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Computation of Geometric Series in Different Ways

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Abstract: This paper present computation of the summation of geometric series in an innovative way. Geometric series is a computational method which is used for numerous applications in science, engineering, and medicine. The geometric series and exponential decay model have already been used as applications in effective medicine dosage [Chinnaraji Annamalai].

MSC Classification codes: 40A05 (65B10)

Keywords: computation, summation, geometric series

1. Introduction

In the earlier days, geometric series served as a vital role in the development of differential and integral calculus and as an introduction to Taylor series and Fourier series. Geometric series have significant applications in science and management. In this article, geometric series [1-7] are computed in a different way.

2. Computation of Geometric Series

Let x be an integer and $N = \{1, 2, 3, \dots\}$ be the set of natural numbers.

$$\begin{aligned} \text{Then, } x^n &= \overbrace{x^{n-1} + x^{n-1} + x^{n-1} + x^{n-1} + \dots + x^{n-1}}^{x \text{ times}} = (x-1)x^{n-1} + x^{n-1} = \\ &= (x-1)x^{n-1} + \overbrace{x^{n-2} + x^{n-2} + x^{n-2} + \dots + x^{n-2}}^{x \text{ times}} = (x-1)x^{n-1} + (x-1)x^{n-2} + x^{n-2}. \end{aligned}$$

Similarly, we can develop this algebraic expression as follows:

$$(x-1)x^{n-1} + (x-1)x^{n-2} + (x-1)x^{n-3} + \dots + (x-1)x^k + x^k = (x-1) \sum_{i=k}^{n-1} x^i + x^k,$$

that is, $x^n = (x-1) \sum_{i=k}^{n-1} x^i + x^k$, where $k \leq n$ and $k, n \in N$. Now, we can conclude that

$$\sum_{i=k}^{n-1} x^i = \frac{x^n - x^k}{x-1} \text{ OR } \sum_{i=k}^n x^i = \frac{x^{n+1} - x^k}{x-1} \text{ OR } \sum_{i=0}^n x^i = \frac{x^{n+1} - 1}{x-1}.$$

For example,

$$\begin{aligned} 3^n &= 3^{n-1} + 3^{n-1} + 3^{n-1} = (3-1)3^{n-1} + 3^{n-1} = (3-1)3^{n-1} + (3-1)3^{n-2} + 3^{n-2} \\ \Rightarrow 3^n &= (3-1)3^{n-1} + (3-1)3^{n-2} + (3-1)3^{n-3} + \dots + (3-1)3^k + 3^k \\ \Rightarrow 3^n &= (3-1) \sum_{i=k}^{n-1} 3^i + 3^k \Rightarrow \sum_{i=k}^{n-1} 3^i = \frac{3^n - 3^k}{3-1} \Rightarrow \sum_{i=0}^{n-1} 3^i = \frac{3^n - 1}{2}. \end{aligned}$$

If x is any number, then we can develop the geometric series as follows:

$$x^n = (x - 1)x^{n-1} + x^{n-1} \Rightarrow (x - 1)x^{n-1} + (x - 1)x^{n-2} + \dots + (x - 1)x^k + x^k$$

$$\Rightarrow x^n = (x - 1) \sum_{i=k}^{n-1} x^i + x^k \Rightarrow \sum_{i=k}^{n-1} x^i = \frac{x^n - x^k}{x - 1} \Rightarrow \sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}.$$

For example,

$$(9.05)^n = (9.05 - 1)(9.05)^{n-1} + (9.05)^{n-1} \Rightarrow \sum_{i=k}^{n-1} (9.05)^i = \frac{(9.05)^n - (9.05)^k}{(9.05 - 1)}.$$

3. Conclusion

In this article, the summation of geometric series is computed in an innovative way. This computing technique for the sum of geometric series can be useful for researchers working in science, engineering, management, and medicine [8].

References

- [1] Annamalai, C. (2009) A novel computational technique for the geometric progression of powers of two. *Journal of Scientific and Mathematical Research*, Vol.3, pp 16-17. <https://doi.org/10.5281/zenodo.6642923>.
- [2] Annamalai, C. (2019) Extension of ACM for Computing the Geometric Progression. *Journal of Advances in Mathematics and Computer Science*, Vol. 31(5), pp 1-3. <https://doi.org/10.9734/JAMCS/2019/v31i530125>.
- [3] Annamalai, C. (2015) A Novel Approach to ACM-Geometric Progression. *Journal of Basic and Applied Research International*, Vol.2(1), pp 39-40. <https://www.ikppress.org/index.php/JOBARI/article/view/2946>.
- [4] Annamalai, C. (2017) Computational Modelling for the Formation of Geometric Series using Annamalai Computing Method. *Jñānābha*. Vol.47(2), pp 327-330. <https://zbmath.org/?q=an%3A1391.65005>.
- [5] Annamalai, C. (2022) Summations of Single Terms and Successive Terms of Geometric Series. SSRN4085922. <http://dx.doi.org/10.2139/ssrn.4085922>.
- [6] Annamalai, C. (2022) Sum of Geometric Series with Negative Exponents. SSRN4088497. <http://dx.doi.org/10.2139/ssrn.4088497>.
- [7] Annamalai, C. (2019) Computation of Series of Series using Annamalai's Computing Model. *OCTOGON MATHEMATICAL MAGAZINE*, Vol. 27(1), pp 1-3. <http://dx.doi.org/10.2139/ssrn.4088497>.
- [8] Annamalai, C. (2010) Application of Exponential Decay and Geometric Series in Effective Medicine. *Advances in Bioscience and Biotechnology*, Vol. 1(1), pp 51-54. <https://doi.org/10.4236/abb.2010.11008>.