



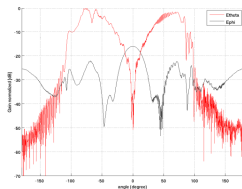
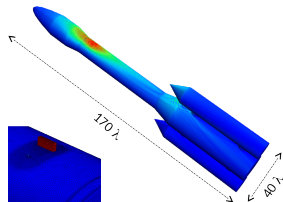
DE LA RECHERCHE À L'INDUSTRIE

## Décomposition de domaines en formulation équations intégrales surfaciques pour des problèmes de grande taille

Francis Collino | Justine Labat | Agnès Pujols | CEA-CESTA

8èmes journées d'études Électromagnétisme et Guerre Électronique  
21-23 juin 2022 à Toulouse

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
- Motivation : Accurate evaluation of **radar cross-sections**
  - ▶ Large-scale objects (in comparison with the wavelength)
  - ▶ Multi-scale phenomena

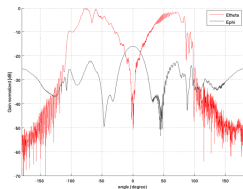
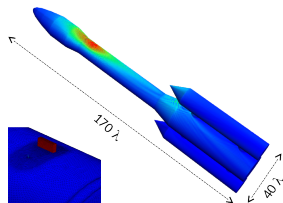


Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

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- Motivation : Accurate evaluation of **radar cross-sections**
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  - Multi-scale phenomena
- Difficulties : **High computational costs**
  - Number of degrees of freedom : 1 650 875
    - Approximately 100 000 000 in volume

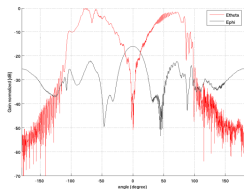
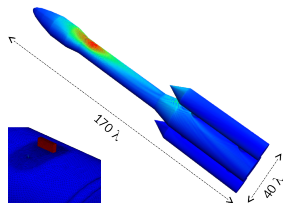
Solver	Number of CPU	Factorization (in CPU·H)
LL <sup>t</sup> (prediction)	15 000	151 435
Block low-rank	800	2 541
H-matrix	200	1 043

Hackbush (1999), Bebendorf (2008), . . .



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- Difficulties : **High computational costs**
- Objectives : Development of a **domain decomposition** method
  - ▶ Robust simulations for a **wide range** of frequencies
    - Solve very large problems (more than 10 000 000 of unknowns)
  - ▶ **Geometry-adaptive** strategies to handle multi-scale structures
    - Allow non-conformal meshes
  - ▶ Simulation-based engineering using **High Performance Computing**



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Adaptation of volumical **discontinuous Galerkin** methods to surface problems

Peng, Hiptmair and Shao (2016), Messai and Pernet (2020)

	Volume	Surface	Comments
Locality	✓	✗	Iterative solution is inevitable
Restrictions	✓	✗	Restrictions of distributions are not easy to define
Traces	✓	✗	No trace theorem in $\mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$

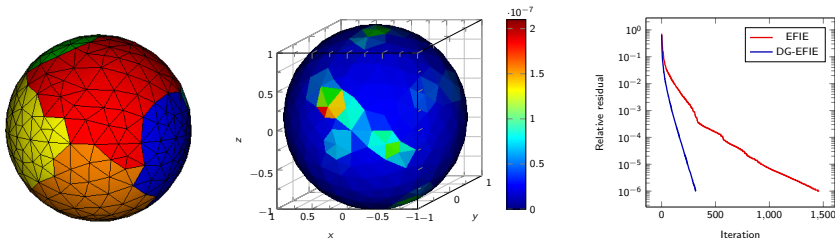
- Adaptation of volumical discontinuous Galerkin methods to surface problems

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- Why not to directly use an iterative solver on BEM ?

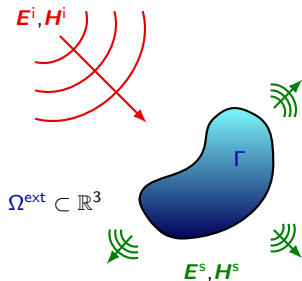
- ▶ Main motivation : non-conformal meshes
- ▶ EFIE simulation preconditioned using domain partitioning on a low-frequency case



- 1 Discontinuous Galerkin-based surface domain decomposition method
  - Model problem
  - Discontinuous formulation
  - Discrete formulation
  
- 2 Numerical results
  - Accuracy of solutions
  - Convergence of the iterative solver
  
- 3 Conclusion and perspectives

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- Using a time-harmonic dependence in  $\exp(+i\omega t)$

$$\nabla \times \mathbf{E}^s + i\kappa Z_0^{-1} \mathbf{H}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times \mathbf{H}^s - i\kappa Z_0 \mathbf{E}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

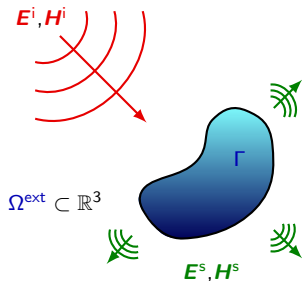
$$\mathbf{n} \times \mathbf{E}^s = -\mathbf{n} \times \mathbf{E}^i \quad \text{on } \Gamma$$

$$\lim_{|\mathbf{x}| \rightarrow \infty} |\mathbf{x}| (Z_0 \mathbf{H}^s \times \hat{\mathbf{x}} - \mathbf{E}^s) = 0 \quad \text{unif. in } \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

$\kappa$  : wave-number,  $Z_0$  : impedance coefficient in vacuum

- $\Omega^{\text{ext}}$  : Exterior domain
- $\Gamma$  : Scattering surface
- $\mathbf{E}^i, \mathbf{H}^i$  : Incident fields
- $\mathbf{E}^s, \mathbf{H}^s$  : Scattered fields
- $\mathbf{E}, \mathbf{H}$  : Total fields

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \quad \mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$$



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- Using the Stratton-Chu formulas for  $\mathbf{x} \in \Omega^{\text{ext}}$

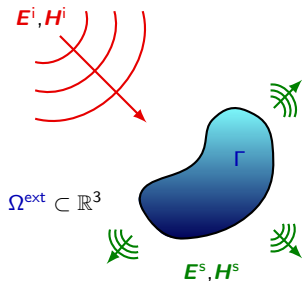
$$\mathbf{E}^s(\mathbf{x}) = -i\kappa \mathcal{T}\mathbf{J}(\mathbf{x}) \quad \text{and} \quad \mathbf{H}^s(\mathbf{x}) = \frac{1}{Z_0} \mathcal{K}\mathbf{J}(\mathbf{x})$$

$\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H})$  : total surface electric current

$$\mathcal{T}\mathbf{J} = \frac{1}{\kappa^2} \nabla (\mathcal{S} \text{div}_\Gamma \mathbf{J}) + \mathcal{S}\mathbf{J}$$

$$\mathcal{K}\mathbf{J} = \nabla \times \mathcal{S}\mathbf{J} \quad \tilde{\mathcal{S}}\lambda(\mathbf{x}) = \int_\Gamma G(\mathbf{x}, \mathbf{y}) \lambda(\mathbf{y}) \, ds_{\mathbf{y}}$$

$G$  : out-going Green function of the Helmholtz equation



- $\Omega^{\text{ext}}$  : Exterior domain
  - $\Gamma$  : Scattering surface
  - $E^i, H^i$  : Incident fields
  - $E^s, H^s$  : Scattered fields
  - $E, H$  : Total fields
- $$E = E^i + E^s \quad H = H^i + H^s$$

- Using a time-harmonic dependence in  $\exp(+i\omega t)$

$$\nabla \times E^s + i\kappa Z_0^{-1} H^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times H^s - i\kappa Z_0 E^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$n \times E^s = -n \times E^i \quad \text{on } \Gamma$$

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- Using the Stratton-Chu formulas for  $\mathbf{x} \in \Omega^{\text{ext}}$

$$E^s(\mathbf{x}) = -i\kappa T J(\mathbf{x}) \quad \text{and} \quad H^s(\mathbf{x}) = \frac{1}{Z_0} K J(\mathbf{x})$$

$J = Z_0(n \times H)$  : total surface electric current

- Jump relations give boundary integral equations

$$\text{(EFIE)} \quad i\kappa T J = n \times (E^i \times n) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2} J - K J = Z_0(n \times H^i) \quad \text{on } \Gamma$$

$$\text{(EFIE)} \quad i\kappa T\mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2}\mathbf{J} - K\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$T : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{curl}_\Gamma, \Gamma)$$

$$T\mathbf{J} = \frac{1}{\kappa^2} \nabla_\Gamma (S \text{div}_\Gamma \mathbf{J}) + S\mathbf{J}$$

$$K : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$$

$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

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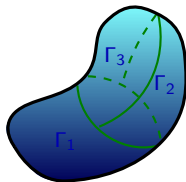
$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

- $N$  : number of subdomains

$$\bullet \Gamma = \bigcup_{n=1}^N \Gamma_n$$

$$\bullet \gamma_{nm} = \Gamma_n \cap \Gamma_m$$

$$\bullet \gamma_n = \partial\Gamma_n$$



- $\boldsymbol{\tau}_{nm}$  : exterior normal vector to  $\gamma_{nm}$

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### 1 Restriction to each subdomain

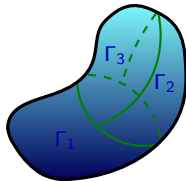
$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

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### 1 Restriction to each subdomain

$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

### 2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

$$\langle \nabla_{\Gamma} (S_{nm} \text{div}_{\Gamma} \mathbf{J}_m), \mathbf{v}_n \rangle_{\Gamma_n} =$$

$$-\langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \text{div}_{\Gamma} \mathbf{v}_n \rangle_{\Gamma_n} + \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n}$$

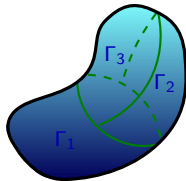
- ▶ which requires **more regularity** ( $\varepsilon > 0$ )

$$\text{div}_{\Gamma} \mathbf{J}_m \in H^{-\frac{1}{2}+\varepsilon}(\Gamma_m) \quad \text{and} \quad \mathbf{v}_n \in \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma}, \Gamma_n)$$

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### 2 Obtention of variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

### 3 Summation over all the subdomains

$$\sum_{n=1}^N \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} =$$

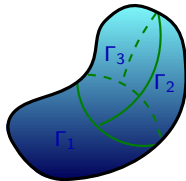
$$\sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

$$[\mathbf{v}]_{\gamma_{nm}} = \boldsymbol{\tau}_{nm} \cdot \mathbf{v}_m + \boldsymbol{\tau}_{mn} \cdot \mathbf{v}_n : \text{jump across } \gamma_{nm}$$

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Find  $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{H}_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$  with  $\varepsilon > 0$  such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$$p_{\gamma}^*(\mathbf{J}, \mathbf{v}) \text{ is function of } [\mathbf{J}]_{\gamma_{nm}} \text{ and } [\mathbf{v}]_{\gamma_{nm}} \quad (\text{penalization})$$

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$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$p_{\gamma}^*(\mathbf{J}, \mathbf{v})$  is function of  $[\mathbf{J}]_{\gamma_{nm}}$  and  $[\mathbf{v}]_{\gamma_{nm}}$  (penalization)

$$(S) \quad \tilde{a}_{\gamma}^{-}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

$$(AS) \quad \tilde{a}_{\gamma}^{+}(\mathbf{J}, \mathbf{v}) = +\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

► Requires more regularity :  $\varepsilon = \frac{1}{2}$

$$\mathbf{J} \text{ and } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

Find  $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$  such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^*(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

- Empirical choice coming from Peng, Hiptmair and Shao (2016) :  $L^2(\gamma)$ -inner product

$$(\mathbf{L}^2) \quad p_{\gamma}^0(\mathbf{J}, \mathbf{v}) = \frac{\beta}{\kappa} \sum_{\gamma_{nm}} \langle [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where  $\beta$  have to be without dimension

- ▶ requires **more regularity** (one more time!) to make sense :  $\mathbf{J}$  and  $\mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$

- A new penalization :  $H^{-\frac{1}{2}}(\gamma)$ -inner product (positive definite bilinear form)

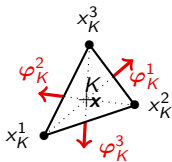
$$(\mathbf{H}^{-\frac{1}{2}}) \quad p_{\gamma}^{-\frac{1}{2}}(\mathbf{J}, \mathbf{v}) = \beta \sum_{\gamma_{nm}} \langle S_{\gamma_{nm}}[\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where  $\tilde{S}_{\gamma} \lambda(\mathbf{x}) = \frac{1}{2\pi} \int_{\gamma} K_0(\kappa|\mathbf{x} - \mathbf{y}|) \lambda(\mathbf{y}) d\sigma_{\mathbf{y}}$  and  $\beta$  have to be without dimension

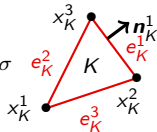
- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \Longleftrightarrow \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$$

where  $\mathbf{J}_m^h(\mathbf{x}) = \sum_{K \in \Gamma_m^h} J_{K,1}^{h,m} \varphi_K^1(\mathbf{x}) + J_{K,2}^{h,m} \varphi_K^2(\mathbf{x}) + J_{K,3}^{h,m} \varphi_K^3(\mathbf{x}) \quad \mathbf{x} \in \Gamma_m^h$  (triangulation of  $\Gamma_m$ )



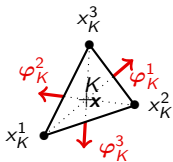
with  $\varphi_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - x_K^i)$  and  $J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i d\sigma$



- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

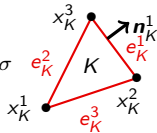
$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \iff \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$$

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with  $\varphi_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - x_K^i)$

and  $J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i d\sigma$



Find  $\mathbf{J}^h \in \mathcal{V}^h$  such that for any  $\mathbf{v}^h \in \mathcal{V}^h$

(DG-EFIE)  $a_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) + a_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) + p_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) = \ell^h(\mathbf{v}^h)$

(DG-MFIE)  $b_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) = \tilde{\ell}^h(\mathbf{v}^h)$

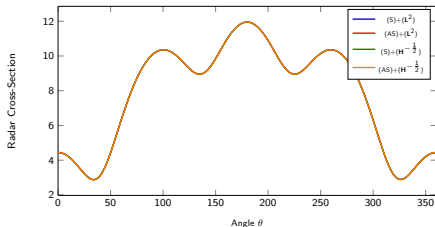
(from continuous MFIE)

(DG-CFIE)  $\alpha$  (DG-EFIE) +  $(1 - \alpha)$  (DG-MFIE)

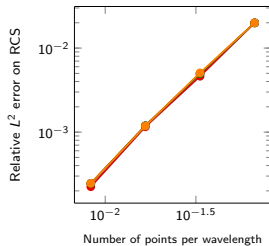
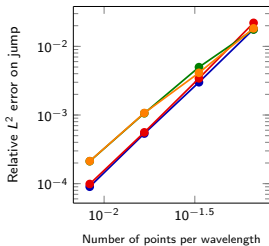
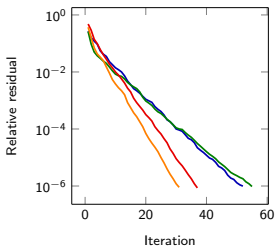
(with  $0 < \alpha < 1$ )

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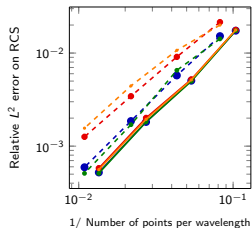
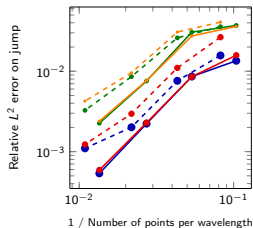
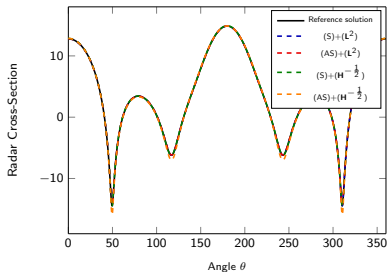
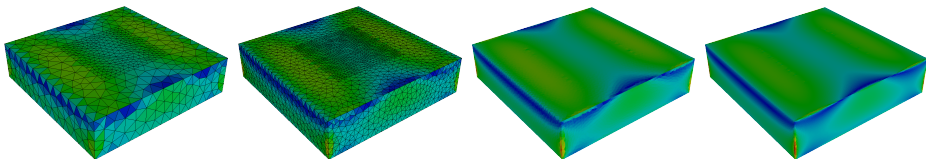
- PEC Sphere of radius 1m at 95MHz : currents and radar cross-section



Convergence history for (DG-CFIE) -  $\alpha = 0.5$

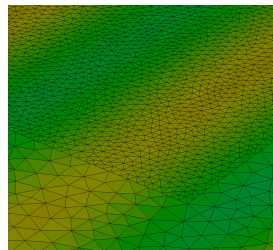
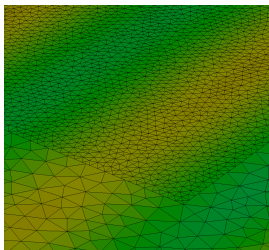
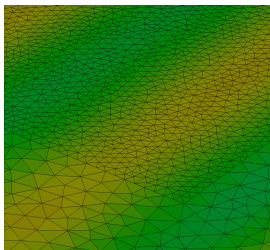
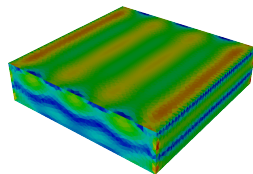
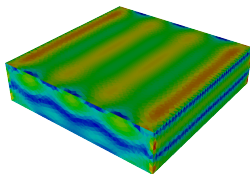
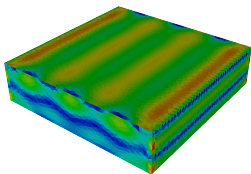


- Surface electric currents : (DG-CFIE) with  $\alpha = 0.5$  at 400 MHz





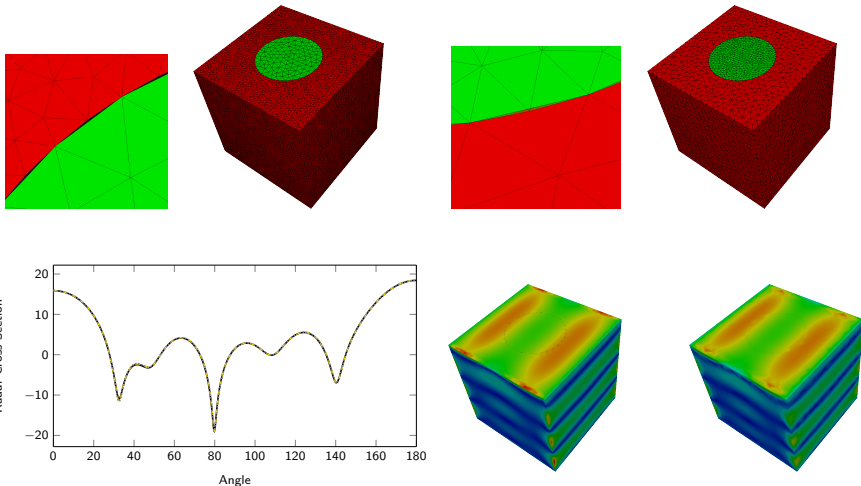
- Surface electric currents : (DG-CFIE) with  $\alpha = 0.5$  at 1GHz



1 triangle in front of 2 triangles    2 triangles in front of 5 triangles

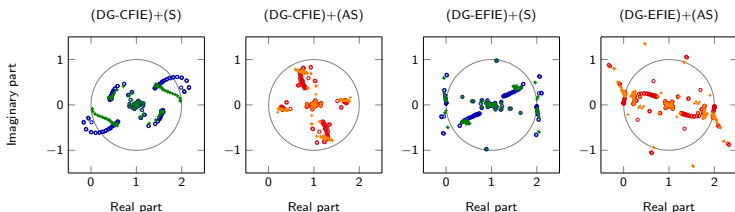
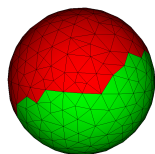
Completely non-regular

- Comparison of electric currents on the PEC cube of side 1 m : Non-conformal (coarse) meshes at 500 MHz using 2 sub-domains

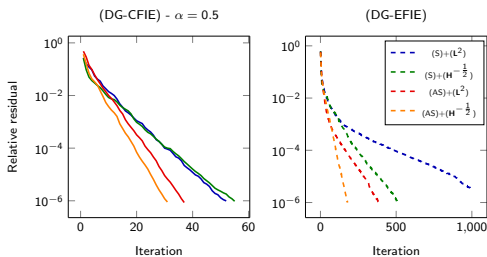


- How to define correctly penalization function on this kind of interfaces?

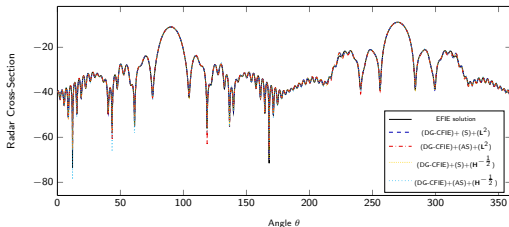
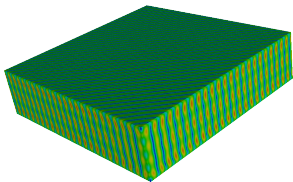
- Sphere of radius 1 m - Frequency  $f = 95$  MHz - Discretization  $h = \frac{\lambda}{15}$  - 2 sub-domains



- Eigenvalue distribution strongly depends on the penalization parameter  $\beta$ 
  - ▶ For  $L^2$ , here  $\beta = \frac{1}{10} |\log \kappa h|$
  - ▶ For  $H^{-\frac{1}{2}}$ , here  $\beta = 1$
- Convergence of EFIE is degraded by using  $\text{restart} = 10$



- Convergence of the iterative solver on the PEC parallelepiped of dimension  $1\text{m} \times 1\text{m} \times 0.25\text{m}$  at 5 GHz enlightened with angle  $(\frac{\pi}{2}, \frac{\pi}{4})$  with  $h_{\text{moy}} = \frac{\lambda}{11}$  and using 4 subdomains
  - $408\,519 + 1\,179 = 409\,698$  degrees of freedom



- Classic simulation duration : Assembling time : 1 750.78s. Factorization time : 5 145.83s.
  - (DG-CFIE) with  $\alpha = 0.5$  : GMRes with restart = 50 and relative residual  $\epsilon = 10^{-3}$

Formulation	$(S)+(L^2)$	$(AS)+(L^2)$	$(S)+(H^{-\frac{1}{2}})$	$(AS)+(H^{-\frac{1}{2}})$
Assembling time (s)	1 762.02	1 767.78	1 780.59	1 767.02
Factorization time (s)	340.41	343.21	343.73	340.61
GMRes time (s)	2 268.18	1 707.54	6 126.59	2 407.10
Number of iterations	13	10	28	14
Jump error	$1.97 \cdot 10^{-3}$	$2.62 \cdot 10^{-3}$	$8.22 \cdot 10^{-3}$	$8.02 \cdot 10^{-3}$
RCS error	$1.82 \cdot 10^{-2}$	$2.94 \cdot 10^{-2}$	$1.85 \cdot 10^{-2}$	$2.93 \cdot 10^{-2}$

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Formulation (S)+(L<sup>2</sup>) (AS)+(L<sup>2</sup>) (S)+(H<sup>-1/2</sup>) (AS)+(H<sup>-1/2</sup>)

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- ▶ (DG-CFIE) with  $\alpha = 0.5$  : GMRes with restart = 50 and relative residual  $\varepsilon = 10^{-6}$

GMRes time (s)	10 167.97	7 349.08	17 625.89	6 149.22
Number of iterations	41	29	73	39
Jump error	$1.90 \cdot 10^{-3}$	$2.58 \cdot 10^{-3}$	$8.18 \cdot 10^{-3}$	$8.01 \cdot 10^{-3}$
RCS error	$1.82 \cdot 10^{-2}$	$2.29 \cdot 10^{-2}$	$1.82 \cdot 10^{-2}$	$2.94 \cdot 10^{-2}$

- ▶ (DG-CFIE) with  $\alpha = 0.9$  : GMRes with restart = 10 and relative residual  $\varepsilon = 10^{-6}$

GMRes time (s)	72 157.85	> 100 000	23 916.78	> 100 000
Number of iterations	415	> 300	140	> 50
Jump error	$1.57 \cdot 10^{-3}$	–	$5.19 \cdot 10^{-3}$	–
RCS error	$3.88 \cdot 10^{-3}$	–	$3.93 \cdot 10^{-3}$	–

- ▶ (DG-EFIE) formulation : GMRes with restart = 50 and relative residual  $\varepsilon = 10^{-3}$

GMRes time (s)	> 100 000	> 100 000	64 010.34	> 100 000
Number of iterations	> 350	> 300	236	> 350
Jump error	–	–	$5.05 \cdot 10^{-3}$	–
RCS error	–	–	$3.79 \cdot 10^{-3}$	–

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## Conclusion

- Development and numerical analysis of a discontinuous Galerkin surface domain decomposition method for electromagnetic scattering by non-penetrable objects
  - ▶ Comparison of *symmetric* and *anti-symmetric* formulations
  - ▶ Comparison of  $L^2$  and  $H^{-\frac{1}{2}}$  interior penalty terms
- Comparison with a boundary element method
  - ▶ Computational costs
  - ▶ Accuracy : Jump *does not pollute* radar cross-sections

## Perspectives

- Integration of *H-matrix* formalism
- Extension to *dielectric* scatterers

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Thank you for your attention !