective function consists of the regulation costs of Gencos and responsive TNLs, the revenue from selling energy to RT TNLs, and the cost/revenue from trading energy with the Disco (non-negative $p_{t,\omega}^{Dis\_RT}$ refers to bid and negative one stands for offer).

$$\text{Minimize} \sum_{t=1}^{T} d_t \left[\begin{array}{l} \sum_{g=1}^{G} \left(C_{g,t}^{UP\_RT}\, rp_{g,t,\omega}^{UP\_RT} - C_{g,t}^{DN\_RT}\, rp_{g,t,\omega}^{DN\_RT} \right) + \\ \sum_{d=1}^{D} \left(C_{d,t}^{DN\_RT}\, rl_{d,t,\omega}^{DN\_RT} - C_{d,t}^{TN\_RT}\, l_{d,t,\omega}^{TN\_RT} \right) - c_{t,\omega}^{Dis\_RT}\, p_{t,\omega}^{Dis\_RT} \end{array} \right] \tag{52}$$

Subject to:

2) Power balance constraint in RT market: Eqs. (53) and (54) satisfy the power balance constraints in DN and TN buses for each scenario, respectively.

$$\sum_{g \in M_n^G} \left(rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) - p_t^{Dis\_RT} = \sum_{r \in \Lambda_n^{TN}} B_{m-r} \left(\theta_{m,t,\omega}^{TN\_RT} - \theta_{m,t}^{TN\_DA} - \theta_{r,t,\omega}^{TN\_RT} + \theta_{r,t}^{TN\_DA} \right) : \lambda_{n,t,\omega}^{TN\_RT} \quad \forall n = m, t, \omega \tag{53}$$

$$\sum_{g \in M_n^G} \left(rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) - \sum_{d \in M_n^D} \left(l_{d,t,\omega}^{TN\_RT} - rl_{d,t,\omega}^{DN\_RT} \right) = \sum_{r \in \Lambda_n^{TN}} B_{n-r} \left(\theta_{n,t,\omega}^{TN\_RT} - \theta_{n,t}^{TN\_DA} - \theta_{r,t,\omega}^{TN\_RT} + \theta_{r,t}^{TN\_DA} \right) \quad : \lambda_{n,t}^{TN\_RT} \quad \forall n \neq m, t, \omega \tag{54}$$

3) Traded power of the Disco with RT market constraint: Eq. (55) limits the exchanged power ($p_{t,\omega}^{Dis\_RT}$) between the Disco and RT market.

$$\underline{P}^{Dis\_TN} \le p_{t,\omega}^{Dis\_RT} \le \overline{P}^{Dis\_TN} \quad : \underline{\mu}_{t,\omega}^{1\_RT}, \overline{\mu}_{t,\omega}^{1\_RT} \qquad \forall t, \omega \tag{55}$$

4) Gencos constraints: Eq. (56) reveals that RT up/down regulation power of each Genco ($rp_{g,t,\omega}^{UP\_RT}/rp_{g,t,\omega}^{DN\_RT}$) should not exceed the associated DA maximum up/down regulation power. Eqs. (57)-(60) limit the power generation of Gencos in RT market considering the ramp-rate limitations (RU_g/RD_g).

$$0 \le rp_{g,t,\omega}^{UP\_RT} \le rp_{g,t}^{UP\_DA} \quad : \underline{\mu}_{g,t,\omega}^{2\_RT}, \overline{\mu}_{g,t,\omega}^{2\_RT} \quad , \quad 0 \le rp_{g,t,\omega}^{DN\_RT} \le rp_{g,t}^{DN\_DA} \quad : \underline{\mu}_{g,t,\omega}^{3\_RT}, \overline{\mu}_{g,t,\omega}^{3\_RT} \qquad \forall g, t, \omega \tag{56}$$

$$\left(p_{g,t-1}^{TN}+rp_{g,t-1,\omega}^{UP\_RT}-rp_{g,t-1,\omega}^{DN\_RT}\right)-\left(p_{g,t}^{TN}+rp_{g,t,\omega}^{UP\_RT}-rp_{g,t,\omega}^{DN\_RT}\right)\le RD_g \quad :\mu_{g,t,\omega}^{4\_RT} \quad \forall g,t>1 \tag{57}$$

$$P_{g,ini}^{TN}-\left(p_{g,t}^{TN}+rp_{g,t,\omega}^{UP\_RT}-rp_{g,t,\omega}^{DN\_RT}\right)\le RD_g \quad :\mu_{g,t,\omega}^{5\_RT} \quad \forall g,t=1 \tag{58}$$

$$\left(p_{g,t}^{TN}+rp_{g,t,\omega}^{UP\_RT}-rp_{g,t,\omega}^{DN\_RT}\right)-\left(p_{g,t-1}^{TN}+rg_{g,t-1,\omega}^{UP\_RT}-rg_{g,t-1,\omega}^{DN\_RT}\right)\le RU_g \quad :\mu_{g,t,\omega}^{6\_RT} \quad \forall g,t>1 \tag{59}$$

$$\left(p_{g,t}^{TN}+rp_{g,t,\omega}^{UP\_RT}-rp_{g,t,\omega}^{DN\_RT}\right)-P_{g,ini}^{TN}\le RU_g \quad :\mu_{g,t,\omega}^{7\_RT} \quad \forall g,t=1 \tag{60}$$

5) TNLs constraints: Eq. (61) limits RT down regulation power ($rl_{d,t,\omega}^{DN\_RT}$) of responsive loads according to the DA maximum down regulation power. Eq. (62) limits the TNLs consumption ($l_{d,t,\omega}^{TN\_RT}$) in RT market.

$$0\le rl_{d,t,\omega}^{DN\_RT}\le rl_{g,t}^{DN\_DA} \quad :\underline{\mu}_{d,t,\omega}^{8\_RT},\overline{\mu}_{d,t,\omega}^{8\_RT} \quad \forall d,t,\omega \tag{61}$$

$$0\le l_{d,t,\omega}^{TN\_RT}\le \overline{L}_{d,t}^{TN}-l_{d,t}^{TN\_DA} :\underline{\mu}_{d,t,\omega}^{9\_RT},\overline{\mu}_{d,t,\omega}^{9\_RT} \quad \forall d,t,\omega \tag{62}$$

6) RT power flow constraints: Eq. (63) represents the capacity limitation ($\overline{F}_{n-r}^{TN}$) of TN line, specifies with sending end of n and receiving end of r.

$$-\overline{F}_{n-r}^{TN}\le B_{n-r}\left(\theta_{n,t,\omega}^{TN\_RT}-\theta_{r,t,\omega}^{TN\_RT}\right)\le\overline{F}_{n-r}^{TN} \quad :\underline{\mu}_{n,r,t,\omega}^{10\_RT},\overline{\mu}_{n,r,t,\omega}^{10\_RT} \quad \forall n,r\in\Lambda_n^{TN},t,\omega \tag{63}$$

7) TN voltage angle constraints: Eqs. (64) and (65) impose bounds on RT TN voltage angle ($\theta_{n,t,\omega}^{TN\_RT}$) in bus *n*.

$$-\frac{\pi}{2}\le\theta_{n,t,\omega}^{TN\_RT}\le\frac{\pi}{2} :\underline{\mu}_{n,t,\omega}^{11\_RT},\overline{\mu}_{n,t,\omega}^{11\_RT} \quad \forall n,t,\omega \tag{64}$$

$$\theta_{n,t,\omega}^{TN\_RT}=0 :\lambda_{n,t,\omega}^{1\_RT} \quad \forall n=slack,t,\omega \tag{65}$$

Eqs. (32)-(51) and (52)-(65) are developed to model the DA and RT market problems, respectively. Of note that, the dual variables of the aforementioned constraints are represented at the right hand side of the equations. The decision variables of the DA and RT problems are $\mathrm{X}_{\mathrm{LL}}^{\mathrm{DA}}=\{p_{g,t}^{TN},l_{d,t}^{TN\_DA},pb_{b,g,t}^{TN},lb_{b,d,t}^{TN},rp_{g,t}^{UP\_DA},rp_{g,t}^{DN\_DA},rl_{d,t}^{DN\_DA}\,p_t^{Dis\_DA},\theta_{n,t}^{TN\_DA}\}$ and $\mathrm{X}_{\mathrm{LL}}^{\mathrm{RT}}=\{rp_{g,t,\omega}^{UP\_RT},rp_{g,t,\omega}^{DN\_RT},rl_{d,t,\omega}^{DN\_RT},l_{d,t,\omega}^{TN\_RT},p_{t,\omega}^{Dis\_RT},\theta_{n,t,\omega}^{TN\_RT}\}$, respectively. In the proposed two-stage stochastic problem, the decisions of the Disco are divided into two parts: 1) before, and 2) after occurring of the scenarios [15, 24]. In this way, the first part decisions are considered as the parameters in the second-stage problem. This means that the first-stage decision variables of

the Disco, including the DA maximum up/down regulation power, power generation of Gencos, DA TNL consumption, and DA TN voltage angle, are considered as the parameters in the second-stage problem, i.e. RT market problem.

D. Mathematical program with equilibrium constraints

The proposed model is a non-linear bi-level problem. One of the appropriate and common solution to deal with the bi-level problem is to replace the LL problem with the KKT conditions [25, 26]. In the bi-level problems, the decision variables of the UL problem, i.e. bids/offers of the Disco to the DA and RT markets, are considered as the parameters of the LL problems. Therefore, the LL problems are linear continuous and convex, and thus could be replaced by the KKT conditions, as described in Appendix B.

Cost of the exchanged power between the Disco and DA market ($\lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA}$) is linearized using the approach of Appendix C. Also of interest, cost/revenue of exchanged power between the Disco and RT market ($\lambda_{m,t,\omega}^{TN\_RT} p_{t,\omega}^{Dis\_RT}$) is linearized using McCormick's relaxation method of Appendix D. Accordingly, the final model of Disco operation in DA and RT markets is formulated as:

$$\text{Minimize} \sum_{\omega=1}^{W}\sum_{t=1}^{T}\left[\begin{array}{l}\left[\begin{array}{l}\sum_{g=1}^{G}\sum_{b=1}^{B}(C_{b,g,t}^{TN} pb_{b,g,t}^{TN}) - \sum_{d=1}^{D}\sum_{b=1}^{B}(C_{b,d,t}^{TN\_DA} lb_{b,d,t}^{TN}) + \\ \sum_{g=1}^{G}\left(C_{g,t}^{UP\_DA} rp_{g,t}^{UP\_DA} + C_{g,t}^{DN\_DA} rp_{g,t}^{DN\_DA}\right) + \sum_{d=1}^{D} C_{d,t}^{DN\_DA} rl_{d,t}^{DN\_DA}\end{array}\right] - [S] + \underline{\mu}_t^{I\_DA}\underline{P}^{Dis\_TN} - \bar{\mu}_t^{I\_DA}\bar{P}^{Dis\_TN} + \\ \left[\begin{array}{l}\sum_{f=1}^{F}\lambda_t^{RES} p_{f,t}^{RES\_DA} + \sum_{k=1}^{K}\lambda_t^{DG} p_{k,t}^{DG} + \sum_{r=1}^{R}\lambda_t^{IL} p_{r,t}^{IL} - \lambda_t^{DNL}(P_t^{DNL\_DA} - p_{r,t}^{IL}) + \\ \sum_{e=1}^{E}\lambda_t^{ESdch} p_{e,t}^{ESdch\_DA} - \sum_{e=1}^{E}\lambda_t^{ESch} p_{e,t}^{ESch\_DA}\end{array}\right] + \\ \left[\begin{array}{l}(v_{t,\omega}^{in} - v_{t,\omega}^{out}) + \sum_{f=1}^{F}\lambda_t^{RES}\left(P_{f,t,\omega}^{RES\_RT} - p_{f,t}^{RES\_DA}\right) + \sum_{e=1}^{E}\lambda_t^{ESdch} p_{e,t,\omega}^{ESdch\_RT} \\ -\sum_{e=1}^{E}\lambda_t^{ESch} p_{e,t,\omega}^{ESch\_RT} - \lambda_t^{DNL}(P_{t,\omega}^{DNL-RT} - P_t^{DNL-DA})\end{array}\right]\end{array}\right] + \beta CVaR \quad (66)$$

Subject to:

Eqs. (4)-(16), (18)-(27), and (28)-(30).

Eqs. (33), (34), (41), (46), (49)-(51), and (3.B)-(27.B).

Eqs. (53), (54), (65), and (30.B)-(49.B).

Eqs. (D.1)-(D.7).

4. Numerical results

The IEEE 6-bus test system is used to investigate the optimal behavior of the Disco as well as the associated impacts on the DA and RT markets. Furthermore, effectiveness of the proposed model is verified on the RTS 24-bus test system. In both the case studies, the Disco is responsible to operate the 33-bus distribution network.

The system data are taken from [15]. The forecasted output power of wind turbines (WTs) and photovoltaic (PV) arrays are taken from [22]. Maximum traded power between the Disco and the market is 80MW. Technical characteristics of RESs, ESs, and DGs are taken from [22]. Moreover, the maximum purchased power from DRA is 7MW. The aggregators' offers are taken from [15, 16, 27] and reported in Table 2. Also, the occurrence probabilities of the scenarios, as calculated in Appendix A, are given in Table 3.

Table 2
The offers of aggregators to the Disco and selling energy price to the DNL ($/MWh)

Time (hour)	λ_t^{RES}	λ_t^{DG}	λ_t^{chES}	λ_t^{dchES}	λ_t^{IL}	λ_t^{DNL}
1	2.00	12	0.745	3.40	1.50	5.760
2	2.90	12	1.11	4.92	2.22	8.940
3	3.00	12	1.18	5.10	2.40	10.52
4	3.50	12	1.31	5.95	3.30	14.40
5	4.50	12	1.68	7.65	4.10	20.16
6	5.30	14	1.98	9.01	4.54	24.48
7	6.00	14	2.69	10.2	5.80	28.80
8	6.50	14	2.92	11.05	6.40	31.68
9	6.80	14	3.05	11.56	6.90	33.12
10	9.00	16	4.04	15.30	8.50	46.08
11	10.0	16	4.49	17.00	9.80	51.84
12	10.6	16	4.75	18.01	10.0	55.30
13	10.0	16	4.49	17.00	9.80	51.84
14	6.50	14	2.92	11.05	6.40	31.68
15	5.50	14	2.47	9.35	5.20	25.92
16	5.80	14	2.60	9.86	5.70	27.65
17	6.50	14	2.92	11.05	6.40	31.68

18	9.30	16	4.20	15.81	9.20	47.52
19	9.50	16	4.27	16.15	9.50	48.96
20	10.0	16	4.49	17.00	9.80	51.84
21	10.5	16	4.72	17.85	9.90	18.16
22	4.50	12	1.68	7.65	4.10	12.60
23	3.10	12	1.15	5.27	2.38	9.090
24	2.50	12	0.94	4.25	1.65	8.640

Table 3
Occurrence probability of scenarios in decision making problem of the Disco

# scenario	1	2	3	4	5
Occurrence probability	0.061	0.049	0.047	0.091	0.051
# scenario	6	7	8	9	10
Occurrence probability	0.085	0.077	0.065	0.065	0.064
# scenario	11	12	13	14	15
Occurrence probability	0.074	0.087	0.067	0.063	0.054

A. 6-bus test system

I. Input data

Structure of the distribution and transmission networks, the input data, and location of DERs in the DN are shown in Fig. 3 and Tables 4-8 [28-30]. While TNL #3, locates at bus 5, is replaced with DN, bus #1 is considered as the reference bus. Moreover, the maximum up/down regulation capacity assumed to be 30MW. As shown in Table 6, two blocks are considered to represent the offers of each Genco. The total energy in DA market is reported for each TNL in Table 7. For simplicity, the total energy consumption is divided into three blocks with equal sizes. Table 8 reports the respective bids of the blocks. The up/down regulation offers of Gencos/responsive TNLs in the RT ($C_{g,t}^{UP\_RT}, C_{g,t}^{DN\_RT}, C_{d,t}^{DN\_RT}$) and DA ($C_{g,t}^{UP\_DA}, C_{g,t}^{DN\_DA}, C_{d,t}^{DN\_DA}$) markets are assumed to be equal. The model, consists of 220344 single equations, 148714 single variables, and 28008 discrete variables, is solved by CPLEX solver with GAMS 24.1.2 software. A personal computer with 6GB RAM running on Intel Core i-5 with a CPU speed of 2.60GHz, 64bits operating system is used to solve the model. Using this system, the computational time to solve the proposed model for the 6-bus system is 314.06 sec.

Table 4
Technical data of generator units, TNLs, and Disco

# Genco	$\overline{P_g}$	$\underline{P_g}$	RD_g/RU_g	$P_{g,ini}^{TN}$	Node location	# TNL	% of system load	$\bar{l}_{b,d,t}^{TN}$	Node location
1	220	0	100	80	1	1	20	$1/3\,(\overline{L}_{d,t}^{TN\_DA})$	3
2	100	0	60	40	2	2	40	$1/3\,(\overline{L}_{d,t}^{TN\_DA})$	4
3	40	0	30	0	6	Disco	-	-	5

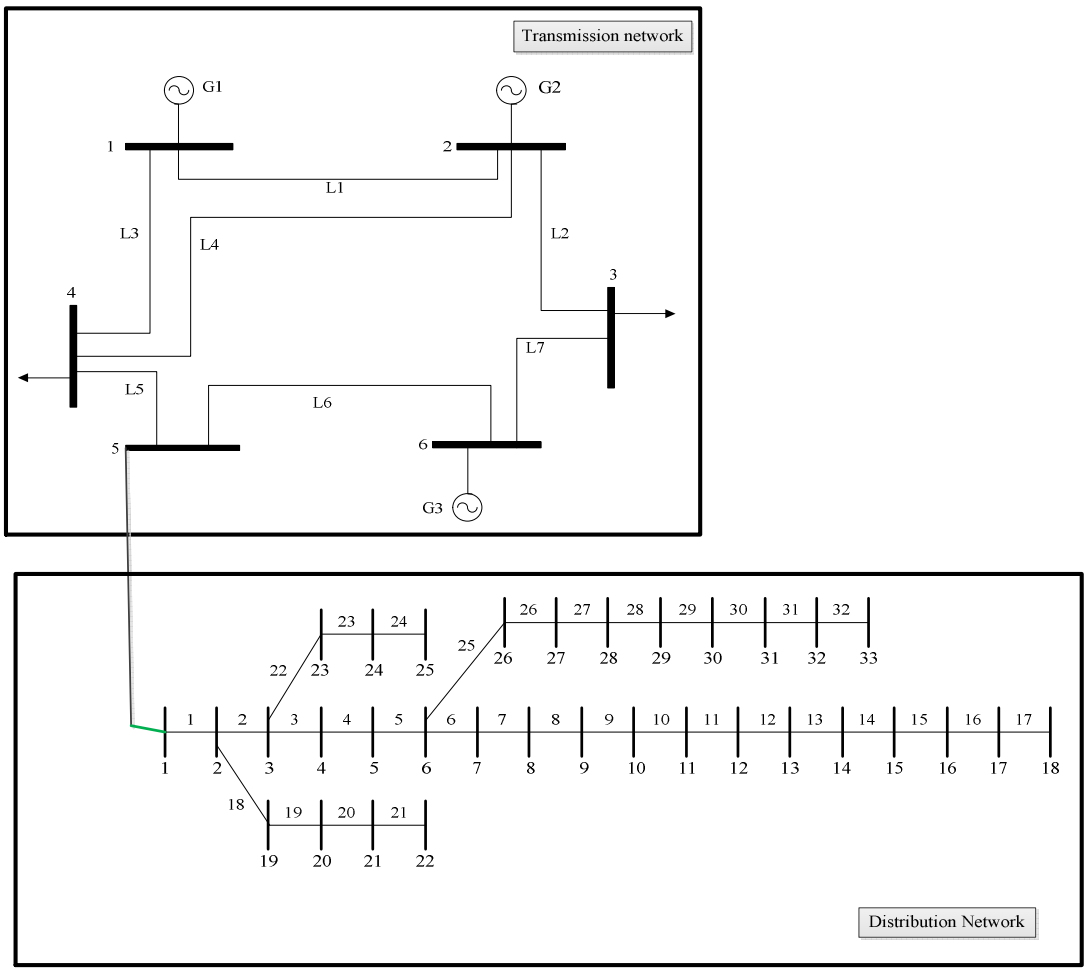

Fig. 3. Structure of the 33-bus distribution network connected to 6-bus power system

Table 5
Location of DERs in the DN

Type of DERs	WT	PV	DG	ES	IL
DN Bus	12, 18, 21, 33	3, 8, 22, 25	18, 24, 29, 33	10, 11, 12, 13	8, 24, 25, 30, 31, 32

Table 6
Size of energy blocks, maximum up/down regulation reserve, and offer costs ($/MWh) submitted by generator units

# Genco	$\overline{PB}_{b,g,t}^{TN}$ (Block1) ($/MW)	$\overline{PB}_{b,g,t}^{TN}$ (Block2) ($/MW)	$C_{b,g,t}^{TN}$ (Block1) ($/MW)	$C_{b,g,t}^{TN}$ (Block2) ($/MW)	$\overline{RG}_g^{UP}$ (MW)	$\overline{RG}_g^{DN}$ (MW)	$C_{g,t}^{UP\_DA}$ ($/MWh)	$C_{g,t}^{DN\_DA}$ ($/MWh)
1	132	88	18.60	20.03	55	55	24	16
2	60	40	19.20	20.32	25	25	25	17

3	24	16	13.89	15.97	10	10	18	11

Table 7
Hourly summation of TNL 1 and 2

Time	1	2	3	4	5	6
(MW)	111.84	105.6	102	99	99	102
Time	7	8	9	10	11	12
(MW)	111.6	130.2	147.6	160.2	166.2	168
Time	13	14	15	16	17	18
(MW)	166.2	168	168	163.2	161.4	161.4
Time	19	20	21	22	23	24
(MW)	156.6	155.4	155.4	156.6	147.6	124.2

Table 8
Bids and maximum down regulation reserve submitted by TNL

# TNL	$C_{b,d,t}^{TN-DA}$ **(Block1)** **($/MW)**	$C_{b,d,t}^{TN-DA}$ **(Block2)** **($/MW)**	$C_{b,d,t}^{TN-DA}$ **(Block3)** **($/MW)**	$\overline{RL_d}^{DN}$ **(MW)**	$C_{d,t}^{DN\_DA}$ **($/MW)**	$C_{d,t}^{TN\_RT}$ **($/MW)**
1	21.58	19.80	18.81	15	23	22
2	23.55	22.37	21.33	15	24	24

II. Results

The Disco total cost and the mean and standard deviations of RT market clearing prices (MCPs) for each scenario are given in Table 9 and Fig. 4, respectively. Fig. 4 reveals that according to the different strategic behaviors of the Disco to manage the uncertainties, the mean and standard deviations of RT prices change.

Table 9
The total cost of Disco in each scenario

# scenario	1	2	3	4	5
Total Cost ($)	-16150	-15380.12	-15731	-15332.24	-16010.4
# scenario	6	7	8	9	10
Total Cost ($)	-14900.84	-15632.36	-16220.52	-15460.19	-15813.42
# scenario	11	12	13	14	15
Total Cost ($)	-14970.73	-15441.28	-15473.1	-16400	-15182.71

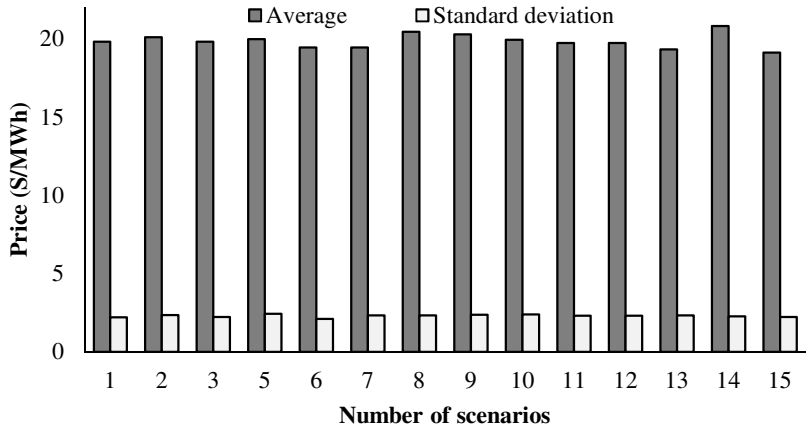

Fig 4. The mean and deviation values of RT MCPs in the operation period in all scenarios

For instance, the operation results of the Disco, for the fourth scenario ($\omega = 4$) with risk aversion parameter of 0 are presented in Figs. 5-10. Fig. 5 reveals that the changes in MCP for the RT market is greater than the DA market. This could be justified by:

- The purchased/sold power in the RT market is less than the DA market.
- Modeling of the responsive TNLs in the RT market besides providing RT down regulation power increase the submitted offers to the RT market.
- The Disco participates as a prosumer player in the RT market in order to face with the power deviation from RESA and DNL.

To this end, the Disco decreases the DA MCP from 18.81$/MWh and 19.2$/MWh to 18.6$/MWh and 18.81$/MWh at hours 8 and 9, respectively by decreasing the purchased power from the DA market. In this way the Disco decides to purchase the required energy from RESA and DGA, as shown in Fig. 7. Moreover, the Disco decreases the purchased power from the DA market by interacting with DGA and DRA at hour 23, as shown in Fig. 7. This in turn leads to decreasing the MCP of the DA market from 19.2$/MWh to 18.81$/MWh.

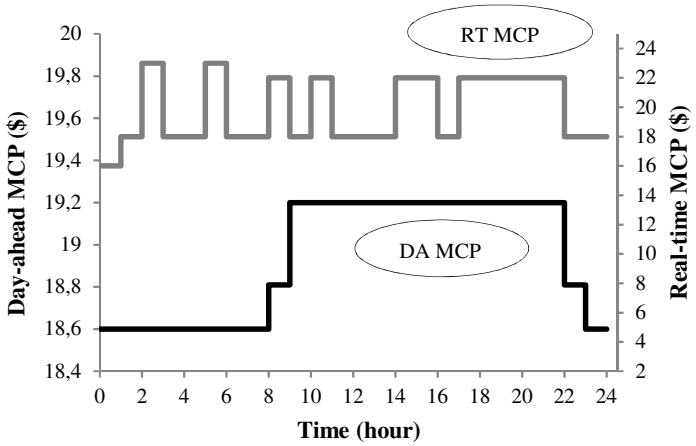

Fig. 5. MCPs in the DA and RT markets

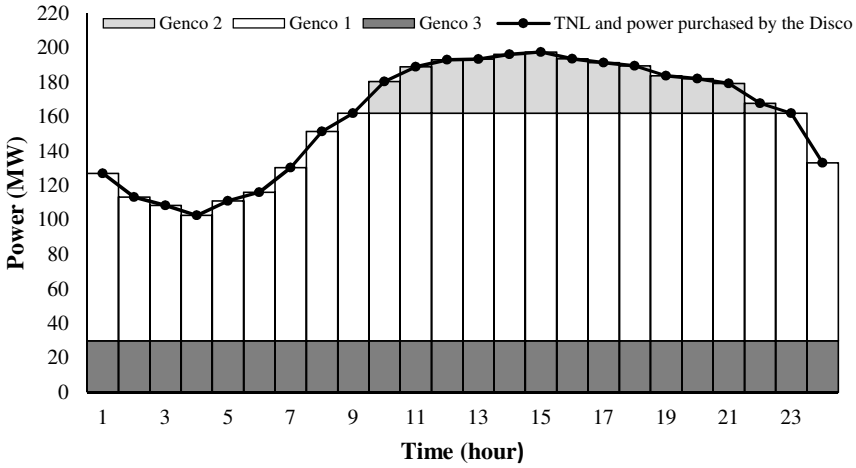

Fig. 6. Share of each Genco to supply day-ahead TNL and DNL

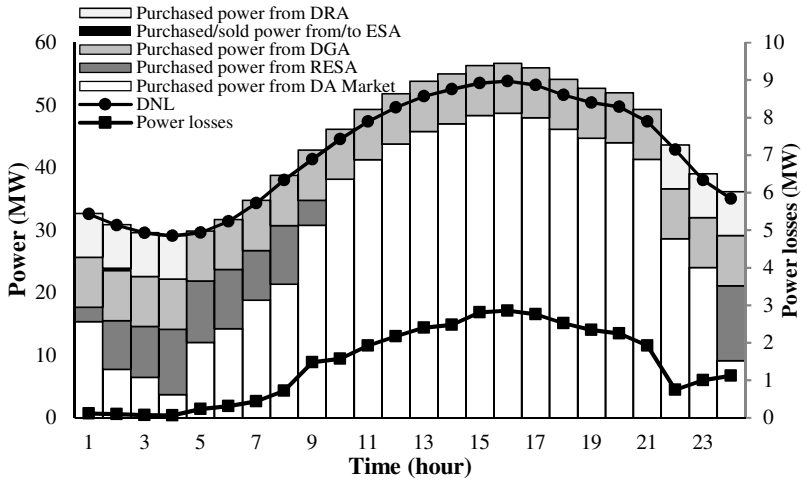

Fig. 7. Share of each Disco's power resources to supply DNL (first-stage power balance)

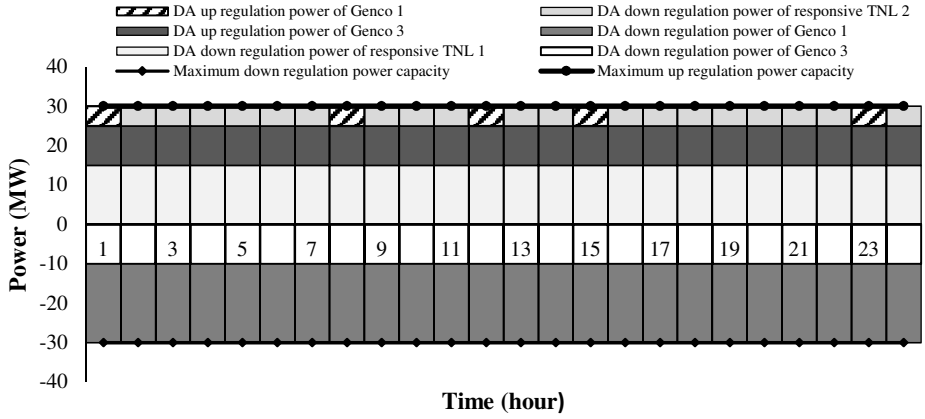

Fig.8. Share of each Genco and responsive TNL to supply maximum up/down regulation power capacity

The maximum up/down regulation capacity is provided by Gencos and responsive TNLs, as represented in Fig. 8. Modeling the responsive TNLs, in the proposed optimization model, increases competitiveness of the DA and RT markets. This in turn has impact on the DA power

generation and DA maximum up regulation power of Gencos in the clearing process. According to Fig. 8, responsive TNLs 1 and 2 provide the main part of maximum up regulation power.

As shown in Fig. 9, in hours 10-18 and 21-23, the TNL of the RT market is provided by the Disco and the Genco # 3. In hours 2-8 and 24, the TNL is zero and thus, the purchased power by the Disco is provided by the Genco # 3 and responsive TNL 1. In hours 9, 19, and 20, the Genco # 3 supplies the required energy of the TNL and Disco. In hour 1, the Genco # 1 re-schedules the power generation according to the maximum down regulation power, which would be determined in the DA market. Thus, the Disco sells energy to the market to compensate for the down regulation power.

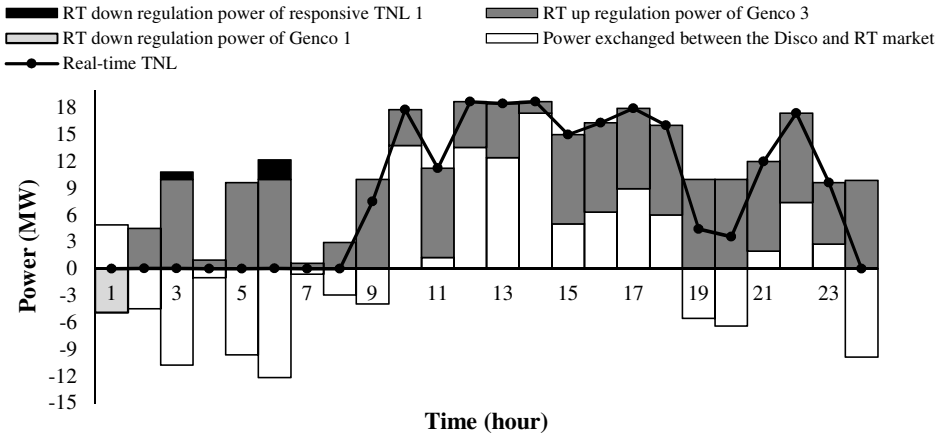

Fig. 9. Share of the Disco, Gencos, and responsive TNLs to supply real-time TNL

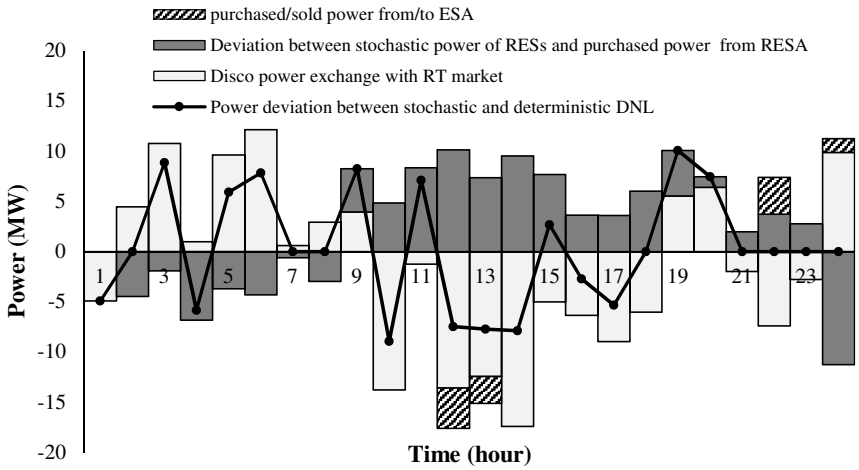

Fig. 10. Share of Disco's resources to supply power deviation between stochastic and deterministic DNL (second-stage power balance)

Fig. 10 reveals that the Disco acts as a prosumer player in the RT market. The Disco behaves as a producer at hours 1, 10-18, and 21-23 and behaves as a consumer at hours 2-9, 19-20, and 24. The decision variables in the first-stage problem, i.e. difference between stochastic and deterministic DNL, difference between stochastic power of RESs and purchased power from RESA, and interaction with the ESA, have direct impacts on the second-stage decisions, including the Disco power transactions with the RT market and the bids/offers to the market. For instance, at hour 19, the differences between stochastic and deterministic DNL, and stochastic power of RESs and the purchased power from RESA are positive. Therefore, the Disco reduces the bid, aimed to decrease the purchased power which in turn causes the real-time MCP decreases from 23$/MWh to 22$/MWh.

III. Comparing different modeling of the ES in Disco problem

In this sub-section, the different operation modes of the ES aggregators in the Disco problem are investigated. For this purpose, three cases are considered: I) modeling the ES in the DA market, II) modelling the ES in the RT market, and III) modeling the ES in the both markets. The results of the cases are shown in Table 10 and Figs. 11 and 12. It could be seen that, the ETC of the Disco decreases for cases II and III. This is because of the optimal decisions of the Disco to trade energy with the market, considering optimal operation of ES. Indeed, when ES is modeled in the second stage, the Disco may charge ES in the case of low prices, and sells the stored energy to the RT market in the case of high prices. For instance, as shown in Fig. 11, for hours 12 and 13, the Disco sells power to the RT market with price of 18$/MWh. On the other hand, for the case I, the Disco sells extra energy to the RT market.

Table 10
The amount of ETC regarding different interacting with the ESs aggregator

Mode of interacting with the ESs aggregator	**Interaction with the ES aggregator in the first-stage**	**Interaction with the ES aggregator in the second-stage**	**Interaction with the ES aggregator in the both stages**
ETC ($)	-15566.159	-15571.47	-15571.661

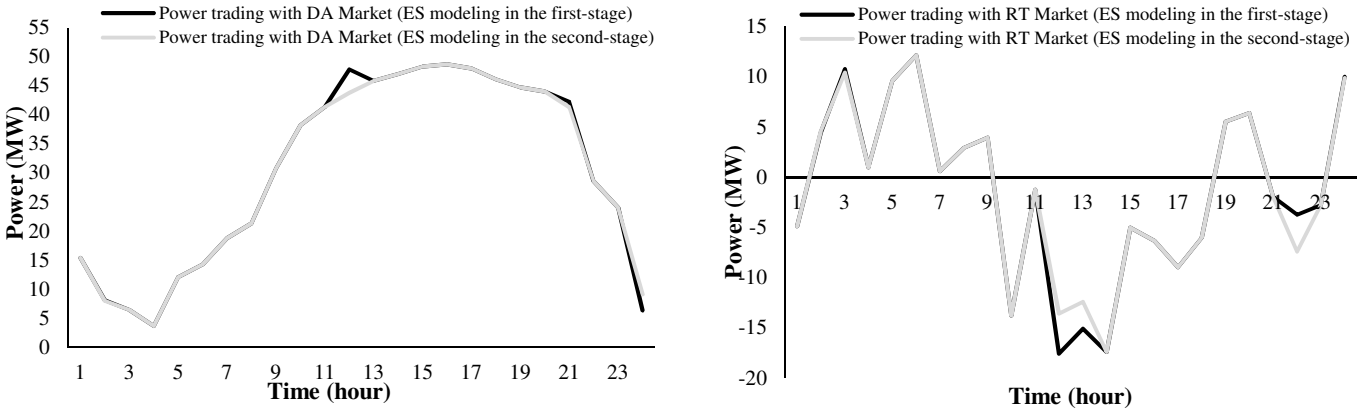

Fig 11. Sensitivity of the Disco DA and RT bids/offers and power trading to the different modeling of ESs

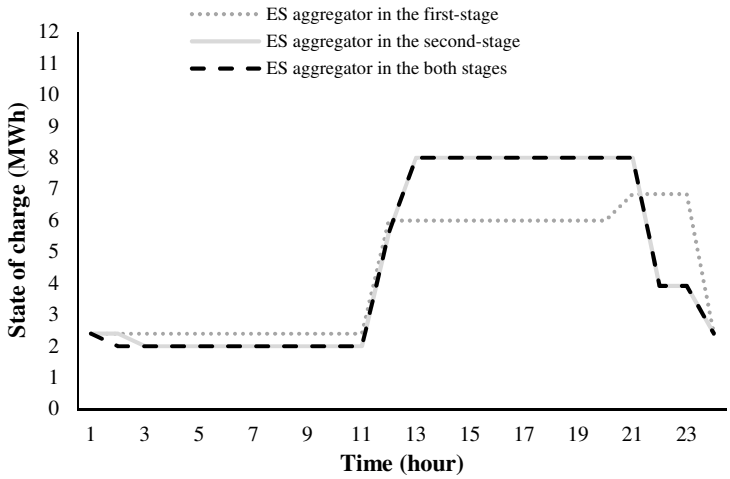

Fig. 12. The state of charge of ESs according to each mode of interacting with the ESs aggregator

IV. Risk analysis

In this sub-section, the impact of risk-aversion parameter on the decisions of the Disco are investigated. In what follows, α sets to 0.8. It means that the Disco trusts to 80 percent of scenarios and tries to manage 20 percent of worst case scenarios, scenarios number 6, 11, and 15 in Table 8. The ETC variations of the Disco, for different values of β, are shown in Fig. 13 in which scenarios are sorted from the worst scenario to the best one. In this way, the worst scenarios, i.e. scenarios # 6, 11, and 15 in Table 9, are renamed as scenarios # 1, 2, and 3, respectively. According to Fig. 13, it is observed that as the risk-aversion parameter increases, the ETC in the worst scenario decreases and difference between the ETCs in the best and worst

scenarios decreases from 1499.16$ to 1248.83$. Indeed, ability of the Disco to control the uncertain behavior of the RESs and DNL increases in the risk-aversion of 100.

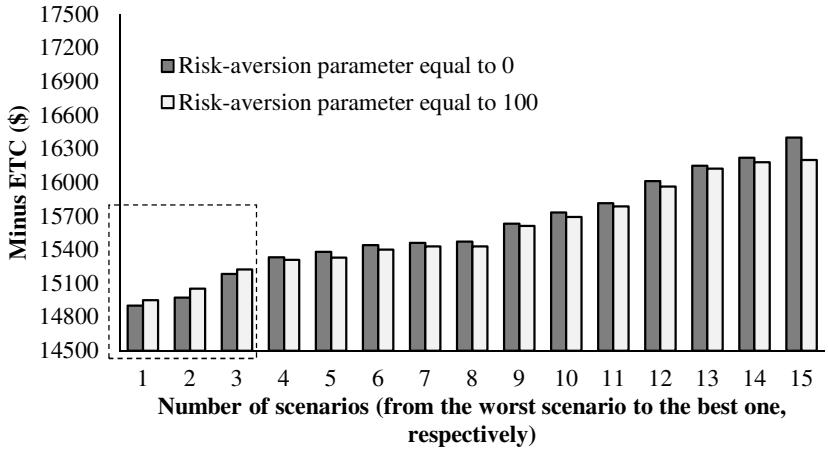

Fig. 13. Sensitivity of the minus TC_{ω} to risk-aversion parameter in each scenario

Fig. 14 represents the relationships between the ETC/CVaR and the risk-aversion parameter. It could be seen that as the risk aversion parameter increases, the ETC of the Disco increases and the CVaR decreases.

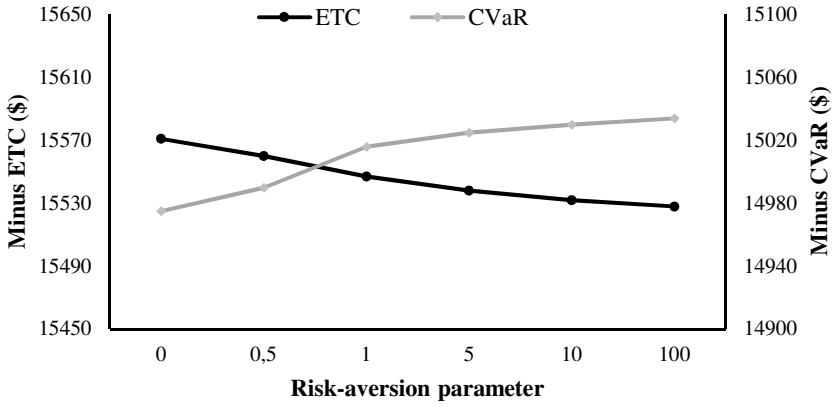

Fig. 14. Sensitivity of the minus ETC and CVaR to risk-aversion parameter

The sensitivity analysis of the Disco first-stage decisions, including bids/offers, exchanged power with the markets, and the difference between stochastic power of RESs and purchased power from RESA, to the risk parameter are reported in Table 11. The results show that the purchased power from the DA market decreases when the risk parameter increases from 0 to 100. For instance, the Disco's bids/offers to DA market decreases from 19.20$/MWh to 18.81$/MWh at hour 22. Moreover, changing decision on purchased power from DA market

and RESA may change difference between stochastic power of RESs and purchased power from RESA and the second-stage decision-variables. As shown in Table 11, power purchased/sold from/to the RT market increases/decreases and bids/offers related to the exchanged power increases from 18.00$/MWh to 22.00$/MWh at hours 7 and 13, respectively. The purchased power from RESA increases from 81.259MW to 102.077MW when the risk-aversion parameter increases from 0 to 100. Generally stated, the risk parameter has significant impacts on the first-stage decisions of the Disco which in turn, changes bids/offers of the Disco in the DA and RT markets.

Table 11
Sensitivity of the Disco DA and RT bids/offers to risk-aversion parameter

Time (hour)	$\beta = 0$		$\beta = 100$		$\beta = 0$		$\beta = 100$	
	Bids/offers to the DA Market	**Disco power exchange with DA market**	**Bids/offers to the DA Market**	**Disco power exchange with DA market**	**Bids/offers to the RT Market**	**Disco power exchange with RT market**	**Bids/offers to the RT Market**	**Disco power exchange with RT market**
1	18.60	15.369	18.60	15369	16.00	-4.887	16.00	-4.887
2	18.60	7.779	18.60	3.546	18.00	4.467	18.00	8.700
3	18.60	6.494	18.60	2.615	23.00	10.783	23.00	14.662
4	18.60	3.700	18.60	2.165	18.00	0.985	18.00	2.520
5	18.60	12.067	18.60	15.401	18.00	9.622	18.00	6.288
6	18.60	14.269	18.60	12.029	23.00	12.160	23.00	14.400
7	18.60	18.818	18.60	14.763	**18.00**	0.600	**22.00**	6.655
8	18.60	21.352	18.60	21.352	18.00	2.946	18.00	2.946
9	18.81	30.800	18.81	30.800	22.00	3.947	22.00	3.947
10	19.20	38.154	19.20	38.154	18.00	-13.771	18.00	-12.770
11	19.20	41.302	19.20	41.302	22.00	-1.248	22.00	-1.248
12	19.20	43.802	19.20	43.802	18.00	-13.575	18.00	-13.575
13	19.20	45.800	19.20	39.187	**18.00**	-12.418	**22.00**	-8.472
14	19.20	46.999	19.20	46.999	18.00	-17.403	18.00	-17.403
15	19.20	48.327	19.20	48.327	22.00	-5.004	22.00	-5.004
16	19.20	48.686	19.20	48.686	22.00	-6.322	22.00	-6.322
17	19.20	47.994	19.20	47.994	18.00	-8.943	18.00	-8.943
18	19.20	46.118	19.20	46.118	22.00	-6.030	22.00	-6.030
19	19.20	44.710	19.20	44.710	22.00	5.549	22.00	5.549
20	19.20	44.000	19.20	44.000	22.00	6.422	22.00	6.422
21	19.20	41.313	19.20	38.487	22.00	-1.980	22.00	0.846
22	**19.20**	28.627	**18.81**	29.627	22.00	-7.400	22.00	-7.400
23	18.81	24.013	18.81	24.013	18.00	-2.775	18.00	-2.775
24	18.60	9.140	18.60	10.369	18.00	9.880	18.00	10.000

As shown in Fig. 15, the risk-averse Disco attempts to control the uncertainties of RESs and DNL to improve the expected cost of $(1-\alpha)\times 100\%$ of the worst scenarios (CVaR). For this reason, the risk-averse Disco trusts to the first-stage decisions to increases the total purchased power from the RESA and change the purchased power from the DA market. Therefore, the difference between stochastic power of RESs and purchased power from RESA $\sum_t(P_{f,t,\omega}^{RES-RT} - P_{f,t}^{RES-DA})$ decreases in each scenario. This in turn decreases the ability of the risk-averse Disco to act as a producer in the RT market.

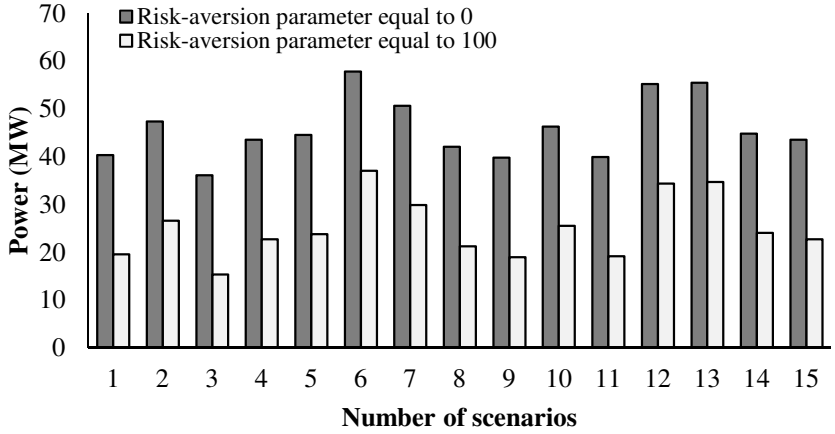

Fig. 15. The total amount of power deviation between the stochastic power of RESs and the purchased power from RESA in each scenario

B. 24-bus test system

I. Input data

RTS 24-bus benchmark is employed to further investigate efficiency of the proposed model. The data of the system are taken from [22, 24]. The developed model, consisting of 927560 single equations, 622635 single variables, and 116110 discrete variables, is solved by CPLEX solver. Using the same personal computer as that of part A, the computational time to solve the model is 1178.18 sec.

II. Results

The numerical results, including power balance of the distribution and transmission networks and MCPs for both DA and RT markets, are presented in Figs. 16-20. As shown in Figs. 16-18, the Disco decreases the MCP of DA market from 10.66$/MWh and 10.66$/MWh

to 10.25$/MWh and 10.25$/MWh in hours 1 and 24, respectively. On the other hands, optimal scheduling of DGs leads to decreasing the DA prices from 18.66$/MWh to 18.2$/MWh in hours 18 and 19. Co-participation of RESA, DRA and Disco to provide demand decreases the prices of MCP from 11.96$/MWh and 11.26$/MWh to 11.72$/MWh, 11.09$/MWh in hours 22 and 23, respectively.

In the RT market and for hour 1, Genco # 9 supplies the required energy of the Disco, as shown in Fig. 19. In hours 12 and 20, the TNL is zero and thus the purchased power by the Disco is provided by the Genco # 1. In hours 9, 10, 13, 15-19, the TNL of the RT market is provided by the Disco. Of note that in hours 1, 3-20, and 23, the generation of RESs is greater than those of purchased by the Disco from RESA in first-stage decision. In this way, the extra power would be sold to the RT market by the Disco (see Fig. 20).

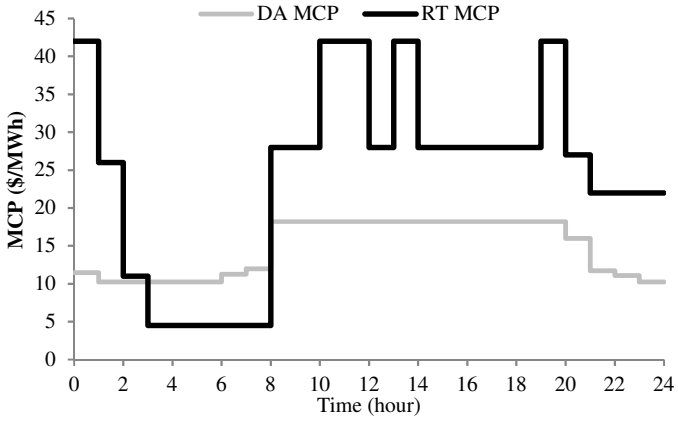

Fig. 16. MCPs of the DA and RT markets

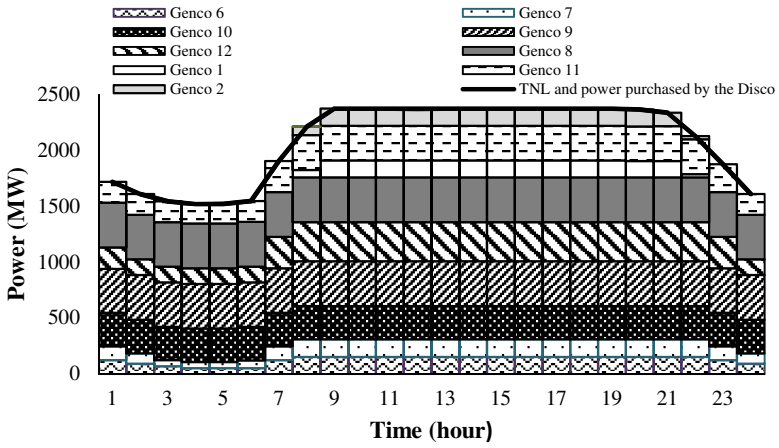

Fig. 17. Share of each Genco to supply day-ahead TNL and DNL

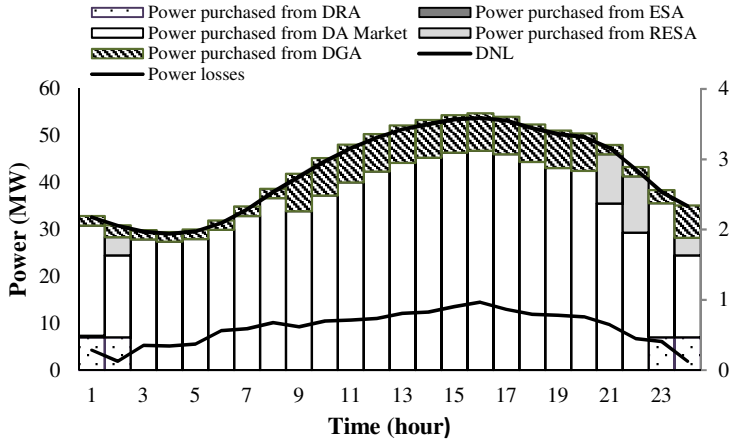

Fig. 18. Share of each Disco's power resources to supply DNL (first-stage power balance)

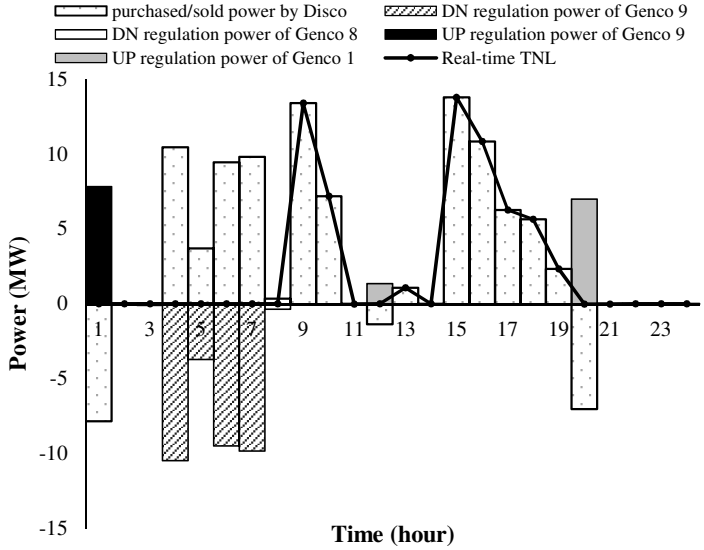

Fig. 19. Share of the Disco, Gencos, and responsive TNLs to supply real-time TNL

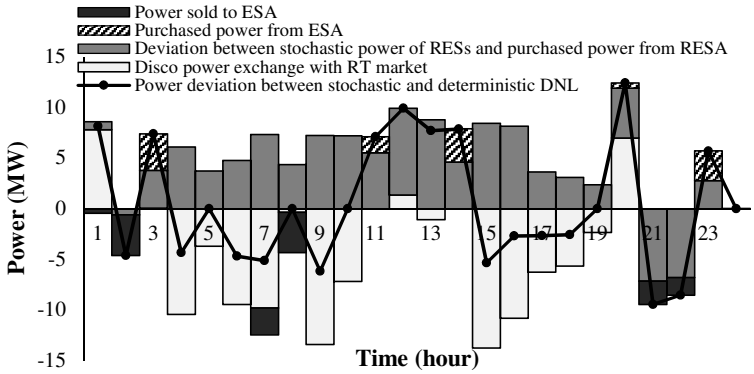

Fig. 20. Share of Disco's resources to supply power deviation between stochastic and deterministic DNL (second-stage power balance)

5. Conclusion

The decision making problem of the Disco to provide the demand through optimal trading with the DER aggregators and participating in the day-ahead and real-time markets is addressed

in this paper. To model such problem, a risk-based two-stage stochastic bi-level optimization approach is developed. The derived model is a nonlinear bi-level problem which may be transformed into the single-level problem using Karush-Kuhn-Tucker conditions. Moreover, the non-linear terms are linearized using the dual theory and McCormick's relaxation methods. To investigate the effectiveness of the proposed model, the distribution network is connected to two test systems. The main conclusions from the results are as follows:

- Interactions with distributed energy resources aggregators besides low level of transactions in the market cause the Disco acts as a prosumer in the real-time market. This in turn has significant effect on the market clearing prices. On the other hand, the Disco can compensate for the power imbalance through participating in the real-time market and interacting with the energy storage aggregators.
- Simultaneous modeling of the Disco in the day-ahead and real-time markets causes the Disco makes the best decisions to interact with the distributed energy resources aggregators, control the renewable energy sources and load uncertainties, and trade energy with the markets. Indeed, these goals are achieved using the proposed two-stage stochastic programming approach.
- The decisions of the Disco in the day-ahead and real-time markets, including bids/offers and purchased/sold power from/to the markets, depend on the associated risk parameter. So, the decisions of the risk-averse Disco can affect the market clearing prices of the day-ahead and real-time markets.

6. References

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7. Appendix

Appendix A: Modeling uncertainties

To model the uncertainties of demand, wind speed, and solar radiation, the normal, weibull, and irradiance distribution models as their probability distribution functions (PDFs), respectively, are discretized into seven and five-intervals as described in [31]. The forecast

amount of these parameters in each time step is considered as the mean value of the related PDFs. The probability of each interval is obtained through the integration of the mentioned PDFs regarding the lower and the upper limitations of each interval. For each parameter, 24000 samples are generated regarding the probability of the intervals. The average value of each interval is multiplied with the forecast value of the parameter to show the value of that parameter in each sample. Then, the scenario tree construction is used to generate different scenarios in the operation time as described in [32, 33]. In this approach, the time steps defined in the problem (i.e. 24 hours in this paper) and the generated samples are used as the scenario tree stages and the nodes, respectively, where a scenario is defined as the path among the nodes. Using the scenario tree method, 1000 scenarios are generated to model the uncertain parameters in the time-period of the operation. To decrease the tractability of the computational problem of the model [22], the generated scenarios are reduced to 15 scenarios using the General Algebraic Modeling System/Scenario Reduction (GAMS/SCENRED) package and the fast-forward scenario reduction technique. Each scenario is consisting of wind speed ($V_{t,\omega}$), solar radiation ($K_{t,\omega}^{AC}$), and demand data for the time-period of the operation problem, i.e. 24 hours. Then, the output power of WT and PV arrays is calculated as follows:

- The output power of WT is calculated using (A.1) [27, 34]:

$$p_{f,t,\omega}^{WT}=\begin{cases}0 & 0\le v_{f,t,\omega}<V_{ci}\\ av_{f,t,\omega}^{3}+bv_{f,t,\omega}^{2}+cv_{f,t,\omega}+d & V_{ci}\le v_{f,t,\omega}<V_{r}\\ P_{r} & V_{r}\le v_{f,t,\omega}\le V_{co}\\ 0 & v_{f,t,\omega}>V_{co}\end{cases} \quad \text{(A.1)}$$

Where P_r is the rated WT output power; $v_{f,t,\omega}$, V_{ci}, V_r, and V_{co} are the forecast wind speed, the cut-in, the rated, and the cut-out wind speeds, respectively; and *a, b, c,* and *d* are the parameters of the WT power curve.

- Equation (A.2) is used to model the output power of PV array [35]:

$$p_{f,t,\omega}^{PV} = \bar{P}_{STC} \frac{K_{t,\omega}^{AC}}{K_{STC}} \left(1 + k \left(T_{t,\omega}^{e} - T_{STC}\right)\right) \quad \text{(A.2)}$$

Where $p_{t,\omega}^{PV}$ is the output power of PV array; $\bar{P}_{STC}$ and K_{STC} which are used to model the maximum output power and the irradiance are calculated in the standard test conditions; T_{STC} and $T_{f,t,\omega}^{e}$ describe the standard temperature and cell temperature, respectively; and k is a temperature coefficient. The equation (A.3) is used to consider the sum of the output power of WT and PV arrays in the RT power balance constraint as follows:

$$p_{f,t,\omega}^{RES\_RT} = P_{f,t,\omega}^{WT\_RT} + P_{f,t,\omega}^{PV\_RT} \quad \text{(A.3)}$$

Appendix B: MPEC

In this Appendix the KKT conditions of the LL problems are described. The KKT conditions consist of four sets of equations which are presented as follows:

B.1. MPEC of the DA market problem

- *Stationarity:* To obtain the stationarity constraints, the lagrangian function is modeled using (B.1) in which $f(x^{DA})$ is objective function, x^{DA} is variable set of the DA problem, $H_i^{DA}(x) = 0$, $\mathrm{i} = 1, 2, \ldots, 7$ describes the equality constraints including (22), (23), (33), (34), and (37)-(39), and $G_j^{DA}(x) \geq 0$, $\mathrm{j} = 1, 2, \ldots, 26$ describes the inequality constraints including (24)-(32) and (35)-(36). By deriving from the proposed function related to each variable, the stationarity constraints are obtained as (B.3)-(B.11).

$$L^{DA} = f\left(x^{DA}\right) + \left(\lambda^{DA}\right)^{T} H_i^{DA}\left(x^{DA}\right) + \left(\mu^{DA}\right)^{T} G_j^{DA}\left(x^{DA}\right) \quad \text{(B.1)}$$

$$\nabla_x f\left(x^{DA}\right) + \left(\lambda^{DA}\right)^{T} \nabla_x h^{DA}\left(x^{DA}\right) + \left(\mu^{DA}\right)^{T} \nabla_x g^{DA}\left(x^{DA}\right) = 0 \quad \text{(B.2)}$$

$$\frac{\partial L}{\partial p_{gt}^{TN}} = -\lambda_{n,t}^{TN\_DA} - \underline{\mu}_{g,t}^{2\_DA} + \overline{\mu}_{g,t}^{2\_DA} - \mu_{g,t}^{3\_DA} - \mu_{g,t}^{9\_DA}\Big|_{t>1} + \mu_{g,t+1}^{9\_DA} - \mu_{g,t}^{10\_DA}\Big|_{t=1} + \mu_{g,t}^{11\_DA}\Big|_{t>1} - \mu_{g,t+1}^{11\_DA} + \mu_{g,t}^{12\_DA}\Big|_{t=1} + \lambda_{g,t}^{1\_DA} = 0 \quad \text{(B.3)}$$

$$\frac{\partial L}{\partial l_{d,t}^{TN\_DA}} = \lambda_{n,t}^{TN\_DA}\Big|_{n \neq m} - \underline{\mu}_{d,t}^{6\_DA} + \overline{\mu}_{d,t}^{6\_DA} - \mu_{d,t}^{7\_DA} + \lambda_{d,t}^{2\_DA} = 0 \quad \text{(B.4)}$$

$$\frac{\partial L}{\partial p_{t}^{Dis\_DA}} = -c_t^{Dis\_DA} + \lambda_{m,t}^{TN\_DA} - \underline{\mu}_{t}^{1\_DA} + \overline{\mu}_{t}^{1\_DA} = 0 \quad \text{(B.5)}$$

$$\frac{\partial \mathrm{L}}{\partial \theta_{n,t}^{TN\_DA}} = \sum_{r\in\Lambda^{TN}} B_{n-r}(\lambda_{n,t}^{TN\_DA} - \lambda_{r,t}^{TN\_DA}) + \sum_{r\in\Lambda^{TN}} B_{n-r}(\overline{\mu}_{n,r,t}^{15\_DA} - \overline{\mu}_{r,n,t}^{15\_DA}) + \sum_{r\in\Lambda^{TN}} B_{n-r}(\underline{\mu}_{r,n,t}^{15\_DA} - \underline{\mu}_{n,r,t}^{15\_DA}) - \underline{\mu}_{n,t}^{16\_DA} + \overline{\mu}_{n,t}^{16\_DA} + \lambda_{n,t}^{5\_DA}\Big|_{n=slack} = 0 \quad \text{(B.6)}$$

$$\frac{\partial L}{\partial pb_{b,g,t}^{TN}} = C_{b,g,t}^{TN} - \underline{\mu}_{b,g,t}^{13\_DA} + \overline{\mu}_{b,gt}^{13\_DA} - \lambda_{g,t}^{1\_DA} = 0 \quad \text{(B.7)}$$

$$\frac{\partial L}{\partial lb_{b,d,t}^{TN}} = -C_{b,d,t}^{TN\_DA} - \underline{\mu}_{b,d,t}^{14\_DA} + \overline{\mu}_{b,d,t}^{14\_DA} - \lambda_{d,t}^{2\_DA} = 0 \quad \text{(B.8)}$$

$$\frac{\partial L}{\partial rp_{g,t}^{UP\_DA}} = C_{g,t}^{UP\_DA} - \underline{\mu}_{g,t}^{2\_DA} + \overline{\mu}_{g,t}^{2\_DA} - \underline{\mu}_{g,t}^{4\_DA} + \overline{\mu}_{g,t}^{4\_DA} + \lambda_{t}^{3\_DA} = 0 \quad \text{(B.9)}$$

$$\frac{\partial L}{\partial rp_{g,t}^{DN\_DA}} = C_{g,t}^{DN\_DA} + \mu_{g,t}^{3\_DA} - \underline{\mu}_{g,t}^{5\_DA} + \overline{\mu}_{g,t}^{5\_DA} + \lambda_{t}^{4\_DA} = 0 \quad \text{(B.10)}$$

$$\frac{\partial L}{\partial rl_{d,t}^{DN\_DA}} = C_{d,t}^{DN\_DA} + \mu_{d,t}^{7\_DA} - \underline{\mu}_{g,t}^{8\_DA} + \overline{\mu}_{g,t}^{8\_DA} + \lambda_{t}^{3\_DA} = 0 \quad \text{(B.11)}$$

- *Primal, dual, and complementary constraints:* These equations are defined as follows:

$$0 \le \underline{\mu}_{t}^{1\_DA} \perp \left(p_{t}^{Dis\_DA} - \underline{P}^{Dis\_TN}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{t}^{1\_DA} \perp \left(\overline{P}^{Dis\_TN} - p_{t}^{Dis\_DA}\right) \ge 0 \quad \text{(B.12)}$$

$$0 \le \underline{\mu}_{g,t}^{2\_DA} \perp (p_{g,t}^{TN} + rp_{g,t}^{UP\_DA} - \underline{P_g}) \ge 0 \;,\; 0 \le \overline{\mu}_{g,t}^{2\_DA} \perp \left(\overline{P_g} - p_{g,t}^{TN} - rp_{g,t}^{UP\_DA}\right) \ge 0 \quad \text{(B.13)}$$

$$0 \le \mu_{g,t}^{3\_DA} \perp \left(p_{g,t}^{TN} - rp_{g,t}^{DN\_DA}\right) \ge 0 \quad \text{(B.14)}$$

$$0 \le \underline{\mu}_{g,t}^{4\_DA} \perp \left(rp_{g,t}^{UP\_DA}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{g,t}^{4\_DA} \perp \left(\overline{RG}_{g}^{UP} - rp_{g,t}^{UP\_DA}\right) \ge 0 \quad \text{(B.15)}$$

$$0 \le \underline{\mu}_{g,t}^{5\_DA} \perp \left(rp_{g,t}^{DN\_DA}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{g,t}^{5\_DA} \perp \left(\overline{RG}_{g}^{DN} - rp_{g,t}^{DN\_DA}\right) \ge 0$$

$$0 \le \underline{\mu}_{d,t}^{6\_DA} \perp \left(l_{d,t}^{TN\_DA}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{d,t}^{6\_DA} \perp \left(\overline{L}_{d,t}^{TN} - l_{d,t}^{TN\_DA}\right) \ge 0 \quad \text{(B.16)}$$

(B.17)

$$0 \le \mu_{d,t}^{7\_DA} \perp \left(l_{d,t}^{TN\_DA} - rl_{d,t}^{DN\_DA}\right) \ge 0 \quad \text{(B.18)}$$

$$0 \le \underline{\mu}_{g,t}^{8\_DA} \perp \left(rl_{d,t}^{DN\_DA}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{g,t}^{8\_DA} \perp \left(\overline{RL}_{d}^{DN} - rl_{d,t}^{DN\_DA}\right) \ge 0 \quad \text{(B.19)}$$

$$0 \le \mu_{g,t}^{9\_DA} \perp \left(RD_{g} - p_{g,t-1}^{TN} + p_{g,t}^{TN}\right) \ge 0 \;,\; 0 \le \mu_{g,t}^{10\_DA} \perp \left(RD_{g} - P_{g,ini}^{TN} + p_{g,t}^{TN}\right) \ge 0 \quad \text{(B.20)}$$

$$0 \le \mu_{g,t+1}^{11\_DA} \perp \left(RU_{g} - p_{g,t}^{TN} + p_{g,t-1}^{TN}\right) \ge 0 \;,\; 0 \le \mu_{g,t}^{12\_DA} \perp \left(RU_{g} - p_{g,t}^{TN} + P_{g,ini}^{TN}\right) \ge 0 \quad \text{(B.21)}$$

$$0 \le \underline{\mu}_{b,g,t}^{13\_DA} \perp \left(pb_{b,g,t}^{TN}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{b,gt}^{13\_DA} \perp \left(\overline{PB}_{b,g,t}^{TN} - pb_{b,g,t}^{TN}\right) \ge 0 \quad \text{(B.22)}$$

$$0 \le \underline{\mu}_{b,d,t}^{14\_DA} \perp \left(lb_{b,d,t}^{TN}\right) \ge 0 \;,\; 0 \le \overline{\mu}_{b,d,t}^{14\_DA} \perp \left(\overline{LB}_{b,d,t}^{TN} - lb_{b,d,t}^{TN}\right) \ge 0 \quad \text{(B.23)}$$

$$0 \le \underline{\mu}_{n,r,t}^{15\_DA} \perp \left(B_{n-r}\left(\theta_{n,t}^{TN\_DA} - \theta_{r,t}^{TN\_DA}\right) - \left(-\overline{F}_{n-r}^{TN}\right)\right) \ge 0 \;,\; 0 \le \overline{\mu}_{n,r,t}^{15\_DA} \perp \left(\overline{F}_{n-r}^{TN} - B_{n-r}\left(\theta_{n,t}^{TN\_DA} - \theta_{r,t}^{TN\_DA}\right)\right) \ge 0 \quad \text{(B.24)}$$

$$0 \le \underline{\mu}_{n,t}^{16\_DA} \perp \left(\theta_{n,t}^{TN\_DA} - \left(-\frac{\pi}{2}\right)\right) \ge 0 \;,\; 0 \le \overline{\mu}_{n,t}^{16\_DA} \perp \left(\left(\frac{\pi}{2} - \theta_{n,t}^{TN\_DA}\right)\right) \ge 0 \quad \text{(B.25)}$$

$$\lambda_{n,t}^{TN\_DA}, \lambda_{g,t}^{1\_DA}, \lambda_{d,t}^{2\_DA}, \lambda_{t}^{3\_DA}, \lambda_{t}^{4\_DA}, \lambda_{n,t|n=Slack}^{5\_DA} \;, \text{Unrestricted} \quad \text{(B.26)}$$

Where each equation is linearized using the big M method as Eq. (B.27). M_1^{DA} and M_2^{DA} are large enough values and U^{DA} is a binary variable.

$$0 \le a^{DA} \perp b^{DA} \ge 0 \Rightarrow a^{DA} \ge 0, \quad b^{DA} \ge 0, \quad a^{DA} \le M_1^{DA} U^{DA}, \quad b^{DA} \le M_2^{DA}(1-U^{DA}) \tag{B.27}$$

B.2. MPEC of the RT market problem

- *Stationarity:* The lagrangian function for the RT market problem is modeled as (B.28) in which $H_i^{RT}(x) = 0$, i = 1, 2,3 describes the equality constraints including (41), (42), and (53), $G_j^{RT}(x) \ge 0$, j = 1, 2, … , 11 describes the inequality constraints including (43)-(52), and x^{RT} is variable set of the RT problem. The stationary equations are obtained as (B.30)-(B.35).

$$L^{RT} = f\left(x^{RT}\right) + \left(\lambda^{RT}\right)^T H_i^{RT}\left(x^{RT}\right) + \left(\mu^{RT}\right)^T G_j^{RT}\left(x^{RT}\right) \tag{B.28}$$

$$\nabla_x f\left(x^{RT}\right) + \left(\lambda^{RT}\right)^T \nabla_x h^{RT}\left(x^{RT}\right) + \left(\mu^{RT}\right)^T \nabla_x g^{RT}\left(x^{RT}\right) = 0 \tag{B.29}$$

$$\frac{\partial L}{\partial rp_{g,t,\omega}^{UP\_RT}} = C_{g,t}^{UP\_RT} - \lambda_{n,t,\omega}^{TN\_RT} - \underline{\mu}_{g,t,\omega}^{2\_RT} + \overline{\mu}_{g,t,\omega}^{2\_RT} - \mu_{g,t,\omega}^{4\_RT}\Big|_{t>1} + \mu_{g,t+1,\omega}^{4\_RT} - \mu_{g,t,\omega}^{5\_RT}\Big|_{t=1} + \mu_{g,t,\omega}^{6\_RT}\Big|_{t>1} - \mu_{g,t+1,\omega}^{6\_RT} + \mu_{g,t,\omega}^{7\_RT}\Big|_{t=1} = 0 \tag{B.30}$$

$$\frac{\partial L}{\partial rp_{g,t,\omega}^{DN\_RT}} = -C_{g,t}^{DN\_RT} + \lambda_{n,t,\omega}^{TN\_RT} - \underline{\mu}_{g,t,\omega}^{3\_RT} + \overline{\mu}_{g,t,\omega}^{3\_RT} + \mu_{g,t,\omega}^{4\_RT}\Big|_{t>1} - \mu_{g,t+1,\omega}^{4\_RT} + \mu_{g,t,\omega}^{5\_RT}\Big|_{t=1} - \mu_{g,t,\omega}^{6\_RT}\Big|_{t>1} + \mu_{g,t+1,\omega}^{6\_RT} - \mu_{g,t,\omega}^{7\_RT}\Big|_{t=1} = 0 \tag{B.31}$$

$$\frac{\partial L}{\partial rl_{d,t,\omega}^{DN\_RT}} = C_{d,t}^{DN\_RT} - \lambda_{n,t,\omega}^{TN\_RT}\Big|_{n\neq m} - \underline{\mu}_{d,t,\omega}^{8\_RT} + \overline{\mu}_{d,t,\omega}^{8\_RT} = 0 \tag{B.32}$$

$$\frac{\partial L}{\partial l_{d,t,\omega}^{TN\_RT}} = -C_{d,t}^{TN\_RT} + \lambda_{n,t,\omega}^{TN\_RT}\Big|_{n\neq m} - \underline{\mu}_{d,t,\omega}^{9\_RT} + \overline{\mu}_{d,t,\omega}^{9\_RT} = 0 \tag{B.33}$$

$$\frac{\partial L}{\partial p_{t,\omega}^{Dis\_RT}} = -c_{t,\omega}^{Dis\_RT} + \lambda_{m,t,\omega}^{TN\_RT} - \underline{\mu}_{t,\omega}^{1\_RT} + \overline{\mu}_{t,\omega}^{1\_RT} = 0 \tag{B.34}$$

$$\frac{\partial \mathrm{L}}{\partial \theta_{n,t,\omega}^{TN\_RT}} = \sum_{r\in\Lambda^{TN}} B_{n-r}\left(\lambda_{n,t,\omega}^{TN\_RT} - \lambda_{r,t,\omega}^{TN\_RT}\right) + \sum_{r\in\Lambda^{TN}} B_{n-r}\left(\overline{\mu}_{n,r,t,\omega}^{10\_RT} - \overline{\mu}_{r,n,t,\omega}^{10\_RT}\right) + \sum_{r\in\Lambda^{TN}} B_{n-r}\left(\underline{\mu}_{r,n,t,\omega}^{10\_RT} - \underline{\mu}_{r,n,t,\omega}^{10\_RT}\right) - \underline{\mu}_{n,t,\omega}^{11\_RT} + \overline{\mu}_{n,t,\omega}^{11\_RT} + \lambda_{n,t,\omega}^{1\_RT}\Big|_{n=slack} = 0 \tag{B.35}$$

- *Primal, dual, and complementary constraints:* These equations are defined as follows:

$$0 \le \underline{\mu}_{t,\omega}^{1\_RT} \perp \left(p_{t,\omega}^{Dis\_RT} - \underline{P}^{Dis\_TN}\right) \ge 0 \ , \ 0 \le \overline{\mu}_{t,\omega}^{1\_RT} \perp \left(\overline{P}^{Dis\_TN} - p_{t,\omega}^{Dis\_RT}\right) \ge 0 \tag{B.36}$$

$$0 \le \underline{\mu}_{g,t,\omega}^{2\_RT} \perp \left(rp_{g,t,\omega}^{UP\_RT}\right) \ge 0 \ , \ 0 \le \overline{\mu}_{g,t,\omega}^{2\_RT} \perp \left(rp_{g,t}^{UP\_DA} - rp_{g,t,\omega}^{UP\_RT}\right) \ge 0 \tag{B.37}$$

$$0 \le \underline{\mu}_{g,t,\omega}^{3\_RT} \perp \left(rp_{g,t,\omega}^{DN\_RT}\right) \ge 0 \ , \ 0 \le \overline{\mu}_{g,t,\omega}^{3\_RT} \perp \left(rp_{g,t}^{DN\_DA} - rp_{g,t,\omega}^{DN\_RT}\right) \ge 0 \tag{B.38}$$

$$0 \le \mu_{g,t,\omega}^{4\_RT} \perp \left(RD_g - \left(p_{g,t-1}^{TN} + rp_{g,t-1,\omega}^{UP\_RT} - rp_{g,t-1,\omega}^{DN\_RT} \right) + \left(p_{g,t}^{TN} + rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) \right) \ge 0 \quad \text{(B.39)}$$

$$0 \le \mu_{g,t,\omega}^{5\_RT} \perp \left(RD_g - P_{g,ini}^{TN} + \left(p_{g,t}^{TN} + rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) \right) \ge 0 \quad \text{(B.40)}$$

$$0 \le \mu_{g,t,\omega}^{6\_RT} \perp \left(RU_g - \left(p_{g,t}^{TN} + rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) + \left(p_{g,t-1}^{TN} + rp_{g,t-1,\omega}^{UP\_RT} - rp_{g,t-1,\omega}^{DN\_RT} \right) \right) \ge 0 \quad \text{(B.41)}$$

$$0 \le \mu_{g,t,\omega}^{7\_RT} \perp \left(RU_g - \left(p_{g,t}^{TN} + rp_{g,t,\omega}^{UP\_RT} - rp_{g,t,\omega}^{DN\_RT} \right) + P_{g,ini}^{TN} \right) \ge 0 \quad \text{(B.42)}$$

$$0 \le \underline{\mu}_{d,t,\omega}^{8\_RT} \perp \left(rl_{d,t,\omega}^{DN\_RT} \right) \ge 0 \;\; , \;\; 0 \le \overline{\mu}_{d,t,\omega}^{8\_RT} \perp \left(rl_{g,t}^{DN\_DA} - rl_{d,t,\omega}^{DN\_RT} \right) \ge 0 \quad \text{(B.43)}$$

$$0 \le \underline{\mu}_{d,t,\omega}^{9\_RT} \perp \left(l_{d,t,\omega}^{TN\_RT} \right) \ge 0 \;\; , \;\; 0 \le \overline{\mu}_{d,t,\omega}^{9\_RT} \perp \left(\overline{L}_{d,t}^{TN} - l_{d,t}^{TN\_DA} - l_{d,t,\omega}^{TN\_RT} \right) \quad \text{(B.44)}$$

$$0 \le \underline{\mu}_{n,r,t,\omega}^{10\_RT} \perp \left(B_{n-r} \left(\theta_{n,t,\omega}^{TN\_RT} - \theta_{r,t,\omega}^{TN\_RT} \right) - \left(-\overline{F}_{n-r}^{TN} \right) \right) \ge 0 \;\; , \; 0 \le \overline{\mu}_{n,r,t,\omega}^{10\_RT} \perp \left(\overline{F}_{n-r}^{TN} - B_{n-r} \left(\theta_{n,t,\omega}^{TN\_RT} - \theta_{r,t,\omega}^{TN\_RT} \right) \right) \ge 0 \quad \text{(B.45)}$$

$$0 \le \underline{\mu}_{n,t,\omega}^{11\_RT} \perp \left(\theta_{n,t,\omega}^{TN\_RT} - \left(-\frac{\pi}{2} \right) \right) \ge 0 \;\; , \;\; 0 \le \overline{\mu}_{n,t,\omega}^{11\_RT} \perp \left(\left(\frac{\pi}{2} - \theta_{n,t,\omega}^{TN\_RT} \right) \right) \ge 0 \quad \text{(B.46)}$$

$$\lambda_{n,t,\omega}^{TN\_RT} , \lambda_{n,t,\omega|n=Slack}^{1\_RT} \quad , \text{Unrestricted} \quad \text{(B.47)}$$

Where each equation is linearized as Eq. (B.48) in which M_1^{RT} and M_2^{RT} are large enough values and U^{RT} is a binary variable.

$$0 \le a^{RT} \perp b^{RT} \ge 0 \Rightarrow a^{RT} \ge 0, \;\; b^{RT} \ge 0, \;\; a^{RT} \le M_1^{RT} U^{RT}, \;\; b^{RT} \le M_2^{RT} (1 - U^{RT}) \quad \text{(B.48)}$$

Appendix C: Linearization of the cost of power exchange with the DA market

The non-linear term of equation (1) ($\lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA}$) is linearized using the dual theory in this appendix. For this purpose, the dual of the DA problem is modeled as follows:

$$\text{Maximize} \sum_{t=1}^{T} d_t \left[\begin{array}{l} \underline{P}^{Dis\_TN} \underline{\mu}_t^{1\_DA} - \overline{P}^{Dis\_TN} \overline{\mu}_t^{1\_DA} - \sum_{g=1}^{G} \overline{P}_g \overline{\mu}_{g,t}^{2\_DA} - \sum_{g=1}^{G} \overline{RG}_g^{UP} \overline{\mu}_{g,t}^{4\_DA} - \sum_{g=1}^{G} \overline{RG}_g^{DN} \overline{\mu}_{g,t}^{5\_DA} \\ - \sum_{d=1}^{D} \overline{L}_{d,t}^{TN} \overline{\mu}_{d,t}^{6\_DA} - \sum_{d=1}^{D} \overline{RL}_d^{DN} \overline{\mu}_{d,t}^{8\_DA} - \sum_{g=1}^{G} RD_g \mu_{g,t}^{9\_DA} \Big|_{t>1} - \sum_{g=1}^{G} \left(RD_g - P_{g,ini}^{TN} \right) \mu_{g,t}^{10\_DA} \Big|_{t=1} \\ - \sum_{g=1}^{G} RU_g \mu_{g,t}^{11\_DA} \Big|_{t>1} - \sum_{g=1}^{G} \left(RU_g + P_{g,ini}^{TN} \right) \mu_{g,t}^{12\_DA} \Big|_{t=1} - \sum_{g=1}^{G} \sum_{b=1}^{B} \overline{PB}_{b,g,t}^{TN} \overline{\mu}_{b,g,t}^{13\_DA} \\ - \sum_{d=1}^{D} \sum_{b=1}^{B} \overline{LB}_{b,d,t}^{TN} \overline{\mu}_{b,d,t}^{14\_DA} - \sum_{n=1}^{N} \sum_{r=1}^{R} \left(\overline{F}_{n-r}^{TN} \underline{\mu}_{n,r,t}^{15\_DA} + \overline{F}_{n-r}^{TN} \overline{\mu}_{n,r,t}^{15\_DA} \right) - \sum_{n=1}^{N} \left(\underline{\mu}_{n,t}^{16\_DA} + \overline{\mu}_{n,t}^{16\_DA} \right) \frac{\pi}{2} \\ + \lambda_t^{3\_DA} P^{RTC\_UP} + \lambda_t^{4\_DA} P^{RTC\_DN} \end{array} \right] \quad \text{(C.1)}$$

The parameter S is used to model all the equations in the bracket represented in (C.1). Based on the strong duality theory $pb_{b,g,t}^{TN}$, $lb_{b,d,t}^{TN}$, $p_t^{Dis\_DA}$, $rp_{g,t}^{UP\_DA}$, $rp_{g,t}^{DN\_DA}$, $rl_{d,t}^{DN\_DA}$ are optimal

solutions of the primal problem and $\mu^{j\_DA}, \forall j = 1, \dots, 26$ and $\lambda_t^{3\_DA}, \lambda_t^{4\_DA}$ are optimal solutions of the dual problem if and only if:

$$\sum_{t=1}^{T} d_t \left[\begin{array}{l} \sum_{g=1}^{G}\sum_{b=1}^{B}(C_{b,g,t}^{TN} pb_{b,g,t}^{TN}) - \sum_{d=1}^{D}\sum_{b=1}^{B}(C_{b,d,t}^{TN\_DA} lb_{b,d,t}^{TN}) - c_t^{Dis\_DA} p_t^{Dis\_DA} + \\ \sum_{g=1}^{G}\left(C_{g,t}^{UP\_DA} rp_{g,t}^{UP\_DA} + C_{g,t}^{DN\_DA} rp_{g,t}^{DN\_DA}\right) + \sum_{d=1}^{D} C_{d,t}^{DN\_DA} rl_{d,t}^{DN\_DA} \end{array} \right] = \sum_{t=1}^{T} D_t [S] \quad \text{(C.2)}$$

Eq. (C.2) can be reformulated as follows:

$$\sum_{t=1}^{T} D_t \left[c_t^{Dis\_DA} p_t^{Dis\_DA} \right] = \sum_{t=1}^{T} D_t \left[\begin{array}{l} \sum_{g=1}^{G}\sum_{b=1}^{B}(C_{b,g,t}^{TN} pb_{b,g,t}^{TN}) - \sum_{d=1}^{D}\sum_{b=1}^{B}(C_{b,d,t}^{TN\_DA} lb_{b,d,t}^{TN}) + \\ \sum_{g=1}^{G}\left(C_{g,t}^{UP\_DA} rp_{g,t}^{UP\_DA} + C_{g,t}^{DN\_DA} rp_{g,t}^{DN\_DA}\right) + \sum_{d=1}^{D} C_{d,t}^{DN\_DA} rl_{d,t}^{DN\_DA} \end{array} \right] - \sum_{t=1}^{T} D_t [S] \quad \text{(C.3)}$$

The equation (B.5) is transformed into (C.4) and (C.5) to obtain the expression of $\lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA}$ from $c_t^{Dis\_DA} p_t^{Dis\_DA}$.

$$-c_t^{Dis\_DA} + \lambda_{m,t}^{TN\_DA} - \underline{\mu}_t^{1\_DA} + \bar{\mu}_t^{1\_DA} = 0 \quad \text{(C.4)}$$

$$c_t^{Dis\_DA} p_t^{Dis\_DA} = \lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA} - \underline{\mu}_t^{1\_DA} p_t^{Dis\_DA} + \bar{\mu}_t^{1\_DA} p_t^{Dis\_DA} \quad \text{(C.5)}$$

Then, (C.6) and (C.7) are obtained from (B.5) regarding which the equation (C.8) is obtained.

$$\bar{\mu}_t^{1\_DA} \left(\bar{P}^{Dis\_TN} - p_t^{Dis\_DA} \right) = 0 \quad \rightarrow \quad \bar{\mu}_t^{1\_DA} \bar{P}^{Dis\_TN} = \bar{\mu}_t^{1\_DA} p_t^{Dis\_DA} \quad \text{(C.6)}$$

$$\underline{\mu}_t^{1\_DA} \left(p_t^{Dis\_DA} - \underline{P}^{Dis\_TN} \right) = 0 \quad \rightarrow \quad \underline{\mu}_t^{1\_DA} \underline{P}^{Dis\_TN} = \underline{\mu}_t^{1\_DA} p_t^{Dis\_DA} \quad \text{(C.7)}$$

$$\rightarrow c_t^{Dis\_DA} p_t^{Dis\_DA} = \lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA} - \underline{\mu}_t^{1\_DA} \underline{P}^{Dis\_TN} + \bar{\mu}_t^{1\_DA} \bar{P}^{Dis\_TN} \quad \text{(C.8)}$$

Finally, the linear form of $\lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA}$ is obtained as (C.9).

$$\sum_{t=1}^{T} \lambda_{m,t}^{TN\_DA} p_t^{Dis\_DA} = \sum_{t=1}^{T} D_t \left[\left[\begin{array}{l} \sum_{g=1}^{G}\sum_{b=1}^{B}(C_{b,g,t}^{TN} pb_{b,g,t}^{TN}) - \sum_{d=1}^{D}\sum_{b=1}^{B}(C_{b,d,t}^{TN\_DA} lb_{b,d,t}^{TN}) + \\ \sum_{g=1}^{G}\left(C_{g,t}^{UP\_DA} rp_{g,t}^{UP\_DA} + C_{g,t}^{DN\_DA} rp_{g,t}^{DN\_DA}\right) + \sum_{d=1}^{D} C_{d,t}^{DN\_DA} rl_{d,t}^{DN\_DA} \end{array} \right] - [S] + \underline{\mu}_t^{1\_DA} \underline{P}^{Dis\_TN} - \bar{\mu}_t^{1\_DA} \bar{P}^{Dis\_TN} \right] \quad \text{(C.9)}$$

Appendix D: Linearization of the cost/revenue of power trading with the RT market

The cost/revenue of traded power with the RT market ($\lambda_{m,t,\omega}^{TN\_RT} p_{t,\omega}^{Dis\_RT}$) is a non-linear term and it cannot be linearized by dual theory. In other words, applying the approach utilized in

Appendix C makes the objective function of the dual of the RT problem (it is similar to Eq. C.1 with respective RT variables and parameters) non-linear. This is because of the DA maximum up/down regulation power of the Gencos and responsive TNLs in the RT market problem as variables in the single-level model. Therefore, an approximate approach is presented to linearize the mentioned non-linear term for comparing the results and models' attributes. The non-linear term $\lambda_{m,t,\omega}^{TN\_RT} p_{t,\omega}^{Dis\_RT}$ can be replaced by $v_{t,\omega}^{in}$ and $v_{t,\omega}^{out}$ as auxiliary variables and under some following constraints regarding [36, 37]. Also, $M^{Mc}$is considered as a big value and the upper limit of the dual variable $\lambda_{m,t,\omega}^{TN\_RT}$.

$$p_{t,\omega}^{Dis\_RT} = p_{t,\omega}^{Dis\_RT\_in} - p_{t,\omega}^{Dis\_RT\_out} \quad \text{(D.1)}$$

$$v_{t,\omega}^{in} \le \bar{P}^{Dis\_TN} \lambda_{m,t,\omega}^{TN\_RT} \;,\; v_{t,\omega}^{in} \le M_1^{Mc} p_{t,\omega}^{Dis\_RT\_in} \quad \text{(D.2)}$$

$$v_{t,\omega}^{in} \ge \bar{P}^{Dis\_TN} \lambda_{m,t,\omega}^{TN\_RT} + M_1^{Mc} p_{t,\omega}^{Dis\_RT\_in} - M_1^{Mc} \bar{P}^{Dis\_TN} \quad \text{(D.3)}$$

$$0 \le \lambda_{m,t,\omega}^{TN\_RT} \le M_1^{Mc} \;,\; 0 \le p_{t,\omega}^{Dis\_RT\_in} \le \bar{P}^{Dis\_TN} \quad \text{(D.4)}$$

$$v_{t,\omega}^{out} \le \bar{P}^{Dis\_TN} \lambda_{m,t,\omega}^{TN\_RT} \;,\; v_{t,\omega}^{out} \le M_2^{Mc} p_{t,\omega}^{Dis\_RT\_out} \quad \text{(D.5)}$$

$$v_{t,\omega}^{out} \ge \bar{P}^{Dis\_TN} \lambda_{m,t,\omega}^{TN\_RT} + M_2^{Mc} p_{t,\omega}^{Dis\_RT\_out} - M_2^{Mc} \bar{P}^{Dis\_TN} \quad \text{(D.6)}$$

$$0 \le \lambda_{m,t,\omega}^{TN\_RT} \le M_2^{Mc} \;,\; 0 \le p_{t,\omega}^{Dis\_RT\_out} \le \bar{P}^{Dis\_TN} \quad \text{(D.7)}$$