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ABSTRACT

So far, multi-objective NK landscapes have been investigated under the assumption of a homogeneous nature of the involved objectives in terms of difficulty. However, we argue that problems with heterogeneous objectives, e.g., in terms of multi-modality, can be challenging for multi-objective evolutionary algorithms, and deserve further considerations. In this paper, we propose a model of multi-objective NK landscapes, where each objective has a different degree of variable interactions (K), as a benchmark to investigate heterogeneous multi-objective optimization problems. We show that the use of a rank-annotated neighborhood network with labeled local optimal solutions, together with landscape metrics extracted from the heterogeneous objectives, thoroughly characterize bi-objective NK landscapes with a different level of heterogeneity among the objectives.

CCS CONCEPTS

• **Theory of computation** → *Optimization with randomized search heuristics*; *Discrete optimization*; • **Applied computing** → *Multi-criterion optimization and decision-making*.

KEYWORDS

Multi-objective optimization, heterogeneous objectives, landscape analysis, NK landscapes.

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1 INTRODUCTION

Multi-objective optimization problems (MOPs) with heterogeneous objectives are those where the objective functions differ in one or several aspects, such as scaling, landscape, evaluation time, or theoretical and practical difficulty [19]. Studies on multi-objective evolutionary algorithms (MOEAs) do not usually pay attention to the heterogeneity of the objectives, and to the way it might influence search difficulty. This is particularly the case for multi-objective benchmark functions where the focus is usually put on global characteristics of the multi-objective landscape, or on the shape of the Pareto front (PF). However, real-world problems might exhibit a significant variability in the objectives' characteristics, and recent research has addressed the question of heterogeneous objectives and proposed multi-objective approaches to deal with them [6]. Much of this previous research has been, however, focused on problems where the heterogeneity arises in evaluation times or latencies, that is, when each objective takes a different amount of time to be evaluated [3, 5, 9, 11].

In this paper, we investigate multi-objective problems with related, but heterogeneous landscapes. We are in particular interested in problems where the multi-modality and the ruggedness significantly differ among the objectives. As a representative class of problems, we select multi-objective NK landscapes [1, 2, 23, 36], and we build on recent results analyzing and characterizing this problem class [13, 14, 24, 25]. The parameter K of the NK model critically influences the characteristics of the fitness landscape. As K increases, the ruggedness and the multi-modality of the landscape also increase. Therefore, we investigate multi-objective NK landscapes where each objective has a different degree of variable interactions K. Such a model serves as a framework to investigate the impact of heterogeneous objectives, by allowing us to evaluate how the difference in the amount of ruggedness among the NK models translates into the difficulty of the MOP. For example, we can quantify how the increment in the number of single-objective local optima for one of the objectives affects the number of multi-objective local optima for MNK landscapes. Although our analysis could be extended to an arbitrary number of objectives, in this paper we focus on bi-objective NK landscapes. Furthermore, we present the analysis from the perspective of optimization strategies based on a Pareto dominant approach.

To evaluate the impact of heterogeneity, we propose to construct a rank-annotated neighborhood network with labeled local optimal solutions. This network is inspired by neighborhood graphs traditionally applied to fitness landscapes [35], and by Pareto local optimal solutions networks [25] previously used to extract informative features from multi-objective NK landscapes. This annotated neighborhood network comprises both local optimal and non-local

optimal solutions. Furthermore, local optimal solutions are labeled to indicate the different classes they belong to: Pareto local optimal solutions [31], and (single-objective) local optima for each objective. In addition, the nodes from the network are layered according to the rank of their corresponding solution with respect to non-dominated sorting [18], similarly to the strategy followed in Pareto plateau connection graphs [17].

Finally, we conduct exhaustive experiments on heterogeneous bi-objective NK landscapes with different degrees of variable interactions per objective, and different degrees of heterogeneity. Using traditional metrics from landscape analysis, and others extracted from the labeled rank-annotated neighborhood network, we analyze the heterogeneous landscapes in terms of multi-modality and of the Pareto set structure. We further analyze to what extent these metrics are impacted by the degree of variable interactions and by the degree of heterogeneity among the objectives.

The paper is organized as follows. Section 2 presents the background on multi-objective NK landscapes and reviews related work. Section 3 introduces the heterogeneous MNK model, proposes ways to quantify heterogeneity, and discusses its usability. Section 4 describes the rank-annotated neighborhood network with labeled local optimal solutions. Section 5 gives the experimental setup, and presents and discusses numerical results. Section 6 concludes the paper and mentions a number of open research lines.

2 BACKGROUND AND RELATED WORK

2.1 NK and MNK Landscapes

Let $\mathbf{X} = (X_1, \ldots, X_N)$ denote a vector of binary variables. We use $\mathbf{x} = (x_1, \ldots, x_N)$ to denote an assignment to the variables. S denotes a set of indices in $\{1, \ldots, N\}$, and X_S (respectively x_S) a subset of the variables of $\mathbf{X}$ (respectively $\mathbf{x}$) determined by the indices in S.

The model for *NK landscapes* was originally introduced to study the effect of different patterns of variable interactions in the landscape [21]. It allows to explore the way in which the structure and the strength of interactions among variables determine the ruggedness of the landscape. For given parameters, the problem consists of finding the global maximum of the function.

An NK landscape is defined with the following components:

- Number of variables, N.
- Number of interactions per variable, K.
- A set of K interactions $\Pi(X_j) \in \mathbf{X}$, for X_j, $j \in \{1, \ldots, N\}$.
- A sub-function f_j defining a real value for each combination of values of X_j and $\Pi(X_j)$, $j \in \{1, \ldots, N\}$.

The objective function F, to be maximized, is defined as follows:

$$F(\mathbf{x}) = \frac{1}{N} \sum_{j=1}^{n} f_j(x_j, \Pi(x_j)). \quad (1)$$

Multi-objective NK (MNK) landscapes [1, 2] provide an extension of the NK model to the multi-objective case. An MNK landscape is defined as a vector function mapping binary vectors into M real numbers $\mathbf{F}(.) = (F_1(.), F_2(.), \ldots, F_M(.)) : \mathcal{B}^N \leftarrow \mathcal{R}^M$, where N is the number of variables, M is the number of objectives, $F_i(.)$ is the i-th objective function, and $\mathcal{B} = \{0, 1\}$. $\mathbf{K} = (K_1, \ldots, K_M)$ is a tuple of integers where K_i is the size of the neighborhood in the i-th landscape[1]. Each $F_i(\mathbf{x})$ is defined similarly to Equation (1).

2.2 Definitions

Let Z denote the mapping of solutions to MNK landscapes in the objective space. Given two objective vectors $z, z' \in Z$, z is dominated by z' iff for all $i \in \{1, \ldots, M\}$ $z'_i \geq z_i$, and there is a $j \in \{1, \ldots, M\}$ such that $z'_j > z_j$. Similarly, given two solutions $x, x' \in X$, x is dominated by x' iff $F(x)$ is dominated by $F(x')$. An objective vector $z^\star \in Z$ is non-dominated if there does not exist any $z \in Z$ such that $z^\star$ is dominated by z. A solution $x^\star \in X$ is Pareto optimal, or non-dominated, if $F(x)$ is non-dominated. The set of Pareto optimal solutions is the Pareto set (PS); its mapping in the objective space is the Pareto front (PF). One of the main challenges in multi-objective optimization is to identify the PS, or a good approximation of it for large-size and complex problems. A number of MOEAs have been designed to this end since the late eighties [12, 15].

2.3 Related Work

Among the research lines related to our work are approaches that study the influence of different factors in MNK landscapes, and works that propose metrics that serve to characterize MOPs, particularly metrics that result from a network-based landscape analysis. In this section, we cover some of the research in these directions.

Aguirre et al. [2] present an exhaustive investigation of how the parameters of MNK landscapes influence several characteristics of the landscape, including the size of the Pareto front and the number of fronts. They extract a number of valuable conclusions related to the effects that the parameters of the MNK landscape have in the complexity of the problems. Since both the variable interactions' structure and the local functions of the NK model influence the ruggedness of the landscape, the impact of these factors have also been investigated from different perspectives. In [36, 37], the NK model has been used to create ρMNK landscapes, where the same variable interactions' structure is used for each objective and the local sub-functions are modified to enforce desired correlations among the objectives. The model has been applied to investigate MOEAs in different scenarios [16, 27]. Knowles and Corne [23] use the MNK model to investigate the effect of augmenting the number of objectives considering different strength of correlation between them. Another way to extend the NK model to the multi-objective domain is by using a single and fixed set of local functions for each objective, and exploring the difficulty of the search by modifying the variable interactions, as proposed in [34]. It is important to notice that all previous analyses of MNK landscapes have considered a fixed K for all the involved objectives. In this paper, we are interested in MNK models with different values for K.

There has been an increasing number of works that propose features which inform about the characteristics or the difficulty of combinatorial MOPs [13, 14, 24, 25]. In addition to providing tools for a better understanding of MOPs, these features have been applied to explain algorithm performance or implement criteria for algorithm selection [13, 24]. We show in this paper how several of these metrics can describe particular aspects of heterogeneous

[1] Although in previous work on MNK landscapes, all K_i's were assumed to be the same, we use this notation to emphasize that this does not always have to be the case.

MOPs. We complement the information provided by traditional features with other metrics extracted from a network representation of the proposed heterogeneous problem.

In landscape analysis, the landscape is often defined by means of a *neighborhood graph*, where there is a vertex for each solution and edges connect neighbors. Local optima networks (LONs) [29] serve as a network characterization of fitness landscapes. Each vertex corresponds to a local optimum, and edges represent basin adjacency between two local optima. They were originally applied to the NK model, and have been later extended to multi-objective NK landscapes in [25]. In the Pareto local optimal solutions network (PLOS-net), vertices correspond to Pareto local optima solutions [30], and edges are constructed between mutually non-dominated neighbors. By definition, a PLOS-net is a sub-graph of the neighborhood graph. Network metrics have been proposed for both LONs and PLOS-nets [10, 20].

There are other approaches that represent the multi-objective landscape as graphs. This is the case of Pareto plateau connection graphs [17], where the nodes from the network are layered according to the rank of their corresponding solution with respect to non-dominated sorting [18]. Also, methods that extract informative network features from the analysis of the interactions between the decision variables have been proposed for characterizing the behavior of multi-objective estimation of distribution algorithms [32] and other MOEAs [28, 33].

3 MULTI-OBJECTIVE NK LANDSCAPES WITH HETEROGENEOUS OBJECTIVES

The parameter K of an NK model critically influences the characteristics of the landscape: the ruggedness and the number of local optima typically increase with K. Therefore, we propose to use MNK models, where each objective is defined using a different parameter K, as a framework to investigate heterogeneous objectives. Such a model allows us to evaluate how the difference in the amount of ruggedness among the single-objective NK models translates into the difficulty of the MOP. We remark that previous research on MNK landscapes has focused on problems where all the objectives share the same parameter K.

Although our analysis could be extended to an arbitrary number of objectives, in this paper we focus on bi-objective MNK landscapes. Furthermore, we present the analysis from the perspective of optimization strategies based on a Pareto dominant approach.

3.1 Constructing Heterogeneous MNK Landscapes

Using K as a way to define the amount of heterogeneity is a simple way to control the characteristics of the problem. However, another important question is the way in which the objectives are related. In this section, we present the way individual objectives have been generated for the heterogeneous MNK model.

We focus on an exhaustive exploration of a small, enumerable, heterogeneous MNK landscape. To reduce the effect that the variability of the variable interactions' structure may have in the characteristics of the landscape, we start from an initial instance with $K = K_{init}$ and progressively construct instances with lower K values. This process is called *NK model reduction* and can be implemented in different ways. For the experiments shown in this paper, we select, for the interactions of variable x_i, the interacting variable with the smallest mean squared distance between each possible configuration of the other variables, that is:

$$x_j = \arg\min_{x_j \in \Pi(x_i)} \sum_{\bar{x} \in x_i \cup \Pi(x_i)/x_j} \left(f_i(x_i, \bar{x} \cup x_j = 1) - f_i(x_i, \bar{x} \cup x_j = 0) \right)^2 \quad (2)$$

where $\bar{x} \in X_i \cup \Pi(X_i)/X_j$ represents a configuration of the interactions of i excluding j, and $f_i(\bar{x} \in X_i \cup \Pi(X_i)/X_j)$ is the value of function f_i for a particular configuration of the interacting variables.

Once the interacting variable to be removed is selected, a new function f_i with 2^{K-1} values is randomly generated from a uniform distribution. This process guarantees that, for any pair of NK models (NK_1, NK_2) generated from a common initial instance and such that $K_1 < K_2$, the $\Pi_{NK_1}(X_j)$ variable interactions of a variable X_j are a subset of $\Pi_{NK_2}(X_j)$.

3.2 Characterizing Heterogeneity

Considering bi-objective MNK landscapes, benchmark parameters K_1 and K_2 configure the heterogeneity of the instance. Using $K_1 = K_2$ to construct a homogeneous instance. We define the total degree of variable interaction as $T = K_1 + K_2$, and the degree of heterogeneity as $D = |K_2 - K_1|$. It is important to notice that, given T and D, the sum $T + D = \max(K_1, K_2) \cdot 2$ must be even in order to ensure feasibility, resulting in only two possible symmetrical pairs (K_1, K_2). Fixing $K_1 < K_2$ reduce the number of possible configurations to one, therefore the total degree of variable interactions T and the degree of heterogeneity D are two alternative parameters to characterize MNK landscapes. To observe the impact of heterogeneity on MNK landscapes, we consider the following scenarios:

- Fixed total degree of variable interactions (T),
- Fixed degree of heterogeneity (D),
- Increasing heterogeneity without fixing T nor D.

4 RANK-ANNOTATED NEIGHBORHOOD NETWORK WITH LABELED LOCAL OPTIMAL SOLUTION

In order to evaluate how heterogeneous objectives influence the properties of the landscape, we focus on metrics that capture the composition of the search space according to the rank of solutions with respect to non-dominated sorting. These metrics attempt to capture the way in which solutions are ranked, and the likelihood for solutions of one rank to have neighbors with a better rank.

4.1 Constructing the Network

We define the neighborhood structure $\mathcal{N}$ as the usual 1-bit flip neighborhood structure; i.e. two solutions are neighbors if their Hamming distance is 1. A Pareto local optimal (PLO) solution is a solution $x \in X$ for which there does not exist any neighboring solution $x' \in \mathcal{N}(x)$ such that x is dominated by x' [30]. Similarly, we use SLO_i to refer to a single-objective local optima with respect to objective F_i.

Solutions in a given rank can be classified into two groups: PLO solutions and solutions that are not locally Pareto optimal. Furthermore, we propose to further classify PLO solutions according to their local optimality for each separate objective. For bi-objective problems, we consider the following five types of solutions:

(t_1) PLO solutions that are SLO_1,
(t_2) PLO solutions that are SLO_2,
(t_3) PLO solutions that are both SLO_1 and SLO_2,
(t_4) PLO solutions that are not local optima of any objective,
(t_5) Solutions that are not PLO.

By construction of MNK landscapes, we assume there is no equivalent neighboring solutions, and therefore a SLO_i for any objective is always a PLO, whereas the opposite is not true. Furthermore, in the previous classification, groups t_1 and t_2 exclude elements from t_3. The rationale behind this classification is to analyze the relationship between SLO and PLO, and to investigate what is the distribution of PLO and non-PLO solutions among solutions with the best ranks. Computing the SLO of a single-objective problem requires the evaluation of only one fitness function. Therefore, it is worth investigating whether the knowledge about the SLO of the objectives that comprise a MOP can be exploited to design more efficient strategies for MOPs. Similarly, an important question is how non-PLO solutions are distributed among the different ranks. While PLO have received most of the attention in the analysis of MOPs, non-PLO solutions can contribute to promising PF approximations.

Since our focus is on Pareto dominant approaches, solutions are further analyzed by organizing them according to non-dominated sorting, as in [17]. As such, non-dominated solutions are ranked 1. For this analysis, we assume that the complete landscape of the MOP is known. This is equivalent to an exhaustive enumeration of the solution space that allows us to associate each solution with its rank. The introduced classification of solutions is used to annotate all the solutions for each rank. As a convenient graphical representation of the search space, we use neighborhood networks augmented with labels that identify the type of solutions in terms of local optimality. The neighborhood network is stratified in layers where all solutions that belong to the same rank are located in the same layer.

Fig. 1 shows an example of a rank-annotated neighborhood network with labeled solutions. In the figure, PLO solutions are represented as circles and non-PLO solutions as squares. The different types of PLO solutions are represented with different colors. To ease the visualization, the example displays only some of the connections between the vertices. Notice that edges indicating the neighborhood relationships have the same interpretation than in the neighborhood network, but they can also be read in terms of rank improvement (or deterioration) when visiting the neighbors of a solution.

4.2 Landscape Characteristics

As previously discussed in Section 2.3, in multi-objective optimization, much research effort has been put to quantify the impact of the number of objectives, their degree of non-linearity, and the correlation among them on general-properties from the landscape. We summarize in Table 1 some existing metrics and introduce additional ones derived from the analysis of the rank-annotated neighborhood network. From the rank-annotated neighborhood

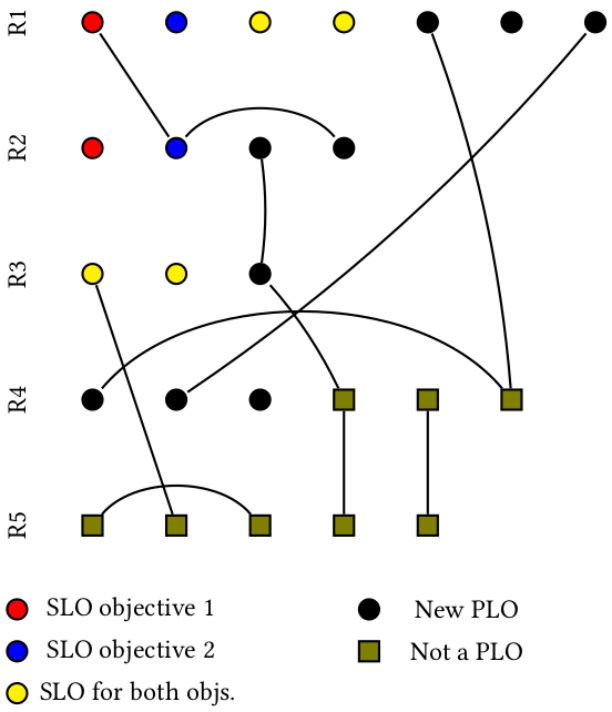

Figure 1: Rank-annotated neighborhood network with labeled local optimal solutions.

network, we have computed features that are further specified by each type of solution t and each rank r. The following features are considered:

- $Q_{t,r}$: Number of solutions of type t and of rank r,
- $L_{t,r}$: Proportion of neighbors from solutions of type t that are connected to solutions with a better rank, whatever its type.
- Number of ranks.

The proportions are computed by first inspecting the ranks of all the neighbors for all solutions of type t, and then dividing by the total number of neighbors. For example, if there are 5 solutions of type t_1 and of rank r, then the total number of neighbors is 30 and this value is used to normalize the number of connections to solutions with lower, equal, and higher ranks. The same value is used at the time of computing the average rank difference $D_{t,r}$ which is also divided by the highest rank in order to scale the average rank change to values in $[-1, 1]$.

The metrics provide information about the composition of the ranks and allow us to identify which type of PLO solutions are more "promising" in terms of improving the quality of the current solutions. For instance, we can evaluate how likely for solutions with sub-optimal ranks is to move to solutions with better ranks, and whether is is better to try first PLO solutions that are SLO for a single objective or, by contrast, whether PLO solutions that are not SLO should be given priority. Our final goal is to determine how the characteristics of the landscape change with heterogeneity,

Table 1: General landscape characteristics extracted from MNK landscapes.

name	description	
PF_size	proportion of solutions in R_1 (best rank)	[2, 22]
supp	proportion of supported solutions in R_1	[22]
hv	hypervolume covered by solution from R_1	[2]
cc	proportion of connected components in R_1	[31]
sing	proportion of isolated solutions (singletons) in R_1	[31]
lcc	prop. size of the largest connected component (lcc) in R_1	[36]
lcc_hv	proportion of hypervolume covered by the lcc in R_1	[26]
plo	proportion of Pareto local optimal solutions (PLO)	[30]
mean_slo	prop. single-objective local optima (SLO) per objective	[26]
slo_i	proportion of SLO for objective f_i	
p_slo_i	same as above, proportional to the number of PLO	
plo_not_slo	proportion of PLO that are *not* SLO for any objective	
p_plo_not_slo	same as above, proportional to the number of PLO	
slo_dev	deviation of the number of SLO per objective: $\mid slo\_1 - slo\_2 \mid$	
p_slo_dev	same as above, proportional to the number of PLO	

and to what extent these metrics can provide information about which search strategy is more promising for different degrees of heterogeneity.

5 EXPERIMENTAL ANALYSIS

In this section, we evaluate the effect of the objectives' heterogeneity in the characteristics of MNK landscapes. We first present the experimental framework, including a description of the instance generation process. Then, we present an initial assessment of traditional metrics used to investigate MNK landscapes. Finally, an analysis of the landscapes based on the augmented neighborhood networks is presented.

5.1 Benchmark Problems

The benchmark used for our experiments have been generated using the reduction method explained in Section 3.1 with $N = 14$ and $K \in \{1, \ldots, 12\}$, such that $K_{init} = 12$. For each K value, we generate 50 random instances, and apply the reduction for each of them obtaining $50 \times 12 = 600$ (single-objective) NK models.

Finally, in order to create bi-objective MNK landscapes, we pair each possible combination of K values for each of the original instances, ending up with 50 instances of the $\frac{12\times11}{2} = 66$ bi-objective problems. For each bi-objective problem, we report average values among the 50 folds.

5.2 Intrinsic Characteristics of Heterogeneous MNK Landscapes

5.2.1 Visual Inspection of the Objective Space. We start by analyzing the impact of heterogeneity on the structure of the objective space. Given a fixed value of $T = K_1 + K_2 = 13$, we show in Fig. 2 three exemplary heterogeneous MNK landscapes in which the level of heterogeneity varies, from slightly heterogeneous objectives (left) to highly heterogeneous objectives (right).

First of all, we observe that the distribution of objective values is different for different levels of heterogeneity. For nearly homogeneous objectives (left), the ellipse surrounding the objective space looks like a circle, where the range of objective values is the same for both objectives, similar to what we observe for homogeneous MNK landscapes [36]. However, as the degree of heterogeneity increases, the position of solutions in the objective space seems to squeeze for the objective with a smaller K. We attribute this to the underlying distribution of objective values for NK models, for which the range is known to increase with K.

Fig. 2 additionally highlights Pareto optimal solutions, SLO for each objective and remaining PLO in different colors. We observe that the PF size and the number of PLO solutions do not seem to significantly fluctuate with the level of heterogeneity. However, the more homogeneous the objectives, the better distributed the SLO among both objectives (left). By contrast, for highly heterogeneous objectives (right), almost all local optima are with respect to the second objective (f_2), resulting in a proportional increase of PLO solutions on top of the objective space.

5.2.2 Structure of the Pareto set. Fig. 3 (top right and center) summarizes some metrics related to the structure of the Pareto set for all considered heterogeneous bi-objective problems: the number of solutions with the best rank (solutions from R_1), and the number of connected components. The analysis of the figure reveals that the number of Pareto optimal solutions does not vary significantly among the considered problems. For a fixed K_1, it seems to slightly decrease with $K_2 > K_1$, which corroborates results from homogeneous MNK landscapes [2, 36]. However, this observation does not hold when fixing K_2 and varying $K_1 < K_2$. This suggests that the "most-difficult" objective has more impact on the cardinality of the Pareto set than the "easier" objective for MNK landscapes. We also observe that there are slightly fewer Pareto optimal solutions as the degree of heterogeneity among the objectives increases. Similarly, the proportion of supported solutions (supp) and the global hypervolume (hv) do not reveal any significant variations (not reported here due to space restriction). They mostly relate to the shape of the PF, which do not reveal significant changes in the benchmark. We can however notice that hv seems to increase with the number of variable interactions, as for homogeneous MNK landscapes [2].

In terms of connectedness of the Pareto set, it is reported to decrease with the number of variable interactions for homogeneous MNK landscapes in [36]. This is what we observe in Fig. 3, where the number of connected components (cc) increases with K_1 and K_2. Although not reported due to space restriction, we observe that related metrics, such as the proportion of isolated Pareto-optimal solutions, and the size and hypervolume of the largest connected component, vary accordingly. Interestingly, we do not observe any significant change with respect to the difference between K_1 and K_2, suggesting that the connectedness is *not* impacted by the level of heterogeneity among the objectives.

Finally, we remark that the number of ranks does not seem to be affected by K_1 and K_2, since it exhibits only very minor variations (not reported here due to space restriction). These initial observations show that, as far as the structure of the Pareto set is concerned, the information that can be extracted from the way in which heterogeneity influences problem difficulty is limited.

5.2.3 Multi-modality. Let us now analyze how the non-linearity of objectives, and their level of heterogeneity influences the multimodality of the landscape. In Fig. 3 (top left, and bottom), we report additional metric values related to the number of local optimal

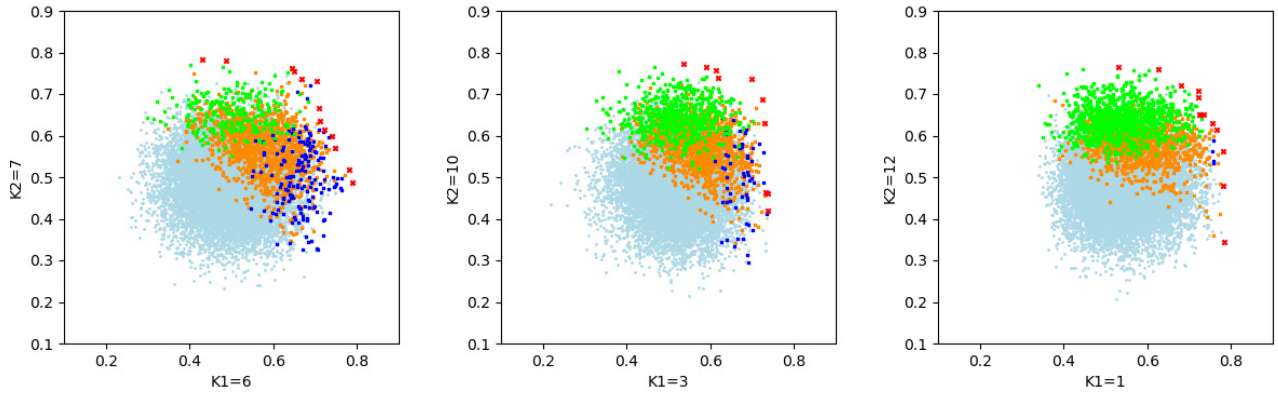

Figure 2: Objective space for three heterogeneous MNK landscapes with fixed total degree of variable interaction $T = K_1 + K_2 = 13$. The degree of heterogeneity increases from left to right. Red points are Pareto optimal solutions, blue points are SLO w.r.t. the first objective (which are not Pareto optimal), green points are SLO w.r.t. the second objective, orange points are PLO solutions (that are not Pareto optimal, nor SLO for any objective), and grey points are non-local optimal solutions.

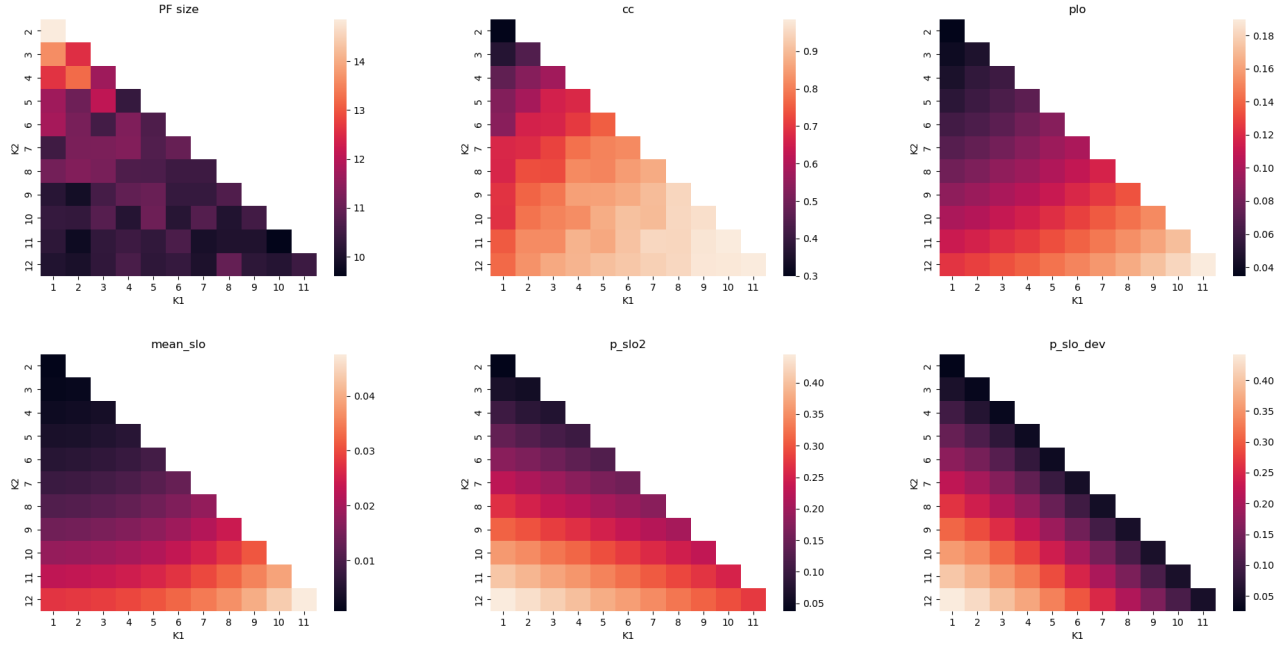

Figure 3: Statistics computed from heterogeneous MNK landscapes.

solutions for the 50 folds of each pair of (K_1, K_2) values. Due to space restriction, only a subset of metrics are reported.

For single-objective NK landscapes, it is expected that higher K values result in more local optima [21]. For homogeneous MNK landscapes, the number of PLO solutions is also known to increase with K [36]. Overall, all metrics related to the multi-modality show a regular variation according to both K values : the larger K, the rugger the landscape. The number of PLO solutions (`plo`, top right) varies significantly and, as expected, it clearly depends on the value of K for both objectives. However, the number of PLO solutions remains constant whatever the level of heterogeneity among the objectives. Similarly, the mean proportion of SLO per objective (`mean_slo`, bottom left) significantly increases along K_1 and K_2. Our results reveal that the variation can be significant. As already illustrated in Fig. 2, the number of SLO_i for objective F_i increases with K_i, and so does its proportion among PLO solutions. Interestingly, we also observe that the latter increases a bit more significantly as

objectives get more heterogeneous, that is when there is a large difference between K_1 and K_2 (see, e.g., p_slo_2, bottom center).

The metrics discussed above, such as plo or mean_slo but also cc, mostly capture this general characteristic of the landscape. Similar to the number of PLO solutions, the number of PLO solutions that are not SLO for any objective plo_not_slo increases with the total degree of interactions T. However, its normalized variant p_plo_not_slo decreases with T. This can be interpreted as follows: for smoother landscapes with ($K_1 = 1, K_2 = 2$), 5% of PLO solutions are also an SLO with respect to one objective, whereas for highly rugged landscapes with ($K_1 = 11, K_2 = 12$), more than 90% of PLO solutions come from SLO solutions with respect to F_1 or F_2.

The total degree of variables interactions T captures the general trend of the ruggedness, but fails to grasp the disparity among the objectives. For a low degree of heterogeneity, we already observed in Fig. 2 that SLO are well-balanced between the objectives. To go further, we compute the deviation between the number of SLO for each objective slo_dev, together with its normalized variant, as a proportion among PLO solutions, p_slo_dev. The latter is reported in Fig. 3 (bottom right) for different K_1 and K_2 values. Increasing the heterogeneity D such as $K_1 < K_2$ result in a strong concentration of SLO on the second objective. In other words, slo_dev increases with the objectives' heterogeneity. Contrary to the proportion of PLO which are not SLO, that decreases with the total degree of interactions $T = K_1 + K_2$, the proportional variant p_slo_dev is mostly impacted by the degree of heterogeneity $D = |K_1 - K_2|$.

5.2.4 Correlation Analysis. To push our analysis further, we report in Fig. 4 the correlation between the 17 considered metrics and (1) the number of variable interactions for the first objective K_1, (2) the number of variable interactions for the second objective K_2, (3) the total number of variable interactions for both objectives $T = K_1 + K_2$, and (4) the degree of heterogeneity among the objectives $D = |K_1 - K_2|$. The correlations reveal five categories of observations. Firstly, as expected, the number of SLO (slo_i) and their proportion within PLO (p_slo_i) are highly correlated with K_i, forming two small clusters in the center. The latter is normalized by the number of PLO solutions, which results in an anti-correlated relationship between p_slo_i and the cluster with K of the other objective. The number of PLO (plo), the average number of SLO per objective (mean_slo) and the number of PLO which are not SLO (plo_not_slo) are all strongly correlated with $T = K_1 + K_2$. So are the proportion of isolated solutions (sing) and the proportion of connected components (cc) among Pareto optimal solutions, forming all together a large cluster in the bottom right of the figure. This cluster is related to both previous clusters, where K_1 and K_2 appear. Two metrics are in conflict with the ones above : as expected, the proportion of PLO which are not SLO (p_plo_not_slo) decreases with $T = K_1 + K_2$. Similarly, the size of the largest connected component (lcc) follows an opposite trend to the number of singletons (sing) and the proportion of connected component (cc). In a fourth cluster, we observe that the deviation of SLO (slo_dev) and its proportion (p_slo_dev) are strongly impacted by the degree of heterogeneity $D = |K_1 - K_2|$, when slo_dev also reveals a medium correlation with other benchmark parameters (K_1, K_2, and $T = K_1 + K_2$). As such, for the same degree of heterogeneity, the range of the deviation of SLO for each objective is impacted by the total degree of variable interactions. Finally, as previously mentioned, the proportion of Pareto-optimal solutions and of supported solutions (supp), together with the hypervolume of the largest connected component (lcc_hv) do not change much with benchmark parameters.

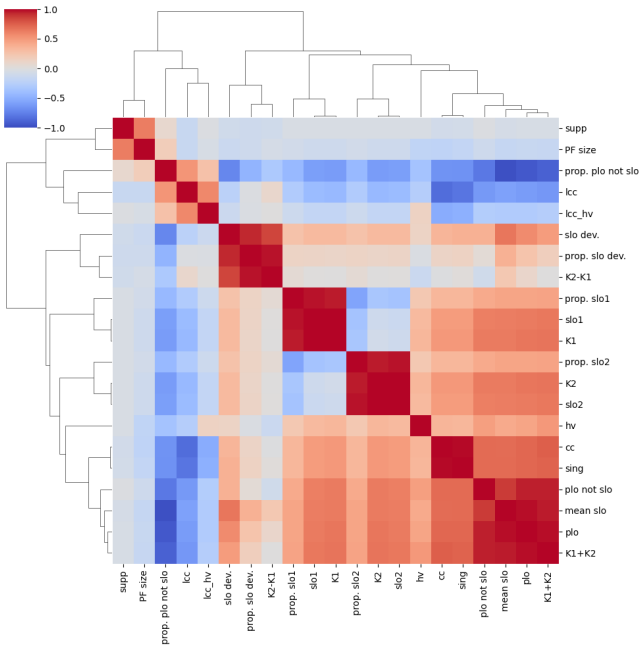

Figure 4: Spearman correlation between landscape metrics, K_1, K_2, the total degree of interactions $K_1 + K_2$, and the degree of heterogeneity $|K_1 - K_2|$.

5.3 Differential Analysis of PLO and non-PLO Solutions

The analysis presented in the previous sections shows that a repertoire of traditional metrics sheds light on how the choice of the parameters K_1 and K_2 shapes the characteristics of heterogeneous MNK landscapes. In this section, we approach the analysis of heterogeneous MNK landscapes using metrics derived from the rank-annotated neighborhood network. These metrics simultaneously provide two perspectives of heterogeneity: the differential role played by the rank, and the distribution of solutions across ranks.

We focus on the three scenarios described in Section 3.2. For each scenario, we select three heterogeneous MNK instances and compute the metrics. Fig. 5 shows, for the 9 instances, and for the set of the first 100 ranks, the proportion of neighbors from solutions in each rank that belong to better ranks. This metric provides a measure of how likely it is to find better solutions by locally exploring solutions of a given rank. In Fig. 5, the first three rows represent the scenario of increasing heterogeneity without fixed T and D, the three rows in the middle correspond to a fixed

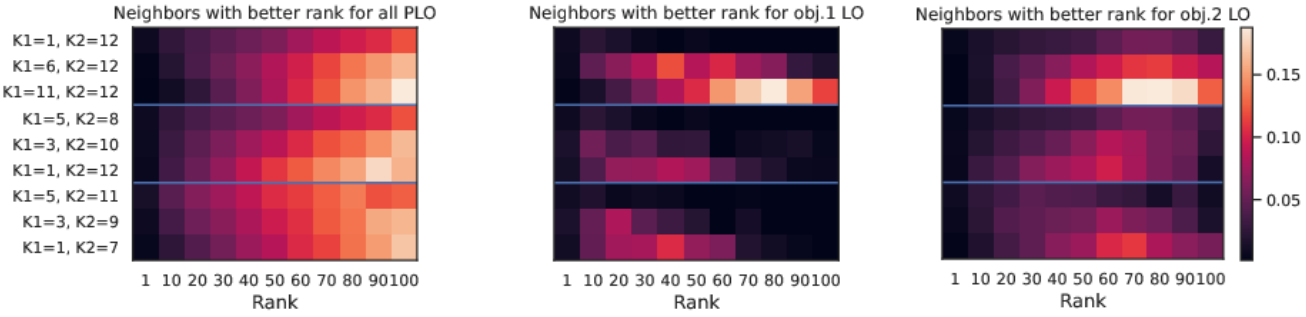

Figure 5: Proportion of neighbors with better rank for different PLO solutions in three different scenarios.

total degree of variable interactions, and the bottom three rows correspond to a fixed degree of heterogeneity.

The analysis of the first scenario in Fig. 5 (left) reveals that, as the total amount of interactions increases, it becomes less likely to improve (jump to a better rank) from a PLO with the best ranks. For the first 50 ranks, the probability of finding better neighbors for $(K_1 = 1, K_2 = 12)$ is higher than for $(K_1 = 11, K_2 = 12)$. It is not only that there are more local optima for the second objective, but also that they are distributed in a different way along the ranks. The analysis of the proportion of better neighbors for SLO_1 and SLO_2 show that, in the first ranks, it seems easier to improve solutions from SLO_2 than those for SLO_1. Notice, that while the second objective is fixed ($K_2 = 12$), the probability of improving a SLO_2 of the different ranks change as both the heterogeneity and the total degree of variable interactions increase.

When the total amount of interactions $T = K_1 + K_2$ is fixed, the proportion of PLO neighbors with a better rank show significant differences for the three MNK instances considered. So does the proportion of SLO_1. For $(K_1 = 5, K_2 = 8)$, it is very difficult to find, in the first ranks, better neighbors for SLO_1. However, as $D = |K_1 - K_2|$ increases, the probability also increases.

When the degree of heterogeneity $D = |K_1 - K_2|$ is fixed, the proportion of neighbors in the first ranks also shows differences among the instances. The increase is related to the increase of the total amount of variable interactions T. A common characteristic for both the second and third scenarios is that the probability to improve SLO_1 solutions is limited to a relatively short range of rank values.

6 CONCLUSIONS

In multi-objective optimization, the heterogeneity among the objectives can be manifested in a variety of ways. One of these ways is the different amount of ruggedness of the landscape induced by each objective. Most previous research on combinatorial MOPs, and in particular on MNK landscapes, have focused on homogeneous objectives. Furthermore, the landscape analysis from the literature of combinatorial MOPs have neither been conceived taking into consideration heterogeneity nor tested on this type of problems. The work presented in this paper aims at providing a better understanding of the impact of heterogeneity on combinatorial MOPs. Our main contribution consists in introducing heterogeneous MNK landscapes with different values of K as a benchmark to investigate this question. We further investigated the impact of heterogeneity by means of existing multi-objective landscape metrics and of new metrics that integrate information about the dominance relation among solutions and their type in terms of local optimality. We also introduced a rank-annotated neighborhood network with labeled local optimal solutions, which allows us to extract a finer description of how heterogeneity influences the dominance relations among solutions. Our detailed analysis about local optima for each objective reveals their impact on heterogeneity in the characteristics of the landscape.

An important direction for future research is how to incorporate information about the heterogeneity, as provided by the problem parameters and metrics, into more efficient optimization methods. In the context of local search algorithms, estimating the heterogeneity of the distribution of local optima could help to take proper decisions on the trade-off of exploration and intensification between all objectives. Different strategies could be followed, such as distributing the search effort with respect to the proportion of local optima per objective. Another way to take the impact of heterogeneity into account is by exploiting the fact that, in different ranks, some types of local optima are more likely to be improved than others.

For the analysis included in this paper, we have considered the case of relatively small K values. It is possible to extend the analysis to large values of K but in this case, it is not feasible to store the fitness values for all sub-functions. Fitness components could be computed as they are called and combined with a *caching* approach that store partial configurations already. Similar approaches have been previously applied in evolutionary computation [7, 8]. Similarly, an aspect that is kept for future work is the investigation of heterogeneous MNK landscapes with more than two objectives. Addressing many-objective problems constitutes a formidable challenge [4] and heterogeneous MOPs do not seem to be an exception in this respect.

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