



**HAL**  
open science

# Identifiability and Estimation of Aircraft Parameters and Delays

Carine Jauberthie, Lilianne Denis-Vidal, Ghislaine Joly-Blanchard

► **To cite this version:**

Carine Jauberthie, Lilianne Denis-Vidal, Ghislaine Joly-Blanchard. Identifiability and Estimation of Aircraft Parameters and Delays. 16th International Symposium on Mathematical Theory of Networks and Systems(MTNS 2004), Jul 2004, Leuven, Belgium. hal-03691820

**HAL Id: hal-03691820**

**<https://hal.science/hal-03691820>**

Submitted on 9 Jun 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Identifiability and Estimation of Aircraft Parameters and Delays

Carine Jauberthie<sup>†</sup>, Lilianne Denis-Vidal<sup>†</sup>, Ghislaine Joly-Blanchard<sup>‡</sup>

<sup>†</sup> Univ. Sciences and Tech. Lille, UFR Math(M3,M2)

59655 Villeneuve d'Ascq, France, carine.jauberthie@lil.fr, denvid@attglobal.net

<sup>‡</sup> University of Technology, BP 20 529, 60205 Compiègne, France

Ghislaine.Joly-Blanchard@utc.fr

**Key words** *Nonlinear systems, Time-delay systems, Identifiability, Differential algebra, Estimation.*

## Abstract

This paper considers an identifiability and estimation problem given by aerospace domain describing aircraft nonlinear dynamics with time delays. The original idea is to use an approximation well in line with the given system and an algebraic approach to analyze identifiability. Then the approximate model is considered to estimate the parameters and delays of the original model. Numerical results are given.

## 1 Introduction

System identification based on physical laws often involves parameter estimation. Before performing estimation problem, it is necessary to investigate its identifiability. It is a mathematical and a priori problem. This paper is concerned with the identifiability of a model given by aerospace domain describing aircraft dynamics. This model is derived from the general equations of motion:

$$\begin{cases} m \frac{dV}{dt} = \sum F_e, \\ \frac{d\mathcal{C}}{dt} = \sum M_e, \end{cases} \quad (1)$$

where  $\frac{dV}{dt}$  denotes the acceleration of the gravity center,  $\mathcal{C}$  the kinetic moment,  $\sum F_e$  the sum of the forces acting on the aircraft and  $\sum M_e$  the moment of these forces. In order to improve the model accuracy, the input which consists here in the turbulence, is supposed to act at three different points of the fuselage. Let  $\sigma_i$  denote the shift operator associated to  $\tau_i > 0$  and defined for some function  $v(t)$

by:

$$(\sigma_i v)(t) := v(t - \tau_i), \quad i = 1, \dots, 3, \quad (2)$$

and let also  $\tau_3 = \tau_2 - \tau_1$ . The projection of equations (1) on the aerodynamic reference frame of the aircraft yields the following nonlinear and retarded equations [4]:

$$\sum^p \begin{cases} \dot{V} = k_0 \sin(\theta - \alpha) + V^2(k_1 + k_2\alpha), \\ V(\dot{\theta} - \dot{\alpha}) = k_0 \cos(\theta - \alpha) + k_3 q V + V^2 [k_4 + p_1(\alpha + u) + p_2(\alpha + \sigma_1 u) + p_3(\alpha + \sigma_2 u) + p_4(\sigma_3 \alpha + \sigma_2 u)], \\ \dot{q} = k_5 q V + V^2 [k_6 + p_5(\alpha + u) + p_6(\alpha + \sigma_1 u) + p_7(\alpha + \sigma_2 u) + p_4 p_7(\sigma_3 \alpha + \sigma_2 u)], \\ \dot{\theta} = q, \end{cases} \quad (3)$$

where  $V$  denotes the speed of the aircraft,  $\alpha$  the angle of attack,  $\theta$  the pitch angle,  $q$  the pitch rate, and  $u$  the input. The set of parameters  $k_i$ ,  $i = 0, \dots, 6$  is assumed a priori known (here for instance  $k_0 = g$ , the gravity coefficient) and doesn't need to be estimated. The state vector is defined by:

$$x := (V, \alpha, q, \theta)^T \quad (4)$$

and is also assumed available from measurements. In this paper we are interested in the identifiability of the set:

$$p = (\underline{p}, \tau_1, \tau_2) \quad \text{with } \underline{p} = (p_1, \dots, p_7). \quad (5)$$

The vector  $p$  is assumed in some subset  $\Omega$  of  $\mathbb{R}^9$ , and the main problem in our case comes from the introduction of the delays in the set of unknown parameters.

The identifiability of linear delay-differential systems described by:

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^r (A_i x(t - \tau_i) + B_i u(t - \tau_i)), \\ 0 = \tau_0 < \tau_1 < \dots < \tau_r, \\ x(s) = x_0(s), \quad s \in [-\tau_r, 0], \end{cases} \quad (6)$$

has been analyzed by [1], [11], [12] and [13]. In this case, the set of unknown parameters is  $\{A_0, \dots, A_r, B_0, \dots, B_r, \tau_1, \dots, \tau_r\}$ . To our knowledge, there is no existing method for solving the identifiability problem for general nonlinear and retarded systems with a priori unknown delays. Unlike the free delay case where the properties one can obtain from the linearized system still hold for the original plant in the vicinity of the linearization point, the extension of such result to the retarded case is still an open problem.

The aim of this paper is to show how a particular approximation derived from the original nonlinear and retarded system may result in different identifiability conclusions and can be used to estimate the parameters and delays of the model. The presentation is organized as follows: An algebraic approach is developed in Section 2, where the nonlinearities are maintained and a particular input (a practicable input) is used with Padé approximation on the retarded terms of state. In Section 3 the parameters and delays of the model are estimated using this identifiable approximated model. Section 4 presents some numerical results. Most of the computations, concerning identifiability, related to the aircraft models are implemented in Maple, a symbolic computation language and in Matlab for the estimation procedures.

## 2 Algebraic approach

### 2.1 Input-output approach for testing Identifiability

The input-output approach can be used to test identifiability of some nonlinear systems. This method is based on differential algebra [8] and consists in rewriting, when it is possible, the nonlinear system as a differential polynomial system that will be completed with  $\dot{p} = 0$  where  $p$  is the vector of parameters of the system. The resulting system  $\Gamma^p$  can be described by the following polynomial system:

$$\Gamma^p \begin{cases} R(\dot{x}, x, u, \dot{u}, p) = 0, \\ S(x, y, p) = 0, \\ \dot{p} = 0. \end{cases} \quad (7)$$

The notion of identifiability is strongly connected to observability. In the 90s Fliess and Diop propose a new approach of nonlinear observability and identifiability based on differential algebra [6]. The initial conditions are ignored as they are in the model  $\Gamma^p$ . A solution of  $\Gamma^p$  is a quadruplet of functions  $(x, y, u, p)$  which satisfies all the equations of the model. Thus, the solution of these equations may

not be unique and some solutions may be degenerate. Therefore the set of non-degenerate solutions  $\bar{x}(p, u)$ ,  $\bar{y}(p, u)$ , corresponding to every possible initial condition, have to be involved in the identifiability definition. Here we adopt the definition introduced in [9].

**Definition 2.1** *The model  $\Gamma^p$  is globally identifiable with respect to  $\Omega$  at  $p$  if for any  $p^* \in \Omega$ ,  $p^* \neq p$  there exists a control  $u$ , such that  $\bar{y}(p, u) \neq \emptyset$  and  $\bar{y}(p, u) \cap \bar{y}(p^*, u) = \emptyset$ .*

The previous definition is well in line with the usual formulation [14] which considers initial conditions: If  $\bar{y}(p, u) = \bar{y}(p^*, u)$  then  $p = p^*$ .

In most models there exist atypical points in  $\Omega$  where the model is unidentifiable. Therefore, the previous definition can also be generically extended so that:

**Definition 2.2**  *$\Gamma^p$  is said to be globally structurally identifiable if it is globally identifiable at all  $p \in \Omega$  except at the points of a subset of zero measure in  $\Omega$ .*

In this approach  $I$  is the radical of the differential ideal generated by the equations of  $\Gamma^p$  and the ranking:

$$[p] \prec [y, u, \dot{u}] \prec [x] \quad (8)$$

is chosen in order to eliminate the state variables. In general,  $I$  should be written as the intersection of regular differential ideals which admit a characteristic presentation. A characteristic presentation [3] is a set of polynomials which is a canonical representant of the ideal and it gives the exhaustive summary of the system which allows us to obtain identifiability results. Generally several characteristic presentations are obtained but only one gives general input-output polynomials and general results of identifiability. The others correspond to particular cases of value of parameters or degenerate solutions. The method is validated by checking the independence of some monomials in  $y$ ,  $u$  and their derivatives occurring in the input-output polynomials.

The computation are achieved with an algorithm implemented in Maple [5]. It is based on the Rosenfeld-Groebner algorithm which has been realized and implemented by F. Boulier in the package Diffalg [3].

Now, when the delays are unknown, the presented approach is unusable unless some modifications. It has been shown in [7], by using this approach, that among possible approximations some expansion in Taylor series of the retarded terms leads to an unidentifiable model when the use of Padé approximants to approximate the retarded terms gives

a global identifiable model. In the next section another idea is considered. To analyze identifiability and also to estimate the parameters and delays of the model (3) only an approximation of the retarded state variable is done and a specific input is used.

## 2.2 Identifiability analysis

The delay expression in frequency domain is given by an exponential function which is approached with Padé approximants at first order by:

$$e^{-s\tau_3} \approx (1 - \frac{s\tau_3}{2}) / (1 + \frac{s\tau_3}{2}).$$

In the time domain, this yields the differential equation:

$$(\sigma_3 v)(t) + \frac{\tau_3}{2} (\sigma_3 \dot{v})(t) = v(t) - \frac{\tau_3}{2} \dot{v}(t). \quad (9)$$

With this approximation, one has to define a new state variable  $z_3(t)$ , which consists in the unknown term  $\sigma_3 \alpha$ . The input  $u$  is given by:

$$u(t) = 5.73e^{kt} \frac{l_1 - l_0}{(e^{kt} + l_0)(e^{kt} + l_1)}, \quad (10)$$

where  $l_0, l_1, k$  are known constants. It is clear from (3) that the identifiability of  $\underline{p}$  (as well as that of  $p$ ) only depends on the second and third equations. Let us define  $z(t) = e^{kt}$  thus  $\dot{z}(t) = kz(t)$  and  $F_1(x, \dot{x}), F_2(x, \dot{x})$  such that:

$$\begin{cases} V^2 F_1(x, \dot{x}) = V(\dot{\theta} - \dot{\alpha}) - k_0 \cos(\theta - \alpha) \\ \quad - k_3 q V - V^2 k_4, \\ V^2 F_2(x, \dot{x}) = \dot{q} - k_5 q V - V^2 k_6. \end{cases} \quad (11)$$

On the other hand:

$$\begin{cases} u(t) = \frac{5.73(l_1 - l_0)z}{(z + l_0)(z + l_1)}, \\ \sigma_1 u(t) = \frac{5.73a_1(l_1 - l_0)z}{(z + a_1 l_0)(z + a_1 l_1)}, \\ \sigma_2 u(t) = \frac{5.73a_2(l_1 - l_0)z}{(z + a_2 l_0)(z + a_2 l_1)}, \end{cases} \quad (12)$$

where  $a_i = e^{k\tau_i}, i = 1, 2$  are parameters to identify (replacing  $\tau_i, i = 1, 2$ ).

Finally the original problem is rewritten in the following rational form (7) where the set of polynomials is given by:

$$D^p \begin{cases} w_1 = p_1(\alpha + u) + p_2(\alpha + \sigma_1 u) + \\ \quad p_3(\alpha + \sigma_2 u) + p_3 p_4(\sigma_2 u + z_3), \\ w_2 = p_5(\alpha + u) + p_6(\alpha + \sigma_1 u) + \\ \quad p_7(\alpha + \sigma_2 u) + p_4 p_7(\sigma_2 u + z_3), \\ \tau_3 \dot{z}_3 = 2(-z_3 + \alpha) - \tau_3 \dot{\alpha}, \\ y_1 = w_1, y_2 = w_2, y_3 = \alpha, y_4 = \dot{\alpha}, y_5 = z_3, \\ \dot{p} = 0. \end{cases} \quad (13)$$

The approach described in the preceding section 2.1 gives the general characteristic presentation which contains two input-output polynomials which are too large to be written here. The exhaustive summary of the parameters in the set  $p$  is analyzed with the same algorithm which concludes to the structural global identifiability of  $D^p$ .

## 3 Parameter and delay estimation

### 3.1 Introduction

The aim of this section is to estimate parameters and delays of the model (3) by using a classical least-square objective function given by:

$$J(p) = \sum_{i=1}^N (z(t_i) - y(t_i, p))^T R^{-1} (z(t_i) - y(t_i, p)), \quad (14)$$

where  $R$  is the measurement noise covariance matrix given by:

$$\begin{bmatrix} 25.10^{-4} & 0 & 0 & 0 \\ 0 & 0.04 & 0 & 0 \\ 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 0.04 \end{bmatrix}.$$

The vector  $z$  represents the data. The measurement noise is assumed to be white gaussian with zero mean. The initial vector  $p_0 = (\underline{p}_0, \tau_{10}, \tau_{20})$  is assumed to be known.

The cost function is minimized with respect to the unknown parameters and delays. This problem is solved by a Quasi-Newton method which is implemented in the toolbox optimization of MATLAB 5.3. It leads to an estimation denoted by  $\hat{p}_{ls}$  the so-obtained parameter and delay vector.

### 3.2 Estimation results

The following study was conducted in simulation with the input  $u(t)$  given by (10) with  $l_0 = e^{k\tilde{t}_1}$  and  $l_1 = e^{k\tilde{t}_2}$  where  $k$  is equal to 100,  $\tilde{t}_1 = 0.3$  seconds and  $\tilde{t}_2 = 0.6$  seconds.

The columns of the Table 1 successively give the true parameter vector  $\bar{p}$ , the initial parameter vector  $p_0$  and the estimated parameter vector  $\hat{p}_{ls}$ . The last column gives the relative errors as indicated.

Parameter	$\bar{p}$	$p_0$	$\hat{p}_{ls}$	$\frac{ \hat{p}_{ls} - \bar{p} }{ \bar{p} }$
$p_1$	0.2440	0.2806	0.4009	0.6430
$p_2$	4.6260	5.3199	5.0489	0.0914
$p_3$	0.8450	0.9717	0.5950	0.2959
$p_4$	0.2550	0.2932	0.2324	0.0886
$p_5$	0.3720	0.4278	0.4448	0.1957
$p_6$	0.2440	0.2806	0.1235	0.4943
$p_7$	-3.0360	-3.4914	-2.9239	0.0370
$\tau_1$	0.0120	0.0138	0.0126	0.0500
$\tau_2$	0.0440	0.0506	0.0451	0.0250

Table 1: Estimation results.

The results presented Table 1 show a good improvement in the estimation of parameters  $p_2$ ,  $p_4$ ,  $p_7$  and delays. The estimation of the other parameters could be improved by using an optimal input design which consists in searching a suitable choice of experimental conditions. Thus a significant increase in accuracy of the parameter estimation can be obtained.

The trajectories presented in the figures 1, 2, 3, 4 are obtained by the following way:  $p = \bar{p}$  (full line) and either  $p = p_0$  (dotted line) (figures 1, 3) or  $p = \hat{p}_{ls}$  (dotted line) (figures 2, 4). They show a good trajectory reconstruction and point out the efficiency of the approximated method.

### 3.3 Relation between the approximated system and original system

The adequacy between the approximate system described previously and the discretized original system has been verified. The obtained trajectories (figures 5, 6, 7, 8) with the both systems are very closed: in fact they are indistinguishable relatively to the measure error of all states. The trajectories presented in the figures 5, 6, 7, 8 are obtained by the following way:  $p = \hat{p}_{ls}$ , the discretized original system is given by full line and the approximate system described previously is given by dotted line.

## 4 Conclusion

This paper presents an approach for the parameter and delay identifiability of aircraft dynamics which consists in a nonlinear and retarded system. In case of unknown delays, the classical methods to test identifiability can't be applied. By using a practicable input and an approximation of the retarded state based on the Padé approximants, a structurally identifiable system is obtained. This

last system is used to estimate parameters and delays of the original nonlinear and retarded model. This procedure gives satisfactory results for the delay estimation and some parameter estimation. The accuracy of the others could be improved by using optimal input design.

## References

- [1] L. Belkoura, Y Orlov. "Identifiability analysis of linear delay-differential systems", *IMA Journal of Math. Control and Information*, **19**, 73-81, (2002).
- [2] R. Bellman, K. E. Cooke. "Differential-difference equations", Academic Press, New York (1963).
- [3] F. Boulier, D. Lazard, F. Ollivier, M. Petitot. "Computing representations for the radicals of a finitely generated differential ideals", *Technical report, LIFL*, Université Lille I.
- [4] P. Coton. "Validation de modèles de représentation du comportement des aérodynes en rafales verticales", *Rapport technique IMFL 81/20* (1981).
- [5] L. Denis-Vidal, G. Joly-Blanchard, C. Noiret, M. Petitot. "An algorithm to test identifiability of non-linear systems", *Proc. 5th IFAC NOLCOS*, St Petersburg, Russia pp 174-178 (2001).
- [6] S. Diop, M. Fliess. "Nonlinear observability, identifiability, and persistent trajectories", *Proc. 30th CDC*, Brighton, pp 714-719 (1991).
- [7] C.Jauberthie, L.Belkoura, L.Denis-Vidal, "Aircraft Parameter and Delay Identifiability", *Proc. 7th ECC*, CD Cambridge,(2003).
- [8] E.R. Kolchin. "Differential algebra and algebraic groups", Academic Press, New York.
- [9] L. Ljung, T. Glad. "On global identifiability for arbitrary model parametrizations", *Automatica* **30-2**, pp 265-276 (1994).
- [10] M. Malek-Zavarei. "Time-delay systems, Analysis, Optimization and Applications", North-Holland Systems and Control Series, **9**, 1987.
- [11] S.I. Nakagiri & M. Yamamoto, "Unique identification of coefficient matrices, time delays and initial functions of functional differential equations", *J. Math. Syst. Estimation Control*, **5**, pp 323-344 (1995).

- [12] Y Orlov, L. Belkoura, J.P. Richard, M. Dambrine. "On identifiability of linear time delay systems", *IEEE Transactions on automatic Control*, **47**, No.8, pp 1319-1325 (2002).
- [13] S.M. Verduyn Lunel, "Parameter identifiability of differential delay equations", *Int. J. Adapt. Cont. Signal Proc.* 15, pp 655-678 (2001).
- [14] E. Walter. "Identifiability of state space models", *Lecture Notes Biomath*, **46**, (1982).

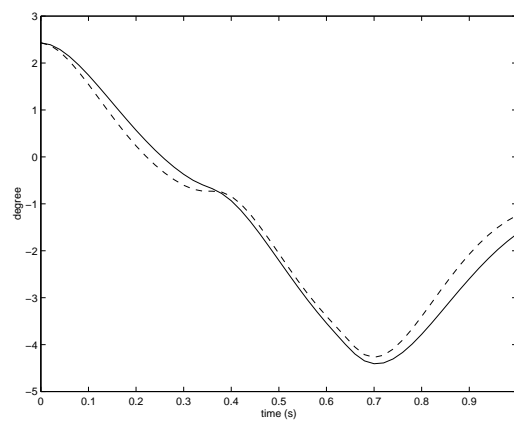


Figure 3: Pitch angle reconstruction with  $p_0$ .

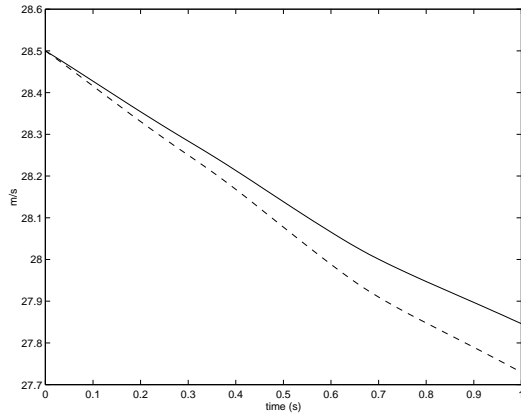


Figure 1: Speed reconstruction with  $p_0$ .

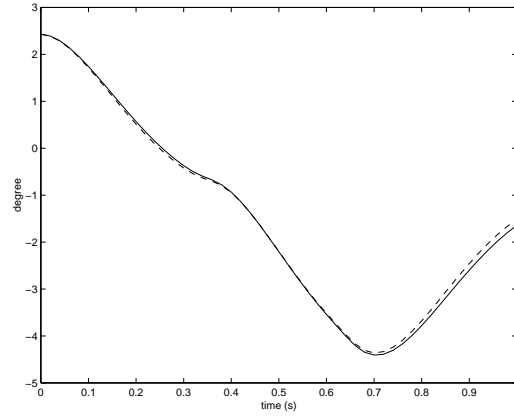


Figure 4: Pitch angle reconstruction with  $\hat{p}_{ls}$ .

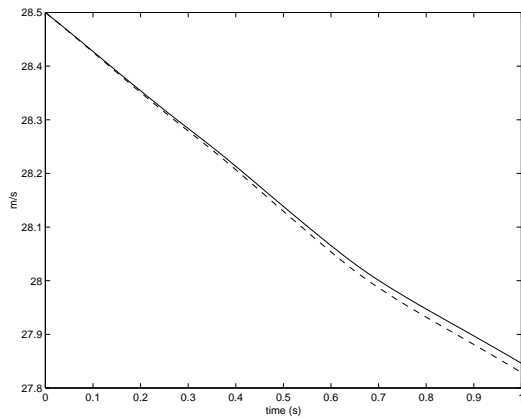


Figure 2: Speed reconstruction with  $\hat{p}_{ls}$ .

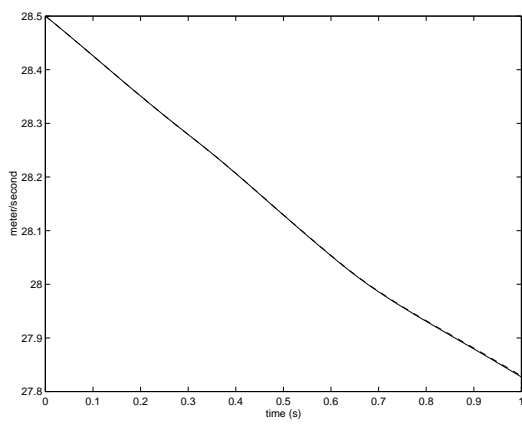


Figure 5: Speed reconstruction (approximate system and discretized original system).

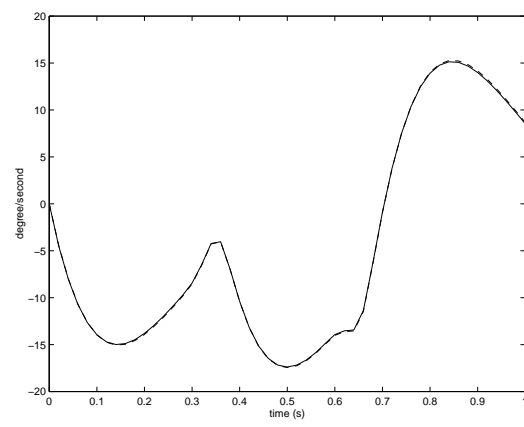


Figure 7: Pitch rate reconstruction (approximate system and discretized original system).

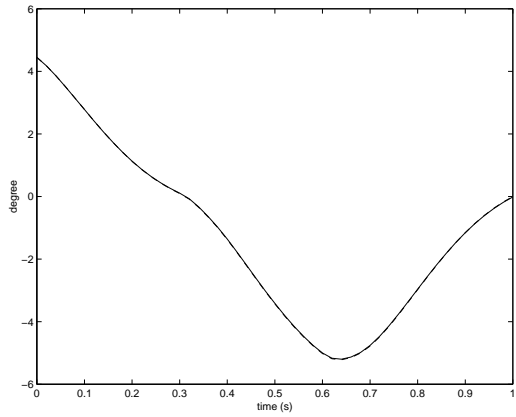


Figure 6: Angle of attack reconstruction (approximate system and discretized original system).

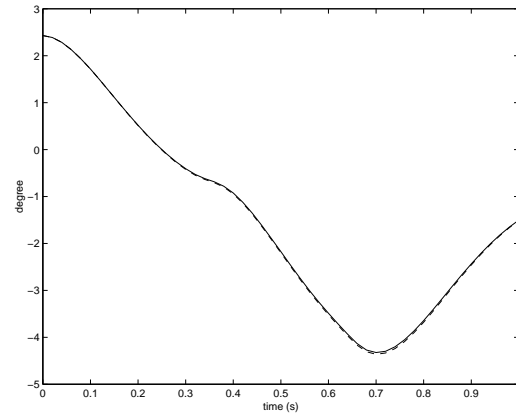


Figure 8: Pitch angle reconstruction (approximate system and discretized original system).