



DE LA RECHERCHE À L'INDUSTRIE

Décomposition de domaine surfacique de type DG pour la diffraction d'ondes électromagnétiques

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- Solution of **electromagnetic** wave scattering problems by **complex** objects in the free space
 - ▶ Very large-scale targets in comparison with the wavelength λ
 - ▶ Highly multiscale phenomena due to the presence of small geometrical details of size $\varepsilon \ll \lambda$
 - ▶ Multilayer structures with high contrast in dielectric constants



Radar applications

Ultra High Frequency
300MHz to 3GHz

Super High Frequency
3GHz to 30GHz



Launcher *Ariane 6*

Dimensions
5m \times 5m \times 65m

In UHF band e.g. 800MHz
 $\sim 10\lambda \times 10\lambda \times 170\lambda$



Aircraft carrier *Charles de Gaulle*

Dimensions
261m \times 64m \times 75m

In UHF band e.g. 400MHz
 $\sim 350\lambda \times 100\lambda \times 100\lambda$

- Evaluation of radar cross-sections σ in the context of **electromagnetic stealth**

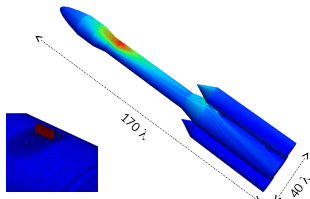
$$\sigma(\theta, \varphi) = \lim_{R \rightarrow \infty} 4\pi R^2 \frac{|\mathbf{E}^s|^2}{|\mathbf{E}^i|^2}$$

- ▶ High attention to **accuracy**

- Solution using a **boundary element method**

- ▶ Take into account the radiation condition at infinity in an exact way
- ▶ Only requires surface meshes (per interface between two homogeneous media)
- ▶ Involves **dense** matrices \implies Dense solvers \implies **High computational costs**

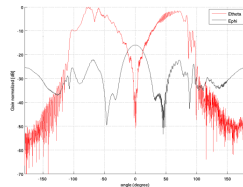
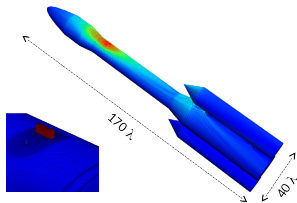
Domain length	Number of mesh points
10λ	10 000
100λ	1 000 000
1000λ	100 000 000
$10\,000\lambda$	10 000 000 000



Number of degrees of freedom : 1 650 875

- Even in the context of modern massively parallel architectures, the most advanced direct solvers reach their limitations in terms of **computational costs**

Monopole antenna
in the presence of
a dielectric object
on a launcher



Solver	Number of CPU	Factorization (in CPU·H)
LL ^t (prediction)	15 000	151 435
Block low-rank	800	2 541
H-matrix	200	1 043

Hierarchical compression allows to consider cases of **few tens of millions of unknowns** on thousands of cores

- Development of **domain decomposition methods** for boundary integral equations



Z. Peng, K.-H. Lim, J.-F. Lee

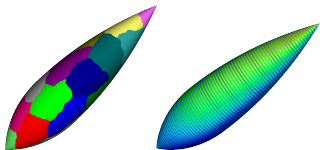
A discontinuous Galerkin surface integral equation method for electromagnetic wave scattering from nonpenetrable targets.
IEEE transactions on antennas and propagation, 2013, vol. 61, no 7, p. 3617-3628.



Z. Peng, R. Hiptmair, Y. Shao, B. MacKie-Mason

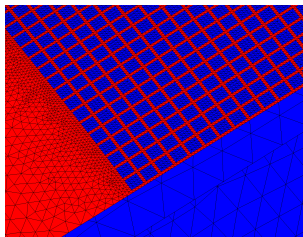
Domain decomposition preconditioning for surface integral equations in solving challenging electromagnetic scattering problems.
IEEE transactions on antennas and propagation, 2015, vol. 64, no 1, p. 210-223.

- Solve larger problems using an iterative solver



- ▶ Factorization of a collection of smaller matrices
- ▶ Flexibility in choosing subdomain solver
- ▶ What about the convergence of the iterative solver ?

- Geometry-adaptative strategies to handle mutliscale structures



- ▶ Avoid local mesh refinement techniques
- ▶ Switch drastically from coarse mesh (λ) to fine mesh (ε)
- ▶ Generate sub-meshes independently can cause holes or overlaps in the overall mesh particularly on curved interfaces

- Simulation-based engineering using High Performance Computing

- Adaptation of volumical discontinuous Galerkin methods to surface problems

	Volume	Surface
Locality	✓	✗
Restrictions	✓	✗
Traces	✓	✗

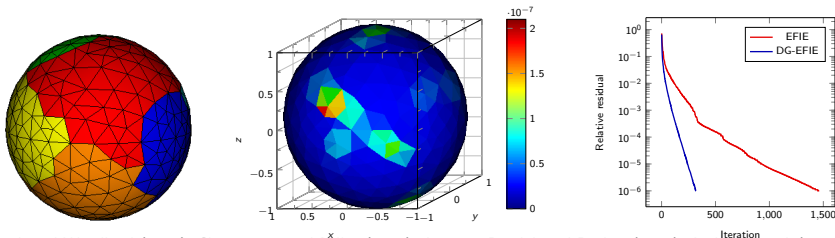


N.-A. Messai, S. Pernet

hp non-conforming *a priori* error analysis of an Interior Penalty Discontinuous Galerkin BEM for the Helmholtz equation.
Computers & Mathematics with Applications, 2020, vol. 80, no 12, p. 2644-2675.

- Off-diagonal dense blocks storage is inevitable in the context of iterative solution
- Theory is not available in the classical mathematical framework associated with surface electric currents
 - Restrictions of distributions are not easy to define
 - No trace theorem in $\mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$

- Preconditioning is a difficult task for classical boundary element methods



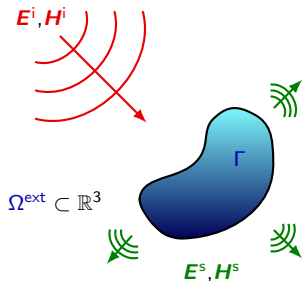
Steinbach and Wendland (1998), Christiansen and Nédélec (2000), Antoine, Bendali and Darbas (2005), Andriulli *et al.* (2008) . . .

- 1 Discontinuous Galerkin-based surface domain decomposition method
 - Model problem
 - Discontinuous formulation
 - Discrete formulation
 - Iterative procedure

- 2 Numerical results
 - Accuracy of solutions
 - Convergence of the iterative solver

- 3 Conclusion and perspectives

- 1 Discontinuous Galerkin-based surface domain decomposition method
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 - Accuracy of solutions
 - Convergence of the iterative solver
- 3 Conclusion and perspectives



- Using a time-harmonic dependence in $\exp(+i\omega t)$

$$\nabla \times \mathbf{E}^s + i\kappa Z_0^{-1} \mathbf{H}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\nabla \times \mathbf{H}^s - i\kappa Z_0 \mathbf{E}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

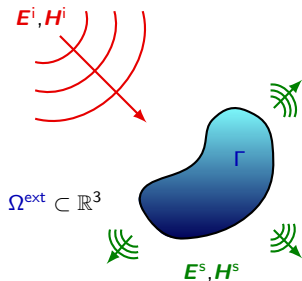
$$\mathbf{n} \times \mathbf{E}^s = -\mathbf{n} \times \mathbf{E}^i \quad \text{on } \Gamma$$

$$\lim_{|\mathbf{x}| \rightarrow \infty} |\mathbf{x}| (Z_0 \mathbf{H}^s \times \hat{\mathbf{x}} - \mathbf{E}^s) = 0 \quad \text{unif. in } \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

κ : wave-number, Z_0 : impedance coefficient in vacuum

- Ω^{ext} : Exterior domain
- Γ : Scattering surface
- $\mathbf{E}^i, \mathbf{H}^i$: Incident fields
- $\mathbf{E}^s, \mathbf{H}^s$: Scattered fields
- \mathbf{E}, \mathbf{H} : Total fields

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \quad \mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$$



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- Using the Stratton-Chu formulas for $\mathbf{x} \in \Omega^{\text{ext}}$

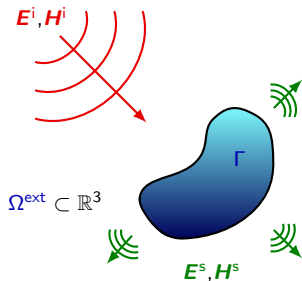
$$\mathbf{E}^s(\mathbf{x}) = -i\kappa \mathcal{T} \mathbf{J}(\mathbf{x}) \quad \text{and} \quad \mathbf{H}^s(\mathbf{x}) = \frac{1}{Z_0} \mathcal{K} \mathbf{J}(\mathbf{x})$$

$\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H})$: total surface electric current

$$\mathcal{T} \mathbf{J} = \frac{1}{\kappa^2} \nabla (\mathcal{S} \text{div}_\Gamma \mathbf{J}) + \mathcal{S} \mathbf{J}$$

$$\mathcal{K} \mathbf{J} = \nabla \times \mathcal{S} \mathbf{J} \quad \tilde{\mathcal{S}} \lambda(\mathbf{x}) = \int_\Gamma G(\mathbf{x}, \mathbf{y}) \lambda(\mathbf{y}) \, ds_{\mathbf{y}}$$

G : out-going Green function of the Helmholtz equation



- Ω^{ext} : Exterior domain
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 - $\mathbf{E}^i, \mathbf{H}^i$: Incident fields
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- Jump relations give boundary integral equations

$$\text{(EFIE)} \quad i\kappa \mathcal{T} \mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2} \mathbf{J} - \mathcal{K} \mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$\text{(EFIE)} \quad i\kappa T\mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$\text{(MFIE)} \quad \frac{1}{2}\mathbf{J} - K\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$T : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{curl}_\Gamma, \Gamma)$$

$$T\mathbf{J} = \frac{1}{\kappa^2} \nabla_\Gamma (S \text{div}_\Gamma \mathbf{J}) + S\mathbf{J}$$

$$K : \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$$

$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

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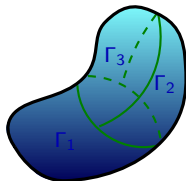
$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

- N : number of subdomains

$$\bullet \Gamma = \bigcup_{n=1}^N \Gamma_n$$

$$\bullet \gamma_{nm} = \Gamma_n \cap \Gamma_m$$

$$\bullet \gamma_n = \partial\Gamma_n$$



- $\boldsymbol{\tau}_{nm}$: exterior normal vector to γ_{nm}
- $\boldsymbol{\tau}_n$: exterior normal vector to γ_n

$$i\kappa \sum_{m=1}^N T_{nm} \mathbf{J}_m = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma_n$$

$$\frac{1}{2} \mathbf{J}_n - \sum_{m=1}^N K_{nm} \mathbf{J}_m = Z_0 (\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma_n$$

$$T_{nm} : \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m) \longrightarrow \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{curl}_{\Gamma_n}, \Gamma_n)$$

$$K_{nm} : \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m) \longrightarrow \tilde{\mathbf{H}}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$T_{nm} \mathbf{J}_m = \frac{1}{\kappa^2} \nabla_{\Gamma_n} (S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m) + S_{nm} \mathbf{J}_m$$

$$K_{nm} \mathbf{J}_m = \text{p.v.} [\mathbf{n} \times (\nabla_{\Gamma_n} \times S_{nm} \mathbf{J}_m)]$$

1 Restriction to each subdomain

- ▶ What about the regularity of restrictions?

2 Obtention of local variational formulations

- ▶ Multiplication by a test-function
- ▶ Integration on partial surfaces
- ▶ Integration by parts (EFIE only)

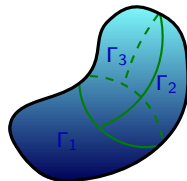
3 Summation over all the subdomains

- N : number of subdomains

$$\bullet \Gamma = \bigcup_{n=1}^N \Gamma_n$$

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- $\boldsymbol{\tau}_{nm}$: exterior normal vector to γ_{nm}

- $\boldsymbol{\tau}_n$: exterior normal vector to γ_n

- 1 Restriction $\mathbf{J}_m = \mathbf{J}|_{\Gamma_m}$ to each subdomain is not trivial

$$\mathbf{J}_m \notin \mathbf{H}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

- 2 Integration by parts governs the required regularity : For

$$\text{div}_{\Gamma} \mathbf{J}_m \in H^{-\frac{1}{2}+\varepsilon}(\Gamma_m) \quad \text{and} \quad \mathbf{v}_n \in \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma}, \Gamma_n)$$

with $\varepsilon > 0$, we have

$$\langle \nabla_{\Gamma} (S_{nm} \text{div}_{\Gamma} \mathbf{J}_m), \mathbf{v}_n \rangle_{\Gamma_n} = - \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \text{div}_{\Gamma} \mathbf{v}_n \rangle_{\Gamma_n} + \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n}$$

- 3 Summation over all the subdomains then shows the jump of the currents at the interfaces

$$\text{For } m = 1, \dots, N \quad \sum_{n=1}^N \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} = \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where $[\mathbf{v}]_{\gamma_{nm}} = \boldsymbol{\tau}_{nm} \cdot \mathbf{v}_m + \boldsymbol{\tau}_{mn} \cdot \mathbf{v}_n$

Find $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{H}_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$ with $\varepsilon > 0$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$$p_{\gamma}^*(\mathbf{J}, \mathbf{v}) \text{ is function of } [\mathbf{J}]_{\gamma_{nm}} \text{ and } [\mathbf{v}]_{\gamma_{nm}} \quad (\text{penalization})$$

Find $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{H}_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$ with $\varepsilon > 0$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

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$p_{\gamma}^*(\mathbf{J}, \mathbf{v})$ is function of $[\mathbf{J}]_{\gamma_{nm}}$ and $[\mathbf{v}]_{\gamma_{nm}}$ (penalization)

$$(S) \quad \tilde{a}_{\gamma}^{-}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

$$(AS) \quad \tilde{a}_{\gamma}^{+}(\mathbf{J}, \mathbf{v}) = +\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

► Requires more regularity : $\varepsilon = \frac{1}{2}$

$$\mathbf{J} \text{ and } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

Find $\mathbf{J} \in \bigoplus_{m=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^*(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N \mathbf{L}_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

- Empirical choice coming from Peng, Hiptmair and Shao (2016) : $L^2(\gamma)$ -inner product

$$(\mathbf{L}^2) \quad p_{\gamma}^0(\mathbf{J}, \mathbf{v}) = \frac{\beta}{\kappa} \sum_{\gamma_{nm}} \langle [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where β have to be without dimension

- requires **more regularity** to make sense : \mathbf{J} and $\mathbf{v} \in \bigoplus_{n=1}^N \mathbf{H}_t^{\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$

- A new penalization : $H^{-\frac{1}{2}}(\gamma)$ -inner product (positive definite bilinear form)

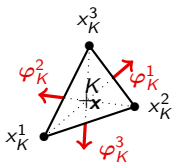
$$(\mathbf{H}^{-\frac{1}{2}}) \quad p_{\gamma}^{-\frac{1}{2}}(\mathbf{J}, \mathbf{v}) = \beta \sum_{\gamma_{nm}} \langle \mathcal{S}_{\gamma_{nm}}[\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where $\tilde{\mathcal{S}}_{\gamma} \lambda(\mathbf{x}) = \frac{1}{2\pi} \int_{\gamma} K_0(\kappa|\mathbf{x} - \mathbf{y}|) \lambda(\mathbf{y}) d\sigma_{\mathbf{y}}$ and β have to be without dimension

- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

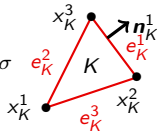
$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \Longleftrightarrow \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$$

where $\mathbf{J}_m^h(\mathbf{x}) = \sum_{K \in \Gamma_m^h} J_{K,1}^{h,m} \varphi_K^1(\mathbf{x}) + J_{K,2}^{h,m} \varphi_K^2(\mathbf{x}) + J_{K,3}^{h,m} \varphi_K^3(\mathbf{x}) \quad \mathbf{x} \in \Gamma_m^h$ (triangulation of Γ_m)



with $\varphi_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - \mathbf{x}_K^i)$ and

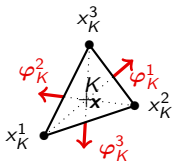
$$J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i \, d\sigma$$



- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \iff \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$$

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with $\varphi_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - x_K^i)$ and

$$J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i \, d\sigma$$

Find $\mathbf{J}^h \in \mathcal{V}^h$ such that for any $\mathbf{v}^h \in \mathcal{V}^h$

(DG-EFIE) $a_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) + a_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) + p_{\gamma}^{*,h}(\mathbf{J}^h, \mathbf{v}^h) = \ell^h(\mathbf{v}^h)$

(DG-MFIE) $b_{\Gamma}^h(\mathbf{J}^h, \mathbf{v}^h) = \tilde{\ell}^h(\mathbf{v}^h)$

(from continuous MFIE)

(DG-CFIE) α (DG-EFIE) + $(1 - \alpha)$ (DG-MFIE)

(with $0 < \alpha < 1$)

- In conformal cases, the discrete jump operator between two adjacent triangles is simply defined by

$$[\mathbf{v}_h]_{e^+} = [\mathbf{v}_h]_{e^-} = \mathbf{v}_h^+ \cdot \boldsymbol{\tau} - \mathbf{v}_h^- \cdot \boldsymbol{\tau} = \frac{1}{|e^+|} (\text{ddl}^+ - \text{ddl}^-)$$

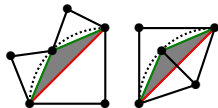
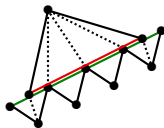
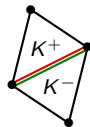
where $\mathbf{v}_h^+ = \mathbf{v}_h|_{K^+}$ and $\mathbf{v}_h^- = \mathbf{v}_h|_{K^-}$

- The discontinuous formulations remain valid when the local triangulations **do not match** at boundaries between adjacent subdomains

$$[\mathbf{v}_h]_e = \sum_{i=1}^{N(K^+)} \left\{ \mathbf{v}_h^+ \cdot \boldsymbol{\tau} - \mathbf{v}_{h,i}^- \cdot \boldsymbol{\tau} \right\} = \sum_{i=1}^{N(K^+)} \left\{ \frac{1}{|e^+|} \text{ddl}^+ - \frac{1}{|e_i^-|} \text{ddl}_i^- \right\}$$

where $N(K^+)$ is the number of neighbors of K^+ and $\mathbf{v}_{h,i}^- = \mathbf{v}_h|_{K_i^-}$

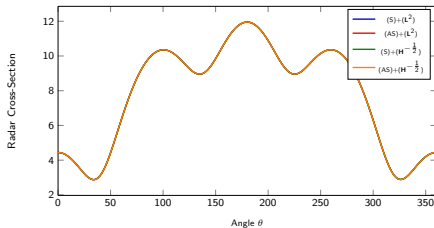
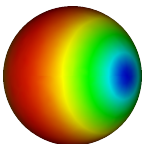
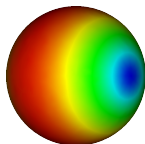
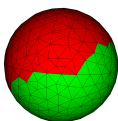
- On **curved** interfaces, the discrete jump operator can cause difficulties to be defined



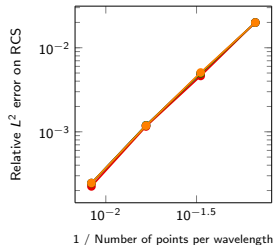
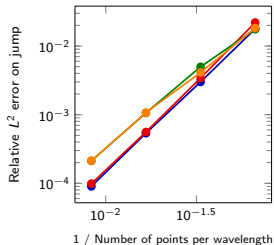
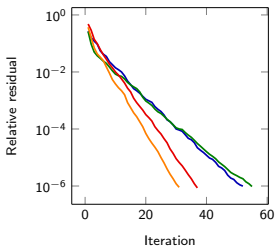
- Use of **GMRes solver** with **block-diagonal Jacobi preconditioning**
 - ▶ Diagonal blocks correspond to matrices for individual subdomain
 - Flexibility in choosing subdomain solvers (LL^t , LU, Block low-rank, H-matrix, . . .)
 - ▶ Off-diagonal blocks correspond to interactions between subdomains
 - Could be sped-up by compression techniques
 - ▶ No theoretical result on the convergence of the iterative solver
 - Condition number of the matrix? Eigenvalue distribution?
 - Impact of physical and numerical parameters?
 - Impact of penalization term?

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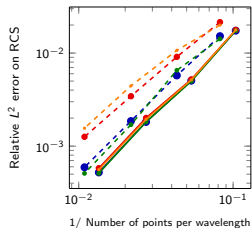
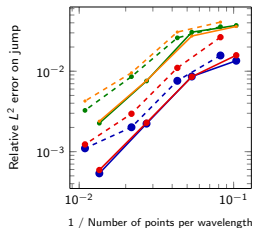
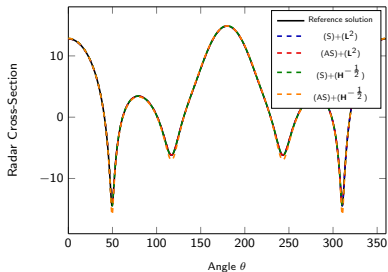
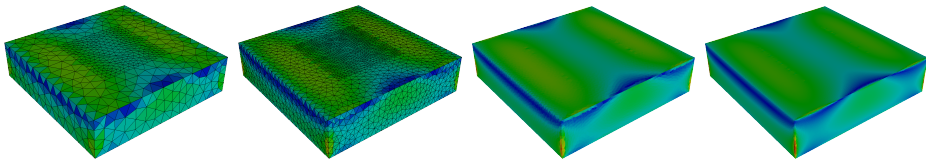
- PEC Sphere of radius 1m at 95MHz : currents and radar cross-section



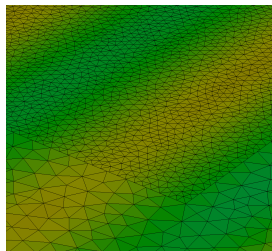
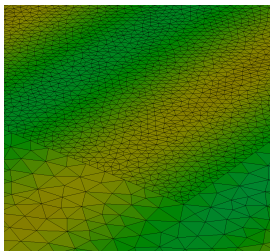
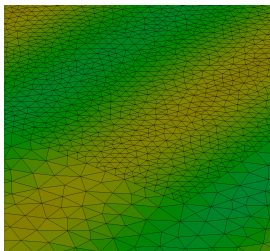
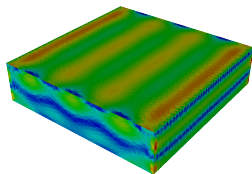
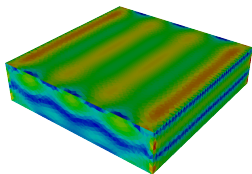
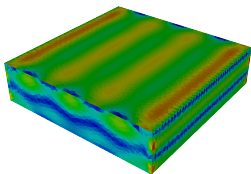
Convergence history for (DG-CFIE) - $\alpha = 0.5$



- Surface electric currents : (DG-CFIE) with $\alpha = 0.5$ at 400 MHz



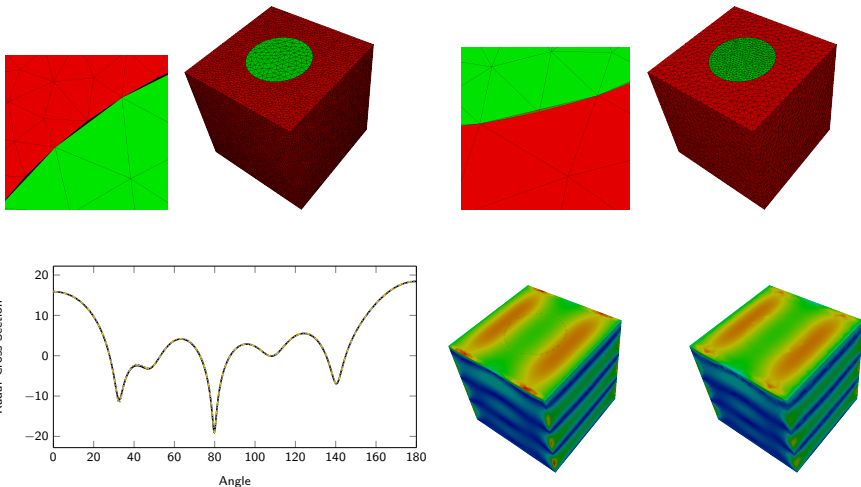
- Surface electric currents : (DG-CFIE) with $\alpha = 0.5$ at 1 GHz



1 triangle in front of 2 triangles 2 triangles in front of 5 triangles

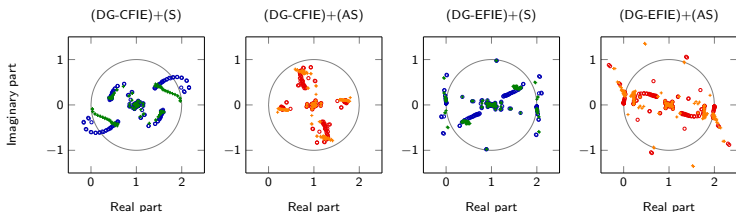
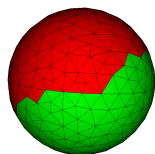
Completely non-regular

- Comparison of electric currents on the PEC cube of side 1 m : Non-conformal (coarse) meshes at 500 MHz using 2 subdomains

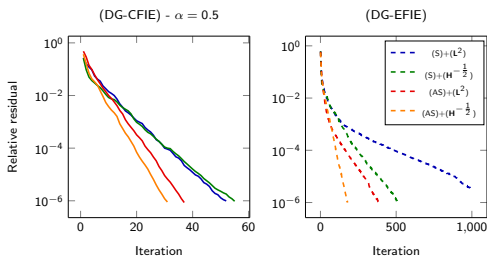


- How to define correctly penalization function on this kind of interfaces ?

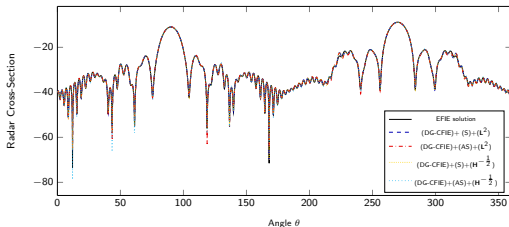
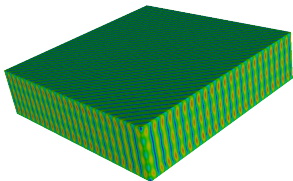
- Sphere of radius 1 m - Frequency $f = 95$ MHz - Discretization $h = \frac{\lambda}{15}$ - 2 subdomains



- Eigenvalue distribution strongly depends on the penalization parameter β
 - ▶ For L^2 , here $\beta = \frac{1}{10} |\log \kappa h|$
 - ▶ For $H^{-\frac{1}{2}}$, here $\beta = 1$
- Convergence of EFIE is degraded by using $\text{restart} = 10$



- Convergence of the iterative solver on the PEC parallelepiped of dimension $1\text{m} \times 1\text{m} \times 0.25\text{m}$ at 5 GHz enlightened with angle $(\frac{\pi}{2}, \frac{\pi}{4})$ with $h_{\text{moy}} = \frac{\lambda}{11}$ and using 4 subdomains
 - $408\,519 + 1\,179 = 409\,698$ degrees of freedom



- Classic simulation duration : Assembling time : 1 750.78s. Factorization time : 5 145.83s.
 - (DG-CFIE) with $\alpha = 0.5$: GMRes with restart = 50 and relative residual $\epsilon = 10^{-3}$

Formulation	$(S)+(L^2)$	$(AS)+(L^2)$	$(S)+(H^{-\frac{1}{2}})$	$(AS)+(H^{-\frac{1}{2}})$
Assembling time (s)	1 762.02	1 767.78	1 780.59	1 767.02
Factorization time (s)	340.41	343.21	343.73	340.61
GMRes time (s)	2 268.18	1 707.54	6 126.59	2 407.10
Number of iterations	13	10	28	14
Jump error	$1.97 \cdot 10^{-3}$	$2.62 \cdot 10^{-3}$	$8.22 \cdot 10^{-3}$	$8.02 \cdot 10^{-3}$
RCS error	$1.82 \cdot 10^{-2}$	$2.94 \cdot 10^{-2}$	$1.85 \cdot 10^{-2}$	$2.93 \cdot 10^{-2}$

- Classic simulation duration : Assembling time : 1 750.78s. Factorization time : 5 145.83s.

Formulation (S)+(L²) (AS)+(L²) (S)+(H^{-1/2}) (AS)+(H^{-1/2})

- ▶ (DG-CFIE) with $\alpha = 0.5$: GMRes with restart = 50 and relative residual $\varepsilon = 10^{-3}$

GMRes time (s)	2 268.18	1 707.54	6 126.59	2 407.10
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- ▶ (DG-CFIE) with $\alpha = 0.5$: GMRes with restart = 50 and relative residual $\varepsilon = 10^{-6}$

GMRes time (s)	10 167.97	7 349.08	17 625.89	6 149.22
Number of iterations	41	29	73	39
Jump error	$1.90 \cdot 10^{-3}$	$2.58 \cdot 10^{-3}$	$8.18 \cdot 10^{-3}$	$8.01 \cdot 10^{-3}$
RCS error	$1.82 \cdot 10^{-2}$	$2.29 \cdot 10^{-2}$	$1.82 \cdot 10^{-2}$	$2.94 \cdot 10^{-2}$

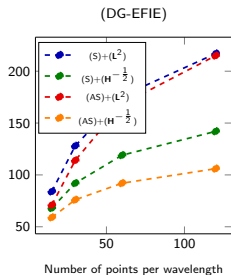
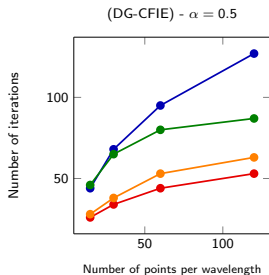
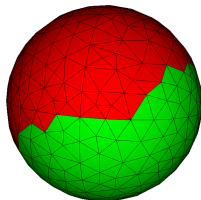
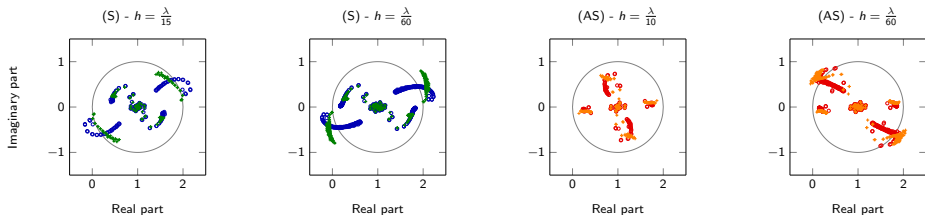
- ▶ (DG-CFIE) with $\alpha = 0.9$: GMRes with restart = 10 and relative residual $\varepsilon = 10^{-6}$

GMRes time (s)	72 157.85	> 100 000	23 916.78	> 100 000
Number of iterations	415	> 300	140	> 50
Jump error	$1.57 \cdot 10^{-3}$	-	$5.19 \cdot 10^{-3}$	-
RCS error	$3.88 \cdot 10^{-3}$	-	$3.93 \cdot 10^{-3}$	-

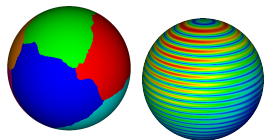
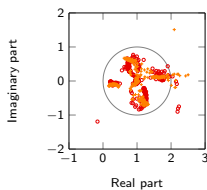
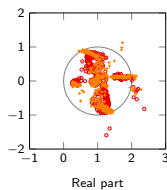
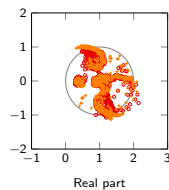
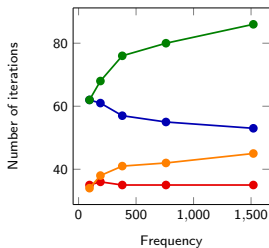
- ▶ (DG-EFIE) formulation : GMRes with restart = 50 and relative residual $\varepsilon = 10^{-3}$

GMRes time (s)	> 100 000	> 100 000	64 010.34	> 100 000
Number of iterations	> 350	> 300	236	> 350
Jump error	-	-	$5.05 \cdot 10^{-3}$	-
RCS error	-	-	$3.79 \cdot 10^{-3}$	-

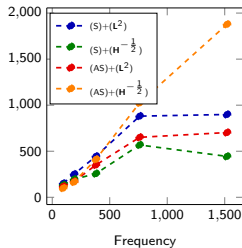
- Sphere of radius 1 m - Frequency $f = 95$ MHz - 2 subdomains - (DG-CFIE) formulation



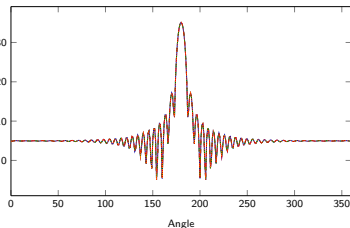
- Sphere of radius 1 m - Discretization $h = \frac{\lambda}{15}$ - 8 subdomains (1.52GHz)

(DG-CFIE)+(AS) - $f = 95$ MHz(DG-CFIE)+(AS) - $f = 190$ MHz(DG-CFIE)+(AS) - $f = 380$ MHz(DG-CFIE) - $\alpha = 0.5$ 

(DG-EFIE)



Radar Cross-Section



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Conclusion

- Development and numerical analysis of a discontinuous Galerkin surface domain decomposition method for electromagnetic scattering by non-penetrable objects
 - ▶ Comparison of *symmetric* and *anti-symmetric* formulations
 - ▶ Comparison of L^2 and $H^{-\frac{1}{2}}$ interior penalty terms
- Comparison with a boundary element method
 - ▶ Computational costs
 - ▶ Accuracy : Jump *does not pollute* radar cross-sections

Perspectives

- Integration of *H-matrix* formalism
- Extension to *dielectric* scatterers

Conclusion

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Thank you for your attention !