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# Multimodal image registration based on geometric similarity term

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## Abstract

In this work, we use the geometric information, such as edges and thin structures, to build a similarity measure for deformable registration models of multi-modality images. The idea is to extract a geometric information from the images and then use it to build a robust and efficient similarity term. In order to extract this information, we use the Blake-Zisserman's energy that is well suited for detecting discontinuities at different scales, i.e. of first and second order. In addition, we present a theoretical analysis of the proposed model. For the numerical solution of the model, we use a gradient descent method and iteratively solve corresponding the Euler-Lagrangian equation. We present some numerical results that demonstrate the efficiency of the proposed model.

## Keywords

Inverse problem; Optimization; Multi-modal images; Fidelity measures; Regularizer; Edge maps; Blake-Zisserman's energy.

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## I INTRODUCTION

Image registration is an active topic of research that is encountered in various fields such as astronomy, optics, life sciences, medical imaging, etc. In the context of medical imaging, quite often, a geometric deformation can occur during the steps of recording, reconstruction and transmission of the images. Therefore, in order to provide medical doctors with images for better diagnosis, the images need to be geometrically aligned which is the goal of image registration. For example, image registration plays a very important role in medicine combining data (fusion) from different modalities, e.g. Computer Tomography (CT) and Positron Emission Tomography (PET), to obtain complete information for monitoring tumor growth in patients. Image registration consists of finding a geometric mapping that align points from one image, called the template image, to corresponding points in another image, called the reference image. Both images represent the same object and could be obtained from different devices (scanner, IRM, etc), or could be taken from the same device at different times. The problem can be described as follows: Given a reference image  $R$  and a template image  $T$  that are represented by the following scalar functions  $R, T: \mathbb{R}^d \supset \Omega \rightarrow \mathbb{R}$ , where  $d$  denotes the spatial dimension of images. The objective here is to align the template image with the reference one by applying a spatial transformation to the template image. Mathematically, deformable variational models

consist in solving the following optimization problem

$$\min_{\mathbf{u} \in (\mathcal{W}(\Omega, \mathbb{R}^d))} \{ \mathcal{J}(\mathbf{u}) = \lambda \mathcal{G}(\mathbf{u}) + \mathcal{S}(T(\varphi(\mathbf{x})), R(\mathbf{x})) \}, \quad (1)$$

where  $\mathcal{W}(\Omega, \mathbb{R}^d)$  is a properly chosen functional space of the solution and  $\varphi(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$  is the deformation field. The first part in (1) represents a regularization term [6] where  $\lambda$  is a positive regularizer parameter. From a mathematical point of view, the regularization term turns an ill-posed problem into a well-posed one, i.e., leads to a unique minimizer and sometimes to a convex objective function. There are several regularizers that have been used in previous works, for example first-order derivatives that are based on total variations (see [10]) and diffusion (see [4]). This turns our attention for how to choose the "best" regularization term that gives the more possible plausible transformations. The second part in (1) is the similarity term which quantifies the deformation between the deformed image  $T$  and the reference image  $R$ . In fact, the goal is to apply a spatial transformation on the template image  $T$  by minimizing or by maximizing the similarity term such that both images are as aligned as possible. Indeed, if we choose a statistical-based similarity term, such as the mutual information or the correlation coefficient, we need to maximize this similarity term instead of minimizing it. On the other hand, if we consider a geometry-based measures, the reformulated optimization problem (1) is a minimization one, In the case where both images are from the same modality, they have same contrasts and similar features (mono-modal images). Then, we must use an effective similarity term that makes the registration between the pixels, such as the well-known choice  $L^2$ -distance (Sum of Squared Differences) between  $R$  and  $T(\mathbf{u})$  i.e.  $S(T, R) = \|T - R\|_2^2$  and the correlation coefficient. Unfortunately, for a pair of multi-modality images (i.e. both images have different contrasts), the two previous terms no longer make sense. The preferred fidelity term of the energy (1) is often given by the "Normalized Gradient Fields" (NGF) (see e.g., [9, 11]) and the "Mutual Information" (MI) (see e.g., [3, 5, 11]), such that the NGF is based on the cosine distance between the two vectors i.e. minimize the angle between the two gradient vectors and the MI consists in comparing the histograms of both images instead of comparing their intensities, i.e. measures the mutual information between  $R$  and  $T(\mathbf{u})$ .

In this paper, we are interested in a deformable registration model for multi-modality images. This model consists on finding a reasonable spatial geometric transformation between two given images that represent the same object and that are taken at different times or that are acquired by using different devices. We are using a geometric approach to measure the similarity in the multi-modality case. The idea is to extract a geometric information from the images and then use it to build a robust and efficient similarity term. Indeed, in most registration model for multi-modality images, the intensities of the same object in different images are not similar, but it is very often that the geometry (edges, gradient, Hessian) is the same. Therefore, this geometric information is useful for defining a fidelity measure. To extract this information, we use the Blake-Zisserman's energy, see e.g., [1, 7, 8]. This energy is well suited for detecting discontinuities at different scales i.e. of first and second order.

## II NEW MODEL

We aim to build a variational approach that contains two parts: a regularizer term which controls the smoothness of the displacement field and a geometric similarity measure which are described in details below.

## 2.1 Geometric similarity measure

We introduce a new similarity term for image registration which is based on the geometric information that is extracted from the images by using the Blake-Zisserman's energy (**BZ**) [1, 8]. Then, for an input image  $T$ , the regularized **BZ** energy is defined in the Sobolev space  $W^{1,2}(\Omega) \times W^{2,2}(\Omega)$  by

$$\begin{aligned}
 BZ_\epsilon(T_s, Z_T) = & \frac{1}{2} \int_{\Omega} \lambda_0(T_s - T)^2 d\mathbf{x} + \alpha \int_{\Omega} Z_T^2 |\nabla^2 T_s|^2 d\mathbf{x} \\
 & + \xi \int_{\Omega} (Z_T^2 + \sigma) |\nabla T_s|^2 d\mathbf{x} + \beta \int_{\Omega} \left( \epsilon |\nabla Z_T|^2 + \frac{(Z_T - 1)^2}{4\epsilon} \right) d\mathbf{x}. \quad (2)
 \end{aligned}$$

When  $\alpha = 0$ , the above energy corresponds to the well-known Mumford-Shah energy [2] which uses only first-order derivatives of  $T$ . But this later has some disadvantages, for example, when the image contains high-order features such as thin structures (points, filaments), so that, these features will not be detectable by first-order derivatives, e.g., see [12]. Whereas, by incorporating high-order derivatives, these high-order features could easily be detected by the variable  $Z_T$ . To illustrate the benefit of using the Blake-Zisserman's energy, we can consider the medical image in the Fig. 1. This figure is characterized by a very sensitive geometrical information such as blood vessels and veins. We see that if we use the Mumford-Shah's energy ( $\alpha = 0$ ), there is a lot of thin structures that are not detected. On the other hand, the Blake-Zisserman's energy ( $\alpha \neq 0$ ) overcomes this limitation.

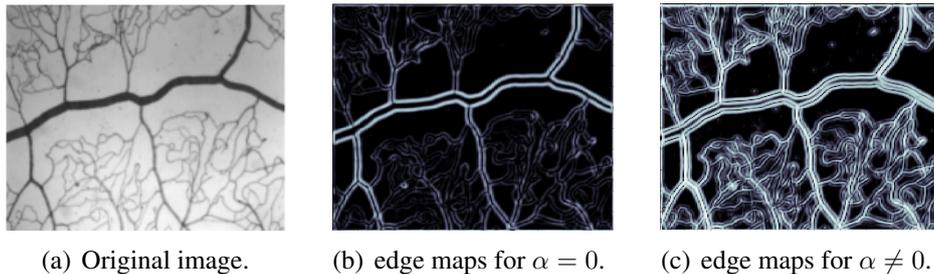


Figure 1: Example of geometric information which are detected by the M-S and B-Z energies

To motivate our proposed new geometric measure, we consider in Fig. 2 a synthetic image of size  $150 \times 150$  pixels that contains sharp and smooth edges. Here, in order to show the performance and the benefits of our model, we compare it with the well-known model denoted by NGF (i.e. using the 'Normalized Gradient Fields' as a fidelity term) which is based on the gradient. We clearly observe that the **NGF** model registers well the outer boundaries of the large square where the edges are sharp. However, for the inner square with smoothed edges and as well as on the diagonals of the large square, the NGF fails completely. In contrast, the new model gives accurate registration in almost all parts of the image.

Motivated by this results, for two given images  $R, T : \Omega \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$  to be registered, we define  $R_s$  and  $T_s$  as their smoothed versions that are obtained by convolution with a Gaussian kernel. Then, we can obtain two edge maps  $Z_T$  and  $Z_R$  by solving the following two optimization problems:

$$\begin{aligned}
 \min_{Z_T} & \left\{ \alpha \int_{\Omega} Z_T^2 |\nabla^2 T_s|^2 d\mathbf{x} + \xi \int_{\Omega} (Z_T^2 + \sigma) |\nabla T_s|^2 d\mathbf{x} + \beta \int_{\Omega} \left( \epsilon |\nabla Z_T|^2 + \frac{(Z_T - 1)^2}{4\epsilon} \right) d\mathbf{x} \right\}, \\
 \min_{Z_R} & \left\{ \alpha \int_{\Omega} Z_R^2 |\nabla^2 R_s|^2 d\mathbf{x} + \xi \int_{\Omega} (Z_R^2 + \sigma) |\nabla R_s|^2 d\mathbf{x} + \beta \int_{\Omega} \left( \epsilon |\nabla Z_R|^2 + \frac{(Z_R - 1)^2}{4\epsilon} \right) d\mathbf{x} \right\}.
 \end{aligned}$$

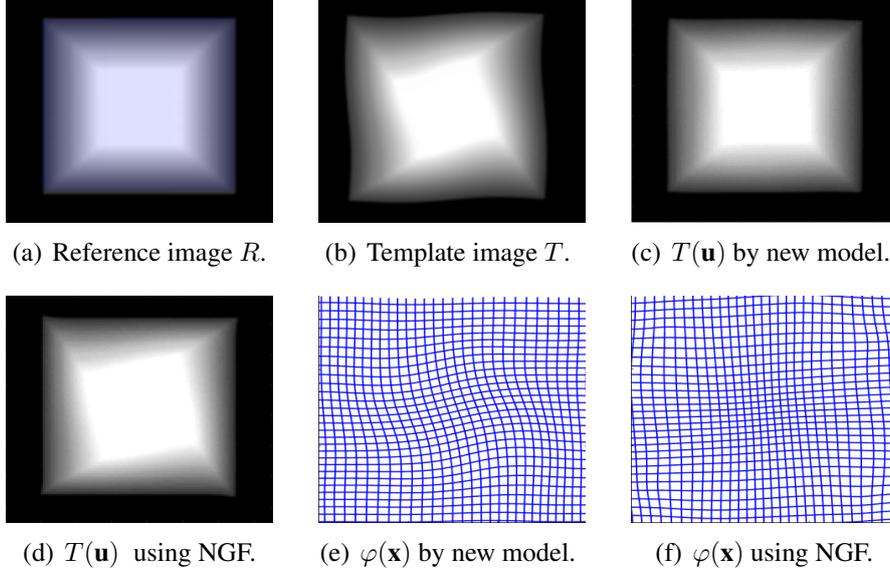


Figure 2: Example of registration of synthetic images of size  $150 \times 150$  by two different models. **New Model** performs well and the **NGF** fails completely in this case because the image is very smooth and the model will miss the edges.

Then, in order to build the similarity measure that will be used in this work, we propose to consider the sum of squared differences between the two edge maps as a similarity term, i.e.

$$\mathcal{S}(T, R) = \|Z_T - Z_R\|_2^2 \quad (3)$$

## 2.2 The full variational model.

As mentioned in the introduction, there are lots of regularizers and hence the choice of a good one is very important. So in order to introduce our variational model, we need to choose a regularization term. More precisely, we propose to register the two functions  $R$  and  $T$  by solving the following minimization problem:

$$\min_{\mathbf{u} \in \mathcal{W}} \left\{ \mathcal{J}(\mathbf{u}) = \frac{1}{2} \|Z_T - Z_R\|_2^2 + \frac{\lambda}{2} \int_{\Omega} |\nabla \mathbf{u}|^2 d\mathbf{x} + \frac{\gamma}{2} \int_{\Omega} |\nabla^2 \mathbf{u}|^2 d\mathbf{x} \right\}, \quad (4)$$

where  $\lambda$  and  $\gamma$  are two regularization parameters and

$$\mathcal{W} = \left\{ \mathbf{u} \in W_0^{1,2}(\Omega) \cap W^{2,2}(\Omega) \text{ such that } \frac{\partial \mathbf{u}}{\partial n} = 0 \text{ on } \partial\Omega \right\}.$$

The regularizer is given by

$$\mathcal{G}(\mathbf{u}) = \frac{\lambda}{2} \int_{\Omega} |\nabla \mathbf{u}|^2 d\mathbf{x} + \frac{\gamma}{2} \int_{\Omega} |\nabla^2 \mathbf{u}|^2 d\mathbf{x}.$$

*Proposition 1:*

The minimization problem (4) admits at least one solution in the space  $\mathcal{W}$ .

*Proof.* The energy  $\mathcal{J}(\mathbf{u})$  in (4) is non-convex with respect to  $\mathbf{u}$  which makes the proof of the weak lower semi-continuity not straightforward. For that reason, we will use the concept of

Carathéodory functions. In addition, the coercivity is easily obtained by using the Poincaré's inequality. Indeed, we have that  $\|\mathbf{u}\|_{\mathcal{W}} = (\|\nabla\mathbf{u}\|_2^2 + \|\nabla^2\mathbf{u}\|_2^2)^{\frac{1}{2}}$  defines a norm in the space  $\mathcal{W}$ . By using the coercivity and the weak lower semi-continuity of  $\mathcal{J}(\cdot)$ , it is easy to show that our energy minimization problem admits at least one solution in the space  $\mathcal{W}$ .  $\square$

### III NUMERICAL EXPERIMENTS

In this section, we present a numerical algorithm to solve the new model. First, we present the gradient descent method that will be used to solve the Euler-Lagrangian equation. Second, we give some numerical tests for multi-modality images registration which are obtained by using Matlab for some examples of cross sections of the human brain of sizes  $200 \times 200$  and  $128 \times 128$ .

#### 3.0.1 Gradient descent method

The gradient descent method is one of the most used methods to solve minimization problems. It is a first-order iterative minimization algorithm used to find the minimum value of a function by replacing the original problem (4) by an approximate gradient.

*Lemma 1:*

The Euler-Lagrangian equation corresponding to the optimization problem (4) writes

$$\left\{ \begin{array}{l} \text{Find } \mathbf{u} \in \mathcal{W} \text{ such that} \\ \frac{\partial \mathcal{J}(\mathbf{u})}{\partial u_1} = \frac{\partial Z_T(\mathbf{u})}{\partial x} (Z_T(\mathbf{u}) - Z_R) - \lambda \Delta u_1 + \gamma \operatorname{div}^2(\nabla^2 u_1) = 0, \\ \frac{\partial \mathcal{J}(\mathbf{u})}{\partial u_2} = \frac{\partial Z_T(\mathbf{u})}{\partial y} (Z_T(\mathbf{u}) - Z_R) - \lambda \Delta u_2 + \gamma \operatorname{div}^2(\nabla^2 u_2) = 0. \end{array} \right. \quad (5)$$

Then, the iterations of the gradient descent applied to the optimization problem (4) are given by

$$\left\{ \begin{array}{l} \mathbf{u}_1 = \mathbf{u}_1^{\text{old}} - \Delta t \left[ \frac{\partial Z_T(\mathbf{u}^{\text{old}})}{\partial x} (Z_T(\mathbf{u}^{\text{old}}) - Z_R) - \lambda \Delta u_1^{\text{old}} + \gamma \operatorname{div}^2(\nabla^2 u_1^{\text{old}}) \right], \\ \mathbf{u}_2 = \mathbf{u}_2^{\text{old}} - \Delta t \left[ \frac{\partial Z_T(\mathbf{u}^{\text{old}})}{\partial y} (Z_T(\mathbf{u}^{\text{old}}) - Z_R) - \lambda \Delta u_2^{\text{old}} + \gamma \operatorname{div}^2(\nabla^2 u_2^{\text{old}}) \right]. \end{array} \right. \quad (6)$$

where  $\Delta t \in \mathbb{R}_+$  is small enough.

### IV RESULTS AND DISCUSSION

In what follows, we display some numerical tests of image registration by using our new variational model. In Figs. 3 and 4, the reference and the template image represent two different MRI cross sections of the brain of size  $128 \times 128$  and  $200 \times 200$ , respectively. For each test, we present both images  $R$  and  $T$ , the edges used for registration, the register image  $T(\varphi)$ , the deformation  $\varphi(\mathbf{u})(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$  which states the smoothness of the transformations. In order to quantify the quality of registration process, we use the relative Sum of Squared Differences

$$\mathcal{E}_{SSD} = \frac{\|Z_R - Z_{T(\varphi)}\|_2^2}{\|Z_R - Z_T\|_2^2}. \quad (7)$$

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**Algorithm III.1** Gradient descend method for the optimization problem (4)
 

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1. Load and prepare reference and template images
  2. Detect the edge map  $Z_R$ .
  3. Choose an initial displacement variable  $u_1^{\text{old}} = u_2^{\text{old}} = 0$ .
  4. Solve alternately the equation (6):
    - for  $j=1..p$
    - $T_s=T$
    - Detect the map of  $T_s$  image.
    - For  $i=1..q$
    - Compute  $\mathbf{u}_1$  and  $\mathbf{u}_2$  solutions of (6).
    - Set  $u_1^{\text{old}} = u_1$  and  $u_2^{\text{old}} = u_2$
    - end
    - $T \leftarrow$  Interpolate the displacement field  $\mathbf{u}=(u_1^{\text{old}}, u_2^{\text{old}})$  in  $T$ .
    - end.
- 

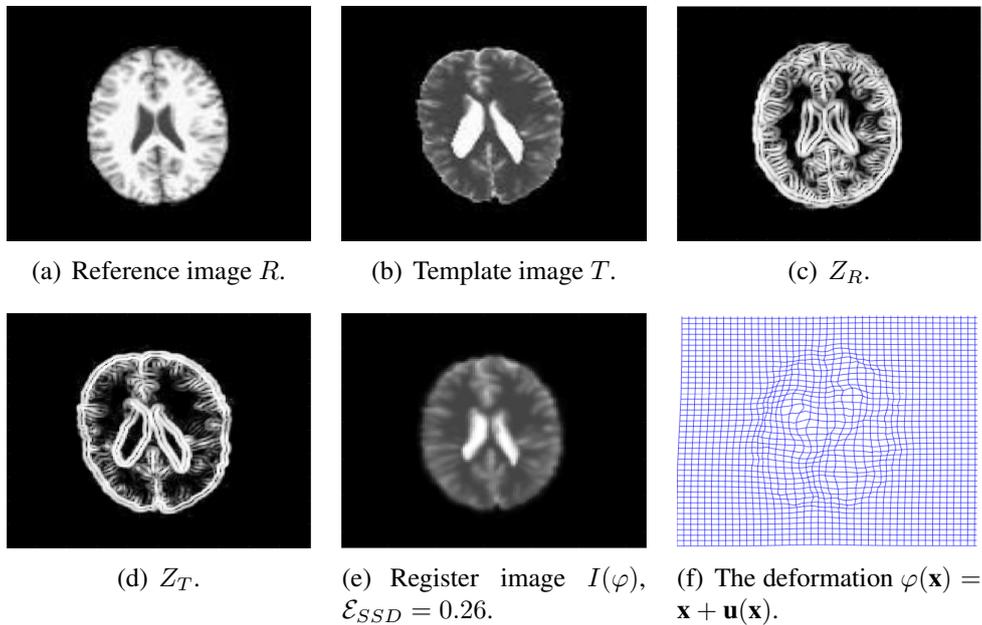


Figure 3: Example 1: Registration of two multi-modal images ( $128 \times 128$ ). These results have been shown for the following regularization parameters:

$$\alpha = 10^{-5}, \beta = 10^{-2}, \varepsilon = 10^{-5}, \xi = 10^{-2}, \Lambda t = 0.3, \lambda = 0.6 \text{ and } \gamma = 0.4.$$

## V CONCLUSION

In conclusion, we have proposed a new geometric similarity term to define a minimization problem for image registration, and then studied this problem theoretically and numerically. We have addressed the benefit of using an energy that is defined by a second-order derivative. The performed numerical experiments confirm that our proposed fidelity measure performs well in multi-modality image registration. Moreover this result have been shown by checking the relative errors.

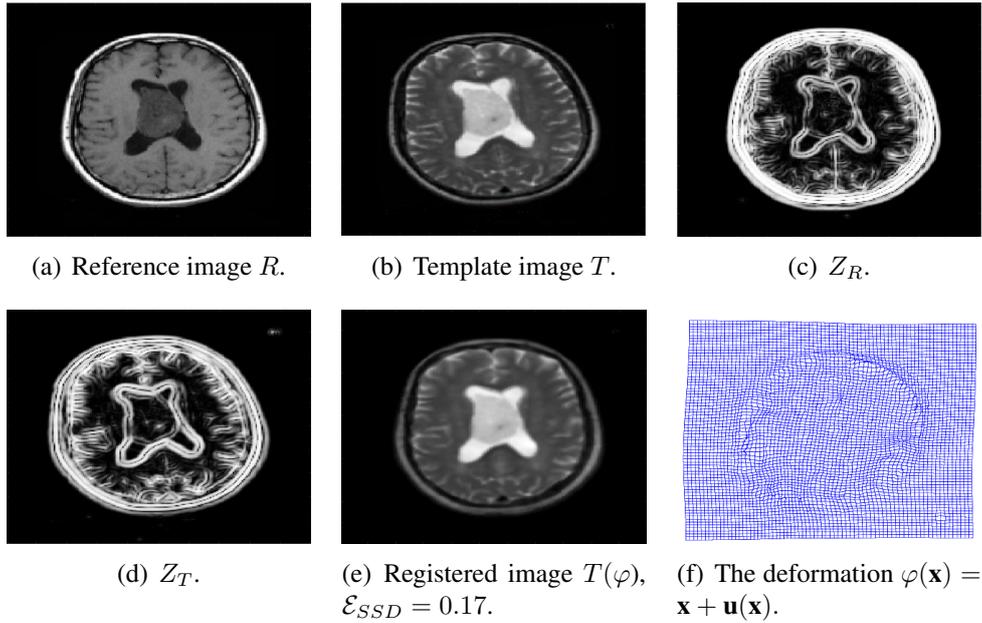


Figure 4: Example 2: Registration of two multi-modal images ( $200 \times 200$ ). These results have been shown for the following regularization parameters:

$$\alpha = 10^{-3}, \beta = 0.1, \varepsilon = 10^{-3}, \xi = 10^{-2}, \Lambda t = 0.43, \lambda = 0.5 \text{ and } \gamma = 0.03.$$

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