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Search via Parallel Lévy Walks on Z^2

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Search via Parallel Lévy Walks on \mathbb{Z}^2

Francesco d'Amore



Joint work with [Andrea Clementi](#), [George Giakkoupis](#),
and [Emanuele Natale](#)

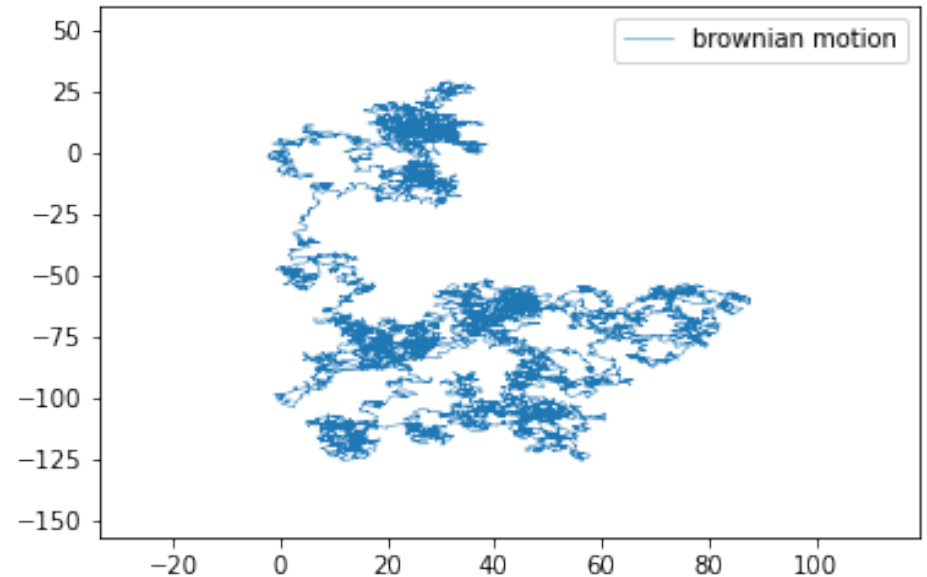
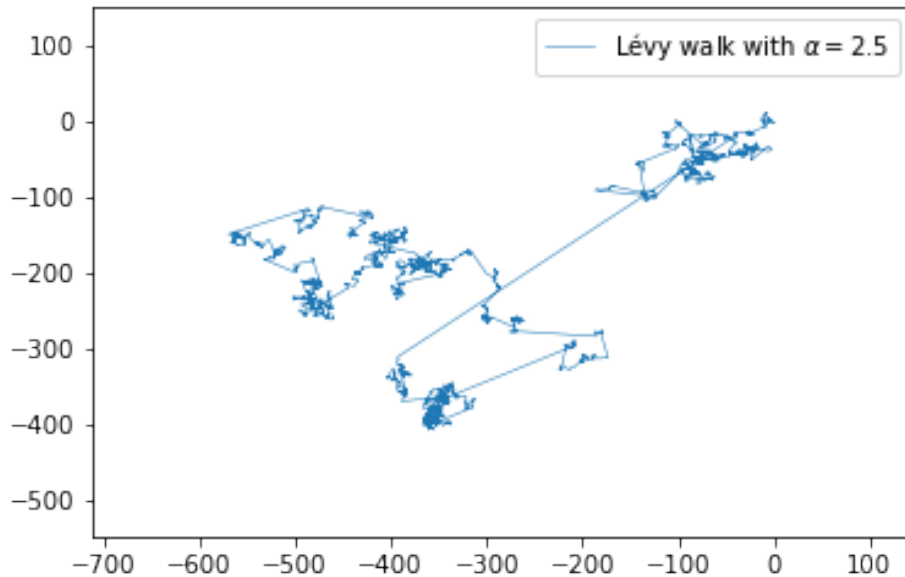
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London, 02 June 2022

What are Lévy walks?



Lévy walk (informal):

A Lévy walk is a random walk whose step-length density distribution is proportional to a power-law, namely, for each $d \in \mathbb{R}^+$, $f(d) \sim 1/d^\alpha$, for some $\alpha > 1$

Note: the **speed** of the walk is **constant**

Movement models and foraging theory

Lévy walks are used to model **movement patterns** [[Reynolds, Biology Open 2018](#)]

Examples:

- T cells within the brain
- swarming bacteria
- midge swarms
- termite broods
- schools of fish
- Australian desert ants
- a variety of molluscs



Rhytidoponera mayri workers. Credit: Associate Professor Heloise Gibb, La Trobe University

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Widely employed in the **Foraging theory**

Optimality results: Lévy walks are **optimal** in different **search problems** [Viswanathan et al., Nature 1999], [Guinard and Korman, Sciences Advances 2021]

The Lévy flight foraging hypothesis

Formulation of an evolutionary hypothesis

The Lévy flight foraging hypothesis [Viswanathan et al., *Physics of Life Reviews* 2008]: since Lévy flights/walks **optimize random searches**, **biological organisms** must have **evolved** to exploit Lévy flights/walks

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We test **this hypothesis** by focusing on a **distributed search problem**:

- the **ANTS** (Ants Nearby Treasure Search) problem



The ANTS problem

Introduced by [Feinerman et al., PODC 2012]

- Setting:
- k (mutually) **independent agents** on \mathbb{Z}^2
 - start moving from the **origin**
 - time is **synchronous** and marked by a global clock
 - one special node $\mathcal{P} \in \mathbb{Z}^2$, the **target**, placed by an **adversary** at unknown (Manhattan) distance ℓ

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Task: **find** the target **as fast as possible**

Lower bound: **hitting time** $\Omega(\ell^2/k + \ell)$
both with **constant probability** and in
expectation

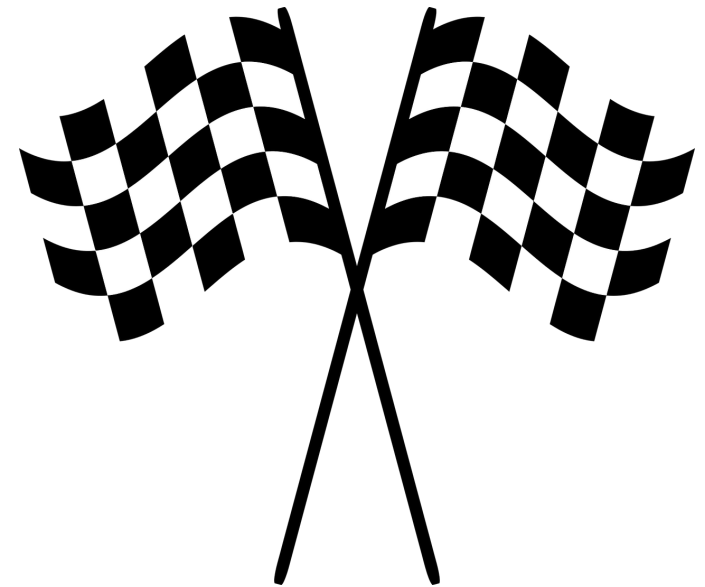


Image by OpenClipart-Vectors from Pixabay

Our contributions

(i) first definition of Lévy walk in the discrete setting in \mathbb{Z}^2 , which is simple and time-homogeneous

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Our contributions

- (i) first definition of Lévy walk in the discrete setting in \mathbb{Z}^2 , which is simple and time-homogeneous
- (ii) to the best of our knowledge, first analysis of the hitting time distribution of k parallel walks
- (iii) Lévy walks can be employed to give a natural, almost-optimal algorithm for the ANTS problem (no knowledge of k , no communication)

Hitting time: $\mathcal{O}\left(\left(\frac{\ell^2}{k} + \ell\right) \text{polylog}\ell\right)$ with high probability

The end

THANK
YOU

To find out more, I invite you to the poster session!

Search via Parallel Lévy Walks on \mathbb{Z}^2
Andrea Chiarant, Francesco Di Biase, Georg Gathka, Evandro Nogueira

Lévy walk
Lévy walks are used to avoid movement patterns that are inefficient.

The ANTS problem
Motivated by the problem of finding a path from the origin to a target point $P \in \mathbb{Z}^2$, the ANTS problem asks for a path from the origin to a target point $P \in \mathbb{Z}^2$ that is shorter than the shortest path from the origin to P .

n-Behavior of Lévy walks
We study the behavior of Lévy walks in the plane for different values of n .

Movement model
Lévy walks are used to avoid movement patterns that are inefficient.

Proposed solution
We propose a solution to the ANTS problem based on the idea of using parallel Lévy walks.

Lévy walk optimality
We study the optimality of Lévy walks in the plane.

Lévy Right Ganging Hypothesis
We propose a hypothesis about the behavior of Lévy walks in the plane.

Discrete Lévy walk
We study the behavior of discrete Lévy walks in the plane.

Hitting time
We study the hitting time of Lévy walks in the plane.

Very far target
We study the behavior of Lévy walks in the plane for very far targets.

Search algorithm
We propose a search algorithm based on parallel Lévy walks.

Lévy walk optimality
We study the optimality of Lévy walks in the plane.

