Search via Parallel Lévy Walks on $Z^2$
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Search via Parallel Lévy Walks on $\mathbb{Z}^2$

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Joint work with Andrea Clementi, George Giakkoupis, and Emanuele Natale

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Lévy walk (informal):
A Lévy walk is a random walk whose step-length density distribution is proportional to a power-law, namely, for each $d \in \mathbb{R}^+$, $f(d) \sim 1/d^\alpha$, for some $\alpha > 1$

Note: the speed of the walk is constant
Movement models and foraging theory

Lévy walks are used to model movement patterns [Reynolds, Biology Open 2018]

Examples:

- T cells within the brain
- swarming bacteria
- midge swarms
- termite broods
- schools of fish
- Australian desert ants
- a variety of molluscs

Rhytidoponera mayri workers. Credit: Associate Professor Heloise Gibb, La Trobe University
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Widely employed in the Foraging theory

Optimality results: Lévy walks are optimal in different search problems [Viswanathan et al., Nature 1999], [Guinard and Korman, Sciences Advances 2021]
The Lévy flight foraging hypothesis

Formulation of an evolutionary hypothesis

The Lévy flight foraging hypothesis [Viswanathan et al., Physics of Life Reviews 2008]: since Lévy flights/walks optimize random searches, biological organisms must have evolved to exploit Lévy flights/walks
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Formulation of an evolutionary hypothesis

**The Lévy flight foraging hypothesis** [Viswanathan et al., Physics of Life Reviews 2008]: since Lévy flights/walks optimize random searches, biological organisms must have evolved to exploit Lévy flights/walks.

We test this hypothesis by focusing on a distributed search problem:
- the **ANTS** (Ants Nearby Treasure Search) problem
The ANTS problem

Introduced by [Feinerman et al., PODC 2012]

Setting:  
• $k$ (mutually) independent agents on $\mathbb{Z}^2$
• start moving from the origin
• time is synchronous and marked by a global clock
• one special node $P \in \mathbb{Z}^2$, the target, placed by an adversary
  at unknown (Manhattan) distance $\ell$
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Task: find the target as fast as possible

Lower bound: hitting time $\Omega \left( \frac{\ell^2}{k} + \ell \right)$ both with constant probability and in expectation
Our contributions

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(ii) to the best of our knowledge, first analysis of the hitting time distribution of $k$ parallel walks

(iii) Lévy walks can be employed to give a natural, almost-optimal algorithm for the ANTS problem (no knowledge of $k$, no communication)

Hitting time: $\mathcal{O}\left(\left(\frac{\ell^2}{k} + \ell\right)\text{polylog}\ell\right)$ with high probability
The end

To find out more, I invite you to the poster session!