

# Adaptive homotopy for organ segmentation

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## Preliminary remarks

This talk is about

- Digital topology: homotopic (possibly homeomorphic) transformations
- Medical imaging: segmentation

It presents strategies for embedding topological priors in “deformable model” paradigms for segmenting organs and tissues, possibly at different scales

This talk has mainly a tutorial value (the results are not new)

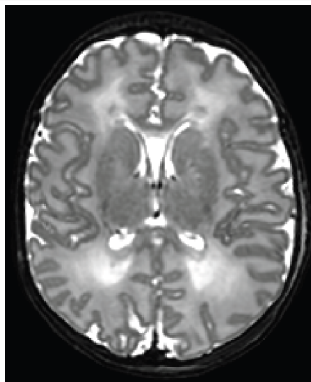
The proposed results derive from published work from two PhD theses:

- Loïc Mazo, *Homotopic deformations in  $n$ -ary digital images*, 2011
- Carlos Tor Díez: *Automatic segmentation of the cortical surface in neonatal brain MRI*, 2019

# Segmentation (in medical imaging)

## Image

- Image = mapping  $I : \Omega \rightarrow \mathbb{V}$
- $\Omega$  = space = subset of  $\mathbb{Z}^d$  modeling a subset of  $\mathbb{R}^d$
- $\mathbb{V}$  = values = subset of  $\mathbb{Z}^k$  modeling a subset of  $\mathbb{R}^k$

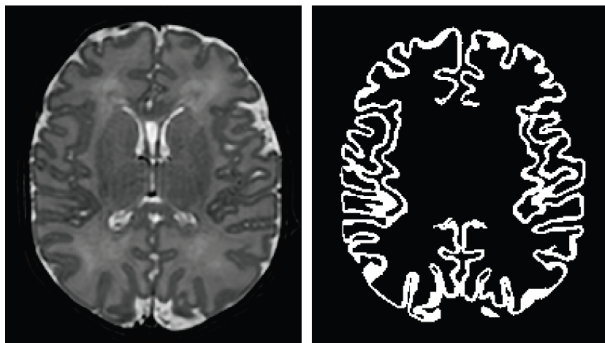


# Segmentation (in medical imaging)

## Segmentation

Segmentation = partition  $S = \{S_1, \dots, S_p\}$  of  $\Omega$

- Binary:  $p = 2$  and  $S = \{S_0, S_1\} = \{B, F\}$  with  $F =$  foreground (object) and  $B =$  background
- Multilabel:  $p \geq 3$  and  $S = \{S_1, \dots, S_p\}$  with each  $S_i$  a given structure

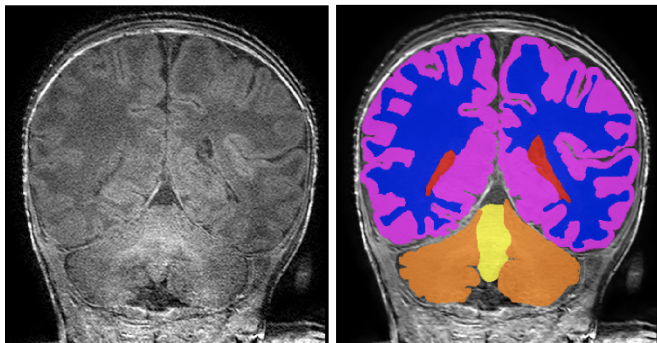


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# Topology

“Object”

Object = topological space = subset  $X \subset \mathbb{R}^d$  or  $\mathbb{Z}^d$

Topology

Topology = the study of the structural properties of such objects, independently of their embedding in  $\mathbb{R}^d$  or  $\mathbb{Z}^d$ .



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## Examples of such properties

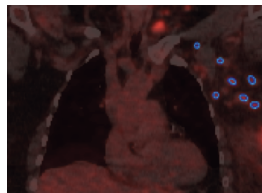
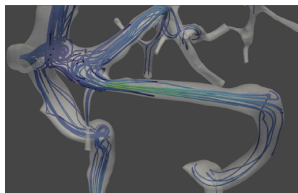
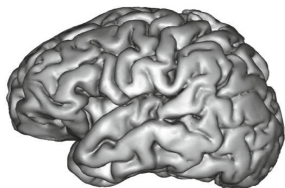
- Connectedness
- Number of tunnels/handles
- Euler Characteristics
- Adjacency tree

# Topology in medical imaging

Why is topology important in medical image segmentation?

Topological properties are especially important for various tasks, e.g.:

- anatomical / biomechanical modelling
- biological / physiological simulation
- morphometric analysis and quantification
- longitudinal studies and interpatient comparison





# Topology in medical imaging

Why is topology important in medical image segmentation?

Organ / tissue segmentation must be as accurate from various points of view:

- morphology: shape, position...
- geometry: size, border properties...
- **topology: connectedness, cavities/holes, adjacency with other structures...**

Important remarks

- Topology is natively designed for binary objects
- The topological properties of an object are intrinsic  
⇒ (in theory) they do not depend of the scale of observation

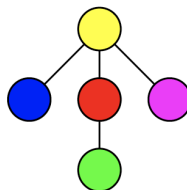
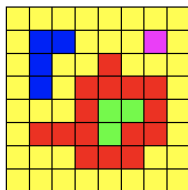
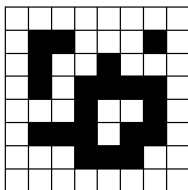
# Two paradigms of topology in medical imaging

## Paradigm 1 – Descriptive topology

Topology provides tools for **observing** / **measuring** structural properties of segmented organs / tissues.

## Descriptive topology tools (examples)

- Low-level: Euler characteristics, connectedness...
- High-level: Homology groups and persistent homology



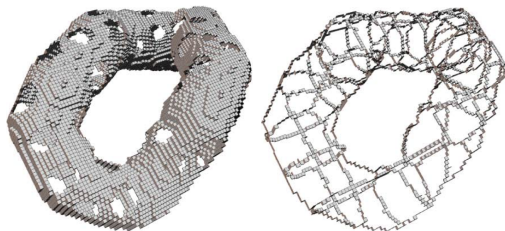
# Two paradigms of topology in medical imaging

## Paradigm 2 – Constructive topology

Topology also provides tools for **modifying** an object **without topological modification**.

### Constructive topology tools (examples)

- Low-level: Region-growing (connectedness preservation)...
- High-level: Homotopy, homeomorphisms



# Homotopic (or homeomorphic) segmentation in medical imaging

## Choices

We focus on the constructive side of topology:

**How to carry out segmentation via a homotopic (or homeomorphic) deformable model paradigm?**

## Related works

Topic widely investigated in the literature, but most often with restrictions:

- Binary segmentation
- Topology determined as a prior
- Monotonic transformations

# Analysis of these topological restrictions

## Binary segmentation

Continuous / digital topology is

- natively defined on binary sets
- **not easy to extend to  $n$ -ary partitions**

## Topology determined a priori

Most of topological concepts, e.g.

- simple points / sets (in  $\mathbb{Z}^n$ )
- collapse sequences (in complexes)

are designed to let the topology unchanged, **not to control / modify it.**

# Simple points: a way to handle homotopy (and homeomorphism)

## Simple points (in $\mathbb{Z}^n$ , $n = 2$ or $3$ )

A simple point  $p$  is:

- a point in  $\mathbb{Z}^n$
- a cell in cubical complexes

that can be added (removed) to (from) a digital object / a complex  $X$  without modifying its homotopy type.

## Simple points and homotopy

- $n = 2$ :  $p$  is a simple point  $\Leftrightarrow \exists$  a homotopy between  $X$  and  $X \setminus \{p\}$
- $n = 3$ :  $p$  is a simple point  $\Rightarrow \exists$  a homotopy between  $X$  and  $X \setminus \{p\}$

(Additional constraints for homeomorphism...)

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- $n = 2$ :  $p$  is a simple point  $\Leftrightarrow \exists$  a homotopy between  $X$  and  $X \cup \{p\}$
- $n = 3$ :  $p$  is a simple point  $\Rightarrow \exists$  a homotopy between  $X$  and  $X \cup \{p\}$

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# Simple points: algorithmic aspects

## Simple points and homotopic transformation (algorithmic sketch)

- Input:  $X \subset \mathbb{Z}^n$
- Output:  $Y \subset \mathbb{Z}^n$
- $Y \leftarrow X$
- Repeat
  - Choose a simple point  $x \in \mathbb{Z}^n$
  - $Y \leftarrow \begin{cases} Y \setminus \{x\} & \text{if } x \in Y \\ Y \cup \{x\} & \text{if } x \notin Y \end{cases}$

Until a termination criterion is satisfied

## Simple point characterization

Local characterization (connectedness)  $\Rightarrow$  Constant time complexity

## Main weakness

Only works for binary objects



## Multilabel simple points (Mazo et al. 2012)

### From binary to multilabel

- Binary object  $X = \text{partition } \{B, F\}$  with  $X = B$  and  $F = \Omega \setminus X$
- Multilabel object  $X = \text{partition } \{S_1, \dots, S_k\}$

**In particular, for  $k = 2$ , both models are compliant.**

### From multilabel to binary

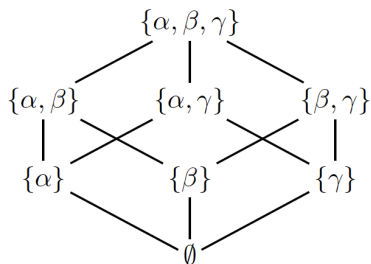
- Set of proto-labels  $L = \{1, \dots, k\}$
- $\forall \ell \in L$ , the set  $S_\ell$  is composed by the points of proto-label  $\ell \in L$
- Let  $\Lambda = 2^L$  the power-set induced by the protolabels of  $L$
- Each element  $\lambda \in \Lambda$  is a label

**In particular, a multilabel image of  $k$  proto-labels induces  $2^k$  binary images.**

# Multilabel simple points (Mazo et al. 2012)

## Example of label set ( $k = 3$ )

- 3 protolabels:  $\alpha, \beta, \gamma$
- $2^3 = 8$  labels, including
  - the  $k = 3$  “protolabels”  $\{\alpha\}, \{\beta\}, \{\gamma\}$
  - other composite labels, from  $\emptyset$  to  $\{\alpha, \beta, \gamma\}$
- Each label  $\lambda$  defines one specific binary object  $X_\lambda = \bigcup_{\ell \in \lambda} S_\ell$  from the multilabel image



# Multilabel simple points (Mazo et al. 2012)

## Multilabel simple point (definition)

Equivalence of the two statements:

- $p$  is a simple point in a multilabel object  $X = \{S_1, \dots, S_k\}$  induced by the protolabels  $L$
- $\forall \lambda \in \Lambda = 2^L$ ,  $p$  is a simple point for the binary image  $X_\lambda$

Less formally

**A simple point in a multilabel image is a point that is simple for all the induced binary images.**

# Towards adaptive, multiscale homotopy

Multilabel simple point: “hard” definition

A simple point in a multilabel image is a point that is simple for **all** the induced binary images.

# Towards adaptive, multiscale homotopy

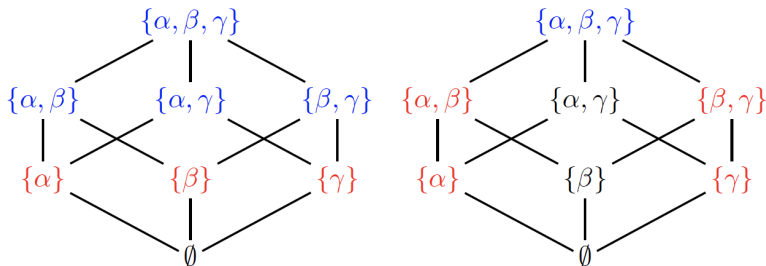
## Multilabel simple point: alternative

A simple point in a multilabel image is a point that is simple for **some of** the induced binary images.

# Towards adaptive, multiscale homotopy

## Partial preservation of the topology

By choosing a subset labels, one can ensure their topology preservation whereas relaxing topological constraints on the other labels.



# Towards adaptive, multiscale homotopy

## Corollaries of this strategy

By considering:

- multilabel simple points
- only certain labels to topologically preserve

one can:

- select the structures to topologically control
- modify these choices depending on the “point of view”

## Towards adaptive, multiscale homotopy

This opens the way to more flexible (partial) homotopic transformations dedicated to specific structures and evolving wrt the scale of observation / resolution of the image

# An illustrative example: brain tissue segmentation

## Input

Segmentation of brain structures from 3D MR images.

- Image  $I : \mathbb{Z}^3 \rightarrow \mathbb{Z}$
- 3 protolabels:  $\alpha$  (White Matter),  $\beta$  (Grey Matter),  $\gamma$  (Cerebrospinal Fluid)
- $2^3 = 8$  induced labels
- 3 scales of observation  $\simeq 3$  resolutions of the MRI



# An illustrative example: brain tissue segmentation

## Topological constraints

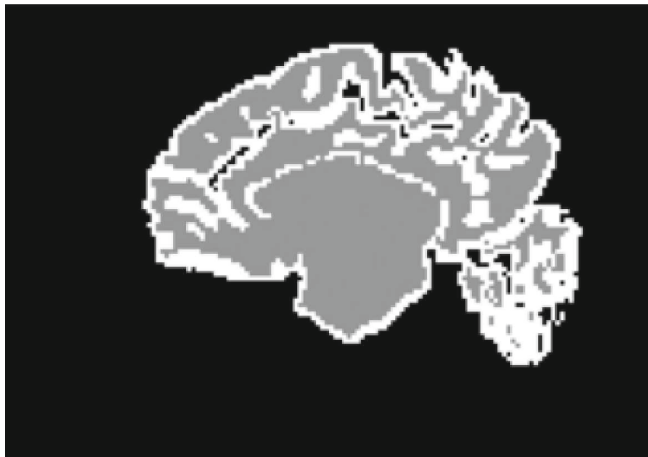
- Initial model – 3 nested layers:
  - $\alpha$  (simply connected)
  - $\beta$  (hollow sphere)
  - $\gamma$  (hollow sphere)
- First level of observation (scales 1 / 2): topological preservation of
  - $\{\alpha\}$ ,  $\{\beta\}$  and  $\{\gamma\}$  (+ side effects)
- Second level of observation (scales 2 / 3): topological preservation of
  - $\{\alpha\}$  (simply connected)
  - $\{\alpha, \beta\}$  (simply connected)
  - $\{\beta, \gamma\}$  (hollow sphere)
  - $\{\gamma\}$  (hollow sphere) (+ side effects)

⇒ **the GM is no longer a hollow sphere**

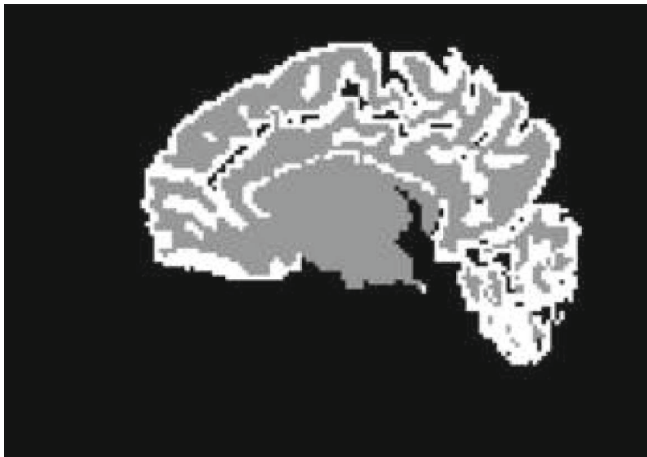
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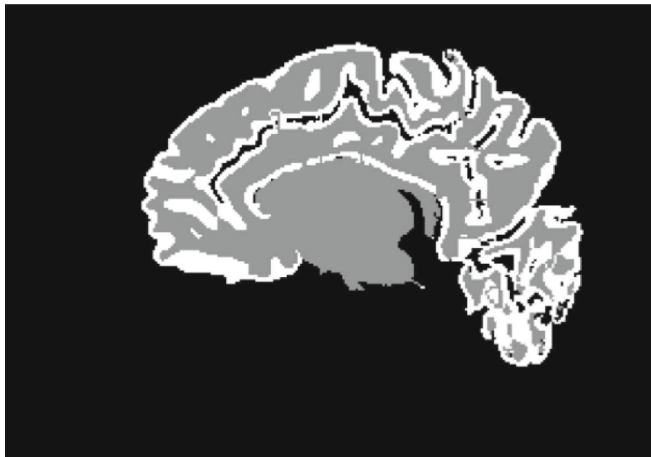
## An illustrative example: brain tissue segmentation



## An illustrative example: brain tissue segmentation



## An illustrative example: brain tissue segmentation



# Discussion

## Positive points

- Inherits the correctness of binary topology tools (simple points)
- Multilabel topology (many tissues)
- Topology viewed as a multiscale descriptor

## Negative points

- Computational cost (mitigated by multiscale)
- Point-wise homotopy can induce deadlocks

## As a conclusion

Multilabel simple points, coupled with multiscale analysis, constitute a promising framework for the design of segmentation of many tissues and organs.

# Conclusion

## Take-home message

- Topology is crucial for many medical image analysis tasks
- (High-level) topological features are generally costly to compute  
→ Need to find strategies to **reduce that cost**
- Useful topological information depends on the scale of observation  
→ Consider the targeted resolution(s) to choose the **right scale(s)**
- Useful topology information depend on the structure(s) of interest  
→ Define the **relevant (clusters of) tissues** to be studied  
→ Determine the **topological information of interest** (not all, in general)

# Thank you for listening!

## Contact

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<https://crestic.univ-reims.fr/fr/nicolas.passat>

## Literature related to these results

- L. Mazo et al. Topology on digital label images. JMIV, 2012
- L. Mazo. A framework for label images. CTIC, 2012
- C. Tor-Díez et al. Multilabel, multiscale topological transformation for cerebral MRI segmentation post-processing. ISMM, 2019

## Funding

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