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Search via Parallel Lévy Walks on Z^2

Andrea Clementi, George Giakkoupis, Emanuele Natale, Francesco d'Amore

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Search via Parallel Lévy Walks on \mathbb{Z}^2

Francesco d'Amore



Joint work with [Andrea Clementi](#), [George Giakkoupis](#),
and [Emanuele Natale](#)

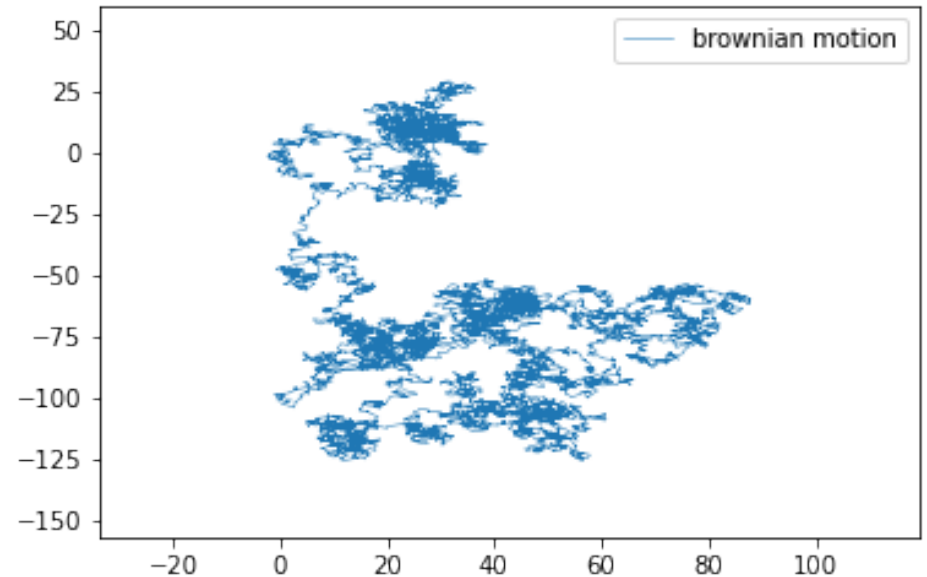
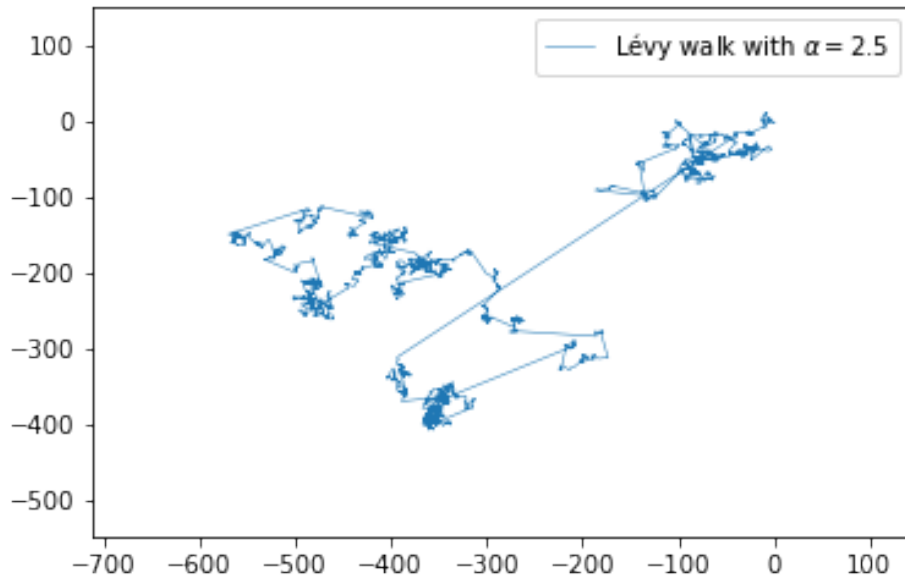
Published in PODC 2021

HALG 2022

London School of Economics

London, 02 June 2022

What are Lévy walks?



Lévy walk (informal):

A Lévy walk is a random walk whose step-length density distribution is proportional to a power-law, namely, for each $d \in \mathbb{R}^+$, $f(d) \sim 1/d^\alpha$, for some $\alpha > 1$

Note: the **speed** of the walk is **constant**

Movement models and foraging theory

Lévy walks are used to model **movement patterns** [[Reynolds, Biology Open 2018](#)]

Examples:

- T cells within the brain
- swarming bacteria
- midge swarms
- termite broods
- schools of fish
- Australian desert ants
- a variety of molluscs



Rhytidoponera mayri workers. Credit: Associate Professor Heloise Gibb, La Trobe University

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Widely employed in the **Foraging theory**

Optimality results: Lévy walks are **optimal** in different **search problems** [Viswanathan et al., Nature 1999], [Guinard and Korman, Sciences Advances 2021]

The Lévy flight foraging hypothesis

Formulation of an evolutionary hypothesis

The Lévy flight foraging hypothesis [Viswanathan et al., *Physics of Life Reviews* 2008]: since Lévy flights/walks **optimize random searches**, **biological organisms** must have **evolved** to exploit Lévy flights/walks

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We test **this hypothesis** by focusing on a **distributed search problem**:

- the **ANTS** (Ants Nearby Treasure Search) problem



The ANTS problem

Introduced by [Feinerman et al., PODC 2012]

- Setting:
- k (mutually) **independent agents** on \mathbb{Z}^2
 - start moving from the **origin**
 - time is **synchronous** and marked by a global clock
 - one special node $\mathcal{P} \in \mathbb{Z}^2$, the **target**, placed by an **adversary** at unknown (Manhattan) distance ℓ

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Task: **find** the target **as fast as possible**

Lower bound: **hitting time** $\Omega(\ell^2/k + \ell)$
both with **constant probability** and in
expectation

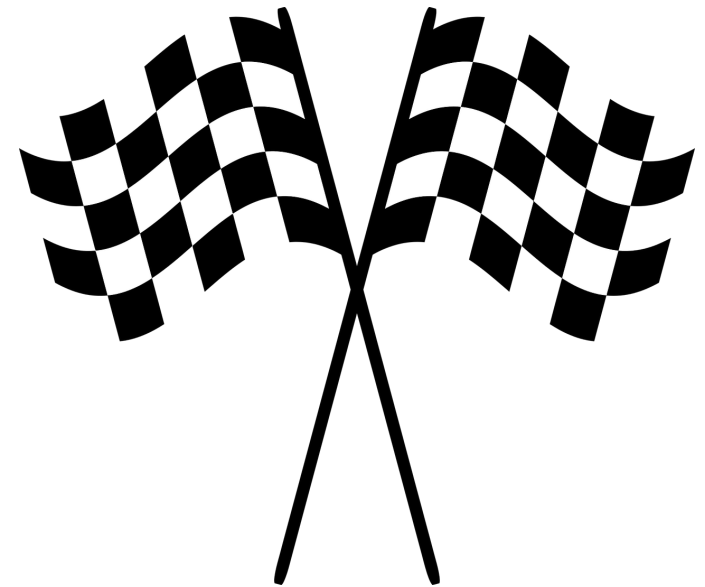


Image by OpenClipart-Vectors from Pixabay

Our contributions

(i) first definition of Lévy walk in the discrete setting in \mathbb{Z}^2 , which is simple and time-homogeneous

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- (i) first definition of Lévy walk in the discrete setting in \mathbb{Z}^2 , which is simple and time-homogeneous
- (ii) to the best of our knowledge, first analysis of the hitting time distribution of k parallel walks
- (iii) Lévy walks can be employed to give a natural, almost-optimal algorithm for the ANTS problem (no knowledge of k , no communication)

Hitting time: $\mathcal{O}\left(\left(\frac{\ell^2}{k} + \ell\right) \text{polylog}\ell\right)$ with high probability

The end

THANK
YOU

To find out more, I invite you to the poster session!

Search via Parallel Lévy Walks on \mathbb{Z}^2
Andrea Chiarant, Francesco Di Biase, Georg Gathka, Evandro Nogueira

Lévy walk
Lévy walks are used to avoid movement patterns that are too regular.

The ANTS problem
Motivated by the problem of finding a path from the origin to a target point $P \in \mathbb{Z}^2$, the ANTS problem asks for a path from the origin to a target point $P \in \mathbb{Z}^2$ that is as short as possible.

n-Behavior of Lévy walks
Let $n \geq 2$ be an integer. A path is called n -regular if it visits every point in \mathbb{Z}^2 at most n times.

Movement model
Lévy walks are used to avoid movement patterns that are too regular.

Proposed solution
We consider a set of n parallel Lévy walks starting from the origin and ending at the target point P .

Lévy walk optimality
We prove that the proposed solution is optimal for the ANTS problem.

Lévy Right Ganging Hypothesis
We conjecture that the proposed solution is optimal for the ANTS problem.

Discrete Lévy walk
We consider a discrete version of the Lévy walk.

Hitting time
We study the hitting time of a Lévy walk to a target point.

Very far target
We study the hitting time of a Lévy walk to a target point that is very far from the origin.

Search algorithm
We describe an algorithm for finding a path from the origin to a target point.

