

Microscopic foundation of the $\mu(I)$ rheology for dense granular flows on inclined planes – Supplementary Information –

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FRICTION

We carry out additional simulations and change the value of the microscopic sliding friction coefficient μ_s , from 0.3 to 1. As shown in Fig S1, we note that it only shifts the threshold values $\mu_c(\mu_s)$ and $\phi_c(\mu_s)$ but has no impact on the scaling laws. By simply subtracting the threshold value, we obtain a collapse of all our data on a single master curve.

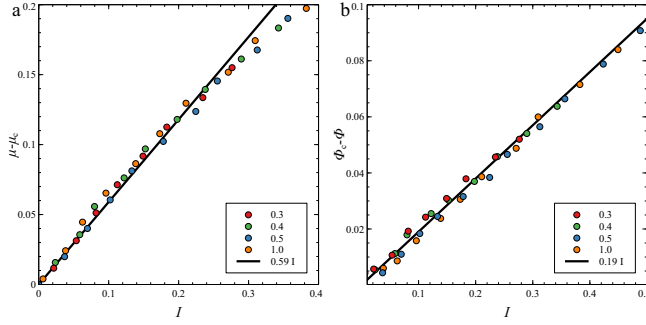


FIG. S1. a) Shifted effective friction coefficient, $\mu - \mu_c$ as a function of the inertial number $I = \dot{\gamma}d\sqrt{\rho_p/P}$, obtained for a layer of grains with an initial layer thicknesses $H_i = 20d$ and various microscopic sliding frictions μ_s as indicated. The solid line indicates a linear fit of the small I values as provided in legend. b) Shifted volume fraction $\phi_c - \phi$ as a function of the inertial number I . The solid line indicates a linear fit as provided in legend.

BAGNOLD-LIKE FLOWS

In order to check the validity of our DEM simulations of granular flows on inclined planes, we study in details the properties of the flow. We observe that the local time-averaged velocity profile is well described by a Bagnold profile $\langle v(z) - v(0) \rangle \sim H^{3/2} - (H - z)^{3/2}$ (see Fig. S2a). Besides, the inertial number I (see Fig. S2b) and the volume fraction ϕ (see Fig. S2c) remain mostly constant throughout the layer (except at both boundaries) for all the studied inclination angles. These observations are in good agreement with previously reported observations [1–5].

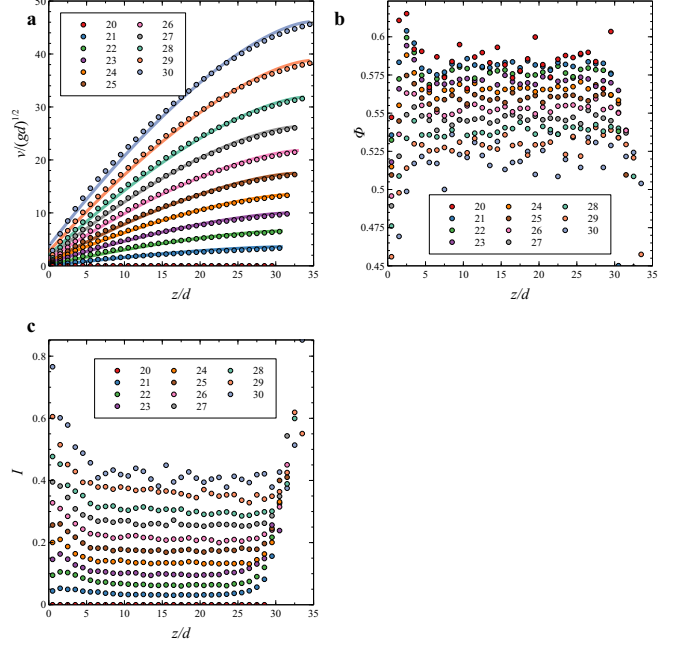


FIG. S2. Evolution of the rescaled local velocity (a), volume fraction ϕ (b) and inertial number I (c) as functions of rescaled normal coordinate z/d for a layer of frictional grains initially characterized by $H_i = 30d$ and various inclination angles θ as indicated. The curves in (a) corresponds to the best fit of the Bagnold profile expression.

VELOCITY CORRELATIONS AND DIFFUSION

As shown in Fig. S3a, we calculate the mean square displacement (Δ_z , MSD) along the z coordinate averaged over all the grains for various inclination angles. From those MSD, the thickness-averaged diffusion coefficient, \overline{D}_z is evaluated by fitting the data in the diffusive regime, *i.e.* where $\Delta_z = 2\overline{D}_z t$. In addition, we also evaluate the time correlation of the z -component of the velocity w , averaged over the layer thickness, for various inclination angles. The thickness averaged correlation times, $\overline{\tau}$ were obtained by fitting the correlation curves with decreasing exponentials of the type $\sim e^{-t/\overline{\tau}}$.

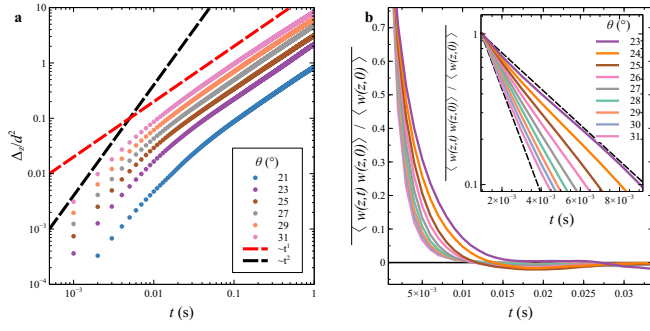


FIG. S3. (a) Mean square displacement Δ_z along the z coordinate averaged over all the grains and (b) time correlations of the velocity following z for a layer of frictional grains initially characterized by $H_i = 30d$ and various inclination angles as indicated. Inset: Log-lin plot of the velocity correlations.

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