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# Integration of experimental and numerical methods to study the mechanical risk conditions for the conservation of unique art pieces: the case of the Mona Lisa

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**International Colloquium - "Virtual Experiments for Wooden Artwork – VirtEx"**  
**- November-5-2021**

**Proceedings**

The VirtEx project was successfully completed in December. The proceedings of our final colloquium on 5.11.2021 are printed.

Based on nine papers by leading research teams, the book offers an up-to-date insight into the state of the art in engineering research in the field of preventive conservation. It is aimed at natural scientists and engineers as well as conservators and art historians and all those who feel responsible for the preservation of cultural heritage. The design concept placed great importance on meeting the high-level scientific content with a very high aesthetic standard. In addition to the illustrations in the papers, numerous other illustrative photos provide insight into the environment of everyday research. The book in A4 format has 152 pages with colour illustrations throughout.

The book itself is free, but there is a charge of €20.00 per book for packaging and shipping. You can order the conference proceedings via the e-mail address [virtex@tu-dresden.de](mailto:virtex@tu-dresden.de). You will receive an invoice with the book.

Title: VIRTEX - Virtual Experiments for Wooden Artwork  
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Institute for Structural Analysis Technische Universität Dresden

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- *Integration of experimental and numerical methods to study the mechanical risk conditions for the conservation of unique art pieces: the case of the Mona Lisa*, Lorenzo Riparbelli, Paolo Dionisi Vici, Luca Uzielli, Joseph Gril
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VIRT EX

VIRTUAL EXPERIMENTS  
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WOODEN ARTWORK

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# IV

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Integration of experimental  
and numerical methods to study  
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the case of the *Mona Lisa*



## Abstract

The engineering approach to analyse the hygro-mechanical behaviour of a panel painting requires a deep knowledge of its actual physical structure, including size, shape, wooden material's anatomy and characteristics, and responses to external actions such as forces or hygrothermal fluctuations. In fact, each panel is unique, different from all the others, because of numerous factors, including the intrinsic variability of the wood, the "cut" of the boards, the construction technique of the artwork, the succession of environments in which it has been maintained, the irreversible processes developed during its existence, and the conservation interventions it has undergone over the centuries. Based on our experience, only a systematic set of experimental investigations, conceived and implemented specifically for the individual artwork, can provide the information necessary to understand its conservation needs. However, experimental investigations alone are not sufficient for a complete assessment of the artwork: numerical simulations integrated with the experimental tests can greatly improve the level of knowledge and provide more extensive and deeper information. Moreover, following such an approach, the simulation can make it possible to explore situations of risk or modifications of the conservation situation in a totally non-invasive and risk-free way. The wooden panel of Leonardo da Vinci's *Mona Lisa* has been studied by an international team with the aim of analys-

ing and possibly improving its conservation conditions by combining a comprehensive set of non-invasively obtained experimental data with an advanced numerical modelling. The development of a Digital Twin of the panel has hence provided a sound knowledge of the mechanical characteristics and behaviour of the panel, including the assessment of the risk of propagation of an ancient crack, and the evaluation of a new framing configuration.



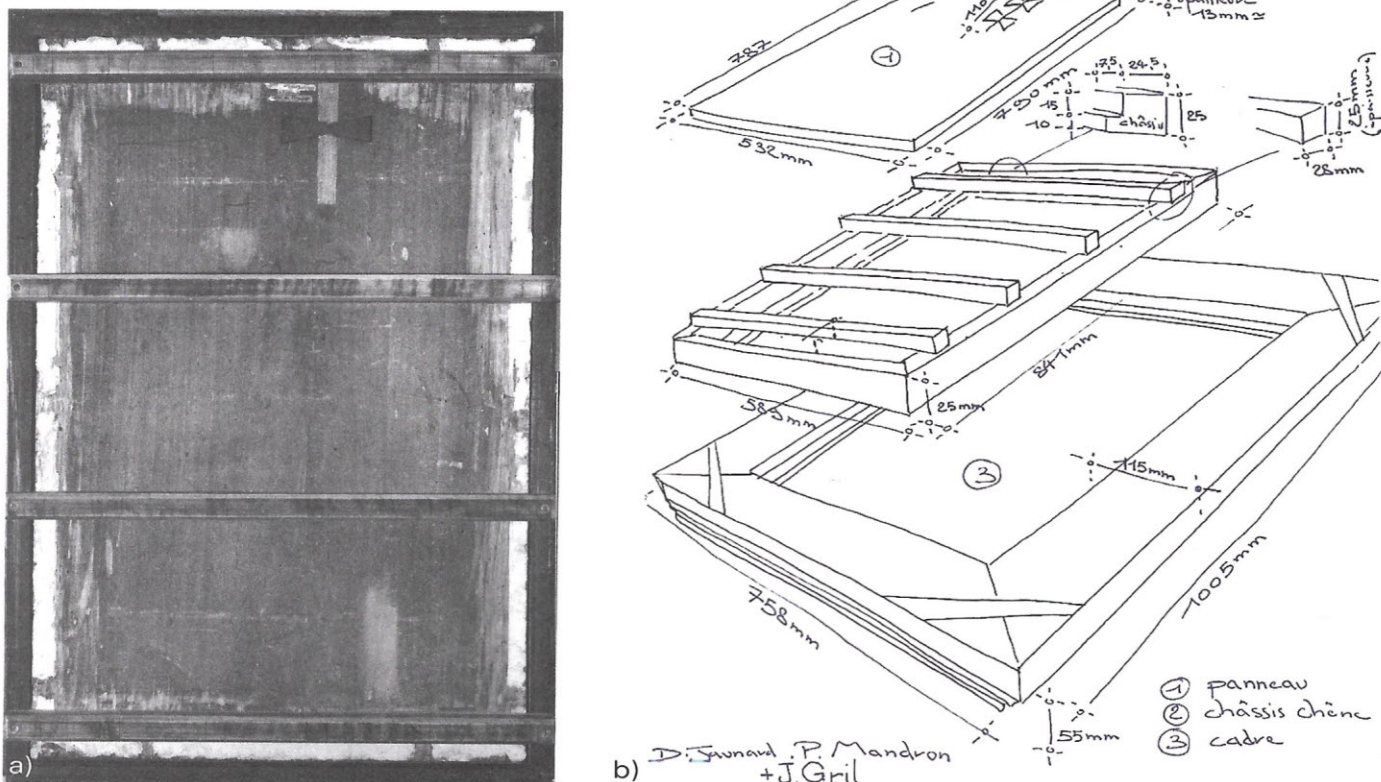


Figure 1:

The assembly of the Panel and its frames in 2004. a) The back of the *Mona Lisa*, housed in the auxiliary frame, with the four crossbeams (cross-section 25×28 mm) screwed on the auxiliary frame. (Photo Centre de recherche et de restauration des musées de France) b) Exploded view and metric survey (by D. Jaunard and P. Mandron, later modified by J. Gril) showing the various elements which made up the *Mona Lisa* assembly before 2005: (1) Panel, with crack and butterflies; (2) auxiliary frame (*châssis-cadre*), formed by battens with L-shaped cross-section, on which the four crossbeams were screwed, only the top and bottom one forcing against the Panel due to its out-of-plane distortion; (3) carved and gilded frame (*cadre*), fully visible from the front.

## 1

### Background

#### 1.1

#### The Panel

The wooden panel on which Leonardo da Vinci painted his *Mona Lisa* (hereinafter “the Panel”) has been studied since 2004 by an international research team, including researchers from French and Italian universities and related scientific institutions (Montpellier, Clermont Auvergne, Poitiers, Florence), to study its mechanical, hygroscopic and shape characteristics and behaviour, to assess its present state of conservation, and to provide suggestions for optimizing its framing and environmental conditions [13, 16].

The Panel’s support is a flat-sawn rectangular board of poplar wood (*Populus alba* L.), 794×534×13 mm, doubly curved (that is, both cupping transversally and bowing longitudinally) convex towards the front face. The paint layers are applied on the external face of the sub-tangential board (that is, the face away from the pith of the log from which it was obtained). The framing assembly until 2004 is shown in Figure 1; in particular, the Panel is pressed against the rim of the intermediate auxiliary frame (*châssis-cadre*) by two crossbeams (the top and the bottom one) screwed onto the auxiliary frame itself; the two central crossbeams do not touch the Panel, due to its longitudinal bowing. The auxiliary frame hosts the Panel, and is in turn hosted and pressed by metal brackets in a carved and gilded external wooden frame. An ancient radial-longitudinal crack runs through the wood thickness from the top

edge of the Panel down to the Lady's forehead; two wooden "butterflies" had been in-laid, possibly during the 19th century, into the Panel's thickness to prevent any longitudinal propagation of the crack; one of them is now missing and has been replaced by a glued canvas strip [15, 16, 20].

Some modifications to the assembly occurred in 2005, when the Panel was moved from its previous location in the *Salle Rose* to its current display case in the *Salle des États*. A new coupling system was then implemented to secure the Panel in its present display situation, and, for this purpose, new parts were installed on the back of the frame; also, the crossbeams were replaced with thinner ones, to fit into the reduced space available. At the same time, a special equipment, described in detail in [16], began being developed and implemented in the framework of the international research team to monitor both the deformations that the Panel undergoes (mainly produced by the inevitable small climatic fluctuations within the display case) and the constraining forces acting on the Panel itself. Such equipment includes load cells hosted in the top and bottom crossbeams, and further instrumentation (three displacement transducers monitoring the Panel's deflection, a data-logger, a wireless communication apparatus, and the supporting electronics) housed in a closed and robust aluminium case fixed on the auxiliary frame at mid-height, facing but not touching the Panel (see also Section 2.2); case and crossbeams were modified and/or replaced several times according to the evolution of the instrumentation. Figure 2 shows the back side of the assembly in 2019.

1.2

## The difficulties in modeling wooden artworks, and the approach followed

The main difficulty that arises when it comes to numerical modelling of wooden artworks belonging to the Cultural Heritage consists in a correct characterisation of their mechanical, hygroscopic, shape and actual boundary conditions. This difficulty is made even greater by the problems that typically arise when dealing with wooden artefacts, including but not limited to the following [1, 6, 18]:

- variability of the physical and mechanical characteristics even within the same wood species, the same tree, the same piece of wood;
- marked dependence of the hygro-mechanical characteristics of a wooden object on the anatomical directions of the wood (e.g. a board's distortion caused

by mechanical stress and/or environmental thermo-hygrometric variations is significantly affected by its "cut", i.e. the sawing pattern according which it has been obtained from the original log);

- phenomena related to aging, particularly dependent also on the environment in which they occurred;
- presence of growth anomalies, and/or consequences of decay by organisms, or other causes of deterioration.

To overcome such difficulties, the choice was made to carry out a systematic series of experimental non-invasive tests, whose results would allow to develop and calibrate a sufficiently realistic numerical model. Accordingly, a general model was built, and was successively optimised and characterised based on the results of the experimental tests so that, when subjected to the same test conditions, it would show the same behaviour and the same deformations as those measured on the original object.

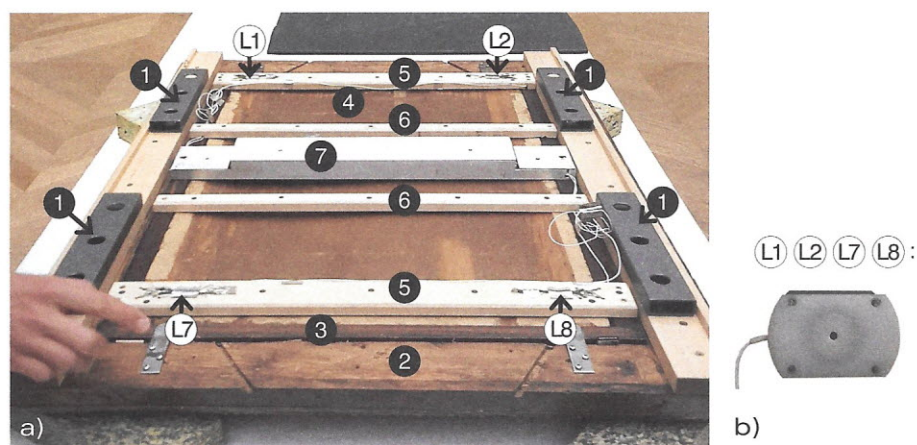


Figure 2:

a) Overall view of the back of the framing assembly in 2019, showing (1) the metal rectangular elements installed in 2005 to hang the assembly in its exhibition position, (2) the carved and gilded frame, (3) the auxiliary frame (*châssis-cadre*), (4) the back face of the Panel (including the ancient crack and the butterflies), and the equipment developed by the team, through various evolutions over the years, for automatically monitoring the mechanical behaviour of the Panel: (5) the top and bottom crossbeams, equipped with four miniature load cells installed in 2013 in the contact locations L1-L2-L7-L8; (6) the two central thinner "dummy" crossbeams, which do not even touch the Panel; (7) the aluminium case, which contains (not visible in the picture) three displacement transducers monitoring the deflection of the Panel, the sensors measuring temperature and relative humidity of the nearby surrounding air, the electronic equipment of the load cells, the data logger, and the apparatus for wireless data transmission.

b) One of the four identical pressers (made of Delrin, a crystalline plastic ensuring frictionless and chemically inert contact with the back face of the Panel) which are free to tilt and adapt to the local inclination of the Panel in their contact area, and transmit to the crossbeam and measure through a miniature load cell (capacity 100 N, not visible in the picture) the force exerted there by the Panel.

This approach made it possible to develop a complete mechanical characterisation of the artwork, through which several important results were obtained, including (see Section 1.1 above) the evaluation of (a) the propagation risk of the ancient crack, (b) the interaction between the residual butterfly and the Panel, and (c) possible improvements to the framing conditions.

## 2

### Experimental tests

#### 2.1

#### Optical measurements

The three-dimensional geometric digital model of the Panel (both free and constrained within the crossbeams-auxiliary frame assembly) was obtained by means of optical methods by the University of Poitiers [2, 3, 4]. The technique of fringe pattern profilometry was used, which consists of projecting onto the object being studied a grid made of parallel lines; the analysis of how the lines are deformed makes it possible to derive the shape of the surface. The measurements are quite accurate, presenting an uncertainty of about 0.03mm with a resolution of 0.45 mm/pixel. Knowing the thickness of the Panel at three points, it is also possible to correctly assemble its front and back faces, thus obtaining its complete geometric representation.

#### 2.2

#### Monitoring forces and deformation, identifying contact zones

As already mentioned (see Section 1.1 and Figure 2), the Panel is forced – flatter than it would stay if unconstrained – between the auxiliary frame and the crossbeams, which are fixed against each other by means of screws. When the surrounding climatic conditions change, the Panel tends to change its shape, whereas the crossbeams tend to prevent any deformation at the top and bottom edges; therefore, the reaction forces and the related internal stresses and strains are also constantly changing.

Both to monitor, the current state of the Panel and to build up and calibrate a numerical model, an accurate monitoring has been implemented. It continuously measures and records the constraining forces and the deformation, and their evolution caused by any

external action, including the climatic fluctuations within the display case.

The equipment briefly described here has been designed and implemented to effectively, safely and non-invasively measure and monitor such forces and deformation even under normal display conditions; a detailed description is given in [16].

The forces exerted by the rim of the auxiliary frame on the front face of the Panel are balanced by the forces exerted by the crossbars on the back face, by the means of the four load cells located in L1-L2-L7-L8 (Figure 2). In other words, the system formed by the Panel, the crossbeams and the auxiliary frame is closed with regards to the contact forces, so that the four load cells are measuring all the significant forces acting on the artwork during the monitoring. Each load cell fits in a presser leaning against the back of the Panel. This system of load cells also allows the adjustment of the individual

forces through the positioning of each load cell.

The auxiliary frame and the crossbeams touch and constrain the Panel only near its top and bottom edges, in addition to a contact near the top end of the left vertical batten [8, 17]. Otherwise, owing to its longitudinal permanent curvature, its central part does not come into contact with either the crossbeams or the auxiliary frame, and is free to move and deform, both transversally (cupping) and longitudinally (bowing). To measure both such deformations, three displacement transducers, already described in [17], are installed on an aluminium case fixed at mid-height on the auxiliary frame, facing but not touching the Panel, also housing further electronics (see Section 1.1 and Figure 2), including an encrypted Bluetooth [11] device for downloading data without opening the case.

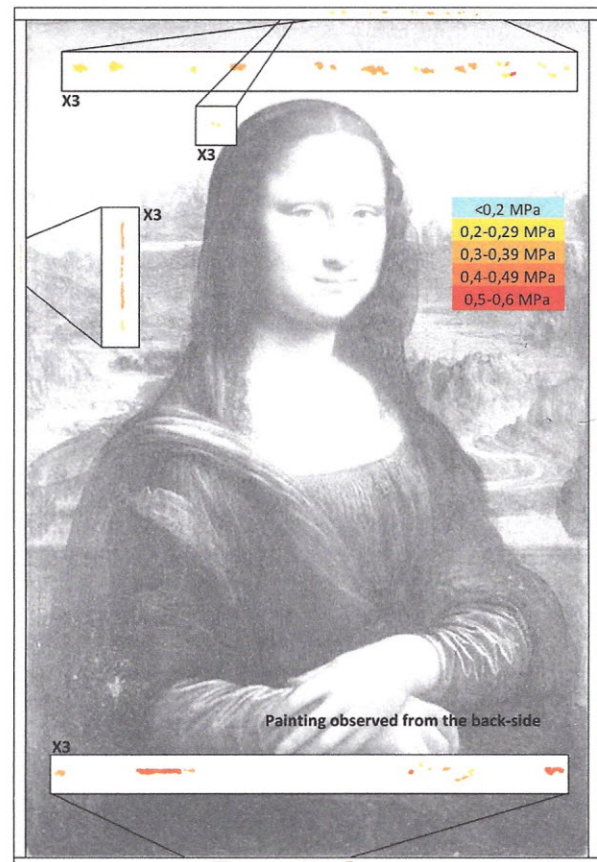


Figure 3:

The impressed film strips of Prescale® pressure sensitive film [8] were scanned in the laboratory and analysed with an imaging software to determine the contact areas and their colour densities (which indicate the contact pressure), as shown on the enlarged white bands.

To identify and evaluate the contact zones between the Panel's front face and the rim of the auxiliary frame, an ad hoc experimental campaign was carried out. This issue is of fundamental importance since the two coupled surfaces have different shapes and roughness, and, hence, determine the location and extent of the contact zones, so that they constitute one of the most essential parameters for the construction of any type of simulation. A totally non-invasive test was

performed using a pressure sensitive film, where strips of film were placed at the interface between Panel and frame, and the system was very carefully kept assembled during a few minutes, according to the instructions: red marks on the film identified the contact areas (Figure 3), the saturation values of the colour indicated the local contact pressures, and entities and locations of the contact forces have been hence computed [8].

2.3

## Wood structure

Due to wood anisotropy, the distribution of anatomical directions within the Panel is of vital importance for any mechanical analysis claiming to be accurate. The localisation and orientation of the wooden board inside the log from which it was obtained had been first studied by Ravaut as follows [15]. The distances between the log's pith and the back face of the Panel, estimated by means of measurements carried out on growth rings

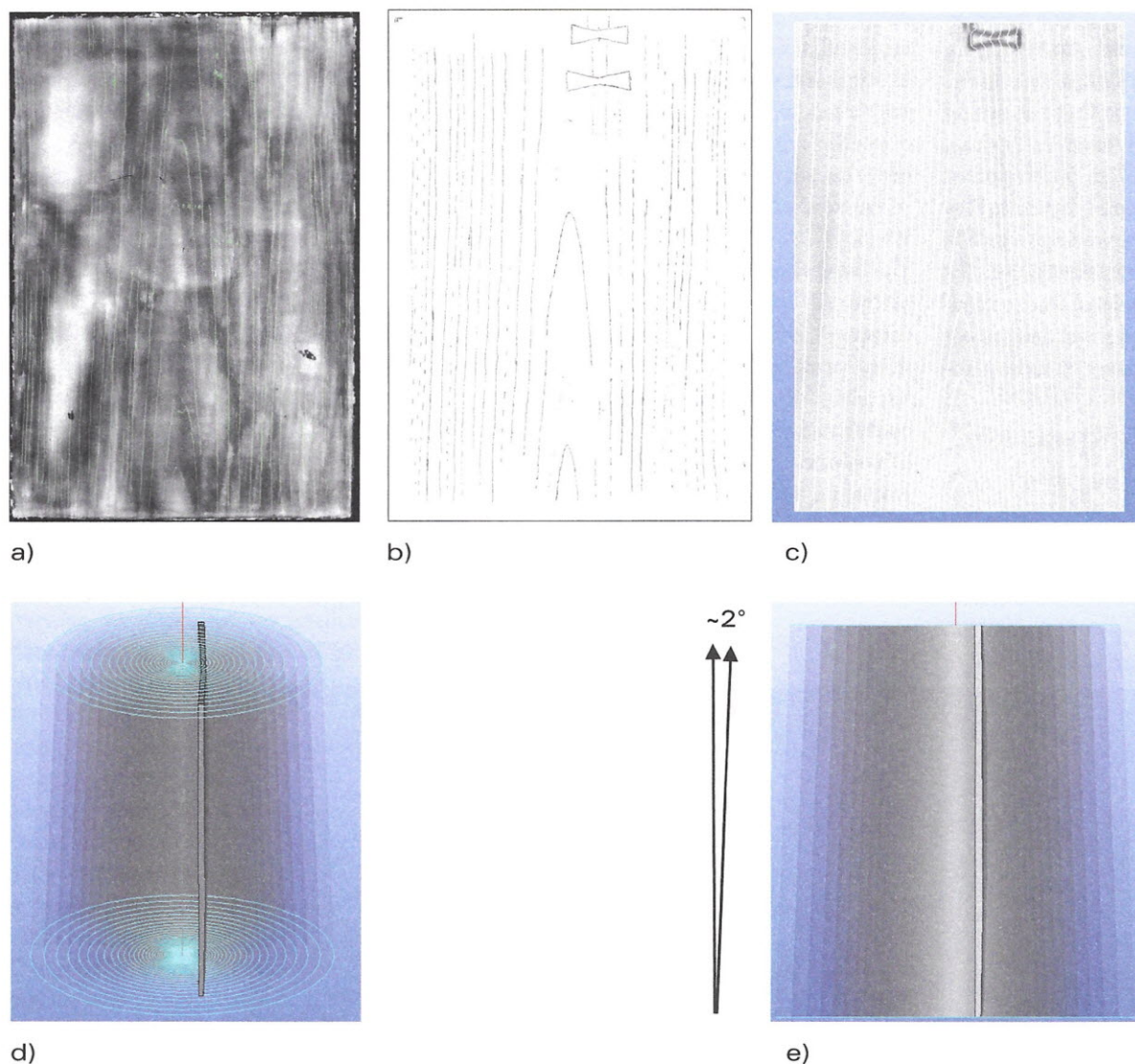


Figure 4:  
 Procedure implemented to estimate the localization and orientation of the wooden panel inside the original log. a) Traces of growth layers observed on an X-ray image, or b) drawn manually on a Melinex sheet placed on the back face, are compared to c) the traces of simulated growth layers, obtained by intersecting the virtual three-dimensional log (formed by coaxial truncated cones representing its growth layers) with the three-dimensional virtual back face of the panel (obtained from the optical measurements). d) Estimated localization and orientation of the wooden board inside the optimized virtual log. e) Projection of the virtual log shape, showing its estimated taper.

visible on the top and bottom edges, was 61 mm at the top edge (at 261 mm from the right top corner), and 46 mm at the bottom edge (at 271 mm from the right bottom corner). Based on the assumption that the growth layers were cylindrical, the difference between these distances led Ravaud to the conclusion that the wooden panel had been cut at a small angle to the pith.

An alternative explanation could be a slightly conical shape of the growth layers (i.e. a tapering log), and the wooden board cut parallel to the pith (i.e. by parallel sawing). Based on these new assumptions, an optimisation procedure has been carried out, using the opensource 3D modelling program Salome-Meca, developed by Electricité de France (EDF)[7]. The best match between the traces of the simulated growth layers (Figure 4c) and the growth rings pattern visible on the back face (Figure 4b) or shown by X-ray images (Figure 4a) taken by Ravaud, was identified visually for an inclination of 4% (equal to two sexagesimal degrees) (Figure 4c). This value, corresponding to an 8% log taper, is plausible for the lower part of a large poplar tree grown isolated. A combination of both effects, conical shape and cutting angle, could possibly be tested to further improve the fitting.

### 3

## Development and use of the “Digital Twin”

### 3.1

#### Development of the predictive finite element model

The multidisciplinary approach that characterizes the entire project obviously includes the numerical modelling of the Panel. The modelling has been developed to obtain deeper levels of information from experimental tests but also to build a predictive model of the hygro-mechanical behaviour of the Panel, able to provide information about its stress and strain states. The availability of such a model, provided it is sufficiently accurate and reliable, is essential for answering many questions concerning the conservation of the artwork, as will be shown in the following.

The numerical analyses reported here were performed using the finite element (FEM) software Code\_Aster, developed in the past thirty years by Electricité de France [7]. It has a high level of industrial validation and contains advanced modules for the solution of non-linear contact problems, an essential feature for the studies presented in this paper. Being an opensource code, it allows developing ad-hoc routines, further tools and au-

tomated procedures, so that reliable analyses could be performed for modelling this artwork; which is a very complex task, reflecting the complexity of the problems to be solved, as it should be for a predictive tool.

This complexity also constitutes the limit for the direct applicability of this experience to other situations; in fact, to reach a satisfactory level of reliability in modelling this kind of artworks, a significant investment is necessary, both for experimental tests and measurements, for choosing and using appropriate modelling tools, and for integrating skills in wood science and in finite element modelling.

The model (Figure 5), which is based on the actual characteristics of the material and has been calibrated against the actual reactions of the Panel to real stresses, can rightly be considered a Digital Twin (DT) of the original Panel, since it reacts to any loading in a way similar to the original; hence it makes it possible to evaluate the stress and strain state of the Panel, and to simulate its reactions to external hygro-mechanical stresses acting on it, without however imposing unacceptable stresses on the real Panel.

Some of the numerical studies performed are outlined in the following, as examples of how the DT can be used to provide answers to some of the basic questions connected to the Panel’s conservation. Having now available such a powerful tool as the DT, numerous additional reliable analyses and simulations can be easily carried out.

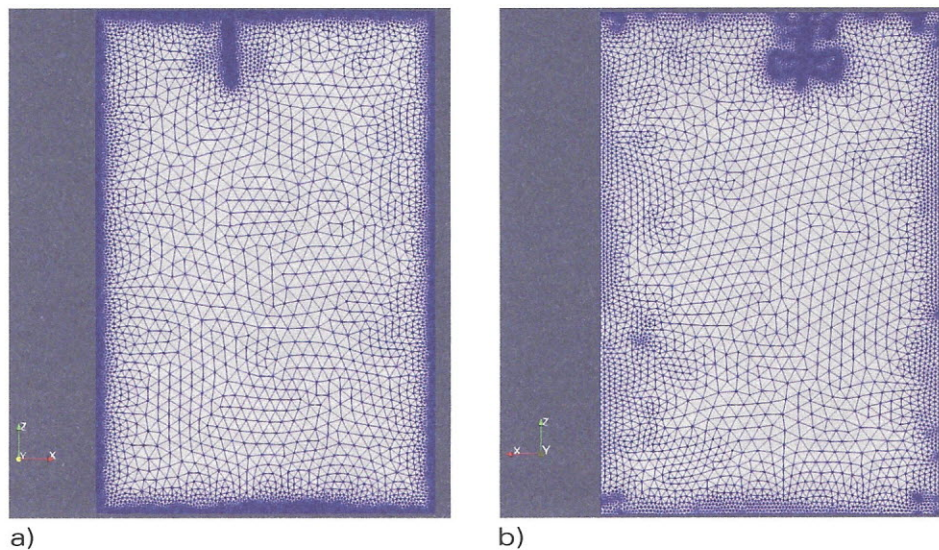


Figure 5:

The numerical discretization of the panel-butterfly assembly, obtained by Netgen algorithm, consists of a tetrahedral second-order mesh, with refinements according to the curvature, and around the contact zones, the crack and the butterfly. The number of degrees of freedom of the numerical solution is almost 1,190,000.

a) Front face. b) Back face.

## Estimating the elastic properties of the Panel

The knowledge of the Panel's elastic properties is of primary importance for its conservation, since if they are known and a reliable FEM model of its hygro-mechanical behaviour is available, the Panel's internal strain and stress state and behaviour can be estimated at any moment.

Typically, an experimental procedure to measure the elastic properties of a wood specimen starts from the measurement of a load-displacement relationship, in the linear range, in one or more locations of the specimen. In the case of the *Mona Lisa*, any displacement of a single load cell produces variations in the readings of all load cells and of the displacement transducers placed in the central area of the Panel. The data obtained in this way (Table 1) have been used to optimize the estimation of the elastic properties of the Panel, and at the same time these estimated parameters have been used to optimize the calibration of the DT.

An accurate knowledge of the geometry of the specimen and of the constraints is essential for the accuracy of the results. For this method to be considered reliable, it is necessary that the simulation model corresponds very accurately to the physical reality of the Panel. To achieve this goal, the shape of the Panel has been reconstructed based on the optical investigations, the contact conditions between the Panel and the auxiliary frame were taken into account as well as the specific features of the Panel's wood macrostructure (e.g. its cylindrical orthotropy, the taper of the original log, and the location and geometry of the crack) identified by its technological analysis. As already mentioned above, it would be also possible to derive shape variations from optical observations of several mechanical states and to use them to validate numerical simulations.

The average elastic properties of the Panel obtained non-invasively through the optimization procedure are given in Table 1. They are comparable with literature data based on correlations between coefficients of elasticity and air-dry density for hardwoods [10] based on a wood density of  $0.45 \text{ g/cm}^3$  as reported in [5].

Source of data	$E_L$ [GPa]	$E_R$ [GPa]	$E_T$ [GPa]
Experimental tests and Numerical analysis by the Team	9.7	1.39	0.83
Data from the literature calculated from density [10]	10.1	1.19	0.58

Table 1:

Elastic properties in the anatomical directions of the wood forming the Panel, estimated by means of the experimental-numerical procedure, and compared with those reported in the literature for poplar wood having the same density.

## 4

### Conservation of the artwork

#### 4.1

#### Identifying relationships between principal strains and craquelure patterns

When the flattening action exerted by the auxiliary frame is simulated, the principal strains close to the face of the Panel, immediately below the preparatory layers (Figure 6), are well matched with the craquelure patterns of the paint and preparatory layers described and analysed by Ravaud [14]. The principal directions of the strains indicate the directions in which the maximum contraction and extension occur. By construction, they are perpendicular to each other and in their own reference system have no tangential component; therefore, they are particularly suitable for comparison with the craquelure. Since the principal directions are strongly influenced by the presence of the crack, this correspondence further supports Ravaud's remark that the occurrence of the crack in the Panel predates the craquelure formation. Although the actual constraining conditions of the Panel when the craquelure formed are unknown, it is reasonable to assume that they were not very different from the current ones; therefore, the match can be considered significant. In this computation, the mechanical action of the butterfly (which is assumed to have been installed much later than the formation of the craquelure) has been omitted.

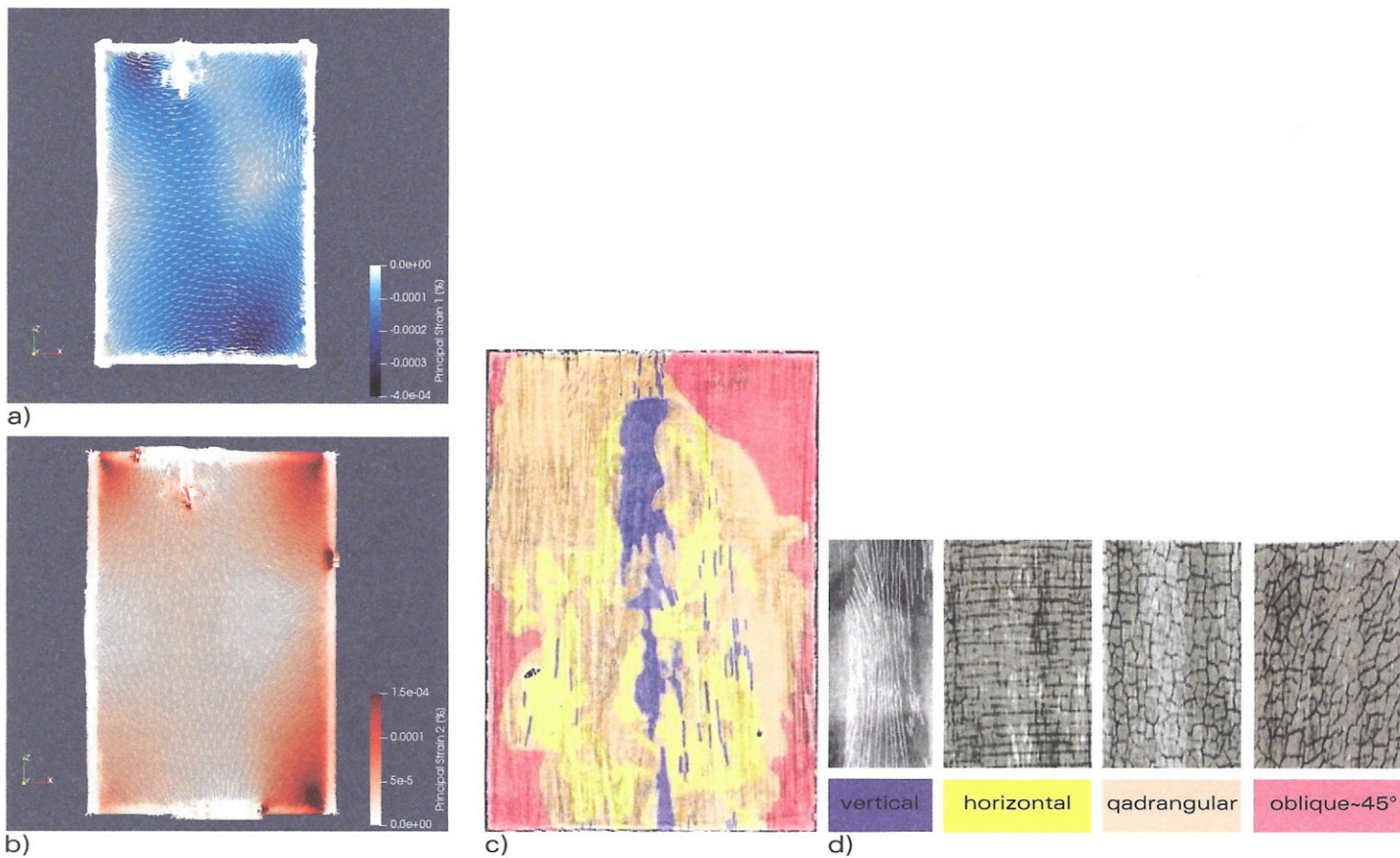


Figure 6: Comparison between the principal directions of the strains in the front face of the Panel (i.e. in the wood layer immediately adjacent to the preparation), computed by means of the DT, and the craquelure patterns of the paint and preparatory layers described and analysed by Ravaud [14]. In a) and b) the white dashed lines indicate the directions of the principal strains; the intensity of the background colour indicates the magnitude of the local strain, shown in the respective colormaps: the negative (i.e. contraction) strains are shown in a) on a blue background, while positive (i.e. extension) strains are shown in b) on a red background. In c), an overview of the various patterns of the craquelure is shown. In d), details are shown of each craquelure pattern, the detail of vertical craquelure was taken from an X-ray image, the others from macro photographs.

Note. Basic meaning of the engineering terms used in this context: "strain" is a deformation (can be a relative extension, contraction or distortion), expressed by a dimensionless number, whereas "stress" is an internal force per unit area (can be tensile, compression or shear), and here is expressed in Megapascal (1 MPa = 1 N/mm<sup>2</sup>).

#### 4.2

### Evaluation of crack propagation risk and of butterfly's contribution to Panel's safety

The crack affecting the Panel, and the probability of its propagation, were analysed in the light of fracture mechanics according to Griffith's theory, which is based on the comparison between the energy required by the crack to propagate and the strain energy potentially released by this propagation; the

identifiable critical value for poplar wood can be represented by the range  $0.1 \text{ kJ/m}^2 \leq G_c \leq 1 \text{ kJ/m}^2$ . In the case of the *Mona Lisa*, the crack is oriented in a radial plane, and its propagation would likely occur in the same plane in the grain direction. In Table 2 we indicate the values calculated for the energy released in the case of an increase in the existing crack; with the aim of highlighting, which of the three modes of rupture is most compatible with a possible propagation, further models have been carried out, releasing only one mode each time.

Confirming a previous study carried out in the context of the team's work [9], a DT

computation shows that in the present and foreseeable state of strains and stresses, the possibility of crack propagation appears remote, provided no extreme climatic variations take place. Figure 7a shows the mechanics of the crack. Strain concentrations around the butterfly, however within safe limits, indicate that the butterfly works inside its recess to keep close to each other the two lips of the crack. Figure 7b points out around the butterfly the stresses perpendicular to wood grain, also within safe limits, indicating that the butterfly is stressed in tension.

Mode of rupture	$\text{kJ} / \text{m}^2$
G	0.059
$G_I$ – Opening Mode	0.012
$G_{II}$ – Sliding Mode	0.018
$G_{III}$ – Tearing Mode	0.048

Table 2:  
Numerical values for the energy release rate G and for the energy release rates related to the three modes of rupture.

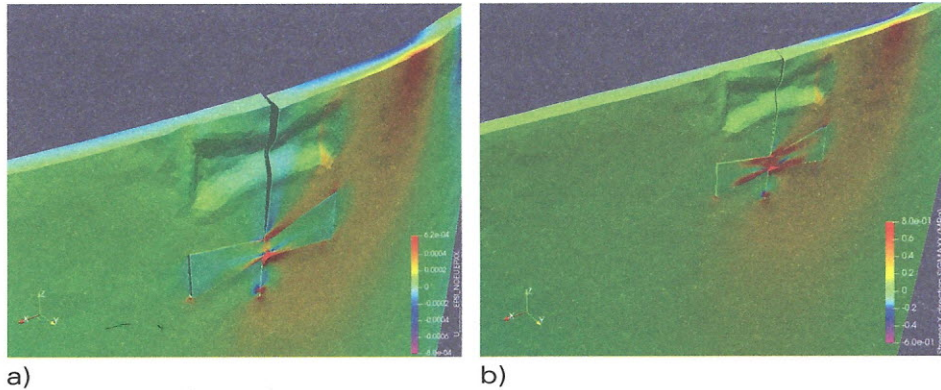


Figure 7:  
Strains a) and stresses b) around the crack on the back surface of the Panel, perpendicular to the grain of the wood, considering the contribution of the butterfly holding the two edges of the crack close together. The larger strains and stresses (red areas) are in the same zone of the Panel, near the “neck” of the butterfly and at the tip of the crack; however the DT computations show that they are within safe limits. Although the crack lies in a longitudinal-radial surface, its actual orientation is poorly perceived in the drawing due to the distortion of the image.

#### 4.3

### Evaluating the effect of inserting a foam padding between Panel and auxiliary frame

The strains and stresses in the wood immediately adjacent to the preparation, while the Panel is constrained by the crossbeams in the current auxiliary frame, are shown in Figure 8. The stresses are all within a safety range [12]; however, concentrations of strains and stresses occur around the crack and at its tip, confirming that this should always be considered a very delicate area. The case of a new possible framing configuration, by insertion of a polyethylene foam padding between the Panel and the auxiliary frame, has been simulated in Figure 8e. The detailed information about the contact zones, acquired by means of the pressure-sensitive film were fed into the hydro-mechanical

model together with the forces that balance them, measured by the load cells on the opposite face of the Panel; hence the actual stresses and strains in the Panel were computed based on actual test results, both in the current and in the possible new condition. To stay on the safe side, the mechanical contribution of the butterfly has been here omitted, so that this computation maximized the potential stresses at the crack tip. Comparison between Figure 8e and Figure 8d shows that the larger contact areas of the possible new configuration lead to quite smaller transverse stresses namely at the tip of the crack, which is a very delicate location. The insertion of this foam padding was firstly recommended by the restorers to reduce the risk of collisions of the paint’s barb against the rim during the manipulations, while also partially filling the irregular voids currently observable between rim and Panel thus improving the aesthetics of the assembly.

In summary, several significant results have been obtained to date:

- the climatic conditions (air temperature and relative humidity) maintained in the current (since 2005) display case are favourable to the conservation of the artwork, since it turned out that no unacceptable stress or deformation took place in the wooden panel while it was exposed to such climate;
  - the risk of crack propagation has been assessed by means of stress analysis and computation of elastic energy release provided by the Digital Twin, calibrated through the knowledge of the Panel’s macrostructure, the contact zones, and the estimation of its mechanical properties; such computation shows that in the present and foreseeable state of strains and stresses, the possibility of crack propagation appears remote, provided no extreme climatic variations take place;
  - the information acquired have provided vital information about desirable and acceptable stress and deformation states of the Panel; two possible improvements to the framing conditions of the Panel have been hence analysed, namely (i) the implementation of a “load limiter” [19] (not discussed here) temporarily allowing for larger deformation in order to prevent dangerous stresses in case of unexpected disturbing factors (e.g. climatic or mechanical shocks), and (ii) the insertion between Panel and auxiliary frame of viscoelastic foam strips (Plastazote®), which seem to be able to bring various advantage;
  - the equipment, developed and improved during the years for monitoring and adjusting the forces acting on the Panel and its deformations, works reliably, and at will can be permanently left in operation, thus allowing for an efficient continuous monitoring of the Panel’s situation.
- It should be emphasized that such results were obtained by a systematic set of experimental investigations, conceived and implemented specifically for the individual artwork, and by integrating numerical simulations with the experimental results, so to greatly improve the level of knowledge that can be obtained from them, and provide more extensive and deeper information: experimental results alone would have provided much less information. Also, the fact that the test results derived from one specific artwork implies that they cannot be usefully applied as they are to the study of a different artwork, however similar it may appear.



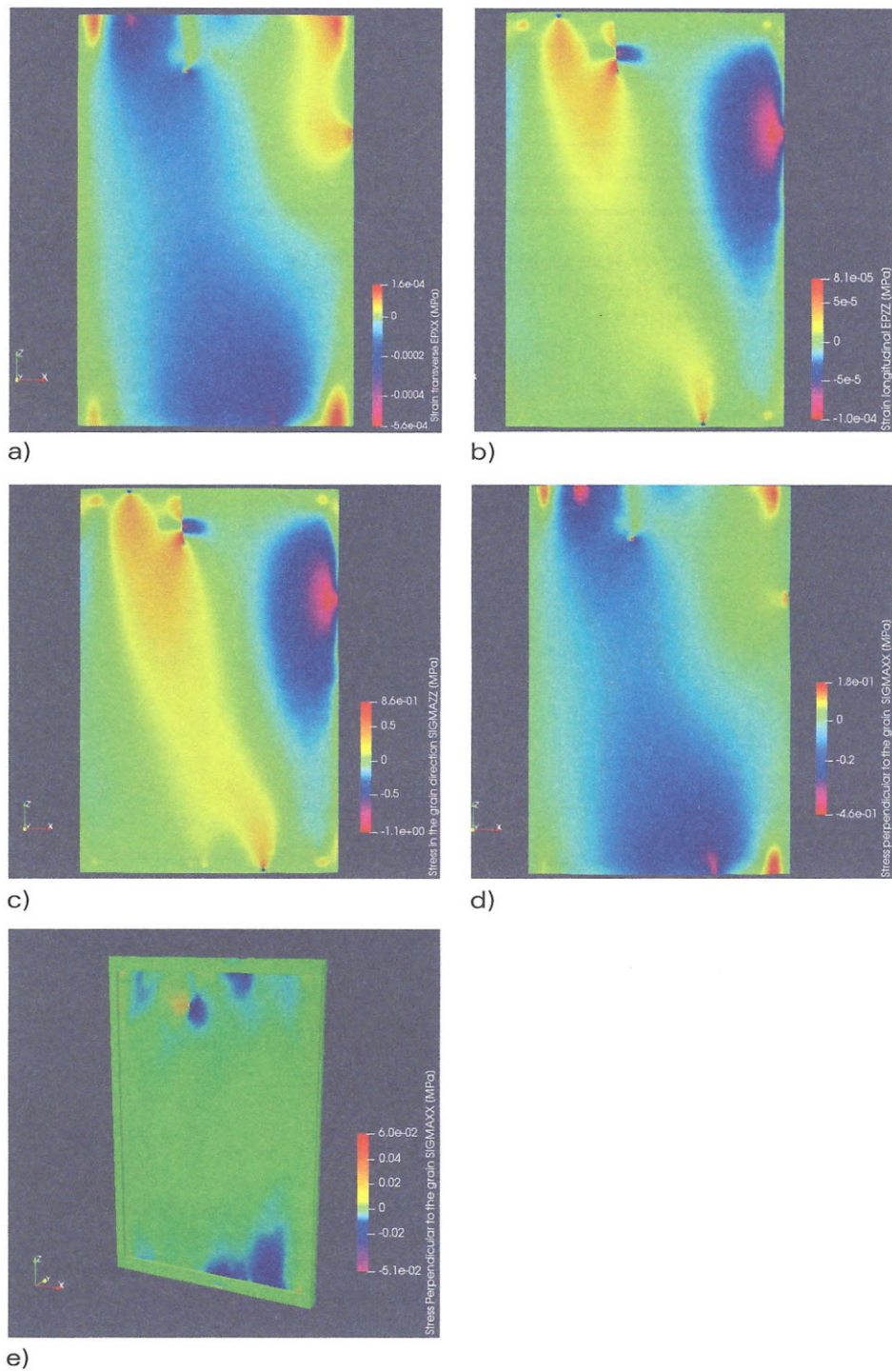


Figure 8:  
 Strains and stresses in the wood layer close to the front surface of the constrained Panel. In the auxiliary frame: strains a)  $\epsilon_{xx}$  perpendicular and b)  $\epsilon_{zz}$  parallel to the grain; stresses c)  $\sigma_{zz}$  parallel and d)  $\sigma_{xx}$  perpendicular to the grain; directions x,z refer to the global reference system. e) Stresses  $\sigma_{xx}$  perpendicular to the grain in case of insertion of a thin strip of closed-pore polyethylene foam between Panel and auxiliary frame B.

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*Florentine Masters of precision machining*, having collaborated with competence and passion in practical design and manufacture

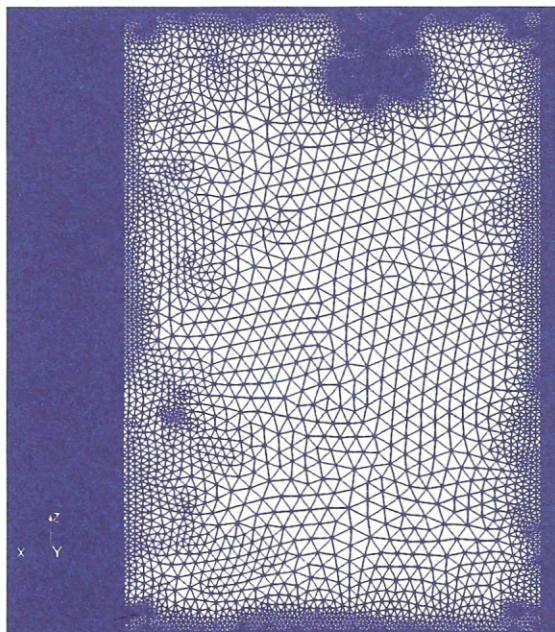
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Backside of the panel of the *Mona Lisa* during examinations 2004;  
photo Lorenzo Riparbelli.



Numerical discretization of the panel-butterfly  
assembly, in the panel of the *Mona Lisa*;  
see page 64.



