Asymptotic expansion of the solution of the transmission problem for the Stokes system

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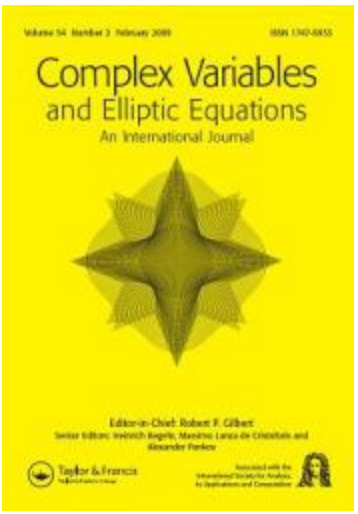

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Asymptotic expansion of the solution of a transmission problem for the Stokes system with a small boundary perturbation for an inclusion

Imen Balloumi [*], and Christian Daveau [†]

Abstract

In this paper, we consider the transmission problem for the Stokes system in three dimensional case. The aim of this work is to derive the development of high order terms in the asymptotic expansion of the solution caused by a small boundary perturbation for an inclusion. The derivation is rigorous and proved by layer potential techniques.

Key words: Stokes system, transmission problem, asymptotic expansions, small perturbation, layer potentials
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1 Introduction

Let $\Omega \subset \mathbf{R}^3$ be a bounded domain with a connected Lipschitz boundary $\partial\Omega$ and suppose that Ω contains an inhomogeneity D with a $\mathcal{C}^3$-boundary ∂D. We assume that there exists a constant $c_0 > 0$ such that $\inf_{x\in D} dist(x, \partial\Omega) > c_0$ which means that D is away from the boundary $\partial\Omega$. Let us consider a boundary value transmission problem for the Stokes system:

$$
(1) \qquad \begin{cases} (-\Delta + \kappa^2)u + \nabla q = 0 & \text{in} \quad \Omega \setminus D \\ (-\Delta + \tilde{\kappa}^2)u + \nabla q = 0 & \text{in} \quad D \\ \nabla \cdot u = 0 & \text{in} \quad \Omega \\ u|_+ - u|_- = 0 & \text{on} \quad \partial D \\ \mu \frac{\partial}{\partial n}(u,q)|_+ - \tilde{\mu}\frac{\partial}{\partial n}(u,q)|_- = 0 & \text{on} \quad \partial D \\ u = g & \text{on} \quad \partial\Omega \\ \displaystyle\int_\Omega q = 0, \end{cases}
$$

where μ and $\tilde{\mu}$ are positive constants related to the physical properties of Ω and D. Here κ^2 and $\tilde{\kappa}^2$ are in $\mathbb{C} \setminus \{z \in \mathbf{R},\ z \leq 0\}$. Let $\kappa = \sqrt{\kappa^2}$ and $\tilde{\kappa} = \sqrt{\tilde{\kappa}^2}$ be particular square roots, which have positive real parts (*i.e.* Re $\kappa > 0$, Re $\tilde{\kappa} > 0$).

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From Gauss's law theorem, we have the compatibility condition:

$$\int_{\partial\Omega} g \cdot \nu_\Omega = 0,$$

where ν_Ω is the outward unit normal to $\partial\Omega$.

Now, we consider ∂D_δ a $\delta-$perturbation of ∂D defined by

$$\partial D_\delta = \{\tilde{x} = x + \delta\nu(x); x \in \partial D, 0 < \delta << 1\} \tag{2}$$

where ν is the outward unit normal to ∂D. We denote by D_δ the domain bounded by ∂D_δ. As ∂D is of class $\mathcal{C}^3$ and in view of the definition (2), the boundary ∂D_δ is of class $\mathcal{C}^2$. We denote by $\frac{\partial}{\partial n}$ the conormal derivative defined on ∂D as follows

$$\frac{\partial}{\partial n}(u,q) := (\nabla u + (\nabla u)^T)\nu - q\nu \tag{3}$$

where $(\nabla u)^T$ denotes the transpose of the matrix $\nabla u := (\partial_i u_k)_{i,k=1..3}$ (see [1], [3], [14] and [17]).

Let $\widetilde{\nu}$ be the outward unit normal to ∂D_δ. Then, the conormal derivative on ∂D is defined by

$$\frac{\partial}{\partial n}(u,q) := (\nabla u + (\nabla u)^T)\widetilde{\nu} - q\widetilde{\nu}. \tag{4}$$

Define (u_δ, q_δ) as the solution to the transmission problem for the Stokes system with a small boundary perturbation D_δ:

$$\begin{cases} (-\Delta + \kappa^2)u_\delta + \nabla q_\delta = 0 & \text{in} \quad \Omega \setminus \overline{D_\delta} \\ (-\Delta + \tilde{\kappa}^2)u_\delta + \nabla q_\delta = 0 & \text{in} \quad D_\delta \\ \nabla \cdot u_\delta = 0 & \text{in} \quad \Omega \\ u_\delta|_+ - u_\delta|_- = 0 & \text{on} \quad \partial D_\delta \\ \mu\frac{\partial}{\partial n}(u_\delta, q_\delta)|_+ - \tilde{\mu}\frac{\partial}{\partial n}(u_\delta, q_\delta)|_- = 0 & \text{on} \quad \partial D_\delta \\ u_\delta = g & \text{on} \quad \partial\Omega \\ \int_\Omega q_\delta = 0. \end{cases} \tag{5}$$

Here and throughout this paper, the subscripts $+$ and $-$ denote the limit from outside and inside D

$$u|_\pm(x) := \lim_{D_\pm \ni y \to x} u(y), \quad x \in \partial D. \tag{6}$$

and similarly it denote the limit from outside and inside D_δ

$$u|_\pm(\widetilde{x}) := \lim_{D_{\delta,\pm} \ni \widetilde{y} \to \tilde{x}} u(\widetilde{y}), \quad \widetilde{x} \in \partial D_\delta. \tag{7}$$

The ultimate objective of this work is to present a schematic way to derive high-order terms in the asymptotic expansions of $(u_\delta - u)|_{\Omega_0}$ and $(u_\delta - u)|_{D_0}$ as δ tends to 0, where D_0 is any close subset of $D \cap D_\delta$ and Ω_0 is any subset of $\Omega \setminus \overline{(D \cup D_\delta)}$. The main ideas are to represent solutions of both initial and perturbed problem via the layer potential operators, to write the Taylor expansion of the integrands and to use properties of the perturbed interface. We have already worked on the derivation of asymptotic expansion for the solution of boundary value perturbation resolvent Stokes system in [9] and on asymptotic behaviors for eigenvalues and eigenfunctions associated to Stokes operator in the presence of small boundary perturbations in [5]. Here we were

inspired by the work of Zribi in [23] to derive asymptotic expansion of the solution of the transmission Stokes system with a small interface perturbation. In connection with this problem we referred to other works on layer potential techniques [2], [18], [19], [20], [22], transmission problems [6], [7], [8] and Stokes system [11], [12], [15], [16].

This paper is organized as follows. In the next section, we give some notations and preliminary results and we describe a layer potential techniques for solving the transmission problem for the Stokes system. In section 3, we establish the existence and uniqueness of the solution of the transmission problem for the Stokes system. Then, in section 4, integral respresentation formulas for the solution of the transmission problem for the Stokes system with a boundary perturbation for an inclusion is given. In section 5, we provide a rigorous derivation of the leading-order term in the asymptotic expansion of u_δ as the parameter of the perturbation δ goes to zero.

2 Definition and preliminary results

2.1 Small perturbation of an interface

Denote by $d\sigma_\delta$ the surface element of ∂D_δ which has the uniform expansion [10]:

$$d\sigma_\delta(\widetilde{x}) = \sum_{n=0}^{\infty} \delta^n \sigma_n(x) d\sigma(x), \qquad x \in \partial D,\ \widetilde{x} \in \partial D_\delta, \tag{8}$$

where σ_n are bounded functions regardless of n. In particular,

$$\sigma_0(x) = 1, \sigma_1(x) = -2H(x), \sigma_2 = K(x),\ x \in \partial D, \tag{9}$$

where H and K denote the mean and the Gaussian curvature of ∂D respectively.

Since ∂D is parallel to ∂D_δ, we can conclude that

$$\widetilde{\nu}(\widetilde{x}) = \nu(x),\ x \in \partial D,\ \widetilde{x} \in \partial D_\delta. \tag{10}$$

We then have for $\widetilde{x}, \widetilde{y} \in \partial D_\delta$:

$$\widetilde{x} - \widetilde{y} = x - y + \delta(\nu(x) - \nu(y)),$$

and

$$|\widetilde{x} - \widetilde{y}|^2 = |x-y|^2(1 + 2\delta \frac{\langle x-y, \nu(x)-\nu(y)\rangle}{|x-y|^2} + \delta^2 \frac{|\nu(x)-\nu(y)|^2}{|x-y|^2}).$$

We also introduce two functions E and G for $x \neq y \in \partial D$ defined by:

$$E(x,y) := \frac{\langle x-y, \nu(x)-\nu(y)\rangle}{|x-y|^2}, G(x,y) := \frac{|\nu(x)-\nu(y)|^2}{|x-y|^2}.$$

Since ∂D is of class $\mathcal{C}^2$, there exists a constant C depending only on ∂D such that:

$$|E(x,y)| + |G(x,y)|^{\frac{1}{2}} \leq C,\ \forall x, y \in \partial D.$$

Hence, we obtain:

$$|\widetilde{x} - \widetilde{y}| = |x-y|\sqrt{1 + 2\delta E(x,y) + \delta^2 G(x,y)} := |x-y| \sum_{n=0}^{\infty} \delta^n L_n(x,y),$$

where the serie converges absolutely and uniformly and the first two terms are:

$$L_0(x,y) = 1, L_1(x,y) = E(x,y).$$

2.2 Layer potential theory for the Stokes resolvent system

2.2.1 Fundamental solution for the Stokes resolvent system

Let consider a bounded Lipschitz domain $\mathfrak{D} \subset \mathbf{R}^3$ with connected boundary $\partial\mathfrak{D}$ and a given complex constant κ^2 in $\mathbb{C}^*$. We denote by $\Gamma^\kappa = (\Gamma^\kappa_{ij})_{i,j=1,2,3}$ and $F^\kappa = (F^\kappa_i)_{i=1,2,3}$ the fundamental tensor and vector of the Stokes resolvent system respectively and they satisfy:

$$\left\{ \begin{array}{ll} (-\Delta + \kappa^2)\Gamma^\kappa_{ij}(x,y) + \partial_j F^\kappa_i(x,y) = \delta_{ij}\delta_y(x) & \text{in } \mathfrak{D}, \\ \partial_i \Gamma^\kappa_{ij}(x,y) = 0 & \text{in } \mathfrak{D}, \end{array} \right. \tag{11}$$

where δ_y is the Dirac distribution with mass at y and δ_{ij} is the Kronecker symbol. The components of $(\Gamma^\kappa, F^\kappa)$ can be obtained by the Fourier transform method in these forms (see [18] [19]):

$$\left\{ \begin{array}{ll} \Gamma^\kappa_{ij}(x,y) & = \frac{1}{4\pi}\left\{ \frac{\delta_{ij}}{|x-y|} e_1(\kappa|x-y|) + \frac{(x-y)_i(x-y)_j}{|x-y|^3} e_2(\kappa|x-y|) \right\}, \\ F^\kappa_i(x,y) & = \frac{1}{4\pi}\frac{(x-y)_i}{|x-y|^3}, \end{array} \right. \tag{12}$$

where

$$e_1(\epsilon) = \sum_{n=0}^{\infty} \frac{(n+1)^2}{(n+2)!}(-\epsilon)^n = exp(-\epsilon)(1+\epsilon^{-1}+\epsilon^{-2}) - \epsilon^{-2},$$

$$e_2(\epsilon) = \sum_{n=0}^{\infty} \frac{1-n^2}{(n+2)!}(-\epsilon)^n = exp(-\epsilon)(-1-3\epsilon^{-1}-3\epsilon^{-2}) + 3\epsilon^{-2}.$$

Now, let introduce for $x, y \in \partial\mathfrak{D}$, the stress tensor $\mathcal{S}$ associated to the fundamental tensors $(\Gamma^\kappa, F^\kappa)$ defined by:

$$\mathcal{S}^\kappa_{ijk}(x,y) = -F^\kappa_j(\hat{x})\delta_{ik} + \frac{\partial\Gamma^\kappa_{ij}(\hat{x})}{\partial\hat{x}_k} + \frac{\partial\Gamma^\kappa_{ik}(\hat{x})}{\partial\hat{x}_j}, \ i,j,k = 1,2,3 \tag{13}$$

where $\hat{x} = x - y = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$ and $r = |\hat{x}|$. Combining (12) and (13), one can obtain:

$$\mathcal{S}^\kappa_{ijk}(x,y) = -\frac{1}{\omega_d}\left\{ \delta_{ik}\frac{\hat{x}_j}{r^d} D_1(\kappa r) + (\delta_{kj}\frac{\hat{x}_i}{r^d} + \delta_{ij}\frac{\hat{x}_k}{r^d}) D_2(\kappa r) + \frac{\hat{x}_i\hat{x}_j\hat{x}_k}{r^{d+2}} D_3(\kappa r) \right\} \tag{14}$$

where

$$D_1(t) = 8\frac{(\frac{t}{2})^{m+1}K_{m+1}(t)}{\Gamma(m)t^2} - \frac{6}{t^2} + 1,$$

$$D_2(t) = 8\frac{(\frac{t}{2})^{m+1}K_{m+2}(t)}{\Gamma(m)t^2} - \frac{6}{t^2} + 2\frac{(\frac{t}{2})^m K_m(t)}{\Gamma(m)},$$

$$D_3(t) = -16\frac{(\frac{t}{2})^{m+2}K_{m+2}(t)}{\Gamma(m)t^2} + \frac{30}{t^2},$$

with $m = \frac{3}{2}$, $\Gamma(m)$ is the Gamma function and K_n for $n \geq 0$ is the modified Bessel function of order n.

Here and throughout this paper we used the Einstein convention for the summation

notation omitting the summation sign for the indices appearing twice.
Moreover, we give components of the pressure tensor $\Lambda^\kappa(x,y)$ defined by

$$\Lambda^\kappa_{ik}(x,y) = -\frac{1}{4\pi}(2\frac{\delta_{ik}}{r^3} - \frac{\delta_{ik}\kappa^2}{r} - 6\frac{\hat{x}_i\hat{x}_k}{r^5}),\ i,k=1,2,3. \tag{15}$$

Notice that $(\mathcal{S}^\kappa, \Lambda^\kappa)$ satisfies:

$$(-\Delta+\kappa^2)\mathcal{S}^\kappa_{ijk}(x,y) + \partial_j\Lambda^\kappa_{ik}(x,y) = 0,\ \partial_{ijk}\mathcal{S}^\kappa_{ij}(x,y) = 0 \text{ for } x\neq y,\ x,y\in\mathfrak{D}. \tag{16}$$

2.2.2 Layer potential operators for the Stokes system

In this section, we present the layer potential operators associated to the Stokes equation in $\mathbf{R}^3$. A special attention is devoted to the invertibility of the potential operators for the standard Stokes equation and to the compactness of the complementary layer potential operators.

In what follows $L^2(\partial\mathfrak{D})$ denotes the Lebesgue space of square integrable functions on $\partial\mathfrak{D}$ with respect to the surface measure $d\sigma$ and $L^2(\partial\mathfrak{D})^3$ denotes the space of vectors whose components belong to $L^2(\partial\mathfrak{D})$.

Let us first introduce a density $\phi = (\phi_1,\phi_2,\phi_3)\in L^2(\partial\mathfrak{D})^3$ and define the single layer potential by $S^\kappa_\mathfrak{D}\phi : \mathbf{R}^3\backslash\partial\mathfrak{D}\to\mathbf{R}^3$ and the pressure potential $V_\mathfrak{D}\phi : \mathbf{R}^3\backslash\partial\mathfrak{D}\to\mathbf{R}$ as follows:

$$S^\kappa_{\mathfrak{D},i}\phi(x) := \int_{\partial\mathfrak{D}} \Gamma^\kappa_{ij}(x-y)\phi_j(y)d\sigma(y),\ i=1,2,3, \tag{17}$$

$$V_\mathfrak{D}\phi(x) := \int_{\partial\mathfrak{D}} F_j(x-y)\phi_j(y)d\sigma(y). \tag{18}$$

Besides, for a given density $\psi = (\psi_1,\psi_2,\psi_3)\in L^2(\partial\mathfrak{D})^3$, we define the double layer potential $\mathcal{D}^\kappa_\mathfrak{D}\psi : \mathbf{R}^3\backslash\partial\mathfrak{D}\to\mathbf{R}^3$, by

$$\mathcal{D}^\kappa_{\mathfrak{D},i}\psi(x) := \int_{\partial\mathfrak{D}} -\mathcal{S}^\kappa_{ijk}(x,y)\nu_k(y)\psi_j(y)d\sigma(y),\ i=1,2,3 \tag{19}$$

where ν is the outward unit normal to $\partial\mathfrak{D}$. Moreover, the associated pressure potential $W^\kappa_\mathfrak{D}\psi : \mathbf{R}^3\backslash\partial\mathfrak{D}\to\mathbf{R}$ is defined by

$$W^\kappa_\mathfrak{D}\psi(x) := \int_{\partial\mathfrak{D}} -\Lambda_{jk}(x,y)\nu_k(y)\psi_j(y)d\sigma(y). \tag{20}$$

We can check that $(S^\kappa_\mathfrak{D}, V^\kappa_\mathfrak{D})$ and $(\mathcal{D}^\kappa_\mathfrak{D}, W^\kappa_\mathfrak{D})$ are solutions of the Stokes system in $\mathbf{R}^3\backslash\partial\mathfrak{D}$,

$$(-\Delta+\kappa^2)S^\kappa_\mathfrak{D}\phi(x) + \nabla V^\kappa_\mathfrak{D}\phi(x) = 0;\ \nabla\cdot S^\kappa_\mathfrak{D}(x) = 0,$$

$$(-\Delta+\kappa^2)\mathcal{D}^\kappa_\mathfrak{D}\psi(x) + \nabla W^\kappa_\mathfrak{D}\psi(x) = 0;\ \nabla\cdot\mathcal{D}^\kappa_\mathfrak{D}(x) = 0.$$

We need to introduce the principal value of the double layer potential, for $\phi = (\phi_1,\phi_2,\phi_3)\in L^2(\partial\mathfrak{D})^3$ by the formula, (see [1]):

$$\mathcal{K}^\kappa_{\mathfrak{D},i}(\phi)(x) := p.v.\int_{\partial\mathfrak{D}} -\mathcal{S}^\kappa_{ijk}(x,y)\nu_k(y)\phi_j(y)d\sigma(y);\ i=1,2,3;\ x\in\partial\mathfrak{D}, \tag{21}$$

and $(\mathcal{K}^{\kappa}_{\mathfrak{D}})^*$ the L^2-adjoint operator of $\mathcal{K}^{\kappa}_{\mathfrak{D}}$

$$(\mathcal{K}^{\kappa}_{\mathfrak{D},i})^*(\phi)(x) := p.v. \int_{\partial\mathfrak{D}} -\mathcal{S}^{\kappa}_{ijk}(x,y)\nu_k(x)\phi_j(y)d\sigma(y);\ i=1,2,3;\ x\in\partial\mathfrak{D}. \tag{22}$$

Here $p.v.$ denotes the Cauchy principal value. The operator $\mathcal{K}^{\kappa}_{\mathfrak{D}}$ is known to be bounded on $L^2(\partial\mathfrak{D})^3$ [4]. We then apply the following decomposition as $r=|x-y|\to 0$:

$$\Gamma^{\kappa}_{ij}(x,y) = \Gamma^{0}_{ij}(x,y) + \Gamma_{ij}(x,y),\ \forall x,y\in\mathfrak{D} \tag{23}$$

$$\mathcal{S}^{\kappa}_{ijk}(x,y) = \mathcal{S}^{0}_{ijk}(x,y) + \mathcal{S}_{ijk}(x,y), \forall x,y\in\mathfrak{D} \tag{24}$$

where $\mathcal{S}^0$ and Γ^0 are weakly singular Stokes tensors. The remaining parts $\mathcal{S}$ and Γ are continuous kernels, so that the continuity behavior of the corresponding surface potential is determined only by the Stokes tensors $\mathcal{S}^0$ and Γ^0. A detailed proof of the above relations can be found in [19] and [20]. In fact, Γ^0 and $\mathcal{S}^0$ are fundamental tensors for the standard Stokes system [14].

According to Theorem 2.27 in [13] p.28, the following lemma holds about the compactness of operators with kernels defined by (23) and (24).

Lemma 1. *Let the operators $\mathcal{K}$, its adjoint $\mathcal{K}^*$ and $\mathcal{S}$ for $x\in\partial\mathfrak{D}$ defined by:*

$$\mathcal{K}_{\mathfrak{D},i}[\phi](x) := p.v. \int_{\partial\mathfrak{D}} -\mathcal{S}_{ijk}(x,y)\nu_k(y)\phi_j(y)d\sigma(y);\ i=1,2,3, \tag{25}$$

$$\mathcal{K}^*_{\mathfrak{D},i}[\phi](x) := p.v. \int_{\partial\mathfrak{D}} -\mathcal{S}_{ijk}(x,y)\nu_k(x)\phi_j(y)d\sigma(y);\ i=1,2,3, \tag{26}$$

$$S_{\mathfrak{D},i}[\phi](x) := \int_{\partial\mathfrak{D}} \Gamma_{ij}(x-y)\phi_j(y)d\sigma(y);\ i=1,2,3, \tag{27}$$

$$\mathcal{S}_{\mathfrak{D}} = (\phi) = S_{\mathfrak{D}}(\phi)|_+ = S_{\mathfrak{D}}(\phi)|_-. \tag{28}$$

These operators have continuous kernel. Moreover, $\mathcal{S}_{\mathfrak{D}}$, $\mathcal{K}_{\mathfrak{D}}$ and $\mathcal{K}^*_{\mathfrak{D}}$ are compact operators on $\mathcal{C}(\partial\mathfrak{D})^3$ and on $L^2(\partial\mathfrak{D})^3$.

We give some results about the trace of the single and double layer potentials on $\partial\mathfrak{D}$, for $(\phi,\psi)\in L^2(\partial\mathfrak{D})^3\times L^2(\partial\mathfrak{D})^3$.

$$\mathcal{D}^{\kappa}_{\mathfrak{D}}(\psi)|_{\pm} = (\mp\frac{1}{2}I + \mathcal{K}^{\kappa}_{\mathfrak{D}})(\psi), \tag{29}$$

$$\frac{\partial}{\partial n}(\mathcal{D}^{\kappa}_{\mathfrak{D}}(\psi),\ W^{\kappa}_{\mathfrak{D}}(\psi))|_+ = \frac{\partial}{\partial n}(\mathcal{D}^{\kappa}_{\mathfrak{D}}(\psi),\ W^{\kappa}_{\mathfrak{D}}(\psi))|_-, \tag{30}$$

$$\frac{\partial}{\partial n}(S^{\kappa}_{\mathfrak{D}}(\phi),\ V^{\kappa}_{\mathfrak{D}}(\phi))|_{\pm} = (\pm\frac{1}{2}I + (\mathcal{K}^{\kappa}_{\mathfrak{D}})^*)(\phi), \tag{31}$$

$$\mathsf{S}^{\kappa}_{\mathfrak{D}}(\psi) = S^{\kappa}_{\mathfrak{D}}(\psi)|_{+} = S^{\kappa}_{\mathfrak{D}}(\psi)|_{-}. \tag{32}$$

The formulas (29), (30), (31) and (32) have been proved in [14] when $\kappa = 0$. Since $\mathcal{D}^{\kappa}_{\mathfrak{D}} - \mathcal{D}^{0}_{\mathfrak{D}}$ and $S^{\kappa}_{\mathfrak{D}} - S^{0}_{\mathfrak{D}}$ are continuous operators and according to (23) and (24), we can extend these equalities to the case $\kappa \neq 0$.

The relations (29) and (31) imply that the single and double layer potentials on $\partial\mathfrak{D}$ satisfy jump relations:

$$\mathcal{D}^{\kappa}_{\mathfrak{D}}(\psi)|_{+} - \mathcal{D}^{\kappa}_{\mathfrak{D}}(\psi)|_{-} = -\psi \text{ on } \partial\mathfrak{D}, \tag{33}$$

$$\frac{\partial}{\partial n} S^{\kappa}_{\mathfrak{D}}(\phi)|_{+} - \frac{\partial}{\partial n} S^{\kappa}_{\mathfrak{D}}(\phi)|_{-} = \phi \text{ on } \partial\mathfrak{D}. \tag{34}$$

Moreover, a result about the decay behavior at infinity of the layer potentials can be found in [18] and in [21].

Lemma 2. *For the single layer potentials $S^{\kappa}_{\mathfrak{D}}[\phi]$, $V^{\kappa}_{\mathfrak{D}}[\phi]$ and the double layer potentials $\mathcal{D}^{\kappa}_{\mathfrak{D}}[\psi]$, $W^{\kappa}_{\Omega}[\psi]$, we have the decay behavior at infinity:*

i)
$$S^{\kappa}_{\mathfrak{D}}[\phi](x) = \mathrm{O}(|x|^{-1}),\ \nabla S^{\kappa}_{\mathfrak{D}}[\phi](x) = \mathrm{O}(|x|^{-2}),\ \text{as } |x| \to \infty.$$

ii)
$$V^{\kappa}_{\mathfrak{D}}[\phi](x) = \mathrm{O}(|x|^{-2}),\ \nabla V^{\kappa}_{\mathfrak{D}}[\phi](x) = \mathrm{O}(|x|^{-3}),\ \text{as } |x| \to \infty.$$

iii)
$$\mathcal{D}^{\kappa}_{\mathfrak{D}}[\psi](x) = \mathrm{O}(|x|^{-2}),\ \nabla \mathcal{D}^{\kappa}_{\mathfrak{D}}[\psi](x) = \mathrm{O}(|x|^{-3}),\ \text{as } |x| \to \infty.$$

iv)
$$W^{\kappa}_{\mathfrak{D}}[\psi](x) = \mathrm{O}(|x|^{-3}),\ \nabla W^{\kappa}_{\mathfrak{D}}[\psi](x) = \mathrm{O}(|x|^{-4}),\ \text{as } |x| \to \infty.$$

Recall that the single layer potential $S^{0}_{\mathfrak{D}}$ defined by $S^{0}_{\mathfrak{D},i}\phi(x) := \int_{\partial\mathfrak{D}} \Gamma^{0}_{ij}(x-y)\phi_j(y)d\sigma(y)$, $i = 1,2,3.$ for $x \in \mathbf{R}^3 \backslash \partial\mathfrak{D}$ corresponds to $\kappa = 0$.

It follows from [14] the theorem.

Theorem 1.

$$\ker(S^{0}_{\mathfrak{D}}) = \{c\nu : c \in \mathbf{R}\}. \tag{35}$$

Now, we give two lemmas about the invertibility of these operators.

Lemma 3. *[1] Let $L^2_0(\partial\mathfrak{D}) := \{f \in L^2(\partial\mathfrak{D})^3; \int_{\partial\mathfrak{D}} f\cdot\nu = 0\}$ and define $H^1_0(\partial\mathfrak{D})$ likewise. We have the next result.*

i) The operator $S^{0}_{\mathfrak{D}} : L^2_0(\partial\mathfrak{D}) \to H^1_0(\partial\mathfrak{D})$ is invertible.

ii) The operator $\lambda I + \mathcal{K}^{0}_{\mathfrak{D}}$ and $\lambda I + (\mathcal{K}^{0}_{\mathfrak{D}})^*$ are invertible on $L^2(\partial\mathfrak{D})^3$ for $|\lambda| > 1/2$.

Lemma 4. *[14] Let $\mathfrak{D} \subset \mathbf{R}^3$ be a bounded domain with boundary $\partial\mathfrak{D}$. Then, we have: $(1/2)I + \mathcal{K}^{0}_{\mathfrak{D}} : L^2_0(\partial\mathfrak{D}) \to L^2_0(\partial\mathfrak{D})$ is invertible and so is $\frac{I}{2} + (\mathcal{K}^{0}_{\mathfrak{D}})^* : L^2_0(\partial\mathfrak{D}) \to L^2_0(\partial\mathfrak{D})$.*

In the next section, with the results given above, we study the existence and uniqueness of the solution of the transmission problem for the Stokes system (1).

3 Existence and uniqueness of the solution of the transmission problem for the Stokes system

We look for a solution $(u,\, q)$ to the transmission problem for the Stokes system (1) with the following representation:

$$(36)\quad \begin{cases} u = S_D^{\tilde{\kappa}}[\phi] & , \quad q = V_D^{\tilde{\kappa}}[\phi] \quad \text{in } D, \\ u = S_D^{\kappa}[\psi] + \mathcal{D}_\Omega^{\kappa}[\theta] & , \quad q = V_D^{\kappa}[\psi] + W_\Omega^{\kappa}[\theta] \quad \text{in } \Omega \setminus \overline{D}, \end{cases}$$

for some triplet $(\phi, \psi, \theta) \in L^2(\partial D)^3 \times L^2(\partial D)^3 \times L^2(\partial\Omega)^3$. According to the equations (29)-(32), (ϕ, ψ, θ) must satisfy the integral equations

$$(37)\quad \begin{pmatrix} S_D^{\tilde{\kappa}}|_+ & -S_D^{\kappa}|_- & -\mathcal{D}_\Omega^{\kappa} \\ \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}_D^{\tilde{\kappa}})^*) & -\mu(\frac{1}{2}I + (\mathcal{K}_D^{\kappa})^*) & -\mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa} \\ 0 & S_D^{\kappa} & \frac{1}{2}I + \mathcal{K}_\Omega^{\kappa} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}.$$

Denote by A^κ the matrix corresponding to the left hand side of (37).

We need to study the existence and uniqueness of the solution of the system (37). This is equivalent to the invertibility of A^κ. We represent A^κ by a sum of two matrices A^{inv} and A^{com} defined by

$$(38)\quad A^{com} = \begin{pmatrix} \mathsf{S}_D & -\mathsf{S}_D & -\mathcal{D}_\Omega^{\kappa} \\ \tilde{\mu}(\mathcal{K}_D)^* & -\mu(\mathcal{K}_D)^* & -\mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa} \\ 0 & S_D^{\kappa} & \mathcal{K}_\Omega \end{pmatrix},$$

and

$$(39)\quad A^{inv} = \begin{pmatrix} S_D^0 & -S_D^0 & 0 \\ \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}_D^0)^*) & -\mu(\frac{1}{2}I + (\mathcal{K}_D^0)^*) & 0 \\ 0 & 0 & \frac{1}{2}I + \mathcal{K}_\Omega^0 \end{pmatrix}.$$

We know that S_D^κ is compact on $L^2(\partial\Omega)^3$ and that $\mathcal{D}_\Omega^\kappa$ and $\frac{\partial}{\partial n}\mathcal{D}_\Omega^\kappa$ are also compact on $L^2(\partial D)^3$ since ∂D and $\partial\Omega$ do not intersect. According to Lemma 1, A^{com} is compact. Hence, it suffices to prove that A^{inv} is invertible and that A^κ is injective to show the invertibility of A^κ according to the Fredholm alternative.

So, we introduce the space $H(\partial D)$ defined by:

$$H(\partial D) := \{(\phi, \psi) \in L^2(\partial D)^3 \times L^2(\partial D)^3; \phi - \psi \in L_0^2(\partial D)\}$$

and we prove the first result.

Lemma 5. *The operator $A^{inv} : H(\partial D) \times L_0^2(\partial\Omega) \to H_0^1(\partial D) \times L^2(\partial D)^3 \times L_0^2(\partial\Omega)$ is invertible.*

proof. Let $(f, h, g) \in H_0^1(\partial D) \times L^2(\partial D)^3 \times L_0^2(\partial\Omega)$, the solution (ϕ, ψ, θ) of the system

$$A^{inv} \begin{pmatrix} \phi \\ \psi \\ \theta \end{pmatrix} = \begin{pmatrix} f \\ h \\ g \end{pmatrix},$$

is given by

$$\begin{array}{ll} \phi & = (S_D^0)^{-1}(f) + \psi \\ \psi & = \frac{1}{\tilde{\mu}-\mu}(-\frac{1}{2}\frac{1+\lambda}{1-\lambda}I + (\mathcal{K}_D^0)^*)^{-1}[h - \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}_D^0)^*)(S_D^0)^{-1}(f)] \\ \theta & = (\frac{1}{2}I + \mathcal{K}_\Omega^0)^{-1}(g) \end{array} \tag{40}$$

where $0 < \lambda = \frac{\mu}{\tilde{\mu}}$. As stated in Lemmas 3 and 4, the operators S_D^0, $-\frac{1}{2}\frac{1+\lambda}{1-\lambda}I + (\mathcal{K}_D^0)^*$ and $\frac{1}{2}I + \mathcal{K}_\Omega^0$ are invertible and the proof is then completed.

Now, we focus on the injectivity of A^κ and we give the following theorem.

Theorem 2. *The operator $A^\kappa : H(\partial D) \times L_0^2(\partial\Omega) \to H_0^1(\partial D) \times L^2(\partial D) \times L_0^2(\partial\Omega)$ is injective .*

proof. Suppose that there exists $(\phi_0, \psi_0, \theta_0) \in H(\partial D) \times L_0^2(\partial\Omega)$ such that $A^\kappa(\phi_0, \psi_0, \theta_0)^T = (0,0,0)^T$ and consider (u, q) defined by

$$\begin{cases} u = S_D^{\tilde{\kappa}}[\phi_0] & \text{with} \quad q = V_D^{\tilde{\kappa}}[\phi_0] & \text{in } D, \\ u = S_D^{\kappa}[\psi_0] + \mathcal{D}_\Omega^{\kappa}[\theta_0] & \text{with} \quad q = V_D^{\kappa}[\psi_0] + W_\Omega^{\kappa}[\theta_0] & \text{in } \Omega \setminus \overline{D}. \end{cases}$$

We notice that the couple (u, q) is a solution of (1) with $g = 0$. Then, using integration by parts [14] on $D_- = D$ and $D_+ = \Omega \setminus \overline{D}$, we obtain

$$2\int_{D_-} Eu : Eu + \tilde{\kappa}^2 \int_{D_-} u.u = \int_{\partial D} \frac{\partial(u,q)}{\partial n}|_-.u|_-,$$

and

$$2\int_{D_+} Eu : Eu + \kappa^2 \int_{D_+} u.u = -\int_{\partial D} \frac{\partial(u,q)}{\partial n}|_+.u|_+ + \int_{\partial D} \frac{\partial(u,q)}{\partial n}|_-.u|_-,$$

where $E(u)$ is the deformation tensor defined by

$$E(u) := \frac{1}{2}(\nabla u + (\nabla u)^T).$$

The system (1) guarantees that $u|_- = 0$ on ∂D, $u|_- = u|_+$ and $\frac{\partial(u,q)}{\partial n}|_+ = \frac{\tilde{\mu}}{\mu}\frac{\partial(u,q)}{\partial n}|_-$ on ∂D. Thus, we have

$$\begin{aligned} 2\int_{D_+} Eu : Eu + \tilde{\kappa}^2 \int_{D_+} u.u &= -\int_{\partial D} \frac{\tilde{\mu}}{\mu}\frac{\partial(u,q)}{\partial n}|_-.u|_- \\ &= -\frac{\tilde{\mu}}{\mu}[\int_{D_-} 2Eu : Eu + \tilde{\kappa}^2 \int_{D_-} u.u]. \end{aligned}$$

Therefore, we get:

$$\begin{array}{llll} u = 0, & q = c_1, & E(u) = 0, & \text{on } D_+, \\ u = 0, & q = c_2, & E(u) = 0, & \text{on } D_-, \end{array} \tag{41}$$

where c_1 and c_2 are real constants. Using the argument that $\int_\Omega q = 0$, we have

$$\int_\Omega q = c_1 \int_{D_-} + c_2 \int_{D_+} = 0. \tag{42}$$

On the other hand, the fifth line of system (1) gives $c_1 = c_2\frac{\mu}{\tilde{\mu}}$. Taking account of (42), we obtain $c_1 = c_2 = 0$.
We also consider the solution to the system:

$$(43)\qquad \begin{cases} (-\Delta + \tilde{\kappa}^2)v + \nabla p = 0 & \text{in} \quad \mathbf{R}^3 \setminus \overline{D} \\ \nabla \cdot v = 0 & \text{in} \quad \mathbf{R}^3 \setminus \overline{D} \\ v = 0 & \text{on} \quad \partial D \\ |v||\nabla v| = o(|x|^{-2}) & \text{as} \quad |x| \to +\infty \\ |v||p| = o(|x|^{-2}) & \text{as} \quad |x| \to +\infty. \end{cases}$$

We know that

$$S_D^{\tilde{\kappa}}(\phi_0)|_+ = S_D^{\tilde{\kappa}}(\phi_0)|_- = u|_- \text{on } \partial D.$$

By (41), we have $S_D^{\tilde{\kappa}}(\phi_0)|_+ = 0$. Then, $(v, p) = (S_D^{\tilde{\kappa}}(\phi_0), V_D^{\tilde{\kappa}}(\phi_0))$ is solution of exterior Dirichlet problem (43) which admits a unique solution [20]. Thus, we have $(v,\, p) = (0,\, 0)$ which implies

$$(S_D^{\tilde{\kappa}}(\phi_0), \mathbf{V}_D^{\tilde{\kappa}}(\phi_0)) = (0, 0) \text{ on } \mathbf{R}^3 \setminus \overline{D}.$$

The jump formula on ∂D yields:

$$\frac{\partial}{\partial n}(S_D^{\tilde{\kappa}}(\phi_0), V_D^{\tilde{\kappa}}(\phi_0))|_- - \frac{\partial}{\partial n}(S_D^{\tilde{\kappa}}(\phi_0), V_D^{\tilde{\kappa}}(\phi_0))|_+ = \phi_0 = 0; \;\; x \in \partial D.$$

On the other hand, let consider the following interior Dirichlet problem:

$$(44)\qquad \begin{cases} (-\Delta + \kappa^2)v + \nabla p = 0 & \text{in} \quad D \\ \nabla \cdot v = 0 & \text{in} \quad D \\ v = 0 & \text{on} \quad \partial D. \end{cases}$$

We have

$$S_D^{\kappa}(\psi_0)|_- + \mathcal{D}_\Omega^{\kappa}(\theta_0)|_- = S_D^{\kappa}(\psi_0)|_+ + \mathcal{D}_\Omega^{\kappa}(\theta_0)|_+ = u|_+ \text{ on } \partial D.$$

Then, we get $S_D^{\kappa}(\psi_0)|_- + \mathcal{D}_\Omega^{\kappa}(\theta_0)|_- = 0$ on ∂D. Finally, we obtain

$$(v, p) = (S_D^{\kappa}(\psi_0) + D_\Omega^{\kappa}(\theta_0),\, V_D^{\kappa}(\psi_0) + W_\Omega^{\kappa}(\theta_0))$$

a solution of (44). We know that interior Dirichlet problem (44) admits a unique solution (v, p) where the unknown p is determined up to an additive constant [20]. Thus, we get

$$(S_D^{\kappa}(\psi_0) + \mathcal{D}_\Omega^{\kappa}(\theta_0),\, V_D^{\kappa}(\psi_0) + W_\Omega^{\kappa}(\theta_0)) = (0, c).$$

Moreover, the trace formula on ∂D give:

$$\begin{aligned} \frac{\partial}{\partial n}(v, p)|_+ - \frac{\partial}{\partial n}(v, p)|_- &= \frac{\partial}{\partial n}(S_D^{\kappa}(\psi_0),\, V^{\kappa}(\psi_0))|_+ + \frac{\partial}{\partial n}(\mathcal{D}_\Omega^{\kappa}(\theta_0), W_\Omega^{\kappa}(\theta_0))|_+ \\ &\quad - \frac{\partial}{\partial n}(S_D^{\kappa}(\psi_0),\, V^{\kappa}(\psi_0))|_- - \frac{\partial}{\partial n}(\mathcal{D}_\Omega^{\kappa}(\theta_0), W_\Omega^{\kappa}(\theta_0))|_- \\ &= \frac{\partial}{\partial n}(S_D^{\kappa}(\psi_0),\, V^{\kappa}(\psi_0))|_+ - \frac{\partial}{\partial n}(S_D^{\kappa}(\psi_0),\, V^{\kappa}(\psi_0))|_- \\ &= \psi_0 = c\nu. \end{aligned}$$

We deduce that $\psi_0 = 0$ on ∂D since $\psi_0 - \phi_0 \in L^2_0(\partial D)$. Therefore, we have $(S^\kappa_D(\psi_0), V^\kappa_D(\psi_0)) = (0,0)$ on $\mathbf{R}^3 \setminus \partial D$ which implies that $(S^\kappa_D(\psi_0), V^\kappa_D(\psi_0)) = (0,0)$ on D_+. Then, we get $(u,q) = (\mathcal{D}^\kappa_\Omega(\theta_0),\, W^\kappa_\Omega(\theta_0)) = (0,0)$ on D_+.

Now, we consider the solution to the following exterior problem :

$$
(45) \qquad \begin{cases} (-\Delta + \tilde{\kappa}^2)v + \nabla p = 0 & \text{in} \quad \mathbf{R}^3 \setminus \overline{\Omega} \\ \nabla \cdot v = 0 & \text{in} \quad \mathbf{R}^3 \setminus \overline{\Omega} \\ \frac{\partial}{\partial n}(v,p) = 0 & \text{on} \quad \partial\Omega \\ |v||\nabla v| = o(|x|^{-2}) & \text{as} \quad |x| \to +\infty \\ |v||p| = o(|x|^{-2}) & \text{as} \quad |x| \to +\infty. \end{cases}
$$

Then, $(\mathcal{D}^\kappa_\Omega(\theta_0),\, W^\kappa_\Omega(\theta_0))$ solves (45) which is an exterior Neumann problem. It has a unique solution. We then deduce that $(v,q) = (0,0)$. We know that $(\mathcal{D}^\kappa_\Omega(\theta_0))|_- - (\mathcal{D}^\kappa_\Omega(\theta_0))|_+ = \theta_0 = 0$ on $\partial\Omega$ which implies that $(\mathcal{D}^\kappa_\Omega(\theta_0),\, W^\kappa_\Omega(\theta_0)) = (0,0)$ in $\mathbf{R}^3 \setminus \partial\Omega$ and particularly in D_+. Finally, we get $(u,q) = (S^\kappa_D[\psi_0], V^\kappa_D[\psi_0])$ on D_+.
We give the result.

Theorem 3. *Let $(\phi, \psi, \theta) \in H(\partial D) \times L^2_0(\partial\Omega)$ be the unique solution of (37). Then (u,q) represented by (36) is the unique solution of (1).*

In the next section, we perturb the transmission problem for the Stokes system.

4 The transmission problem for the Stokes system with a boundary perturbation for an inclusion

Let (u_δ, q_δ) be the solution to problem (5). Using Theorem 3, (u_δ, q_δ) can be represented by:

$$
\begin{cases} u_\delta = S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta] & \text{with} \quad q_\delta = V^{\tilde{\kappa}}_{D_\delta}[\phi_\delta] \quad \text{in } D_\delta, \\ u_\delta = S^{\kappa}_{D_\delta}[\psi_\delta] + \mathcal{D}^\kappa_\Omega[\theta_\delta] & \text{with} \quad q_\delta = V^{\kappa}_{D_\delta}[\psi_\delta] + W^\kappa_\Omega[\theta_\delta] \quad \text{in } \Omega \setminus \overline{D_\delta}, \end{cases}
$$

where $(\phi_\delta, \psi_\delta, \theta_\delta) \in H(\partial D_\delta) \times L^2_0(\partial\Omega)$ is the unique solution of the system

$$
(46) \qquad \begin{cases} S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta] - S^\kappa_{D_\delta}[\psi_\delta] - \mathcal{D}^\kappa_\Omega[\theta_\delta] = 0 & \text{on} \quad \partial D_\delta \\ \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}^{\tilde{\kappa}}_{D_\delta})^*)[\phi_\delta] - \mu(\frac{1}{2}I + (\mathcal{K}^\kappa_{D_\delta})^*)[\psi_\delta] - \mu\frac{\partial}{\partial n}\mathcal{D}^\kappa_\Omega[\theta_\delta] = 0 & \text{on} \quad \partial D_\delta \\ S^\kappa_{D_\delta}[\psi_\delta] + (\frac{1}{2}I + \mathcal{K}^\kappa_\Omega)[\theta_\delta] = g & \text{on} \quad \partial\Omega. \end{cases}
$$

Let $\Psi_\delta(x) = x + \delta\nu(x)$ be the diffeomorphism from ∂D to ∂D_δ. The following estimates hold.

Lemma 6. *There exists a constant C depending only on D such that for any functions $(\phi_\delta, \psi_\delta, \theta_\delta) \in H(\partial D_\delta) \times L^2_0(\partial\Omega)$, we have:*

i)

$$
\|S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta] \circ \Psi_\delta - S^{\tilde{\kappa}}_D[\phi_\delta \circ \Psi_\delta]\|_{L^2(\partial D)^3} \leq C\delta\|\phi_\delta\|_{L^2(\partial D_\delta)^3}.
$$

ii)

$$\|S^{\kappa}_{D_\delta}[\psi_\delta]\circ\Psi_\delta - S^{\kappa}_{D}[\psi_\delta\circ\Psi_\delta]\|_{L^2(\partial D)^3} \leq C\delta\|\psi_\delta\|_{L^2(\partial D_\delta)^3}.$$

iii)

$$\|(\mathcal{K}^{\tilde{\kappa}}_{D_\delta})^*[\phi_\delta]\circ\Psi_\delta - (\mathcal{K}^{\tilde{\kappa}}_{D})^*[\phi_\delta\circ\Psi_\delta]\|_{L^2(\partial D)^3} \leq C\delta\|\phi_\delta\|_{L^2(\partial D_\delta)^3}.$$

iv)

$$\|(\mathcal{K}^{\kappa}_{D_\delta})^*[\phi_\delta]\circ\Psi_\delta - (\mathcal{K}^{\kappa}_{D})^*[\phi_\delta\circ\Psi_\delta]\|_{L^2(\partial D)^3} \leq C\delta\|\phi_\delta\|_{L^2(\partial D_\delta)^3}.$$

proof The two last inequalities have been proven in Lemma 3.1 in [9]. Following the same steps, we can easily obtain the first and the second ones.

5 Derivation of the asymptotic expansion

5.1 Asymptotic expansion of the density functions

First, we consider the asymptotic behavior of $S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta]$, $S^{\kappa}_{D_\delta}[\psi_\delta]$, $\mathcal{D}^{\kappa}_{\Omega}[\theta_\delta]$, $(\mathcal{K}^{\tilde{\kappa}}_{D_\delta})^*[\phi_\delta]$ and $(\mathcal{K}^{\kappa}_{D_\delta})^*[\psi_\delta]$ on ∂D_δ, and $S^{\kappa}_{D_\delta}[\psi_\delta]$ on $\partial\Omega$ as $\delta\to 0$.
Denote by $\widetilde{\phi}=\phi_\delta\circ\Psi_\delta$ the vectorial function with components $\tilde{\phi}_j$, $(j=1,2,3)$.
Besides, for all $\widetilde{x}\in\partial D_\delta$ and $x-y\neq 0$, we define the integral operators for $i=1,2,3$:

$$S^{\tilde{\kappa}}_{D_\delta,i}[\phi_\delta](\widetilde{x}) = \int_{\partial D_\delta}\Gamma^{\tilde{\kappa}}_{ij}(\widetilde{x}-\widetilde{y})\phi_{\delta,j}(\widetilde{y})d\sigma_\delta(\widetilde{y}),$$

$$S^{\kappa}_{D_\delta,i}[\psi_\delta](\widetilde{x}) = \int_{\partial D_\delta}\Gamma^{\kappa}_{ij}(\widetilde{x}-\widetilde{y})\psi_{\delta,j}(\widetilde{y})d\sigma_\delta(\widetilde{y}),$$

$$(\mathcal{K}^{\tilde{\kappa}}_{D_\delta,i})^*[\phi_\delta](\widetilde{x}) = \int_{\partial D_\delta}-\mathcal{S}^{\tilde{\kappa}}_{ijk}(\widetilde{x},\widetilde{y})\widetilde{\nu}_k(\widetilde{x})\phi_{\delta,j}(\widetilde{y})d\sigma_\delta(\widetilde{y}),$$

$$(\mathcal{K}^{\kappa}_{D_\delta,i})^*[\psi_\delta](\widetilde{x}) = \int_{\partial D_\delta}-\mathcal{S}^{\kappa}_{ijk}(\widetilde{x},\widetilde{y})\widetilde{\nu}_k(\widetilde{x})\psi_{\delta,j}(\widetilde{y})d\sigma_\delta(\widetilde{y}),$$

$$D^{\kappa}_{\Omega,i}[\theta_\delta](\widetilde{x}) = \int_{\partial\Omega}-\mathcal{S}^{\kappa}_{ijk}(\widetilde{x},y)\nu_k(y)\theta_{\delta,j}(y)d\sigma(y),$$

and for $x\in\partial\Omega$

$$(S^{\kappa}_{D_\delta})_i[\psi_\delta](x) = \int_{\partial D_\delta}\Gamma^{\kappa}_{ij}(x-\widetilde{y})\psi_{\delta,j}(\widetilde{y})d\sigma_\delta(\widetilde{y}).$$

By Taylor expansions of $\Gamma^{\tilde{\kappa}}_{ij}(\widetilde{x}-\widetilde{y})$, $\Gamma^{\kappa}_{ij}(x-\widetilde{y})$, $\mathcal{S}_{ijk}(\widetilde{x},\widetilde{y})\widetilde{\nu}_k(\widetilde{x})$ and $\mathcal{S}_{ijk}(\widetilde{x},y)\nu_k(y)$, we obtain:

$$\Gamma^{\tilde{\kappa}}_{ij}(\widetilde{x}-\widetilde{y}) = \Gamma^{\tilde{\kappa}}_{ij}(x-y) + \sum_{n=1}^{\infty}\delta^n\underbrace{\sum_{|\alpha|=n}\frac{(\nu(x)-\nu(y))^\alpha}{\alpha!}\nabla^\alpha\Gamma^{\tilde{\kappa}}_{ij}(x-y)}_{:=\Gamma^{\tilde{\kappa},n}_{ij}(x,y)},$$

$$\Gamma^{\kappa}_{ij}(x-\tilde{y}) = \Gamma^{\kappa}_{ij}(x-y) + \sum_{n=1}^{\infty}\delta^n\underbrace{\sum_{|\alpha|=n}\frac{(-\nu(y))^\alpha}{\alpha!}\nabla^\alpha_y\Gamma^{\kappa}_{ij}(x-y)}_{:=\Gamma'^{\kappa,n}_{ij}(x,y)},$$

$$-\mathcal{S}(\widetilde{x},y)\nu_k(y)=\underbrace{-\mathcal{S}_{ijk}(x,y)\nu_k(y)}_{:=\mathcal{H}_{ij}(x,y)}-\sum_{n=1}^{\infty}\delta^n\underbrace{\sum_{|\alpha|=n}\frac{(-\nu(x))^\alpha}{\alpha!}\nabla_x^\alpha\mathcal{S}_{ijk}(x,y)\nu_k(y)}_{:=\mathcal{H}'^n_{ij}(x,y)},$$

$$-\mathcal{S}(\widetilde{x},\widetilde{y})\widetilde{\nu}_k(\widetilde{x})=\underbrace{-\mathcal{S}_{ijk}(x,y)\nu_k(x)}_{:=\mathcal{H}_{ij}(x,y)}-\sum_{n=1}^{\infty}\delta^n\underbrace{\sum_{|\alpha|=n}\frac{(\nu(x)-\nu(y))^\alpha}{\alpha!}\nabla_x^\alpha\mathcal{S}_{ijk}(x,y)\nu_k(y)}_{:=\mathcal{H}^n_{ij}(x,y)}.$$

We then introduce the operators using (8) for $i=1,2,3$ and for $\widetilde{x}\in\partial D_\delta$:

$$S^{\tilde{\kappa},n}_{D,i}[\phi_\delta](\widetilde{x})=\sum_{m+q=n}\int_{\partial D}\Gamma^{\kappa,m}_{ij}(x,y)\sigma_q(x)\tilde{\phi}_j(y)d\sigma(y),\quad n\geqslant 0,$$

$$S'^{\kappa,n}_{D,i}[\psi_\delta](\widetilde{x})=\sum_{m+q=n}\int_{\partial D}\Gamma'^{\kappa,m}_{ij}(x,y)\sigma_q(x)\tilde{\psi}_j(y)d\sigma(y),\quad n\geqslant 0,$$

$$\mathcal{D}^{\kappa,n}_{\Omega,i}[\theta_\delta](\widetilde{x})=\int_{\partial\Omega}\mathcal{H}'^m_{ij}(x,y)\theta_{\delta,j}(y)d\sigma(y),\quad n\geqslant 0,$$

$$\mathcal{K}^{\kappa,n}_{D,i}[\psi_\delta](\widetilde{x})=p.v.\sum_{m+q=n}\int_{\partial D}\mathcal{H}^m_{ij}(x,y)\sigma_q(x)\tilde{\psi}_j(x)d\sigma(x),\quad n\geqslant 0.$$

where $\tilde{\phi}(x)=\phi_\delta\circ\Psi(x)$ and $\tilde{\psi}(x)=\psi_\delta\circ\Psi(x)$

Note that $\mathcal{S}^{\kappa,0}_D=\mathcal{S}'^{\kappa,0}_D=\mathcal{S}^\kappa_D$, $\mathcal{D}^{\kappa,0}_D=\mathcal{D}^\kappa_D$, $\mathcal{K}^{\kappa,0}_D=(\mathcal{K}^\kappa_D)^*$ and that $\sigma_q=0$ for all $q\geq 3$.

We can obtain the following theorem whose demonstration is similar to that of Lemma 6.

Theorem 4. *Let $N\in\mathbf{N}$, there exists C depending on N and D such that for any $(\phi_\delta,\psi_\delta,\theta_\delta)\in H(\partial D_\delta)\times L^2_0(\partial\Omega)$:*

$$\|(\mathcal{K}^\kappa_{D_\delta})^*[\phi_\delta]\circ\Psi_\delta-(\mathcal{K}^\kappa_D)^*[\tilde{\phi}]-\sum_{n=1}^{N}\delta^n\mathcal{K}_D{}^{\kappa,n}[\widetilde{\phi}]\|_{(L^2(\partial D))^3}\leq C\delta^{N+1}\|\widetilde{\phi}\|_{(L^2(\partial D))^3}$$

$$\|S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta]\circ\Psi_\delta-S^{\tilde{\kappa}}_D[\tilde{\phi}]-\sum_{n=1}^{N}\delta^n S^{\tilde{\kappa},n}_D[\widetilde{\phi}]\|_{(L^2(\partial D))^3}\leq C\delta^{N+1}\|\widetilde{\phi}\|_{(L^2(\partial D))^3}$$

$$\|S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta]-S'^{\tilde{\kappa}}_D[\tilde{\phi}]-\sum_{n=1}^{N}\delta^n S'^{\tilde{\kappa},n}_D[\widetilde{\phi}]\|_{(L^2(\partial D))^3}\leq C\delta^{N+1}\|\widetilde{\phi}\|_{(L^2(\partial D))^3}$$

$$\|\mathcal{D}^\kappa_{D_\delta}[\theta_\delta]-\mathcal{D}^\kappa_D[\theta_\delta]-\sum_{n=1}^{N}\delta^n\mathcal{D}^{\kappa,n}_D[\theta_\delta]\|_{(L^2(\partial\Omega))^3}\leq C\delta^{N+1}\|\theta_\delta\|_{(L^2(\partial\Omega))^3}$$

where $\widetilde{\phi}:=\phi_\delta\circ\Psi_\delta$, $\widetilde{\psi}:=\psi_\delta\circ\Psi_\delta$.

Let $(\phi^{(n)}, \psi^{(n)}, \theta^{(n)}), n \geq 1$ be the solution of the following system:

(47)
$$\begin{cases} \mathsf{S}_D^{\tilde{\kappa}}[\phi^{(n)}] - \mathsf{S}_D^{\kappa}[\psi^{(n)}] - \mathcal{D}_\Omega^{\kappa}[\theta^{(n)}] = \\ \sum_{m=0}^{n-1} -\mathsf{S}_D^{\tilde{\kappa},n-m}[\phi^{(m)}] + \mathsf{S}_D^{\kappa,n-m}[\psi^{(m)}] + \mathcal{D}_\Omega^{\kappa,n-m}[\theta^{(m)}] & \text{on} \quad \partial D \\ \\ \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}_D^{\tilde{\kappa}})^*)[\phi^{(n)}] - \mu(\frac{1}{2}I + (\mathcal{K}_D^{\kappa})^*)[\psi^{(n)}] - \mu\frac{\partial}{\partial n} D_\Omega^{\kappa}[\theta^{(n)}] = \\ \sum_{m=0}^{n-1} -\tilde{\mu}((\mathcal{K}_D^{\tilde{\kappa},n-m})^*)[\phi^{(m)}] + \mu((\mathcal{K}_D^{\kappa,n-m})^*)[\psi^{(m)}] + \mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa,n-m}[\theta^{(m)}] & \text{on} \quad \partial D \\ \\ S_D^{\kappa}[\psi^{(n)}] + (\frac{1}{2}I + \mathcal{K}_\Omega^{\kappa})[\theta^{(n)}] = \\ \delta_{0n} g + \sum_{m=0}^{n-1} -S_D^{\prime\kappa,n-m}[\psi^{(m)}] - \mathcal{K}_\Omega^{\kappa,n-m}[\theta^{(m}] & \text{on} \quad \partial\Omega. \end{cases}$$

We recursively construct the sequence $(\phi^{(n)}, \psi^{(n)}, \theta^{(n)})$ $\forall n \in \mathbf{N}^*$.
Let define

$$\phi^N = \sum_{n=0}^{N} \delta^n \phi^{(n)}, \psi^N = \sum_{n=0}^{N} \delta^n \psi^{(n)}, \theta^N = \sum_{n=0}^{N} \delta^n \theta^{(n)}, \tag{48}$$

where $(\phi^{(0)}, \psi^{(0)}, \theta^{(0)}) = (\phi, \psi, \theta)$ is the unique solution of (37). Taking into account Theorem 4, one deduces from system (47) that:

$$\begin{cases} \sum_{n=0}^{N} \delta^n \mathsf{S}_D^{\tilde{\kappa},n}[\tilde{\phi} - \phi^N] - \sum_{n=0}^{N} \delta^n \mathsf{S}_D^{\kappa,n}[\tilde{\psi} - \psi^N] - \sum_{n=0}^{N} \delta^n \mathcal{D}_\Omega^{\kappa,n}[\theta_\delta - \theta^N] = O(\delta^{N+1}) & \text{on} \quad \partial D \\ \\ \sum_{n=0}^{N} \tilde{\mu}(-\frac{\delta_{0n}}{2}I + \delta^n(\mathcal{K}_D^{\tilde{\kappa},n})^*)[\tilde{\phi} - \phi^N] - \sum_{n=0}^{N} \mu(\frac{\delta_{0n}}{2}I + \delta^n(\mathcal{K}_D^{\kappa,n})^*)[\tilde{\psi} - \psi^N] \\ -\sum_{n=0}^{N} \mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa,n}[\theta_\delta - \theta^N] = O(\delta^{N+1}) & \text{on} \quad \partial D \\ \\ \sum_{n=0}^{N} \delta^n S_D^{\prime\kappa,n}[\tilde{\psi} - \psi^N] + \mathcal{D}_\Omega^{\kappa}[\theta_\delta - \theta^N] = O(\delta^{N+1}) & \text{on} \quad \partial\Omega. \end{cases}$$

Then we get the lemma.

Lemma 7. *Let $N \in \mathbf{N}$, there exists C depending only on N and Ω such that*

$$\|\tilde{\phi} - \sum_{n=0}^{N} \delta^n \phi^{(n)}\|_{(L^2(\partial D))^3} \leq C\delta^{N+1},$$

$$\|\tilde{\psi} - \sum_{n=0}^{N} \delta^n \psi^{(n)}\|_{(L^2(\partial D))^3} \leq C\delta^{N+1},$$

$$\|\theta_\delta - \sum_{n=0}^{N} \delta^n \theta^{(n)}\|_{(L^2(\partial\Omega))^3} \leq C\delta^{N+1}$$

where $(\phi^{(n)}, \psi^{(n)}, \theta^{(n)})$ are defined by the recursive relations (47).

5.2 Asymptotic expansion for the solution of perturbed problem

In this section, we develop the asymptotic behavior of $u_\delta - u$ as $\delta \to 0$. The boundary integral representation of the solution can be expressed as

$$u_\delta - u = \begin{cases} S^{\tilde{\kappa}}_{D_\delta}[\phi_\delta] - S^{\tilde{\kappa}}_{D}[\phi] & \text{in } D_0, \\ S^{\kappa}_{D_\delta}[\psi_\delta] - S^{\kappa}_{D}[\psi] + \mathcal{D}^{\kappa}_{\Omega}[\theta_\delta] - \mathcal{D}^{\kappa}_{\Omega}[\theta] & \text{in } \Omega_0, \end{cases} \tag{49}$$

where D_0 is any closed subset of $D \cap D_\delta$ and Ω_0 is any subset of $\Omega \setminus \overline{(D \cup D_\delta)}$.

We obtain for $i = 1, 2, 3$ with Taylor expansion of $\Gamma^{\tilde{\kappa}}_{ij}(x - \tilde{y})$ with respect to y and through the change of variables $\tilde{x} = \Psi_\delta(x)$ with (8) and with Lemma (7):

$$S^{\tilde{\kappa}}_{D_{\delta,i}}[\phi_\delta](x) = \int_{\partial D_\delta} \Gamma^{\tilde{\kappa}}_{ij}(x - \tilde{y})\tilde{\phi}_j(y) d\sigma_\delta(\tilde{y}) \tag{50}$$

$$= \int_{\partial D} \left(\Gamma^{\tilde{\kappa}}_{ij}(x,y) + \sum_{n=1}^{N} \delta^n \Gamma'^{\tilde{\kappa},n}_{ij}(x,y) \right) \left(\phi_j(y) + \sum_{n=1}^{N} \delta^n \phi^{(n)}_j(y) \right) \left(1 + \sum_{n=1}^{N} \delta^n \sigma_n(x) \right) d\sigma(y) + O(\delta^{3N+1})$$

$$= \int_{\partial D} \Gamma^{\tilde{\kappa}}_{ij}(x,y)\phi_j(y) d\sigma(y) + \sum_{n=1}^{3N} \delta^n \sum_{m+q+k=n} \int_{\partial D} \Gamma'^{\tilde{\kappa},m}_{ij}(x,y)\sigma_q(y)\phi^{(k)}_j(y) d\sigma(y) + O(\delta^{3N+1})$$

$$= S^{\tilde{\kappa}}_{D,i}[\phi](x) + \sum_{n=1}^{3N} \delta^n \sum_{m+q+k=n} \int_{\partial D} \Gamma'^{\tilde{\kappa},m}_{ij}(x,y)\sigma_q(y)\phi^{(k)}_j(y) d\sigma(y) + O(\delta^{3N+1}), \ \forall x \in \partial D.$$

By a similar computation, $S^{\kappa}_{D_{\delta,i}}[\psi_\delta](x)$ for $x \in \partial D$ can be rewritten as follows:

$$S^{\kappa}_{D_{\delta,i}}[\psi_\delta](x) = \int_{\partial D_\delta} \Gamma^{\kappa}_{ij}(x - \tilde{y})\tilde{\psi}_j(y) d\sigma(\tilde{y}), \ i = 1, 2, 3 \tag{51}$$

$$= S^{\kappa}_{D,i}[\psi](x) + \sum_{n=1}^{N} \delta^n \sum_{m+q+k=n} \int_{\partial D} \Gamma'^{\kappa,m}_{ij}(x,y)\sigma_q(y)\psi^{(k)}_j(y) d\sigma(y) + O(\delta^{N+1}).$$

Similarly, we have, for $i = 1, 2, 3$

$$\mathcal{D}^{\kappa}_{\Omega,i}[\theta_\delta](x) = \int_{\partial \Omega} -\mathcal{S}^{\kappa}_{ijk}(x,y)\nu_k(y)\theta_{\delta,j}(y) d\sigma(y) \tag{52}$$

$$= \int_{\partial \Omega} -\mathcal{S}^{\kappa}_{ijk}(x,y)\nu_k(y) \left(\theta_j(y) + \sum_{n=1}^{N} \delta^n \theta^{(n)}_{\delta,j}(y) \right) d\sigma(y) + O(\delta^{N+1}), x \in \Omega_0.$$

Then, we get the result.

Theorem 5. *Define for $n \in \mathbf{N}$ the vector-valued functions $u^n = (u^n_i)_{i=1,2,3}$ as follows:*

$$u^n_i(x) = \begin{cases} \displaystyle\sum_{m+q+k=n} \int_{\partial D} \Gamma'^{\tilde{\kappa},m}_{ij}(x,y)\sigma_q(y)\phi^{(k)}_j(y) d\sigma(y) & \forall x \in D_0; \\ \displaystyle\sum_{m+q+k=n} \int_{\partial D} \Gamma'^{\kappa,m}_{ij}(x,y)\sigma_q(y)\psi^{(k)}_j(y) d\sigma(y) & \\ \displaystyle + \int_{\partial \Omega} -S^{\kappa}_{ijl}(x,y)\nu_l(y)\theta^{(n)}_j(y) d\sigma(y) & \forall x \in \Omega_0. \end{cases}$$

Then the next formula holds uniformly for $x \in \Omega_0 \cup D_0$:

$$u_\delta(x) - u(x) = \sum_{n=1}^{N} \delta^n u^n(x) + O(\delta^{N+1}). \tag{53}$$

The remainder $O(\delta^{N+1})$ depends on N, Ω and Ω_0.

5.3 Computation of the first order approximation

Let us compute the first order approximation of u_δ explicitly. If we put $(\phi^{(0)}, \psi^{(0)}, \theta^{(0)}) = (\phi, \psi, \theta)$ defined by (36), $(\phi^{(1)}, \psi^{(1)}, \theta^{(1)})$ is then defined by the following system:

$$\begin{cases} \mathsf{S}_D^{\tilde{\kappa}}[\phi^{(1)}] - \mathsf{S}_D^{\kappa}[\psi^{(1)}] - \mathcal{D}_\Omega^{\kappa}[\theta^{(1)}] = \\ -\mathsf{S}_D^{\tilde{\kappa},1}[\phi^{(0)}] + \mathsf{S}_D^{\kappa,1}[\psi^{(0)}] + \mathcal{D}_\Omega^{\kappa,1}[\theta^{(0)}] & \text{on} \quad \partial D \\ \\ \tilde{\mu}(-\frac{1}{2}I + (\mathcal{K}_D^{\tilde{\kappa}})^*)[\phi^{(1)}] - \mu(\frac{1}{2}I + (\mathcal{K}_D^{\kappa})^*)[\psi^{(1)}] - \mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa}[\theta^{(1)}] = \\ -\tilde{\mu}((\mathcal{K}_D^{\tilde{\kappa},1})^*)[\phi^{(0)}] + \mu((\mathcal{K}_D^{\kappa,1})^*)[\psi^{(0)}] + \mu\frac{\partial}{\partial n}\mathcal{D}_\Omega^{\kappa,1}[\theta^{(0)}] & \text{on} \quad \partial D \\ \\ S_D^{\kappa}[\psi^{(1)}] + (\frac{1}{2}I + \mathcal{K}_\Omega^{\kappa})[\theta^{(1)}] = \\ -S_D^{\prime\kappa,1}[\psi^{(0)}] - \mathcal{K}_\Omega^{\kappa,1}[\theta^{(0)}] & \text{on} \quad \partial\Omega. \end{cases}$$

Let consider the vector-valued function $u^1 = (u_i^1)_{i=1,2,3}$ defined by:

$$u_i^1(x) = \begin{cases} \displaystyle\sum_{m+q+k=1} \int_{\partial D} \Gamma'^{\tilde{\kappa},m}_{ij}(x,y)\sigma_q(y)\phi_j^{(k)}(y)d\sigma(y) & \text{in } D_0; \\ \displaystyle\sum_{m+q+k=1} \int_{\partial D} \Gamma'^{\kappa,m}_{ij}(x,y)\sigma_q(y)\psi_j^{(k)}(y)d\sigma(y) & \\ \displaystyle+\int_{\partial\Omega} -S^{\kappa}_{ijl}(x,y)\nu_l(y)\theta_j^{(1)}(y)d\sigma(y) & \text{in } \Omega_0. \end{cases}$$

Therefore, u_i^1 takes the form:

$$u_i^1(x) = -2\int_{\partial D} \Gamma'^{\tilde{\kappa}}_{ij}(x-y)H(y)\phi_j^{(0)}(y)d\sigma(y) + \int_{\partial D} \Gamma'^{\tilde{\kappa}}_{ij}(x-y)\phi_j^{(1)}(y)d\sigma(y)$$

$$+\int_{\partial D} \Gamma'^{\tilde{\kappa},1}_{ij}(x,y)\phi_j^{(0)}(y)d\sigma(y); \;\; x \in D_0$$

and more explicitly:

$$u_i^1(x) = -2\int_{\partial D} \Gamma'^{\kappa}_{ij}(x-y)H(y)\phi_j^{(0)}(y)d\sigma(y) + \int_{\partial D} \Gamma'^{\kappa}_{ij}(x-y)\phi_j^{(1)}(y)d\sigma(y)$$

$$+\int_{\partial D} \Gamma'^{\kappa,1}_{ij}(x,y)\phi_j^{(0)}(y)d\sigma(y) + \int_{\partial\Omega} -S^{\kappa}_{ijl}(x,y)\nu_l(y)\theta_j^{(1)}(y)d\sigma(y); \;\; x \in \Omega_0.$$

with H denotes the mean curvature defined in (9).

Thus, we get the following formula for $x \in \Omega_0 \cup D_0$

$$u_\delta(x) - u(x) = \delta u^1(x) + O(\delta^2).$$

6 Conclusion

In this paper, we present a result about the construction and justification of the asymptotic expansion of the solution of the transmission problem for the Stokes system with a boundary perturbation for an inclusion. We derive high-order terms in the asymptotic expansion of the solution with layer potential techniques. Some results concerning a reconstruction procedure to determine the localization of the inhomogeneity and the shape of the inclusion based on boundary measurements will be discussed in a forthcoming work.

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