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► **To cite this version:**

Ming Liu, Zhongzheng Liu, Feng Chu, Alexandre Dolgui, Chengbin Chu, et al.. An optimization approach for multi-echelon supply chain viability with disruption risk minimization. *Omega*, 2022, 112, pp.102683. 10.1016/j.omega.2022.102683 . hal-03692701

HAL Id: hal-03692701

<https://hal.science/hal-03692701v1>

Submitted on 22 Jul 2024

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An optimization approach for multi-echelon supply chain viability with disruption risk minimization

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Abstract

The outbreak of extraordinary disruptive events, e.g., the COVID-19 pandemic, has greatly impacted the orderly operation in global supply chains (SCs), and may lead to the SC breakdown. Regulatory actions, such as government interventions during the pandemic, can greatly mitigate the disruption propagation (i.e., the ripple effect) and improve SC viability. However, existing works that focus on the disruption propagation management have not considered the possibility of such interventions. Motivated by the fact, in this study, we investigate a new disruption propagation management problem in a multi-echelon SC with limited intervention budget. The aim is to minimize disruption risk measured by the disrupted probability of target participants in the SC. For the problem, a novel approach, combining the Causal Bayesian Network (CBN), the do-calculus and the mathematical programming, is developed. Specially, two mixed-integer non-linear programming models are constructed to determine appropriate interventions. To enhance the proposed mathematical models, two valid inequalities are proposed. Then, a problem-specific genetic algorithm (GA) is developed for handling large-scale problem instances. Numerical experiments on a case study and randomly generated instances are conducted to evaluate the efficiency of the proposed models, the valid inequalities and the GA. Based on experiment analysis, managerial insights are drawn.

Keywords: Disruption risk; Ripple effect; Supply chain viability; Causal Bayesian Network; Do-calculus; Mathematical programming

1. Introduction

The outbreak of the COVID-19 pandemic, as an extraordinary disruptive event, has caused long-term and unpredictable scaling impacts, and exposed the vulnerability of supply chains (SCs) (Queiroz et al., 2020; Ivanov, 2020; Ivanov and Dolgui, 2021a; Burgos and Ivanov, 2021; Chowdhury et al., 2021; Sawik, 2022). Consequently, the issue of improving SC adaptability and survivability under extraordinary disruptive events has received great attentions from both scholars and practitioners (Ivanov, 2020; Ivanov and Dolgui, 2021b). Specifically, Ivanov (2020) introduces

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the concept of SC viability to describe the ability of SC to survive under disruptive events, either by reinforcing and reconfiguring its structure to adapt the environment change due to long-term disruptive events, or by external interventions (e.g., government intervenes SC participants to mitigate the impact of disruptive events). SC viability is becoming an interesting topic for the SC, especially for the multi-echelon SC, to deal with its disruption propagation. Generally, the multi-echelon SC has a complicated structure and multiple collaborations between SC participants, thus it is more vulnerable and difficult to survive in the face of extraordinary disruptions at local nodes and ripple effect (Schmitt et al., 2017; Ivanov, 2020; Ivanov and Dolgui, 2021a; Dolgui et al., 2020; Sawik, 2020). Ivanov et al. (2016a) describe the ripple effect as “the disruption propagation, the impact of a disruption on SC performance and the disruption-based scope of changes in the SC structures.” For example, Mercedes-Benz restarted an off-road vehicle plant in Alabama in June 2020, but soon stopped production due to a shortage of components made in Europe due to the COVID-19 pandemic (REUTERS news, 2020). In January 2021, due to the delay caused by the epidemic, Honda suspended production at its Swindon plant in the United Kingdom, mainly responsible for the production of vehicles such as the type of the civic hatchback (BBC news, 2021).

To maintain the SC viability under disruptive events, it is important to analyze the disruption propagation mechanism and evaluate the disruption risk. To portray the dependence relationships of disruption propagations among SC participants, Bayesian network (BN) is introduced by Hosseini and Ivanov (2019). BN encapsulates probability informations into a directed acyclic graph (DAG), of which nodes indicate random variables and arcs express dependence relationships (Koller and Friedman, 2009; Hosseini and Ivanov, 2019; Hosseini and Ivanov, 2020). Especially, Hosseini et al. (2019b) propose a dynamic method based on BN to study the dynamics of the disruption propagation in a SC over a time horizon. Due to the difficulty of obtaining perfect information in data scarce environment, Liu et al. (2021) develop a novel robust approach that combines the BN and the mathematical programming. However, existing works focus on the SC disruption propagation assessment, and have not investigated the possible interventions that can reduce considerable SC disruption propagations and maintain the SC viability.

Ivanov (2020) delineates the importance of the control of adaptive mechanisms in viable SCs. The author points out that a visibility control system can establish the SC robustness and help it recover from disruptions, e.g., severe natural disasters, which may temporarily, adversely affect demand fulfillment continuity. Specially, Ivanov (2020) highlights the important role of the government to strengthen the SC viability against dramatic disruptions. In practice, under the COVID-19 pandemic, government typically intervenes the SC to ensure essential material flows, by hedging against the disruption propagation. For example, China addressed the issue of the supply of non-woven fabric to intervene the production of melt-blown polypropylene. One hundred and three companies were involved in stepping up production so that mask manufacturers would not face a shortage for their key input (OECD news, 2020). Intervention actions of government are helpful to improve the SC viability. However, intervention actions are typically costly, and the intervention budget is always limited. Thus, it is important to choose appropriate interventions and evaluate their performance. However, the proposed approaches in the literature fail to choose and evaluate the impacts of interventions. To bridge the gap between theory and practice, we investigate a new multi-echelon SC viability problem under limited intervention budget that can be considered as the government’s intervention

budget. Specifically, we consider a multi-echelon SC with several suppliers and one manufacturer under the ripple effect. For ease of quantification, the disruption risk in the SC is measured by the disruption probability of the manufacturer. The problem is to determine appropriate interventions to minimize the disruption probability of the SC participant in the last echelon (i.e., the manufacturer). For the problem, a new approach that combines the Causal Bayesian Network (CBN), the do-calculus and the mathematical programming, is designed. The CBN is a particular BN with causal assumptions to appropriately portray dependence relationships (Pearl, 2009). The do-calculus, proposed by Pearl (2009), is introduced to quantify the impact of interventions. The mathematical programming model is constructed to formulate the studied problem. The main contributions of the work include:

- (1) A new multi-echelon SC viability problem, subjected to limited intervention budget, is investigated.
- (2) For the problem, a novel approach, combining the CBN, the do-calculus, and the mathematical programming, is designed. Especially, two mixed-integer non-linear programming models are constructed, and two valid inequalities are proposed to enhance the models.
- (3) A problem-specific genetic algorithm (GA) is designed to solve large-scale problem instances.
- (4) Based on experiment analysis, managerial insights are drawn.

The remainder of the paper is organized as follows. Literature review is presented in Section 2. In Section 3, the studied problem is stated. The CBN and the do-calculus are introduced to quantify the impacts of interventions on SC disruption propagation. In Section 4, two mixed-integer non-linear programming approaches are developed to optimally determine interventions. Two valid inequalities are further proposed based on problem property analysis. In Section 5, a problem-specific GA is designed to solve the problem. In Section 6, numerical experiments on a case study and randomly generated instances are conducted to evaluate the efficiency of the proposed models, the valid inequalities, and the problem-specific GA. Then, managerial insights are drawn. Finally, Section 6 summarizes this paper and suggests future research directions.

2. Literature review

SC disruption risk arises from disruptive events, such as natural disaster or/and man-made ones, and has attracted researchers' attention due to increasing uncertainties and disruptions (Dubey et al., 2019; Hosseini et al., 2019; Sawik, 2020; Choi, 2020; Azadegan and Dooley, 2021). The COVID-19 pandemic causes tremendous consequences to public health and global economy for more than two years (Bonaparte, 2020; Beck, 2020; Ivanov and Das, 2020; Sohrabi et al., 2020). Thus, designing and reconfiguring an efficient and viable SC is vital for companies, especially in the COVID-19 pandemic context. Ivanov (2020) sheds light on the importance of the SC viability under long-term disruptive events. We study the SC viability issue from the perspective of SC disruption propagation quantification and mitigation. In this section, we focus on reviewing the most related literatures about the SC disruption propagation management, of which the approaches can be mainly classified into two categories: mathematical models and BN-based approaches.

2.1. SC disruption propagation management via mathematical models

Various mathematical modelling approaches are introduced to address the SC disruption propagation management problem (Snyder et al., 2016). In the following, optimal control methods and operational research methods are reviewed in details.

The advantage of optimal control methods is to portray the dynamic characteristic of the SC disruption risk under the ripple effect. Tomlin (2006) investigates mitigation strategies for SC disruption risk management problem. A mathematical model is proposed for a single-product setting. Qi and Lee (2015) generalize the research of Tomlin (2006) by considering expedited shipping, and propose a mathematical model. Yu et al. (2009) consider a supplier selection problem under SC disruption risks. The authors present a modelling approach based on expected profit functions. Ivanov et al. (2013) treat a production distribution planning problem under the ripple effect. The objective is to optimize the production distributions under the ripple effect. The authors propose an optimal programme control (OPC) model integrating the control theory and the linear programming. Ivanov et al. (2014) consider a multi-period and multi-commodity distribution (re)planning problem under the ripple effect. The authors establish an OPC model to optimize multiple good distributions. Ivanov et al. (2015) study an integrated aggregate distribution and transportation planning problem with limited transportation capacity under the ripple effect. For the problem, an OPC model is established to balance supply and demand. Ivanov et al. (2016b) investigate a perishable product SC resilience problem under the ripple effect in Australia dairy industry. For the problem, they propose an OPC model based on reactive recovery policies. The work of Sokolov et al. (2016) aims to quantify the ripple effect in the SC from the structural perspective. The authors propose a multi-criteria approach, combining the OPC and the mixed-integer linear programming, to assess the SC resilience.

Due to the uncertainty of SC disruptive events, many scholars focus on applying stochastic optimization models to investigate the SC disruption propagation management problem. Sawik (2010) investigates a SC supplier selection problem in a make-to-order environment, and proposes a novel portfolio-based approach. Sawik (2011) considers a supply portfolio selection problem under disruption risks. Sawik (2014) considers a joint supplier selection and customer order scheduling problem under disruption risks. The author proposes a stochastic mixed-integer programming formulation utilizing conditional value-at-risk. Sawik (2016) studies an integrated supply, production and distribution problem under disruption risks. The author proposes a bi-objective stochastic mixed-integer programming model. Sawik (2021) considers a supplier selection problem in multi-echelon SC networks. The author proposes an efficient scenario-based stochastic mixed-integer programming model. Gholami-Zanjani et al. (2021) study a SC location-inventory problem under the ripple effect, and develop a two-stage stochastic programming model to maximize the SC total profit. Özçelik et al. (2021) consider a disruption propagation problem in a reverse SC. The study first quantifies the impact of the ripple effect on the system performance, and then a robust optimisation model is introduced.

2.2. Disruption propagation management via BNs

BN approach is a probabilistic graphical model, encapsulating probability information into a directed acyclic graph (DAG), of which nodes indicate random variables and arcs express conditional probability relationships between nodes. In the SC, the disruption risks of upstream SC participants to their successors are

often characterized by probabilities, and thus BN is introduced by Hosseini and Ivanov (2019) for assessing the SC disruption propagation. Hosseini et al. (2019a) review several approaches for measuring the SC resilience, and justify the significance of applying BN approach to assess the SC disruption risk. Hosseini and Ivanov (2021) investigate the quantification of the SC disruption risk in the context of the COVID-19 pandemic, and propose a multi-echelon BN model to portray the forward and backward disruption propagations. Hosseini et al. (2016) consider the design of resilient SCs, and propose a BN model to quantify the SC resilience based on a generic conceptual framework. Hosseini et al. (2020) focus on SC resilience in an open-system context, and propose a useful BN-based decision-making support approach to mitigate disruption propagation and improve recovery capability simultaneously. Hosseini et al. (2019b) investigate the dynamics of the disruption propagation in a two-echelon SC over a time horizon, and propose a dynamic BN.

Few researchers study the disruption propagation problem by integrating the BN approach and the mathematical programming model. This integrated method can characterize the SC disruption propagation via the BN, and can provide decision support by mathematical programming models. Hosseini et al. (2019c) consider a SC supplier selection problem under the ripple effect. The authors apply a BN approach to portray the disruption propagation, and propose a bi-objective stochastic mixed-integer linear programming model to optimize supplier selection. Liu et al. (2021) study a disruption risk assessment problem, and propose a robust BN approach integrated with the mixed-integer non-linear programming to evaluate the worst-case disruption risk in data scarce environment.

The above studies only focus on the SC disruption risk assessment without considering possible interventions to mitigate disruption propagations in the SC and maintain the SC viability. In practice, interventions, such as regulatory actions of government, can greatly reduce the disruption propagation. To the best of our knowledge, the benefit of interventions upon disruption propagations in the SC has not been quantitatively studied. In addition, the traditional BN cannot analyze the impact of interventions on disruption propagations. Causal Bayesian Network (CBN) is a particular BN with causal assumptions (Pearl, 2009). The do-calculus is proposed by Pearl (2009) to quantify the impact of an intervention for a CBN. The do-calculus approach has been widely applied in transportation (Chatterjee et al., 2019), machine learning (Barber, 2012; Brownlee et al., 2022), causal inference (Zečević et al., 2021), and econometrics (Pearl, 2015; Pearl, 2019; Heckman and Pinto, 2022). In this work, a novel approach that combines the CBN, the do-calculus and the mathematical programming is designed for the studied problem. Especially, the CBN is applied to describe the disruption propagation with interventions, the do-calculus to quantify the impact of interventions on disruption propagations, and the mathematical programming to optimally determine the interventions. This novel approach is also one of the main contributions of our work. Table 1 summarizes main differences between related existing studies and our work.

3. Problem statement, CBN and do-calculus

In this section, the studied problem is described, and the principles of the CBN and the do-calculus are presented. Section 3.1 presents the considered problem. In Section 3.2, the CBN is applied to describe the disrupted multi-echelon SC. In Section 3.3, the do-calculus is introduced to quantify interventions for the disrupted SC.

Table 1: Comparison of related studies on the SC disruption propagation management

Literature	Problem setting			Approach		
	supply chain structure		interventions	mathematical model	BN approach	do-calculus
	two-echelon	multi-echelon				
Tomlin (2006)	✓			✓		
Qi and Lee (2015)	✓			✓		
Ivanov et al. (2013)		✓		✓		
Ivanov et al. (2014a)		✓		✓		
Ivanov et al. (2015)		✓		✓		
Ivanov et al. (2016b)		✓		✓		
Sokolov et al. (2016)		✓		✓		
Sawik (2011)	✓			✓		
Sawik (2014)	✓			✓		
Sawik (2016)	✓			✓		
Sawik (2021)		✓		✓		
Levner and Ptuskin (2018)		✓		✓		
Kinra et al. (2020)		✓		✓		
Li and Zobel (2020)	✓			✓		
Gholami-Zanjani et al. (2021)		✓		✓		
Özçelik et al. (2021)		✓		✓		
Hosseini and Ivanov (2019)		✓			✓	
Hosseini et al. (2019b)	✓				✓	
Liu et al. (2021)	✓			✓	✓	
Hosseini et al. (2019c)		✓		✓	✓	
Our work		✓	✓	✓	✓	✓

3.1. Multi-echelon SC and disruption propagation

Consider a n -echelon SC network with a set of participants \mathcal{I} in which \mathcal{N}^j denotes the set of participants in the j th echelon, $j = 1, \dots, n$. Specially, $\mathcal{N}^1 = \{1, \dots, |\mathcal{N}^1|\}$, $\mathcal{N}^j = \{|\mathcal{N}^1| + \dots + |\mathcal{N}^{j-1}| + 1, \dots, |\mathcal{N}^1| + \dots + |\mathcal{N}^j|\}$ for $j = 2, \dots, n-1$, and $\mathcal{N}^n = \{|\mathcal{I}|\}$. Especially, $i, i \in \{1, \dots, |\mathcal{I}|-1\}$, denotes a supplier, and $|\mathcal{I}|$ a manufacturer. In the study, we consider that a participant i in the j th echelon can supply materials to multiple participants in the next echelon. There is no supply relationship among participants in the same echelon. In Figure 1, a 3-echelon SC network is illustrated, in which $\mathcal{N}^1 = \{1, 2\}$, $\mathcal{N}^2 = \{3, 4, 5\}$ and $\mathcal{N}^3 = \{6\}$. The set of predecessors of participant i is denoted as $\delta(i)$. For the example, we have $\delta(1) = \delta(2) = \emptyset$, $\delta(3) = \{1\}$, $\delta(4) = \{1\}$, $\delta(5) = \{2\}$, and $\delta(6) = \{3, 4, 5\}$. In line with Hosseini et al. (2019b) and Liu et al. (2020), we assume that suppliers in the first echelon can suffer diverse disruptive events, and disruptions of these SC participants can cause disruptions of the downstream SC participants.

Let \mathcal{S}_i be the set of possible states of SC participant $i \in \mathcal{I}$, and $x_i^s \in \mathcal{S}_i$ denotes the s th state of participant i , where s denotes a state index, $s \in \{1, \dots, |\mathcal{S}_i|\}$. Especially, let the first state x_i^1 and the last state $x_i^{|\mathcal{S}_i|}$ denote the operational and the fully disrupted states of i , respectively. The aim of interventions is to minimize the disruption risk in the entire SC. For interventions to SC participants, we have the following assumptions: 1) the intervention budget is limited to be B ; 2) an intervention can impose the state of participant i to be one of states in \mathcal{S}_i , and thus can affect the disruption propagation in the SC; 3) the intervention cost for a particular participant to be the first state (i.e., the operational state) is more expensive than other states; 4) intervention can be applied to several SC participants at different echelons, except for the manufacturer, as we attempt to analyze the disruption propagation; 5) when the parents of the manufacturers are in the operational state, the disruption risk of the manufacturer is minimum.

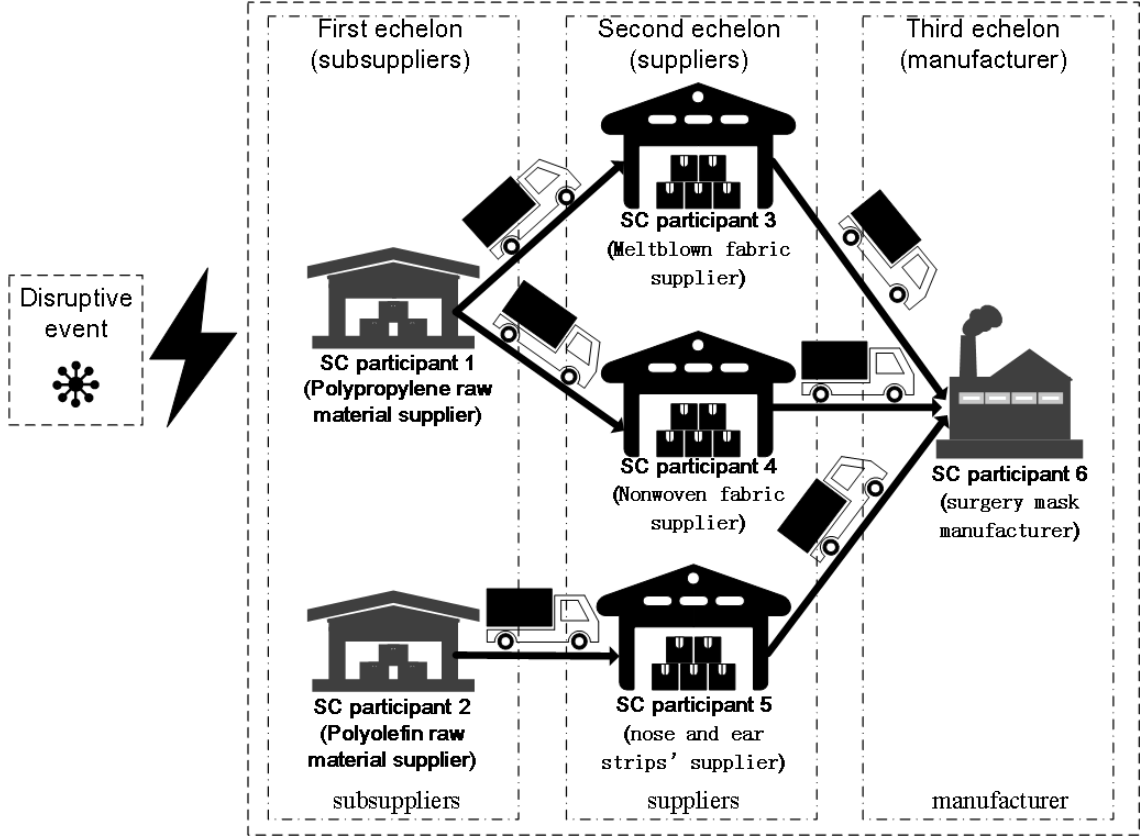


Figure 1: The structure of a 3-echelon SC network and disruptive events

3.2. Causal Bayesian network

In this work, CBN is introduced for the first time to portray the disruption propagation. The CBN for the SC disruption propagation without intervention is illustrated in Figure 2 in which the states of SC participants and disruption propagation relationships are represented by nodes and arcs, respectively. The arcs in Figure 2 represent the disruption propagations from SC participants in j th echelon to SC participants in the $(j + 1)$ th echelon, where $j = 1, \dots, n - 1$.

For example in Figure 1, suppose that participant i , $i = 1, \dots, 6$, has two states x_i^1 and x_i^2 , where x_i^1 and x_i^2 represent the operational and the fully disrupted states, respectively. Clearly, in this example, $|\mathcal{S}_i| = 2$ for $i = 1, \dots, 6$. The probability distribution of participant i , $i \in \mathcal{N}^1$, are both $(p_{i1}, p_{i2})^\top = (0.96, 0.04)^\top$, and the CPT $_i$ of participant i , $i \in \mathcal{N}^j$, $j = 2, 3$, are presented in Figure 3. For example, participant 6 in Figure 3 has the parent set $\delta(6) = \{3, 4, 5\}$ and the sets of possible states of its parent nodes can be represented as $\{x_i^1, x_i^2\}$, $i \in \{3, 4, 5\}$. The set of combinations of participant 6, \mathcal{G}_6 , can be represented as $\{(x_3^1, x_4^1, x_5^1), (x_3^1, x_4^1, x_5^2), (x_3^1, x_4^2, x_5^1), (x_3^1, x_4^2, x_5^2), (x_3^2, x_4^1, x_5^1), (x_3^2, x_4^1, x_5^2), (x_3^2, x_4^2, x_5^1), (x_3^2, x_4^2, x_5^2)\}$.

To establish the probability distribution of participant i , $i \in \mathcal{N}^j$, $j = 2, \dots, n$, let $G_i(\cdot)$ be a unique bijection mapping $\mathcal{G}_i \xrightarrow{G_i(\cdot)} \{1, \dots, g, \dots, |\mathcal{G}_i|\}$ where g denotes a state-combination-index. $G_i^{-1}(\cdot)$ denotes the inverse mapping of $G(\cdot)$, i.e., $\{1, \dots, g, \dots, |\mathcal{G}_i|\} \xrightarrow{G_i^{-1}(\cdot)} \mathcal{G}_i$, which maps a state-combination-index of SC participant i to a state combination of its parent participants. Especially, $G_i^{-1}(g)(i')$ represents the state index of parent node i' , $i' \in \delta(i)$, given the state-combination-index g of SC participant i , $i \in \mathcal{N}^j$, $j \in \{2, \dots, n\}$. Taking Figure 3 as example, $G_6^{-1}(1)(3) = 1$, which results in the first state index (i.e, the value 1 of the right-

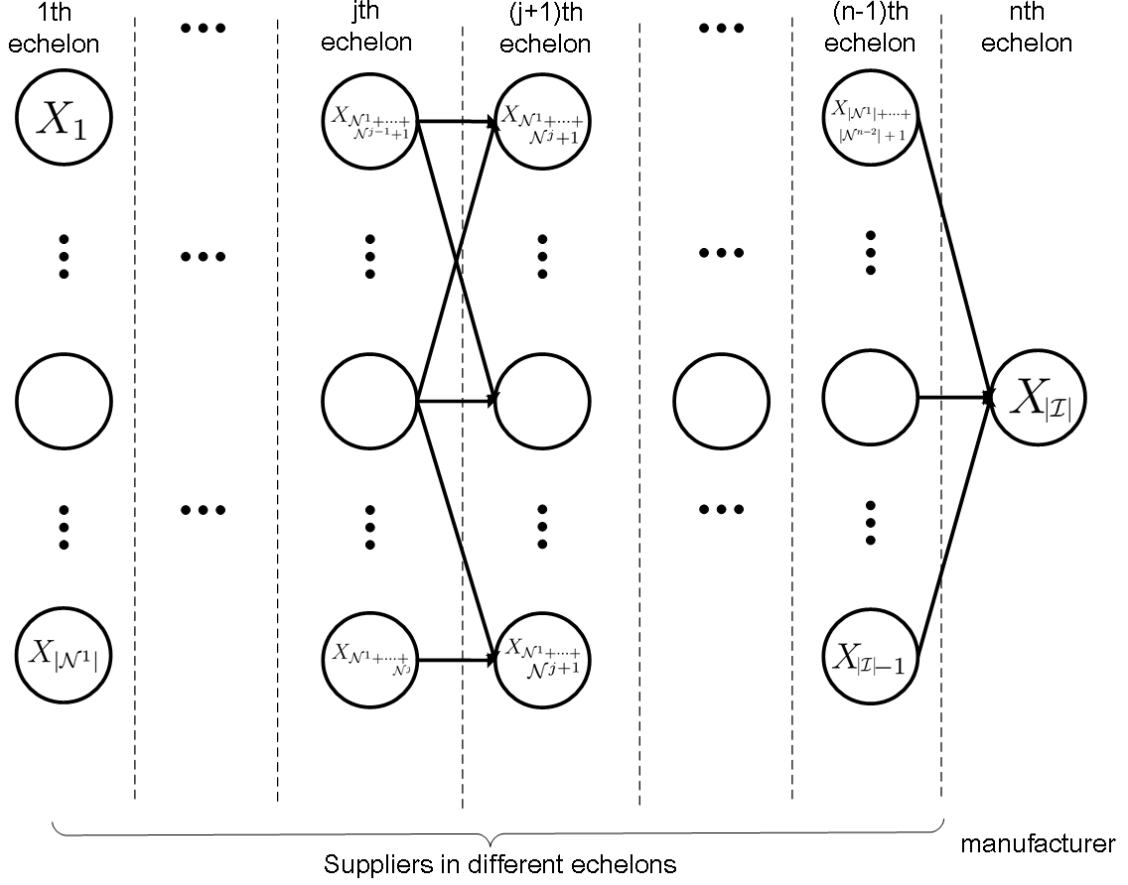


Figure 2: An illustrative CBN structure of the multi-echelon SC

hand side of this equation) of parent node 3 given the state-combination-index 1 of participant 6.

Given the above notations, the probability of participant i in the s th state, i.e., $p_{is} = \mathbb{P}\{X_i = x_i^s\}$, can be calculated as follows.

$$p_{is} = \sum_{g=1}^{|\mathcal{G}_i|} c_g^{i,s} \prod_{i' \in \delta(i)} p_{i', G_i^{-1}(g)(i')}, \quad i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}. \quad (1)$$

For example, in Figure 3, the probability distributions of SC participants 3, 4 and 5, can be calculated via Equation (1) as:

$$\begin{aligned} p_{31} &= c_1^{3,1} \cdot p_{11} + c_2^{3,1} \cdot p_{12} = 0.98 \cdot 0.96 + 0.30 \cdot 0.04 = 0.95, \\ p_{32} &= c_1^{3,2} \cdot p_{11} + c_2^{3,2} \cdot p_{12} = 0.05, \\ p_{41} &= c_1^{4,1} \cdot p_{11} + c_2^{4,1} \cdot p_{12} = 0.93, \\ p_{42} &= c_1^{4,2} \cdot p_{11} + c_2^{4,2} \cdot p_{12} = 0.07, \\ p_{51} &= c_1^{5,1} \cdot p_{11} + c_2^{5,1} \cdot p_{12} = 0.95, \\ p_{52} &= c_1^{5,2} \cdot p_{11} + c_2^{5,2} \cdot p_{12} = 0.05. \end{aligned}$$

Similarly, the probability distribution of the manufacturer in Figure 3 can be calculated via Equation (1) as:

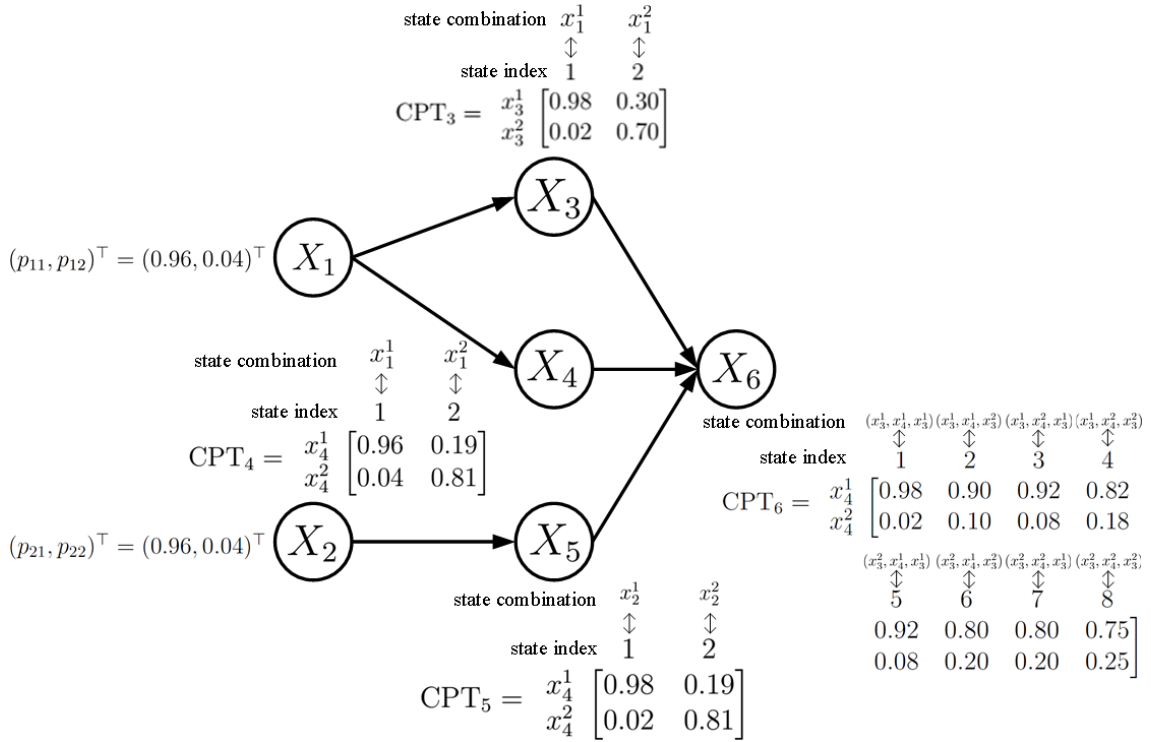


Figure 3: The CBN, the probability distributions, and the CPTs for the example in Figure 1

$$\begin{aligned}
p_{61} &= c_1^{6,1} \cdot p_{31} \cdot p_{41} \cdot p_{51} + c_2^{6,1} \cdot p_{31} \cdot p_{41} \cdot p_{52} + c_3^{6,1} \cdot p_{31} \cdot p_{42} \cdot p_{51} \\
&\quad + c_4^{6,1} \cdot p_{31} \cdot p_{42} \cdot p_{52} + c_5^{6,1} \cdot p_{32} \cdot p_{41} \cdot p_{51} + c_6^{6,1} \cdot p_{32} \cdot p_{41} \cdot p_{52} \\
&\quad + c_7^{6,1} \cdot p_{32} \cdot p_{42} \cdot p_{51} + c_8^{6,1} \cdot p_{32} \cdot p_{42} \cdot p_{52} = 0.97, \\
p_{62} &= c_1^{6,2} \cdot p_{31} \cdot p_{41} \cdot p_{51} + c_2^{6,2} \cdot p_{31} \cdot p_{41} \cdot p_{52} + c_3^{6,2} \cdot p_{31} \cdot p_{42} \cdot p_{51} \\
&\quad + c_4^{6,2} \cdot p_{31} \cdot p_{42} \cdot p_{52} + c_5^{6,2} \cdot p_{32} \cdot p_{41} \cdot p_{51} + c_6^{6,2} \cdot p_{32} \cdot p_{41} \cdot p_{52} \\
&\quad + c_7^{6,2} \cdot p_{32} \cdot p_{42} \cdot p_{51} + c_8^{6,2} \cdot p_{32} \cdot p_{42} \cdot p_{52} = 0.03.
\end{aligned}$$

When an intervention is conducted on participant i , the disruption propagation along the SC can be affected. To quantify the impact of interventions, we introduce the do-calculus in the next section.

3.3. The do-calculus based on CBN

In this work, each intervention is represented by $do(\cdot)$. Specifically, $do(X_i = x_i^s)$ denotes that an intervention imposes the state of participant i to be x_i^s . Consequently, the probability of this state changes to be one, i.e., $\mathbb{P}\{X_i = x_i^s\} = 1$. In other words, the state of i is fixed as x_i^s , i.e., $X_i = x_i^s$, due to the intervention $do(X_i = x_i^s)$. The do-operator $do(\cdot)$ can be included in the probability notations (Pearl, 2009). $\mathbb{P}(X_6 | do(X_4 = x_4^1))$ means the probability distribution of X_6 under an intervention to participant 4 with $do(X_4 = x_4^1)$.

To mathematically detail the procedure of the do-calculus, for each participant $i \in \mathcal{I}$, we first introduce binary decision variables z_i and y_{is} , where z_i is an intervention decision and y_{is} is a state-fixing decision. If an intervention is imposed to participant i , $z_i = 1$; $z_i = 0$, otherwise. If i 's state is imposed (or intervened) to

be x_i^s , then $y_{is} = 1$, i.e., $do(X_i = x_i^s)$; $y_{is} = 0$, otherwise. Clearly, if $z_i = 0$, then $y_{is} = 0$, for $s \in \{1, \dots, |\mathcal{S}_i|\}$ as y_{is} is the state-fixing decision (no intervention, no fixing). The probability of i in the s th state, $i \in \mathcal{I}, s \in \{1, \dots, |\mathcal{S}_i|\}$, conditional on its parent nodes is updated with interventions according to the following logic.

$$\mathbb{P}\{X_i = x_i^s | \delta(X_i)\} \leftarrow \begin{cases} \mathbb{P}\{X_i = x_i^s | \delta(X_i)\}, & \text{if } z_i = 0; \\ 1, & \text{if } z_i = 1 \text{ and } y_{is} = 1; \\ 0, & \text{if } z_i = 1 \text{ and } y_{is} = 0. \end{cases} \quad (2)$$

The do-calculus-based updated probability distribution of participant i , $i \in \mathcal{N}^1$, is denoted as $(P_{i1}, \dots, P_{is}, \dots, P_{i|\mathcal{S}_i|})^\top$. Based on the above notations, formula (2) can be mathematically represented as follows.

$$P_{is} = \begin{cases} p_{is}, & \text{if } z_i = 0, \\ 1, & \text{if } z_i = 1 \text{ and } y_{is} = 1, \\ 0, & \text{if } z_i = 1 \text{ and } y_{is} = 0, \end{cases} \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\}. \quad (3)$$

Note that, $(P_{i1}, \dots, P_{i|\mathcal{S}_i|})^\top$, $i \in \mathcal{N}^j$, $j \in \{2, \dots, n\}$, needs to be calculated echelon by echelon, which serve as auxiliary variables.

The do-calculus-based updated CPT of participant i is denoted as CPT_i^{do} , and the elements of CPT_i^{do} are denoted as $C_g^{i,s}$, $\forall i \in \mathcal{N}^j$, $j \in \{2, \dots, n\}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$, $g \in \{1, \dots, |\mathcal{G}_i|\}$. Therefore, based on the above notations, under state-combination-index of i , say, $g \in \{1, \dots, |\mathcal{G}_i|\}$, formula (2) can be mathematically represented as follows.

$$C_g^{i,s} = \begin{cases} c_g^{i,s}, & \text{if } z_i = 0, \\ 1, & \text{if } z_i = 1 \text{ and } y_{is} = 1, \\ 0, & \text{if } z_i = 1 \text{ and } y_{is} = 0, \end{cases} \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}. \quad (4)$$

For example, in Figure 4, the states of SC participants 4 and 5 are intervened to be states x_4^1 and x_5^1 respectively (i.e., $do(X_4 = x_4^1)$ and $do(X_5 = x_5^1)$ are performed), and CPT_4 and CPT_5 are updated to be CPT_4^{do} and CPT_5^{do} respectively. After the interventions, the probability distributions of SC participants 3, 4 and 5, can be calculated via formulas (3) and (4) as:

$$\begin{aligned} P_{31} &= c_1^{3,1} \cdot P_{11} + c_2^{3,1} \cdot P_{12} = 0.98 \cdot 0.96 + 0.30 \cdot 0.04 = 0.95, \\ P_{32} &= c_1^{3,2} \cdot P_{11} + c_2^{3,2} \cdot P_{12} = 0.05, \\ P_{41} &= c_1^{4,1} \cdot P_{11} + c_2^{4,1} \cdot P_{12} = 1, \\ P_{42} &= c_1^{4,2} \cdot P_{11} + c_2^{4,2} \cdot P_{12} = 0, \\ P_{51} &= c_1^{5,1} \cdot P_{11} + c_2^{5,1} \cdot P_{12} = 1, \\ P_{52} &= c_1^{5,2} \cdot P_{11} + c_2^{5,2} \cdot P_{12} = 0, \end{aligned}$$

Similarly, the probability distribution of the manufacturer in Figure 4 can be calculated via formulas (3) and (4) as:

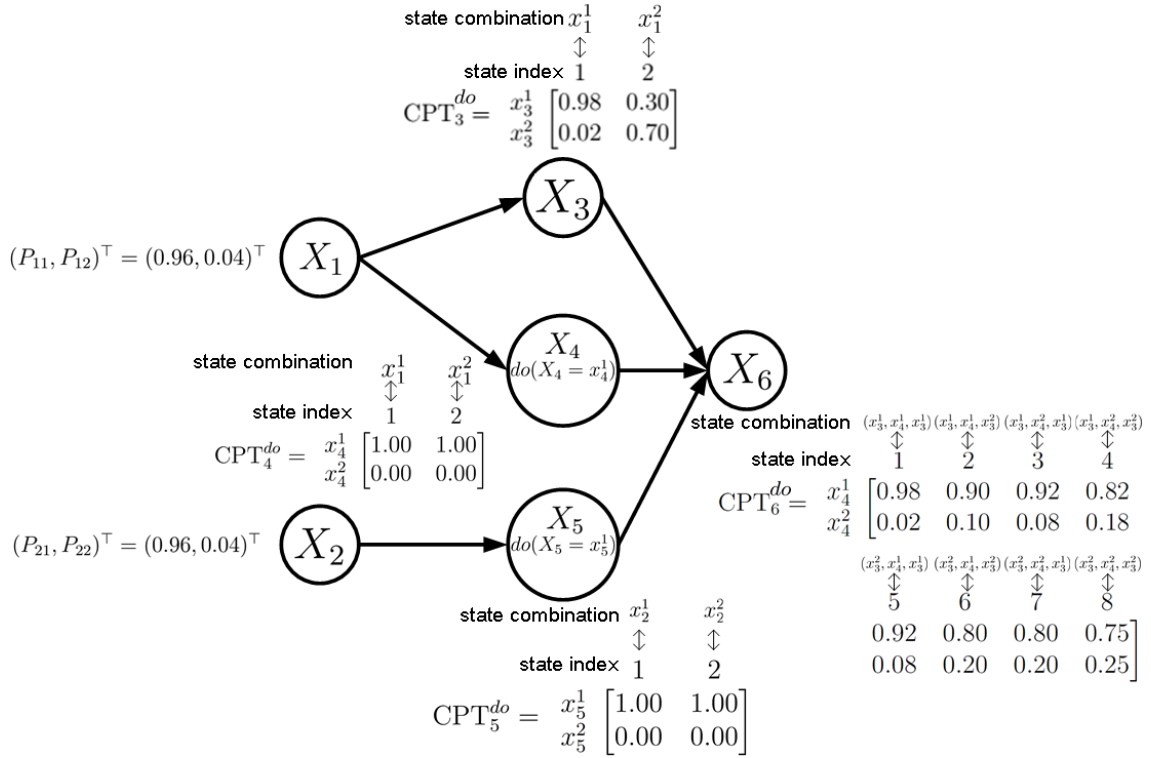


Figure 4: The updated probability distributions and CPTs^{do} for the CBN (after the interventions “do($X_4 = x_4^1$)” and “do($X_5 = x_5^1$)” are carried out on the state variable of SC participants 4 and 5)

$$\begin{aligned}
P_{61} &= C_1^{6,1} \cdot P_{31} \cdot P_{41} \cdot P_{51} + C_2^{6,1} \cdot P_{31} \cdot P_{41} \cdot P_{52} + C_3^{6,1} \cdot P_{31} \cdot P_{42} \cdot P_{51} \\
&\quad + C_4^{6,1} \cdot P_{31} \cdot P_{42} \cdot P_{52} + C_5^{6,1} \cdot P_{32} \cdot P_{41} \cdot P_{51} + C_6^{6,1} \cdot P_{32} \cdot P_{41} \cdot P_{52} \\
&\quad + C_7^{6,1} \cdot P_{32} \cdot P_{42} \cdot P_{51} + C_8^{6,1} \cdot P_{32} \cdot P_{42} \cdot P_{52} = 0.98, \\
P_{62} &= C_1^{6,2} \cdot P_{31} \cdot P_{41} \cdot P_{51} + C_2^{6,2} \cdot P_{31} \cdot P_{41} \cdot P_{52} + C_3^{6,2} \cdot P_{31} \cdot P_{42} \cdot P_{51} \\
&\quad + C_4^{6,2} \cdot P_{31} \cdot P_{42} \cdot P_{52} + C_5^{6,2} \cdot P_{32} \cdot P_{41} \cdot P_{51} + C_6^{6,2} \cdot P_{32} \cdot P_{41} \cdot P_{52} \\
&\quad + C_7^{6,2} \cdot P_{32} \cdot P_{42} \cdot P_{51} + C_8^{6,2} \cdot P_{32} \cdot P_{42} \cdot P_{52} = 0.02.
\end{aligned}$$

After mathematically quantifying the impacts of interventions, in the next section, we resort to mathematical programming tools to optimally determine the combination of interventions.

4. Non-linear programming models

Generally, the government intervenes a SC with the objective of minimizing the disruption risk of the target/key manufacturer. However, in practice, it is a challenging work how to choose SC participants to intervene and achieve the best result for reducing disruption propagation. Intervention actions are costly in practice, and the allocated budget for interventions is limited. Thus, in the section, we propose two mathematical programming models to select the best interventions.

In the following, the problem notations and decision variables are defined first. Then, two mixed-integer non-linear programming models, namely F_1 and F_2 , are established. Two valid inequalities are further proposed based on problem analysis.

Parameters:

\mathcal{I} : the set of SC participants, $\mathcal{I} = \{1, \dots, |\mathcal{I}|\}$, indexed by i ;

\mathcal{S}_i : the set of states of the SC participant i , i.e., $\mathcal{S}_i = \{x_i^1, \dots, x_i^{|\mathcal{S}_i|}\}$, where $s \in \{1, \dots, |\mathcal{S}_i|\}$;

\mathcal{N}^j : the set of SC participants in the j th echelon, $j \in \{1, \dots, n\}$;

$\delta(i)$: the set of parent nodes of SC participant i , $i \in \mathcal{I}$; note that, $\delta(i) = \emptyset$, for $i \in \mathcal{N}^1$;

$c_g^{i,s}$: the probability of participant i in state x_i^s conditional on the g th state combination of its parent nodes (i.e., an element in CPT_i), where $i \in \mathcal{I}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$, $g \in \{1, \dots, |\mathcal{G}_i|\}$;

$G_i^{-1}(g)(i')$: the state index of parent node i' , $i' \in \delta(i)$, for SC participant i , $i \in \mathcal{I}$, given state-combination-index g , $g \in \{1, \dots, |\mathcal{G}_i|\}$;

a_{is} : the intervention cost of SC participant i to be the s th state, where $i \in \mathcal{I} \setminus \{|\mathcal{I}|\}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$;

B : the total budget for executing interventions.

Decision variables:

z_i : equal to 1 if SC participant i , $i \in \mathcal{I}$, is intervened, 0 otherwise;

y_{is} : equal to 1 if SC participant i , $i \in \mathcal{I}$, is intervened to be state $x_i^s \in \mathcal{S}_i$ (i.e., $\text{do}(X_i = x_i^s)$), 0 otherwise;

P_{is} : the probability of participant i in state x_i^s after interventions, where $i \in \mathcal{I}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$;

$C_g^{i,s}$: the probability of participant i in state x_i^s given the state-combination-index g of its parent nodes after interventions (i.e., an element in CPT_i^{do}), where $i \in \mathcal{I}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$, $g \in \{1, \dots, |\mathcal{G}_i|\}$.

4.1. Non-linear programming model \mathbf{F}_1

In this subsection, we establish the first mixed-integer non-linear programming model, denoted by \mathbf{F}_1 , based on the do-calculus.

$$[\mathbf{F}_1] : \quad \min \quad P_{|\mathcal{I}|, |\mathcal{S}_{|\mathcal{I}|}} \quad (5)$$

$$\text{s.t.} \quad \sum_{i=1}^{|\mathcal{I}|-1} \sum_{s=1}^{|\mathcal{S}_i|} a_{is} y_{is} \leq B \quad (6)$$

$$\sum_{s=1}^{|\mathcal{S}_i|} y_{is} = z_i, \quad \forall i \in \mathcal{I} \quad (7)$$

$$P_{is} \leq 2 - y_{is}, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (8)$$

$$P_{is} \geq y_{is}, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (9)$$

$$P_{is} \leq 1 + y_{is} - z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (10)$$

$$P_{is} \leq p_{is} + z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (11)$$

$$P_{is} \geq p_{is} - z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (12)$$

$$C_g^{i,s} \leq 2 - y_{is}, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (13)$$

$$C_g^{i,s} \geq y_{is}, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (14)$$

$$C_g^{i,s} \leq 1 + y_{is} - z_i, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (15)$$

$$C_g^{i,s} \leq c_g^{i,s} + z_i, \quad i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (16)$$

$$C_g^{i,s} \geq c_g^{i,s} - z_i, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (17)$$

$$P_{is} = \sum_{g=1}^{|\mathcal{G}_i|} C_g^{i,s} \prod_{i' \in \delta(i)} p_{i', G_i^{-1}(g)(i')}, \quad i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (18)$$

$$0 \leq P_{is} \leq 1, \quad \forall i \in \mathcal{I}, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (19)$$

$$0 \leq C_g^{i,s} \leq 1, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (20)$$

$$z_{|\mathcal{I}|} = 0 \quad (21)$$

$$z_i \in \{0, 1\}, \quad \forall i \in \mathcal{I} \quad (22)$$

$$y_{is} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (23)$$

The objective function (5) minimizes the fully disrupted probability of SC participant $|\mathcal{I}|$ (i.e., the probability of fully disrupted state $x_{|\mathcal{I}|}^{|\mathcal{S}_i|}$) after interventions. Constraint (6) ensures that the total cost of all interventions imposed on SC participants does not exceed the limited budget. Constraints (7) guarantee that an intervened SC participant, if any, is only intervened to be a specific state. Constraints (8-12) ensure that the probability distribution of participant $i \in \mathcal{N}^1$, is updated based on formula (3). Specifically, constraints (8-9) ensure that the probability P_{is} is updated to be 1 if participant $i, i \in \mathcal{N}^1$, is intervened to be the s th state; constraints (10) guarantee that P_{is} is updated to be 0 if participant $i, i \in \mathcal{N}^1$, is intervened but not to be the s th state; and constraints (11-12) ensure that P_{is} keeps the same as before if SC participant $i, i \in \mathcal{N}^1$, is not intervened. Constraints (13-17) ensure that $CPT_i, i \in \mathcal{I}$, is updated to be CPT_i^{do} after interventions. Specifically, constraints (13-14) ensure that $C_g^{i,s}$ is updated to be 1 if SC participant i is intervened to be the s th state given the state-combination-index g ; constraints (15) ensure that $C_g^{i,s}$ is updated to be 0 if SC participant i is intervened but not to be the s th state given the state-combination-index g ; and constraints (16-17) ensure that $C_g^{i,s}$ keeps the same as before if SC participant i is not intervened given the state-combination-index g . These constraints are established to satisfy formula (4). Constraints (18) are established to calculate the probability distribution of each SC participant $i, i \in \mathcal{I}$, which are based on formula (1). **In constraint (18), index i' denotes the index for parent nodes of SC participant $i, i \in \mathcal{N}^1$.** Constraints (19-23) define the domains of variables.

4.2. Non-linear programming model \mathbf{F}_2

We attempt to reduce the number of variables and constraints used in \mathbf{F}_1 , and a more compact mixed-integer non-linear programming model, denoted by \mathbf{F}_2 , is established in this subsection.

The additional notations are defined as follows. To formalize the one-to-one correspondence relationship between a state combination of participants $1, \dots, |\mathcal{I}| - 1$ and a state-combination-index, we define the following mapping. Let $L(\cdot)$ be a unique bijection mapping $\mathcal{S}_1 \times \dots \times \mathcal{S}_{|\mathcal{I}|-1} \xrightarrow{L(\cdot)} \{1, \dots, l, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\}$, which maps a state combination of all suppliers to a state-combination-index $l, l \in \{1, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\}$. Note that, “ \times ” denotes the Cartesian product. Taking Figure 3 as example, the state combination of all suppliers is $\{(x_1^1, \dots, x_5^1), \dots, (x_1^{|\mathcal{S}_1|}, \dots, x_5^{|\mathcal{S}_5|})\}$ and $(x_1^1, x_2^1, x_3^1, x_4^1, x_5^1)$ corresponds to the state-combination-index $l = 1$. $L^{-1}(\cdot)$ denotes the inverse mapping of $L(\cdot)$, i.e., $\{1, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\} \xrightarrow{L^{-1}(\cdot)} \mathcal{S}_1 \times \dots \times \mathcal{S}_{|\mathcal{I}|-1}$ which maps a state-combination-index to a state combination of all suppliers. Conversely, state-combination-index $l = 1$ corresponds to state combination $(x_1^1, x_2^1, x_3^1, x_4^1, x_5^1)$. $L^{-1}(l)(i)$ represents the state index of SC participant $i, i \in \{1, \dots, |\mathcal{I}| - 1\}$, given the state-combination-index l . For the example in Figure 3, $L^{-1}(1)(2) = 1$ represents that given the state-combination-index 1 SC participant 2 is in the state with index 1 (in state x_1^1).

Similarly, let $F_i(\cdot)$ be a mapping $\{1, \dots, l, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\} \xrightarrow{F_i(\cdot)} \{1, \dots, g, \dots, |\mathcal{G}_i|\}$ which maps a state-combination-index of all suppliers to a state-combination-index of parent nodes of SC participant i . For example, in Figure 3, $F_6(1) = 1$ denotes that the state-combination-index 1 of all suppliers 1, 2, 3, 4, and 5 (corresponding to the state combination $(x_1^1, x_2^1, x_3^1, x_4^1, x_5^1)$) is mapped into the state-combination-index 1 of the parent nodes for participant 6 (corresponding to the state combination (x_3^1, x_4^1, x_5^1)).

Based on the above notations, the probability in state $x_{|\mathcal{I}|}^{|\mathcal{S}_i|}$ for the target manufacturer (with notation $|\mathcal{I}|$) after interventions can be described as:

$$P_{|\mathcal{I}|, |\mathcal{S}_i|} = \sum_{l=1}^{\prod_{i'=1}^{|\mathcal{I}|-1} |\mathcal{S}_{i'}|} \left(\prod_{i \in \mathcal{N}^1} P_{i, L^{-1}(l)(i)} \cdot \prod_{i \in \mathcal{N}^j, j \in \{2, \dots, n\}} C_{F_i(l)}^{i, L^{-1}(l)(i)} \right), \quad (24)$$

where $P_{i, L^{-1}(l)(i)}$ denotes the probability of i in the $L^{-1}(l)(i)$ th state; $C_{F_i(l)}^{i, L^{-1}(l)(i)}$ represents the probability of i in the $L^{-1}(l)(i)$ th state conditional on state-combination-index $F_i(l)$.

First, the additional notations are defined as follows. Then, the second mixed-integer non-linear programming model \mathbf{F}_2 is established, which is more concise than \mathbf{F}_1 .

New additional parameters:

$L^{-1}(l)(i)$: the state index of SC participant $i, i \in \{1, \dots, |\mathcal{I}| - 1\}$, for a given state-combination-index $l, l \in \{1, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\}$;

$F_i(l)$: the state-combination-index of parent nodes of SC participant $i, i \in \mathcal{N}^j, j \in \{2, \dots, n\}$, for a given state-combination-index $l, l \in \{1, \dots, \prod_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i|\}$.

We are now ready to present the second mixed-integer non-linear programming model.

$$[\mathbf{F}_2]: \quad \min \sum_{l=1}^{\prod_{i'=1}^{|\mathcal{I}|-1} |\mathcal{S}_{i'}|} \left(\prod_{i \in \mathcal{N}^1} P_{i, L^{-1}(l)(i)} \cdot \prod_{i \in \mathcal{N}^j, j \in \{2, \dots, n\}} C_{F_i(l)}^{i, L^{-1}(l)(i)} \right) \quad (25)$$

$$\text{s.t.} \quad \sum_{i=1}^{|\mathcal{I}|-1} \sum_{s=1}^{|\mathcal{S}_i|} a_{is} y_{is} \leq B \quad (26)$$

$$\sum_{s=1}^{|\mathcal{S}_i|} y_{is} = z_i, \quad \forall i \in \mathcal{I} \quad (27)$$

$$P_{is} \leq 2 - y_{is}, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (28)$$

$$P_{is} \geq y_{is}, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (29)$$

$$P_{is} \leq 1 + y_{is} - z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (30)$$

$$P_{is} \leq p_{is} + z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (31)$$

$$P_{is} \geq p_{is} - z_i, \quad \forall i \in \mathcal{N}^1, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (32)$$

$$C_g^{i,s} \leq 2 - y_{is}, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (33)$$

$$C_g^{i,s} \geq y_{is}, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (34)$$

$$C_g^{i,s} \leq 1 + y_{is} - z_i, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (35)$$

$$C_g^{i,s} \leq c_g^{i,s} + z_i, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (36)$$

$$C_g^{i,s} \geq c_g^{i,s} - z_i, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (37)$$

$$0 \leq C_g^{i,s} \leq 1, \quad \forall i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_i|\}, g \in \{1, \dots, |\mathcal{G}_i|\} \quad (38)$$

$$z_{|\mathcal{I}|} = 0 \quad (39)$$

$$z_i \in \{0, 1\}, \quad \forall i \in \mathcal{I} \quad (40)$$

$$y_{is} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, s \in \{1, \dots, |\mathcal{S}_i|\} \quad (41)$$

The objective function (25) minimizes the fully disrupted probability of SC participant $|\mathcal{I}|$ after interventions. Constraint (26) ensures that the total cost of all interventions imposed on SC participants does not exceed the limited budget. Constraints (27) guarantee that an intervened SC participant, if any, is only intervened to be a specific state. Constraints (28-32) ensure that the probability distribution of participant $i \in \mathcal{N}^1$, is updated based on formula (3). Specifically, constraints (28-29) ensure that the probability P_{is} is updated to be 1 if participant $i, i \in \mathcal{N}^1$, is intervened to be the s th state; constraints (30) guarantee that P_{is} is updated to be 0 if participant $i, i \in \mathcal{N}^1$, is intervened but not to be the s th state; and constraints (31-32) ensure that P_{is} keeps the same as before if SC participant $i, i \in \mathcal{N}^1$, is not intervened. Constraints (33-37) ensure that $\text{CPT}_i, i \in \mathcal{I}$, is updated to be CPT_i^{do} after interventions. Specifically, constraints (33-34) ensure that $C_g^{i,s}$ is updated to be 1 if SC participant i is intervened to be the s th state given the state-combination-index g ; constraints (35) ensure that $C_g^{i,s}$ is updated to be 0 if SC participant i is intervened but not to be the s th state given the state-combination-index g ; and constraints (36-37) ensure that the $C_g^{i,s}$ keeps the same as before if SC participant i is not intervened given the state-combination-index g . These constraints are constructed to satisfy formula (4). Constraints (38-41) define the domains of variables.

The number of decision variables in model \mathbf{F}_2 is less than that of model \mathbf{F}_1 by $\sum_{i=1}^{|\mathcal{I}|-1} |\mathcal{S}_i| + 1$. The number of constraints in model \mathbf{F}_2 is less than that of model \mathbf{F}_1 by $\sum_{i=1}^{|\mathcal{I}|} |\mathcal{S}_i| + 1$. To obtain an approximate solution quickly in larger-scale instances, we can use the following heuristics: 1) In the mixed-integer nonlinear programming

model, the integer variables can be relaxed to be continuous variables, and this model is transformed into a nonlinear programming model, which can be solved quickly. 2) Then, the obtained the solutions of relaxed integer variables are rounded to be integer variables. In the above way, we can quickly obtain an approximate solution.

4.3. Problem analysis

In this subsection, we present two problem properties by Propositions 1 and 2 to establish valid inequalities, leading to the enhanced formulations.

The additional notations are defined as follows.

New additional parameters:

$suc(i)$: the set of successors of i , $i \in \mathcal{N}^j, j \in \{1, \dots, n-2\}$, indexed by i' ;

κ : binary parameter, equal to 1 if the total budget B is not less than the total cost for intervening participants in the $(n-1)$ th echelon to be operational state (i.e., the first state); 0 otherwise.

In the following, we present two propositions and the corresponding valid inequalities are established.

Proposition 1. *When all successors of participant i , $i \in \mathcal{N}^j, j \in \{1, \dots, n-2\}$, are intervened, there is an optimal solution of models \mathbf{F}_1 and \mathbf{F}_2 such that participant i does not need to be intervened.*

Proof: see Appendix.

Based on **Proposition 1**, a valid inequality, i.e., VI1, can be proposed as follows.

$$\text{VI1: } z_i \leq |suc(i)| - \sum_{i' \in suc(i)} z_{i'}, \quad \forall i \in \mathcal{N}^j, j = 1 \dots, n-1.$$

The valid inequality VI1 means that if all successors of participant i are intervened, i.e., $z_{i'} = 1, \forall i' \in suc(i)$, there is an optimal solution of models \mathbf{F}_1 and \mathbf{F}_2 in which this participant needs not to be intervened, i.e., $z_i = 0, i \in \mathcal{N}^j, j \in \{1, \dots, n-2\}$.

Proposition 2. *When budget B is not less than the total intervention cost of all participants in the $(n-1)$ th echelon to be the operational state, i.e., $B \geq \sum_{i \in \mathcal{N}^{n-1}} a_{i1}$ (and thus $\kappa = 1$), then in an optimal solution of models \mathbf{F}_1 and \mathbf{F}_2 all these participants in the $(n-1)$ th echelon are intervened to be operational state.*

Proof: see Appendix.

Based on **Proposition 2**, another valid inequality, i.e., VI2, can be proposed as follows.

$$\text{VI2: } y_{i1} \geq \kappa, \quad \forall i \in \mathcal{N}^{n-1}.$$

The valid inequality VI2 implies that if $\kappa = 1$, i.e., $B \geq \sum_{i \in \mathcal{N}^{n-1}} a_{i1}$, then $y_{i1} = 1, \forall i \in \mathcal{N}^{n-1}$.

5. Solution method for large-scale problems

Genetic Algorithm (GA), a popular heuristic algorithm, can be applied to solve mixed-integer nonlinear optimization problems (Hua and Huang, 2006). In this section, a problem-specific GA algorithm is designed to cope with large-scale problems. The following subsections discuss the chromosome representation and initialization, crossover and mutation, fitness calculation, and selection operation. Given the mathematical programming models in Section 3, commercial solvers cannot solve large-scale problems, therefore a problem-specific GA is developed.

5.1. Chromosome representation and initialization

In the chromosome, the number p at the i th position represents that the SC participant i , $i \in \mathcal{I}$, is intervened into state p . The length of chromosome is $|\mathcal{I}|$, where \mathcal{I} denotes the set of SC participants. Specially, $p = 0$ denotes the SC participant is not intervened, and the last bit in the chromosome represents the fitness value. The gene structure of an example is illustrated in Figure 5. In this example, SC participants include 1, 2, 3, 4, 5, and 6. SC participants 1, 2, 5 are intervened into states 1, 2, 1, respectively. In the chromosome, SC participants 3 and 4 are not intervened, which are set to be 0. The fitness value 0.23 is coded in the 6th bit.

For each initial individual, the intervened SC participants are generated according to the limited budget B . The chromosome initialisation steps are illustrated in Algorithm 1 for intervening SC participants. Now we have coded the $|\mathcal{I}|$ bits for a chromosome. Given the above information, we calculate its objective value by using BN inference approach, which serves as the reciprocal of the fitness value for this chromosome (coded in the last bit).

Algorithm 1 Intervention of SC participants in the chromosome initialization

function Initialize SC participants intervention (a_{ij}, B)

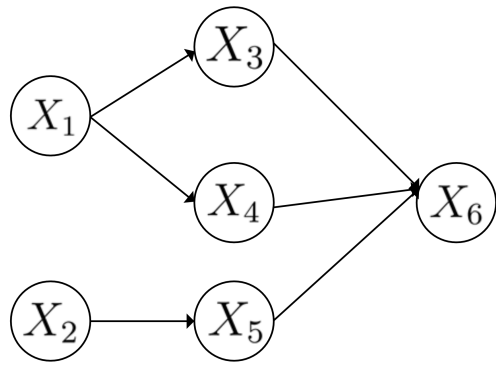
% a_{ij} : the intervention cost of SC participant i to be the s th state; B : the total budget for executing interventions
 $U = 0$; $X = \emptyset$; % U denotes the used intervention cost; X denotes the SC participants' intervention
while $U \geq B$ **do**
 randomly select an SC participant i_c and fix its state to be s_c ,
 and $U = U + a_{i_c s_c}$;
 $X = X \cup \{i_c \mapsto s_c\}$; % intervene SC participant i_c into state s_c
return: SC participants' intervention X .

5.2. Crossover and mutation

To generate offsprings, efficient crossover and mutation are commonly applied. In this work, one-point crossover and two different mutation operators are designed.

To satisfy the restriction of the limited budget, we conduct the following operations: (i) randomly selecting a crossover point; (ii) swapping the tails of two parents to generate two children; (iii) calculating the fitness values of two children; (iv) if the restriction is satisfied, output two children; otherwise, adjusting the intervened SC participants and their states to satisfy the restriction. Figure 6 illustrates the procedure.

The mutation operator can expand the searching space of the GA, and improve the performance of the GA. Therefore, two mutation operators are designed in this work, which include: (i) randomly modifying an intervened SC participant and its state; (ii) randomly swapping of two intervened SC participants in the chromosome.



(a) the SC structure of the example

1	2	0	0	1	0.23
---	---	---	---	---	------

Fitness
value

(b) the chromosome of the GA for the example

Figure 5: Chromosome representation

For example in Figure 6, the first mutation operator consists of (i) copying the chromosome to an offspring; (ii) randomly selecting a free SC participant (say $i1$); (iii) judging which SC participants could be intervened to replace SC participant $i1$ conforming with the limited budget; (iv) randomly modifying the state for the SC participant based on step (iii), and calculating the fitness value of the child. The second mutation operator is illustrated in Figure 7, which includes: (i) coping the chromosome to an offspring individual; (ii) randomly selecting two intervened SC (say, participant $r1$ in state $i1$, and participant $r2$ in state $i2$); (iii) judging whether SC participant $r1$ can be intervened into state $i2$ conforming with the limited budget B , and further judging whether SC participant $r2$ can be intervened to state $i1$ conforming with the limited budget B ; (iv) if both yes, swapping and calculating the fitness value of the offspring; otherwise, not swapping.

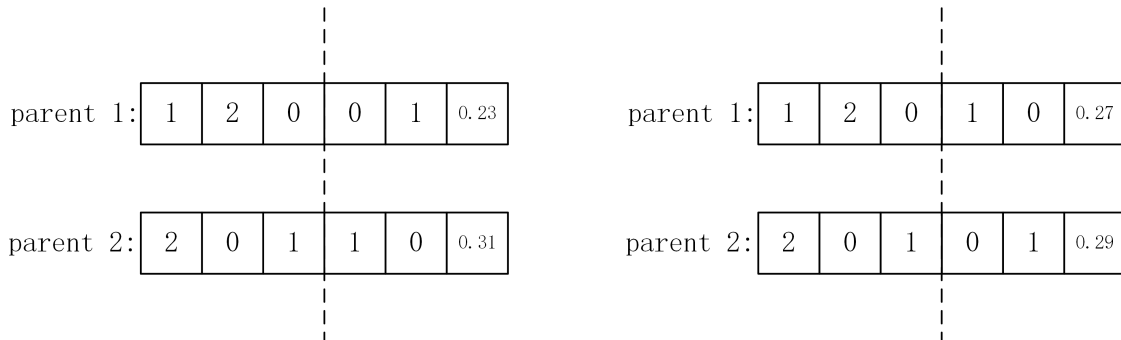


Figure 6: Crossover



Figure 7: Mutation

5.3. Fitness Calculation

To calculate the fitness value of the chromosome, the BN inference approach can be applied. Based on the notation of do-calculus in Section 3.3, the intervened SC participants and their states adjust the probability informations in the CBN, and the structure of CBN cannot be changed. Before calculating the fitness value, the probability informations can be updated according to logic (2) in Section 3.3. The BN approach can be directly applied to calculate the fitness value, which is expressed as follows.

$$fitness_c = \sum_{l=1}^{\prod_{i'=1}^{|Z|} |S_{i'}|} \left(\prod_{i \in \mathcal{N}^1} P_{i, L^{-1}(l)(i)} \cdot \prod_{i \in \mathcal{N}^j, j \in \{2, \dots, n\}} C_{F_i(l)}^{i, L^{-1}(l)(i)} \right). \quad (42)$$

5.4. Selection operator

The chromosome is selected according to probability pr_c , which is calculated by fitness as follows.

$$pr_c = \frac{fitness_c}{\sum_{i=1}^{sizepop} fitness_i}$$

where $sizepop$ is the number of chromosomes in the population. We will examine the effectiveness of our propose GA for addressing large-scale problems in the next section.

6. Numerical experiments

In this section, experiments on a case study and 30 randomly generated instances are conducted to test the proposed models that are solved by internal solver BMIBN-B in YALMIP. In Section 5.1, a case study is conducted and managerial insights are drawn. In Section 5.2, the efficiency of the proposed valid inequalities are evaluated. All the numerical experiments are conducted on a personal computer with Core I5 and 3.60GHz processor and 8GB RAM under Windows 7 Operating System. The computation time of the internal solver BMIBNB in YALMIP is limited as 3600s.

6.1. Case study

In this subsection, we test different 3-echelon SCs with 6 participants (see Figure 8). Three states, i.e., fully operational, semi-fully operational, and fully disrupted, are considered. Suppose that the intervention costs of SC participants 1, 2, 3, 4, 5 to be the fully operational state are 45, 78, 57, 66, 67, respectively. The intervention costs of SC participants 1, 2, 3, 4, 5 to be the semi-fully operational state are 50, 48, 47, 56, 57, respectively. Different intervention budgets B equal to 0, 50, 100, 150, 200, 250, and 300, are tested to evaluate their impacts on the disruption risk of the manufacturer. The case study is modelled by the proposed models F_1 and F_2 . Internal solver BMIBNB is applied by exploiting the optimization infrastructure in YALMIP.

Table 2 reports the average results of 5 instances under the same parameter combination. That is, each combination, i.e., one row in the Table 2, is tested with 5 randomly generated instances. The first column in Table 2 gives different SC structures. The second column presents different intervention budgets. The third and fourth columns report computation time of models F_1 and F_2 in seconds (s for short). The fifth and sixth columns report the intervened participants and the

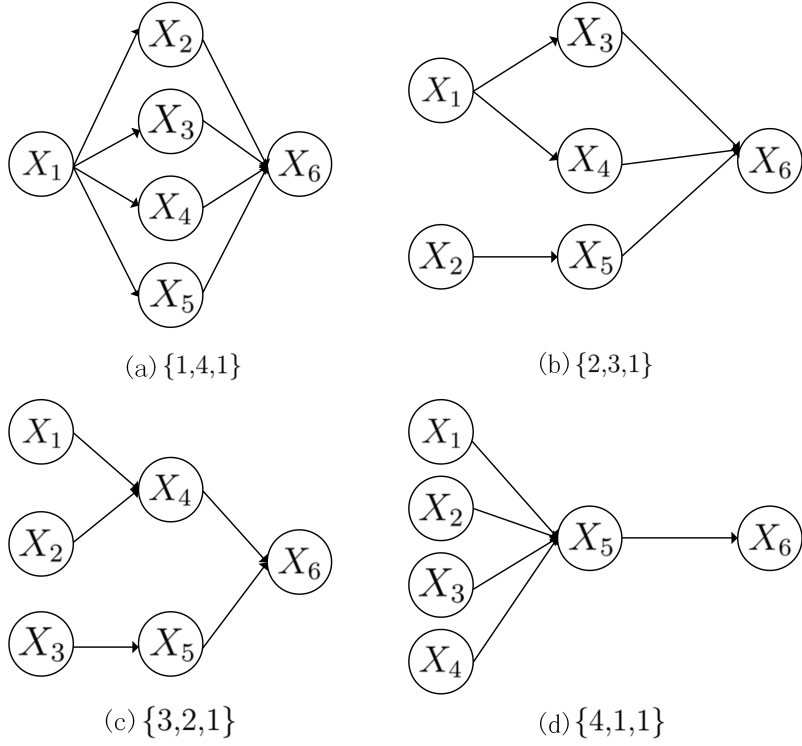


Figure 8: Different SC structures in the case study

minimum disruption risk of the manufacturer under a given B , respectively. The seventh column represents the used budget. From the 3rd and the 4th columns of Table 2, we can observe that models F_1 and F_2 can be solved in long computation time, thus it is necessary to design a fast heuristic algorithm. From the 5th and the 6th columns, we can see that optimal interventions are performed on the participants in the echelon close to the manufacturer's position, if budget B reaches a threshold. For example, in combination $\{4,1,1\}$, when budget B reaches 100, the increase of budget B does not further impact the disruption risk of the manufacturer. We can observe from the 6th column of Table 2, the allocated budgets are not all consumed. The reason is that, based on Proposition 2, this threshold can be obtained as $\sum_{i \in \mathcal{N}^{n-1}} a_{i,1}$, i.e., the sum of SC participants in the $(n-1)$ -echelon intervened to be state $s = 1$. Generally, when B increases, the disruption probability of the manufacturer decreases in general.

In view of the above observations, we can have the following managerial insights:

1. The disruption probability of the manufacturer may decrease with the increase of budget. Thus, both an appropriate budget and an optimal intervention scheme are important to reduce the SC disruption propagation.
2. SC participants that are close to the manufacturer are often extremely important in reducing the disruption probability of the manufacturer. Therefore, the manufacturer needs to pay close attention to the operations of preceding suppliers in the SC.
3. When all SC participants in the $(n-1)$ -echelon are intervened into fully operational state, the disruption probability of the manufacturer remains unchanged no matter how other participants are intervened. This result can be derived from Proposition 2. Thus, it is important to determine the budget of the interventions. Therefore, an excessive intervention budget is unnecessary.

Table 2: The intervention decisions under different SC structures and budgets

$\{ \mathcal{N}^1 , \mathcal{N}^2 , \mathcal{N}^3 \}$	Total budget B	Model \mathbf{F}_1 time(s)	Model \mathbf{F}_2 time(s)	Intervened participants	Disruption risk of the manufacturer	Used budget
$\{1,4,1\}$	0	5.50	61.35	None	0.0849	0
	50	51.80	127.22	1	0.0782	45
	100	663.98	718.96	1	0.0782	45
	150	653.40	1359.51	1	0.0782	45
	200	625.67	941.16	1	0.0782	45
	250	619.38	1813.29	1	0.0782	45
	300	651.93	659.73	2,3,4,5	0.0300	268
$\{2,3,1\}$	0	0.78	0.89	None	0.0327	0
	50	7.74	4.04	1	0.0279	45
	100	32.21	20.36	4	0.0271	66
	150	95.87	49.61	4,5	0.0228	133
	200	35.30	49.75	3,4,5	0.0200	190
	250	37.73	66.72	3,4,5	0.0200	190
	300	113.35	77.30	3,4,5	0.0200	190
$\{3,2,1\}$	0	12.37	1.82	None	0.2954	0
	50	15.49	7.00	5	0.1716	17
	100	71.73	88.30	3,4,5	0.0800	86
	150	84.09	116.95	4,5	0.0200	133
	200	217.02	118.20	4,5	0.0200	133
	250	123.06	146.24	4,5	0.0200	133
	300	118.31	216.85	4,5	0.0200	133
$\{4,1,1\}$	0	10.31	14.83	None	0.0624	0
	50	542.86	491.59	None	0.0624	0
	100	723.52	657.21	5	0.0400	67
	150	601.89	628.62	5	0.0400	67
	200	623.76	1004.47	5	0.0400	67
	250	629.78	925.39	5	0.0400	67
	300	644.52	1113.02	5	0.0400	67

6.2. Valid inequalities

In this subsection, the performances of two valid inequalities, i.e., VI1 and VI2, are tested. The number of states and the number of echelons are set to be 2 and 3, respectively. Different numbers of SC participants, $|\mathcal{I}| \in \{5, 6, 7, 8, 9, 10\}$, are tested. The intervention cost a_{is} is randomly generated on the interval $[50, 75]$, where $i \in \mathcal{I} \setminus \{|\mathcal{I}|\}$, $s \in \{1, \dots, |\mathcal{S}_i|\}$, conforming with the problem assumption. The budget B is randomly generated between $[50 \cdot \frac{|\mathcal{I}|}{4}, 75 \cdot \frac{|\mathcal{I}|}{2}]$, where $|\mathcal{I}|$ is the number of participants. The probability informations are randomly generated with respect to the second Kolmogorov axiom of probability (Durrett, 2019), i.e., $\sum_{s=1}^{|\mathcal{S}_i|} p_{is} = 1$, $i \in \mathcal{N}^1$, and $\sum_{s=1}^{|\mathcal{S}_i|} c_g^{i,s} = 1$, where $g \in \{1, \dots, |\mathcal{G}_i|\}$, $i \in \mathcal{N}^j$, $j = 2, \dots, n$. Each participant in the j th echelon randomly link to one to two participants in the $(j+1)$ th echelon, where $j = 1, \dots, n-1$. Five instances with the same $|\mathcal{I}|$ are randomly generated and tested. In total, 30 random instances are tested.

The average computation results for \mathbf{F}_1 and valid inequalities are presented in Table 3. The first column in Table 3 represents the number of participants, i.e., $|\mathcal{I}|$. The 2nd to the 9th columns represent the objective value and the running time of models \mathbf{F}_1 , \mathbf{F}_1 +VI1, \mathbf{F}_1 +VI2, and \mathbf{F}_1 +VI1+VI2, respectively. We can observe from Table 3 that the average running time of models \mathbf{F}_1 +VI1, \mathbf{F}_1 +VI2, and \mathbf{F}_1 +VI1+VI2 are 1496.89s, 822.02s, and 709.77s respectively, which are only 92.57%, 50.84% and 43.89% of that of needed by model \mathbf{F}_1 . We can remark that the running time of model \mathbf{F}_1 +VI2 for $|\mathcal{I}| = 8$ is 697.13s, which is faster than that proposed by model \mathbf{F}_1 +VI1+VI2. The reason may be that more inequalities consume more computation time, and VI1 is not very efficient for the instances.

Table 3: Evaluation of the efficiency of VI1 and VI2 for F_1

\mathcal{I}	F_1		F_1+VI1		F_1+VI2		$F_1+VI1+VI2$	
	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)
5	0.23	120.61	0.23	106.66	0.23	17.65	0.23	18.29
6	0.20	385.45	0.20	334.50	0.20	284.40	0.20	283.59
7	0.17	879.04	0.17	786.37	0.17	653.25	0.17	649.35
8	0.22	1116.77	0.22	983.09	0.22	697.13	0.22	763.67
9	0.29(5)*	3600.00	0.28(5)*	3600.00	0.22(2)*	1889.27	0.23(2)*	1451.37
10	0.26(5)*	3600.00	0.26(5)*	3170.74	0.24(1)*	1390.43	0.24(1)*	1092.36
Average	0.23	1616.98	0.23	1496.89	0.22	822.02	0.22	709.77

“(number of instances)*”: the number of instances that be solved to optimum in the time limit.

Table 4: Evaluation of the efficiency of VI1 and VI2 for F_2

\mathcal{I}	F_2		F_2+VI1		F_2+VI2		$F_2+VI1+VI2$	
	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)
5	0.23	56.70	0.23	49.82	0.23	10.23	0.23	10.96
6	0.20	242.81	0.20	204.09	0.20	101.47	0.20	152.23
7	0.17	958.46	0.17	565.25	0.17	346.03	0.17	472.42
8	0.24(2)*	2470.21	0.22	1779.00	0.22	1548.90	0.22	1478.40
9	0.20(4)*	3444.68	0.20(5)*	3223.27	0.17(1)*	1914.11	0.13	1380.12
10	0.36(5)*	3600.00	0.31(5)*	3600.00	0.23(1)*	1536.35	0.22(1)*	1078.72
Average	0.23	1795.48	0.22	1570.24	0.20	909.52	0.20	762.14

“(number of instances)*”: the number of instances that be solved to optimum in the time limit.

With the increase of the number of participants, the computation time of model F_1 increases from 120.61s to 3600s. Especially, model F_1 cannot obtain optimal solution for all five instances. In conclusion, the computation results show that: (i) both valid inequalities are efficient for reducing computation times; (ii) model $F_1+VI1+VI2$ is of the most efficiency in term of computation time.

The average computation results for F_2 and valid inequalities are presented in Table 4. We can observe from Table 4 that the average running time of models F_2+VI1 , F_2+VI2 , and $F_2+VI1+VI2$ are 1570.24s, 909.52s, and 762.14s respectively, which are only 87.46%, 50.66% and 42.45% of that required by model F_2 . Specifically, when the number of participants is 10, model F_2 cannot obtain optimal solution for all 5 instances. In conclusion, the computation results show that: (i) both valid inequalities are efficient for reducing the computation times of model F_2 ; (ii) model $F_2+VI1+VI2$ is of the most efficiency in term of computation time.

We can observe from Tables 3 and 4 that the efficiencies of models $F_1+VI1+VI2$ and $F_2+VI1+VI2$ are comparable. Especially, the average computation time of $F_1+VI1+VI2$ is 93.13% of that needed by $F_2+VI1+VI2$, while $F_1+VI1+VI2$ and $F_2+VI1+VI2$ cannot find optimal solution for 3 and 1 (out of 5) instances, respectively.

6.3. Experimental results

To evaluate the performance of the problem-specific GA, the comparison is made with the two models ($F_1+VI1+VI2$ and $F_2+VI1+VI2$, solved with internal solver BMIBNB by the optimization infrastructure in YALMIP) and the GA, respectively. Computational results of the problem-specific GA and the models are reported in Table 5. The first column in Table 5 represents the number of participants, i.e.,

Table 5: Comparison of computational results between models and GA

\mathcal{I}	$F_1+VI1+VI2$		$F_2+VI1+VI2$		GA		
	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)	Improvement(%)
4	0.26	4.23	0.26	3.52	0.26	0.01	0
5	0.40	11.36	0.40	8.63	0.40	0.02	0
6	0.18	23.94	0.18	17.30	0.18	0.02	0
7	0.21	48.18	0.21	79.15	0.22	2.05	-4.76
8	0.25	118.45	0.25	347.42	0.26	3.16	-4.00
9	0.16	277.19	0.16	749.59	0.17	4.51	-6.25
10	0.34	526.74	0.34	2747.85	0.34	20.49	0
11	0.26	2081.60	0.28	3600.00	0.22	131.84	15.38
12	0.32	3600.00	0.32	3600.00	0.30	413.85	6.25
13	0.34	3600.00	0.34	3600.00	0.26	443.12	25.00
14	0.43	3600.00	0.43	3600.00	0.36	427.54	16.28
15	-	3600.00	-	3600.00	0.39	426.78	-
16	-	3600.00	-	3600.00	0.41	573.03	-
17	-	3600.00	-	3600.00	0.45	1490.51	-
Average*	0.29	1262.88	0.29	1668.50	0.27	131.51	6.90

“-”: no instance can be solved to feasible solutions in the time limit.

“Improvement(%)”: is calculated via $\frac{\min\{\text{model values}\}-\text{GA value}}{\min\{\text{model values}\}} \times 100\%$.

“*”: the average value for these instances that can obtain feasible solutions.

| \mathcal{I} |. The 2nd to the 7th columns represent the objective value and the running time of the models, and those of the GA, respectively. The 8th column represents the improvement of the GA. The improvement value between the GA and the two models is calculated by $\frac{\min\{\text{model values}\}-\text{GA value}}{\min\{\text{model values}\}} \times 100\%$.

From Table 5, we can observe that with increase of the size of instances, the computation time of two models increase much faster than the GA. The average running time of the GA is 131.51, which is only 10.41% (i.e., $\frac{131.51}{1763.69} \times 100\%$) of model $F_1+VI1+VI2$. The average improvement value of the GA is 6.90%.

The experimental results show that: (i) our proposed models are time-consuming; (ii) the problem-specific GA is very efficient, as it can obtain a feasible solution with significantly shorter time than two models in addressing large-scale problems.

7. Conclusion

In this paper, we study the disruption propagation management problem in a multi-echelon SC under limited intervention budget. For the problem, a novel approach, combining the CBN, the do-calculus and the mathematical programming, is developed. Then, we propose two mixed-integer non-linear programming models to solve the problem. The proposed models allow for minimizing disruption risk under limited intervention budget. To improve the efficiencies of proposed models, we exploit the problem properties and develop two valid inequalities. Besides, we investigate a case study, and managerial insights are drawn. Finally, the numerical experiments demonstrate that the efficiencies of models and valid inequalities.

Our future researches include the following issues. First of all, to portray the dynamic fluctuation of the disruption risk, a hybrid model integrating Markov chain with static or dynamic BN may be further designed to extend our study. Second, different intervention costs from a state to another state can be considered. Third, the study can be further extended to data scarce environment in which only partial distribution probability information is known. Fourth, some uncontrollable suppliers can be considered, and a more general model can be further developed. Fifth, to

solve large-scale problems faster and more effectively, more efficient approaches, e.g., heuristics, may be designed. Finally, it may be interesting to combine supplier selection decisions into the studied problem.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (NSFC) under Grants 71531011, 71771048, 71432007, 71832001, 71871159 and 71571134.

Appendix

Proposition 1. *When all successors of participant i , $i \in \mathcal{N}^j$, $j \in \{1, \dots, n-2\}$, are intervened, there is an optimal solution of models \mathbf{F}_1 and \mathbf{F}_2 such that participant i does not need to be intervened.*

PROOF. For each participant i , $i \in \mathcal{N}^j$, $j \in \{1, \dots, n-2\}$, the probability distribution of subsequent (or immediately succeeding) participant i' , $i' \in \text{suc}(i)$, can be calculated via equation (1) as:

$$p_{i's} = \sum_{g=1}^{|\mathcal{G}_{i'}|} c_g^{i',s} \prod_{i \in \delta(i')} p_{i, G_i^{-1}(g)(i')}, \quad i' \in \text{suc}(i), i \in \mathcal{N}^j, j \in \{1, \dots, n-2\}, s \in \{1, \dots, |\mathcal{S}_{i'}|\}.$$

When participant i' , $i' \in \text{suc}(i)$, is intervened to be state $x_{i'}^{s'}$, the elements of $\text{CPT}_{i'}$ are updated via Equation (4) as: $C_g^{i',s} = 0, \forall s \in \{1, \dots, |\mathcal{S}_{i'}|\} \setminus \{s'\}, g \in \{1, \dots, |\mathcal{G}_{i'}|\}$, and $C_g^{i',s'} = 1, g \in \{1, \dots, |\mathcal{G}_{i'}|\}$. Based on the above informations, the elements of the probability distribution of participant i' are $p_{i's} = 0, i \in \mathcal{N}^j, j \in \{2, \dots, n\}, s \in \{1, \dots, |\mathcal{S}_{i'}|\} \setminus \{s'\}$, and $p_{i's'} = 1$. Thus, the probability distribution of participant i' , $i' \in \text{suc}(i)$, is not relate to the probability distribution of participant i . Thus, participant i does not need to be intervened if all subsequent participants of participant i , $i \in \mathcal{N}^j$, $j \in \{1, \dots, n-2\}$, are intervened. \square

Proposition 2. *When budget B is not less than the total intervention cost of all participants in the $(n-1)$ th echelon to be in the operational state, i.e., $B \geq \sum_{i \in \mathcal{N}^{n-1}} a_{i1}$ (and thus $\kappa = 1$), then in an optimal solution of models \mathbf{F}_1 and \mathbf{F}_2 all these participants in the $(n-1)$ th echelon are intervened to be operational state.*

PROOF. According our assumptions of the problem, when the parents nodes of the manufacturer are in the operational state, the disruption risk of the manufacturer is minimum. Mathematically, speaking $c_g^{|Z|, |\mathcal{S}_{|Z|}|}, c_1^{|Z|, |\mathcal{S}_{|Z|}|} = \min_{g \in \mathcal{G}_i} \{c_g^{|Z|, |\mathcal{S}_{|Z|}|}\}$. The disruption risk of the manufacturer can be calculated according to formula (1) as:

$$\begin{aligned}
p_{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} &= \sum_{g=1}^{|\mathcal{G}_{|\mathcal{I}|}|} c_g^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} \prod_{i' \in \delta(|\mathcal{I}|)} p_{i',G_{|\mathcal{I}|}^{-1}(g)(i')} \\
&\geq \sum_{g=1}^{|\mathcal{G}_{|\mathcal{I}|}|} c_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} \prod_{i' \in \delta(|\mathcal{I}|)} p_{i',G_{|\mathcal{I}|}^{-1}(g)(i')} \quad (\text{as } c_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} = \min\{c_g^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} | g \in \mathcal{G}_i\}) \\
&= c_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} \sum_{g=1}^{|\mathcal{G}_{|\mathcal{I}|}|} \prod_{i' \in \delta(|\mathcal{I}|)} p_{i',G_{|\mathcal{I}|}^{-1}(g)(i')} \\
&= c_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}, \quad (\text{as } \sum_{g=1}^{|\mathcal{G}_{|\mathcal{I}|}|} \prod_{i' \in \delta(|\mathcal{I}|)} p_{i',G_{|\mathcal{I}|}^{-1}(g)(i')} = 1)
\end{aligned}$$

where the last equal sign holds when $p_{i',G_{|\mathcal{I}|}^{-1}(1)(i')} = 1, \forall i' \in \delta(|\mathcal{I}|)$. Because the manufacturer $|\mathcal{I}|$ is not intervened (i.e., $z_{|\mathcal{I}|} = 0$) according to our assumptions of the problem, $P_{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} = p_{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}$, due to formula (3). Similarly, $C_g^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|} = c_g^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}$, due to formula (4). Clearly, $C_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}$ is a lower bound on $P_{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}$, and this lower bound can be attained in the condition that $p_{i',G_{|\mathcal{I}|}^{-1}(1)(i')} = 1, \forall i' \in \delta(|\mathcal{I}|)$. That is to say, if all participants in the $(n - 1)$ th echelon are intervened to be the operational state, the lower bound can be achieved. Due to the condition of this proposition that budget B is not less than the total intervention cost of all participants in the $(n - 1)$ th echelon to be in the operational state, we can attain the lower bound $C_1^{|\mathcal{I}|,|\mathcal{S}_{|\mathcal{I}|}|}$. Thus, the proposition holds. \square

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