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# Rational solutions of order $N$ to the KPI equation with multi-parameters and the explicit case of order 3

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## Abstract

We present multiparametric rational solutions to the Kadomtsev-Petviashvili equation (KPI). These solutions of order  $N$  depend on  $2N - 2$  real parameters.

Explicit expressions of the solutions at order 3 are given. They can be expressed as a quotient of a polynomial of degree  $2N(N + 1) - 2$  in  $x$ ,  $y$  and  $t$  by a polynomial of degree  $2N(N + 1)$  in  $x$ ,  $y$  and  $t$ , depending on  $2N - 2$  real parameters. We study the patterns of their modulus in the  $(x,y)$  plane for different values of time  $t$  and parameters.

PACS numbers :

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## 1 Introduction

The Kadomtsev-Petviashvili equation (KPI) is a well-known nonlinear partial differential equation in two spatial and one temporal coordinates which can be written in the following form :

$$(4u_t - 6uu_x + u_{xxx})_x = 3u_{yy}, \quad (1)$$

with subscripts  $x$ ,  $y$  and  $t$  denoting partial derivatives.

The KP equation first appeared in 1970, in a paper written by Kadomtsev and Petviashvili [65]. The discovery of the KP equation happened almost simultaneously with the development of the inverse scattering transform (IST) as it is

explained in Manakov et al. [68]. In 1974 Dryuma showed how the KP equation could be written in Lax form [5], and Zakharov extended the IST to equations in two spatial dimensions, including the KP equation, and obtained several exact solutions to the KP equation.

In 1981 Dubrovin constructed for the first time [7] the solutions to KPI given in terms of Riemann theta functions in the frame of algebraic geometry .

From the 1980's, a lot of methods have been found to solve that equation. We can quote the nonlocal Riemann-Hilbert problem, the d-bar problem or inverse scattering problem using integration in the complex plane. More details can be found in the book by Ablowitz and Clarkson published in 1991 [1].

We can cite in particular the works of Krichever [67], Satsuma and Ablowitz in 1979 [73], Matveev in 1979 [69], Freeman and Nimmo in 1983 [8, 9], Pelinovsky and Stepanyants in 1993 [71], Pelinovsky in 1994 [72], Ablowitz and Villarroel [2, 75] in 1997-1999, Biondini and Kodama [3, 66, 4] in 2003-2007.

This paper is part of a program of research of rational solutions of partial differential equations. New solutions of the KPI equation are presented here. We express rational solutions in terms of a quotient of a polynomial of degree  $2N(N+1) - 2$  in  $x, y$  and  $t$  by a polynomial of degree  $2N(N+1)$  in  $x, y$  and  $t$  depending on  $2N - 2$  real parameters. This representation allows to obtain an infinite hierarchy of solutions to the KPI equation, depending on  $2N - 2$  real parameters .

That provides an effective method to construct an infinite hierarchy of rational solutions of order  $N$  depending on  $2N - 2$  real parameters. We present here only the rational solutions of order 3, depending on 4 real parameters, and the representations of their modulus in the plane of the coordinates  $(x, y)$  according to real parameters  $a_1, b_1, a_2, b_2$  and time  $t$ .

## 2 Rational solutions of order $N$ to the KPI equation depending on $2N - 2$ real parameters

We consider the matrix  $M$  defined by :

$$m_{ij} = \sum_{k=0}^i c_{i-k} \left( \frac{\sqrt{p^2-4}}{3} \partial_p \right)^k \sum_{l=0}^j c_{j-l} \left( \frac{\sqrt{q^2-4}}{3} \partial_q \right)^l \times \left( \frac{1}{p+q} \exp \left( \frac{1}{2}(p+q)(-x + \frac{3}{4}t) - \frac{1}{4}(p^2 - q^2)iy \right) \right)_{p=q=-1}. \quad (2)$$

The coefficients  $c_j$  are defined by :

$$c_{2j} = 0, \quad c_{2j+1} = a_j + ib_j \quad 1 \leq j \leq N-1, \quad (3)$$

where  $a_j$  and  $b_j$  are arbitrary real numbers.

Then we have the following result :

**Theorem 2.1** *The function  $v$  defined by*

$$v(x, y, t) = -2 \partial_x^2 (\ln \det(m_{2i-1, 2j-1})_{1 \leq i, j \leq N}) \quad (4)$$

*is a solution to the KPI equation (1), depending on  $2N - 2$  parameters  $a_k, b_k$ ,  $1 \leq k \leq N - 1$ .*

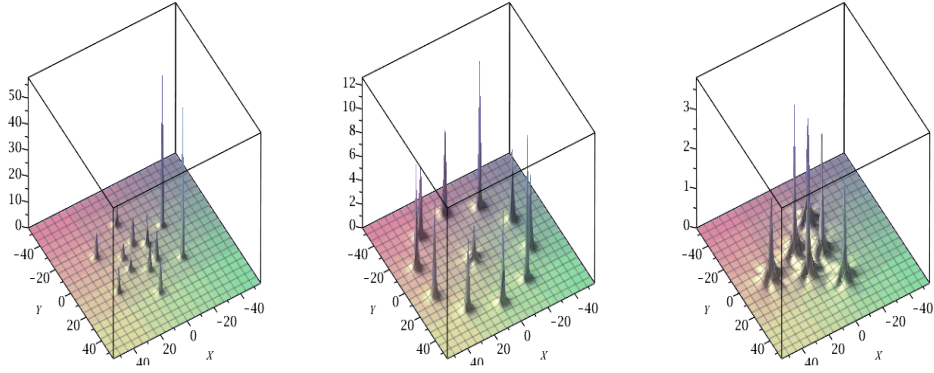
### 3 Rational solutions of order 3 depending on 4 parameters

In the following, we explicitly construct rational solutions to the KPI equation of order 3 depending on 4 parameters.

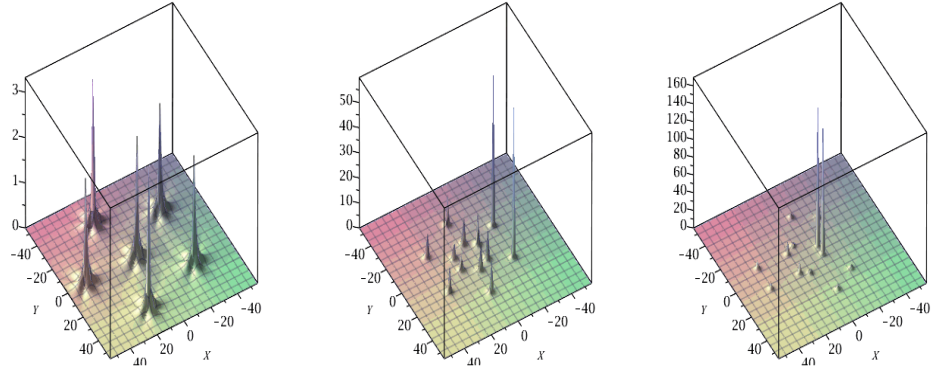
Because of the length of the expression of the solution, we only give the expression without parameters and we present it in the appendix.

We give patterns of the modulus of the solutions in the plane  $(x, y)$  of coordinates in functions of parameters  $a_1, b_1, a_2, b_2$  and time  $t$ .

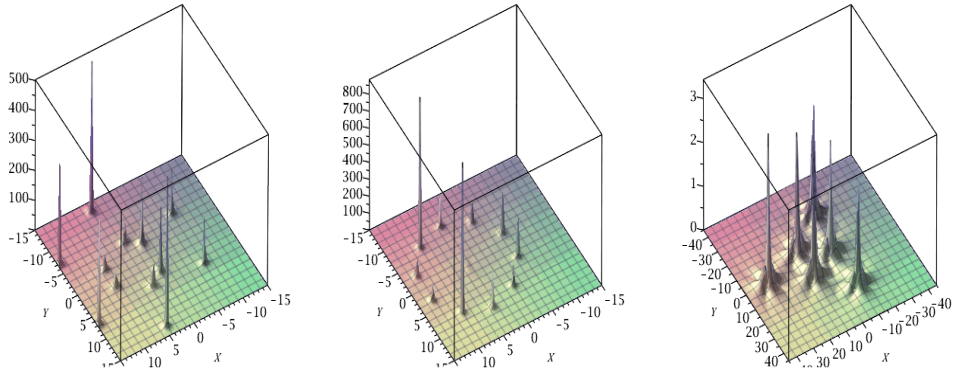
In all the following figures, if the parameters are not quoted then there are equal to 0.



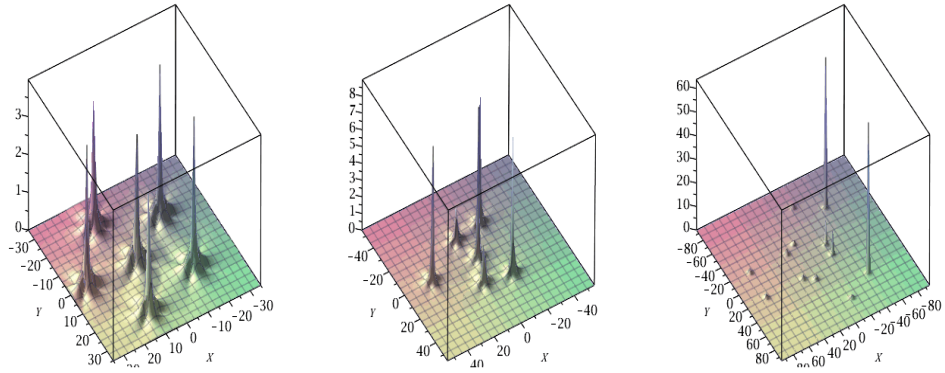
**Figure 1.** Solution of order 3 to KPI, on the left for  $t = 0$ ,  $a_1 = 10^4$ ; in the center for  $t = 0$ ,  $a_2 = 10^8$ ; on the right for  $t = 0$ ,  $b_1 = 10^4$ .



**Figure 2.** Solution of order 3 to KPI, on the left for  $t = 0$ ,  $b_2 = 10^8$ ; in the center for  $t = 0$ ,  $a_1 = 10^4$ ,  $a_2 = 10^4$ ; on the right for  $t = 0$ ,  $a_1 = 10^4$ ,  $a_2 = 10^4$ ,  $b_1 = 10^4$ ,  $b_2 = 10^8$ .

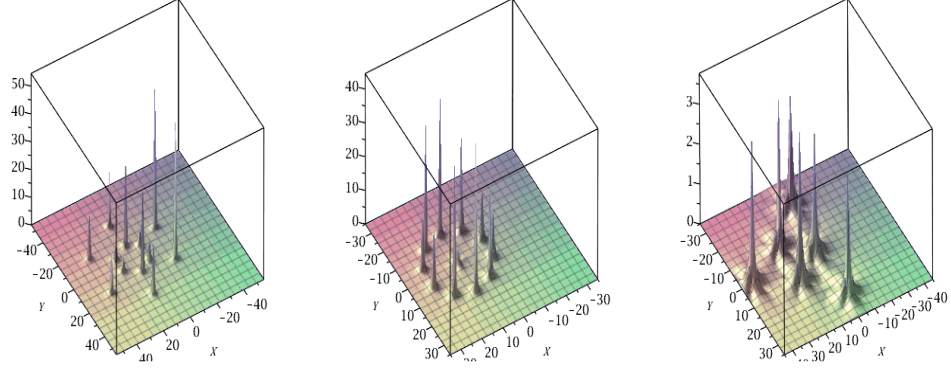


**Figure 3.** Solution of order 3 to KPI, on the left for  $a_1 = 10^3$ ; in the center for  $a_2 = 10^5$ ; on the right for  $b_1 = 10^4$ ; here  $t = 1$ .

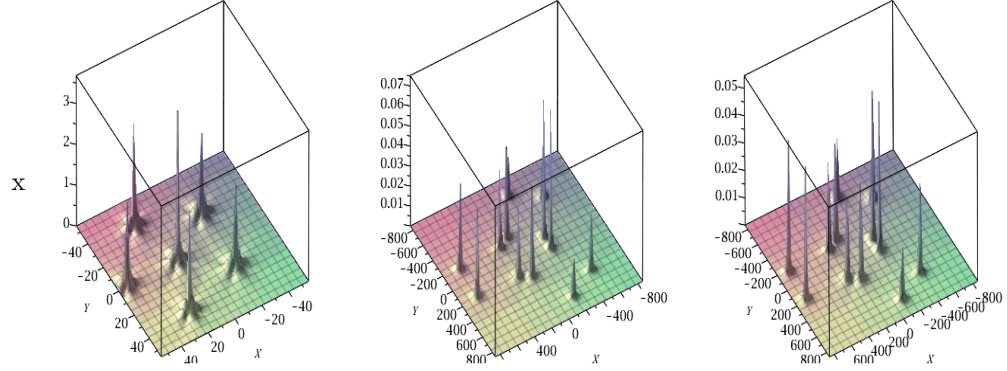


**Figure 4.** Solution of order 3 to KPI, on the left for  $t = 1$ ,  $b_2 = 10^7$ ; in the

center for  $t = 1$ ,  $a_1 = 10^3$ ,  $a_2 = 10^5$ ,  $b_1 = 10^4$ ,  $b_2 = 10^7$ ; on the right for  $t = 1$ ,  
 $a_1 = 10^5$ ,  $a_2 = 10^5$ ,  $b_1 = 10^5$ ,  $b_2 = 10^5$ .



**Figure 5.** Solution of order 3 to KPI, on the left for  $t = 10$ ,  $a_1 = 10^4$ ; in the center for  $t = 10$ ,  $a_2 = 10^4$ ; on the right for  $t = 10$ ,  $b_1 = 10^4$ .



**Figure 6.** Solution of order 3 to KPI, on the left for  $t = 10$ ,  $b_1 = 10^8$ ; in the center for  $t = 10$ ,  $a_1 = 10^8$ ,  $a_2 = 10^8$ ,  $b_1 = 10^8$ ,  $b_2 = 10^8$ ; on the right for  $t = 10^2$ ,  $a_1 = 10^8$ ,  $a_2 = 10^8$ ,  $b_1 = 10^8$ ,  $b_2 = 10^8$ .

The previous study shows the appearance of different types of configurations.  
 If  $a_1 \neq 0$  and the other parameters equal to 0, we get 12 peaks on two concentric rings, 6 on the first and 6 on the second one.  
 For  $a_2 \neq 0$  and the other parameters equal to 0, we get 10 peaks on a ring and a peak in the center of the ring.  
 If  $b_1 \neq 0$  and the other parameters equal to 0, we get 6 peaks on a triangle.  
 For  $b_2 \neq 0$  and the other parameters equal to 0, we get 5 peaks on a ring and one peak in the center of the ring.  
 In the case where two parameters  $a_1$  and  $a_2$  are not equal to 0, the other parameters being equal to 0, for the same values of parameters, we get 12 peaks on two concentric rings, which shows the predominance of the parameter  $a_1$  over

the parameter  $a_2$  on the structure of the solutions.

In the case where two parameters  $b_1$  and  $b_2$  are not equal to 0, the other parameters being equal to 0, for the same values of parameters, we get 6 peaks on a ring with a peak in the center of the ring, which shows also the predominance of the parameter  $b_1$  over the parameter  $b_2$  on the structure of the solutions.

In the case where all parameters  $a_1, a_2, b_1, b_2$ , are not equal to 0, for the same values of parameters, we get 6 couples of 2 peaks on two rings.

## 4 Conclusion

In this article, rational solutions to the KPI equation have been built in terms of quotients of a polynomial of degree  $2N(N+1)-2$  in  $x, y$  and  $t$  by a polynomial of degree  $2N(N+1)$  in  $x, y$  and  $t$ , depending on  $2N-2$  parameters.

Other approaches to build solutions of KPI equation had been realized, and we can mention those most significant. In 1990, Hirota and Ohta [62] built, the solutions as particular case of a hierarchy of coupled bilinear equations given in terms of Pfaffians. In 1993, the Darboux transformations was used to obtain among others the solutions of the multicomponent KP hierarchy [70], but no explicit solutions were given. More recently, in 2013, wronskians identities of bilinear KP hierarchy were given [64]. In 2014, solutions of KPI equation were constructed [76]; an explicit solution at order 1 was built and only one asymptotic study has been carried out for order higher than 2.

In 2016, three types of representations of the solutions were given in [42], in terms of Fredholm determinants, wronskians and degenerate determinants.

The structures of the solutions given in this paper looks like to these given in [42] which were deduced from the solutions to the NLS equation [15, 32, 39].

But, to the best of my knowledge, some of the structures of these solutions had never been presented. It will be relevant to find explicit solutions for higher orders and try to describe the structure of these rational solutions.

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**Appendix** The general solution depending on 4 parameter being too large, we present only the solution without parameters. In can be written as :

$$v(x, y, t) = -2 \frac{n_3(x, y, t)}{(d_3(x, y, t))^2}$$

with

$$\begin{aligned} n_3(x, y, t) = & -3377699720527872x^{22} + (55732045388709888t + 272467777455915008)x^{21} + (-4291367494930661376t - \\ & 438889857436090368t^2 - 30399297484750848y^2 - 10655516718358593536)x^{20} + (2194449287180451840t^3 + \\ & 4559894622711262720ty^2 + 32185256211979960320t^2 + 268346983796871004160 + 2229281815548395520y^2 + \\ & 159832750775378903040t)x^{19} + (-118219490218475520y^4 - 79015655462215352320y^2 - 152879967006904811520t^3 - \\ & 3823944519105411809280t - 31767265871564636160ty^2 - 1138808349274574684160t^2 - 4877083144468331233280 - \\ & 7817725585580359680t^4 - 3248924918682746880t^2y^2)x^{18} + (25811625503961529712640t^2 + 1796373301367406592000y^2 + \\ & 214429044633061294080t^2y^2 + 14620162134072360960t^3y^2 + 5124637571735586078720t^3 + 1595963117949419520ty^4 + \\ & 67958991866047800279040 + 21107859081066971136t^5 + 515969888648303738880t^4 + 7802486354419384320y^4 + \\ & 65840622450322471649280t + 1066711348739907256320ty^2)x^{17} + (-109699408391836501278720t^3 - \\ & 419733968120805756764160t^2 - 866477146292109453557760t - 44854200547267313664t^6 - 253327479039590400y^6 - \\ & 16334782259907180625920t^4 - 99481701018847150080ty^4 - 753040926423114370252800 - 29326224801537279918080y^2 - \\ & 22903759592434434048000ty^2 - 10174264876927549440t^2y^4 - 911323439690510499840t^3y^2 - 46601766802355650560t^4y^2 - \\ & 1315723216053174534144t^5 - 6800284848216908759040t^2y^2 - 247945677484882657280y^4)x^{16} + (6793167708394783548375040 + \\ & 27201139392867635036160t^3y^2 + 76892915223886823424t^7 + 596890206113082900480t^2y^4 + 2975348129818591887360ty^4 + \\ & 40697059507710197760t^3y^4 + 351914697618447359016960ty^2 + 5027548408018779504640y^4 + 111844240325653561344t^5y^2 + \\ & 5198862877752656721346560t^2 + 329098225175509503836160t^4 + 364818141054811661926400y^2 + \\ & 39203477423777233502208t^5 + 3039929748475084800ty^6 + 1678935872483223027056640t^3 + 14861878770322636800y^6 + \\ & 2631446432106349068288t^6 + 2733970319071531499520t^4y^2 + 9036491117077372443033600t + 137422557554606604288000t^2y^2)x^{15} + \\ & (-5083026253360219992064000t^2 - 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$878612385407285267005440 y^6 + 149690696374667315207208960 t^6 + 780562219030984128253132800 t^5 +$   
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55985264646653214720  $t^{13}y^2$  + 158897749172567408640  $t^{11}y^4$  + 217600403473170432000  $t^9y^6$  + 13926425822829076480  $t^7y^8$  +  
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$269227656793028793729024000 ty^8 - 220340245987131707243888640000 t^3 y^4 - 18162086942448288264093696000 ty^6 -$   
 $505733005816807589029085184000 t^5 y^2 - 20997919425127156555972608000 t^2 y^6 - 220720092677277959967473664000 t^6 y^2 -$   
 $7478798023495086243840 t^{16} + 1082260026453403893760 y^{16} - 123510757980454697041920 t^{18} - 1610117289489375166464000 t^{14} +$   
 $53321718868141198540800 y^{14} - 16945643270565049820774400 t^{13} - 146045391709087386619084800 t^{12} +$   
 $2007508807297976500224000 y^{12} - 1039535983849557302496460800 t^{11} - 30054915247890496613253120000 t^9 +$   
 $286525580463801171695370240000 y^2 + 222833259647112425861283840000 ty^2 - 405944005177171906784133120000 t^2 y^2 -$   
 $836604972631021841075404800000 t^4 y^2 - 237824755887806465074790400000 t^2 y^4 - 889258215913747488807321600000 t^3 y^2 -$   
 $147294685333798089287270400000 ty^4 - 2233841645927259462759874560000 t - 35120825374851878690488320000 y^4 -$   
 $3986467961252812558392360960000 t^4 - 4983442874343911411511459840000 t^3 - 7436528181162099345457152000 y^6 -$   
 $1097638598629726343857176576000 t^6 - 2372733777586505215106875392000 t^5 - 6136367371351683373832601600 t^{10} -$   
 $2474333788050432 t^{20} + 30399297484750848 y^{20} - 241934859276042240 t^{19} - 11413815730179932160 t^{18} +$   
 $10876193100099747840 y^{18} - 344932651030991339520 t^{17} - 121780707923783080828993536000 t^{16} -$   
 $405541361521209436737421312000 t^7 + 503944783021321689759744000 y^8 - 494598582966379249737400320000 +$   
 $56688844024154013931929600 y^{10} - 93435591545812569135513600 t^{11} y^2 - 192070244572932354342912000 t^9 y^4 -$   
 $173539843278170077790208000 t^7 y^6 - 57932838755947928302387200 t^5 y^8 + 1096627297394650565836800 t^3 y^{10} +$   
 $1806861284179627003084800 ty^{12} - 681909710127675723900518400 t^{10} y^2 - 1156970395249340942647296000 t^8 y^4 -$   
 $808751685160277789324083200 t^6 y^6 - 168827940293308116369408000 t^4 y^8 + 15759615961763528520499200 t^2 y^{10} -$   
 $4029472318954882713255936000 t^2 y^2 - 5535839694918346950574080000 t^7 y^4 - 2883836144196514247344128000 t^5 y^6 -$   
 $3011638295360871010271232000 t^3 y^8 + 48731433729963093865267200 ty^{10} - 62703977873675806310400 t^{14} y^2 -$   
 $209641174169767693516800 t^{12} y^4 - 347608944841805329858560 t^{10} y^6 - 278220528182280742502400 t^8 y^8 -$   
 $6464054608593800028160 t^6 y^{10} + 38130408821950827724800 t^4 y^{12} + 18671248515133970841600 t^2 y^{14} -$   
 $9100434940397939695104000 t^{13} y^2 - 2620638560988164417126400 t^{11} y^4 - 3628368442567587948134400 t^9 y^6 -$   
 $2292614856700493561856000 t^7 y^8 - 335596780371574448455680 t^5 y^{10} + 204658520425853091840000 t^3 y^{12} +$   
 $46864570312408065638400 ty^{14} - 10324877158861619016499200 t^{12} y^2 - 25298499092918279602176000 t^{10} y^4 -$   
 $28653217782398733149798400 t^8 y^6 - 13633115071511098766131200 t^6 y^8 - 84346793280337814016000 t^4 y^{10} +$   
 $766723481336014700544000 t^2 y^{12} - 2865514177399357440 t^{17} y^2 - 14180478216422031360 t^{15} y^4 - 37073145663848448000 t^{13} y^6 -$   
 $51408095320536514560 t^{11} y^8 - 25777278565283266560 t^9 y^{10} + 24996148911789834240 t^7 y^{12} + 44437598065404149760 t^5 y^{14} +$   
 $23511956648361984000 t^3 y^{16} + 3901243177209692160 ty^{18} - 121167121902820392960 t^{16} y^2 - 530145982993032806400 t^{14} y^4 -$   
 $1203936050511756656640 t^{12} y^6 - 1415406932130099363840 t^{10} y^8 - 577467849291690147840 t^8 y^{10} +$   
 $454026570132891893760 t^6 y^{12} + 577539153307946188800 t^4 y^{14} + 187107676018641469440 t^2 y^{16} - 3264783981861666816000 t^{15} y^2 -$   
 $12541283463751689830400 t^{13} y^4 - 24495178509700489543680 t^{11} y^6 - 24034057555237090099200 t^9 y^8 -$   
 $7670030420629813985280 t^7 y^{10} + 5042994083482986086400 t^5 y^{12} + 4245641884964015308800 t^3 y^{14} +$   
 $709874261764740218880 ty^{16} - 32562661106810880 t^{18} y^2 - 181284522653122560 t^{16} y^4 - 541653102231552000 t^{14} y^6 -$   
 $876274352054599680 t^{12} y^8 - 527262516108066816 t^{10} y^{10} + 639106080130990080 t^8 y^{12} + 1514918115866050560 t^6 y^{14} +$   
 $1202315964973056000 t^4 y^{16} + 398990779487354880 t^2 y^{18} x^2 + (1675381234445444597069905920000 t^2 +$   
 $82627592245174390216458240000 t^4 y^4 + 5727707949089956036608000000 t^6 y^6 - 201920742594771595296768000 t^2 y^8 +$   
 $1535653183055440379330560000 t^4 y^4 + 13775987840151160614813696000 t^6 y^2 + 4100899425044483095920640000 t^5 y^4 +$   
 $47297162716559562850172928000 t^7 y^2 + 10498959712563578277986304000 t^3 y^6 - 755917174531982534639616000 ty^8 +$   
 $118912377943903232537395200000 t^3 y^4 + 11154792271743149018185728000 ty^6 + 250981491789306552322621440000 t^5 y^2 +$   
 $13621565206836216198070272000 t^2 y^6 + 126433251454201897257271296000 t^6 y^2 + 1157913306667627847680 t^{16} -$   
 $2089096018147420405760 y^{16} + 161011728948937516646400 t^{15} + 1815604636131969623654400 t^{14} -$   
 $85546775641828045619200 y^{14} + 16851391351048544609894400 t^{13} + 129941997981194662812057600 t^{12} -$   
 $3151060122520116422246400 y^{12} + 836777368820684096431718400 t^{11} + 20296784653963846804832256000 t^9 -$   
 $264454466614155319978229760000 y^2 - 429788370695701757543055360000 ty^2 - 167124944735334319395962880000 t^2 y^2 +$   
 $333471830967655308302745600000 t^4 y^2 + 110471014000348566965452800000 t^2 y^4 + 202972002588585953392066560000 t^3 y^2 +$   
 $52681238062277818035732480000 ty^4 + 741897874449568874606100480000 t + 4589971349874067636224000000 y^4 +$   
 $1868791077878966779316797440000 t^4 + 2152492683035299645331865600000 t^3 + 4133858506987413474115584000 y^6 +$   
 $593183444396626303776718848000 t^6 + 1195940388375843767517708288000 t^5 + 4508237287183574491987968000 t^{10} +$   
 $176738127717888 t^{21} + 18145114445703168 t^{20} - 222928181554839552 y^{20} + 901090715540520960 t^{19} +$   
 $2874438758591594960 t^{18} - 33349155240678522880 y^{18} + 659893943249566433280 t^{17} + 76039005283520519388266496000 t^8 +$   
 $235205268992084216540823552000 t^7 - 768412131911995125399552000 y^8 + 117773106967986435753246720000 -$   
 $85529414940804476462694400 y^{10} + 92987687744683053259161600 t^{11} y^2 + 192828399208223490441216000 t^9 y^4 +$   
 $173303932534345240569446400 t^7 y^6 + 5064838208792434910822400 t^5 y^8 - 7879807980881764260249600 t^3 y^{10} -$   
 $3011263210946964750336000 ty^{12} + 604420847843232406988390400 t^{10} y^2 + 1037969942797190053232640000 t^8 y^4 +$   
 $120959036049128561836032000 t^6 y^6 + 113114398260326628851712000 t^4 y^8 - 36548575297472320398950400 t^2 y^{10} +$   
 $3205417958829884555919360000 t^2 y^2 + 4480517614084764651749376000 t^7 y^4 + 2325188969569593772631654400 t^5 y^6 +$   
 $104228293402051397812224000 t^3 y^8 - 85033266036231020897894400 ty^{10} + 97504660075686395904000 t^{14} y^2 +$   
 $327579820123520552140800 t^{12} y^4 + 544255266385138192220160 t^{10} y^6 + 429865285631342542848000 t^8 y^8 +$   
 $83899195092893612113920 t^6 y^{10} - 76746945159694909440000 t^4 y^{12} - 35148427734306049228800 t^2 y^{14} +$   
 $1191331979868648348057600 t^{13} y^2 + 3449795330852492673024000 t^{11} y^4 + 4775536297066455524966400 t^9 y^6 +$   
 $2921381801038092592742400 t^7 y^8 + 253040379841001344204800 t^5 y^{10} - 383361740668007350272000 t^3 y^{12} -$   
 $79982578302211797811200 ty^{14} + 11679448943226571141939200 t^{12} y^2 + 28810536685939853151436800 t^{10} y^4 +$   
 $32538720614656889585664000 t^8 y^6 + 14483209688986982075596800 t^6 y^8 - 411235236522993962188800 t^4 y^{10} -$   
 $1355145963134720252313600 t^2 y^{12} + 10691216638484152320 t^{17} y^2 + 53014598299303280640 t^{15} y^4 +$   
 $138915698135971921920 t^{13} y^6 + 19301003619959004160 t^{11} y^8 + 96244641548615024640 t^9 y^{10} - 97291407885619691520 t^7 y^{12} -$   
 $1343708942544823910400 t^5 y^{14} + 3061897313712561192960 t^3 y^{16} - 16314289650149621760 ty^{18} + 306073498299531264000 t^{16} y^2 +$   
 $1438130703868090122240 t^8 y^{10} - 1260748520870746521600 t^6 y^{12} - 1592115706861505740800 t^4 y^{14} -$

$$\begin{aligned}
& 532405696323555164160 t^2 y^{16} + 6270397787367580631040 t^{15} y^2 + 24189366250357810790400 t^{13} y^4 + \\
& 47401219751155272253440 t^{11} y^6 + 46370088030380123750400 t^9 y^8 + 13851545583984385720320 t^7 y^{10} - \\
& 11439122646585248317440 t^5 y^{12} - 9335624257566985420800 t^3 y^{14} - 1623390039680105840640 t y^{16} + \\
& 2570736403169280 t^{19} y^2 + 15995693175275520 t^{17} y^4 + 54165310223155200 t^{15} y^6 + 101108579083223040 t^{13} y^8 + \\
& 71899434014736384 t^{11} y^{10} - 106517680021831680 t^9 y^{12} - 324625310542725120 t^7 y^{14} - 360694789491916800 t^5 y^{16} - \\
& 199495389743677440 t^3 y^{18} - 45598946227126272 t y^{20} + 238792848116613120 t^{18} y^2 + 1329419832789565440 t^{16} y^4 + \\
& 3972122749698048000 t^{14} y^6 + 6426011915067064320 t^{12} y^8 + 3866591784792489984 t^{10} y^{10} - 4686777920960593920 t^8 y^{12} - \\
& 11109399516351037440 t^6 y^{14} - 8816983743135744000 t^4 y^{16} - 2925932382907269120 t^2 y^{18} - 3405391301709054049517568000 t^3 y^6 + \\
& 576309098933996344049664000 t y^8 - 3100393880240560105586688000 t y^6 - 50020774645148296245411840000 t^5 y^2 - \\
& 275211702918588327977287680000 t^2 - 27617753500087141471363200000 t^3 y^4 - 196855494605670927122432000 t^4 y^6 + \\
& 283468940449493450489856000 t^2 y^8 - 5126123678130560386990080000 t^6 y^4 - 4434109004677459017203712000 t^8 y^2 - \\
& 12394138836776158532468736000 t^5 y^4 - 13546419798664488991850496000 t^7 y^2 - 403592378069118683499724800000 t^4 - \\
& 418845308211361149267476480000 t^3 - 800800884399037728423936000 y^6 - 149492548546980470939713536000 t^6 - \\
& 280318661681845016897519616000 t^5 - 1522258849047288510362419200 t^{10} - 6025163444928 t^{22} - \\
& 22296070864481856100761600000 t^4 y^4 + 3377699720527872 y^{22} - 648039801632256 t^{21} - 32790901832769536 t^{20} + \\
& 580964356059833796028000640 y^{20} - 1134646878391418880 t^{19} - 27495580968731934720 t^{18} + 46071824188000174080 y^{18} - \\
& 510844127676512993280 t^{17} - 22050493968007895300702208000 t^8 - 63555369042495675404648448000 t^7 + \\
& 493536726717588129710080000 y^8 - 4183047101903680881819648000 t^2 y^2 - 31372686473663319040327680000 t^6 y^2 - \\
& 754742479481446092800 t^{16} + 1840193325741721518080 y^{16} - 9078023180659848182720 t^{15} - 902753108091886318387200 t^{14} + \\
& 66281277155862537830400 y^{14} - 7496653729684307469926400 t^{13} - 5229858551292756026982400 t^{12} + \\
& 2341367234189406398054400 y^{12} - 307379815035243715362816000 t^{11} - 6336583773626709949022208000 t^9 + \\
& 97174929823379754602987520000 y^2 + 19834084996061648983672320000 t y^2 + 161170639010888159078645760000 t^2 y^2 - \\
& 38057250485359866261012480000 t^4 y^2 - 19755464273354181763399680000 t^2 y^4 + 41781236183833579848990720000 t^3 y^2 - \\
& 3442478512405550727168000000 t y^4 - 88329830225989826814935040000 t + 3896811218593394060165120000 t^9 y^4 + \\
& 590933560598337960280006400 y^{10} - 41210512352947664112844800 t^{11} y^2 - 86497495233099171102720000 t^9 y^4 - \\
& 77245611005263774482432000 t^7 y^6 - 16967159739048994327756800 t^5 y^8 + 9137143824368080099737600 t^3 y^{10} + \\
& 2363295091890087316684800 t y^{12} - 240406346912241341693952000 t^{10} y^2 - 420048526320446686101504000 t^8 y^4 - \\
& 290648621196199221578956800 t^6 y^6 - 19542805012884637089792000 t^4 y^8 + 31887474763586632836710400 t^2 y^{10} - \\
& 1147998986679263384567808000 t^3 y^2 - 1645342696130940040642560000 t^7 y^4 - 859156192363493405491200000 t^5 y^6 + \\
& 504801856486928988241920000 t^3 y^8 + 64147061205603357347020800 t y^{10} - 63821356064391875788800 t^{14} y^2 - \\
& 215612208178280792064000 t^{12} y^4 - 358165222279984164372480 t^{10} y^6 - 273879543847321180569600 t^8 y^8 - \\
& 31630047480125168025600 t^6 y^{10} + 71880326375251378176000 t^4 y^{12} - 29993466863329424179200 t^2 y^{14} - \\
& 673814362109225258188800 t^{13} y^2 - 1964354774041353623961600 t^{11} y^4 - 2711560051221407465472000 t^9 y^6 - \\
& 1551772466677176650956800 t^7 y^8 + 61685285478449094328320 t^5 y^{10} + 338786490783680063078400 t^3 y^{12} + \\
& 64160081731371034214400 t y^{14} - 5811730484042690828697600 t^{12} y^2 - 14462129940616761783091200 t^{10} y^4 - \\
& 16247243675094866303385600 t^8 y^6 - 6331047760999054363852800 t^6 y^8 + 1477463996415330798796800 t^4 y^{10} + \\
& 1129223704105111781376000 t^2 y^{12} - 13503242572038144000 t^{17} y^2 - 67185447127241195520 t^{15} y^4 - \\
& 176647921944955453440 t^{13} y^6 - 245802861360379330560 t^{11} y^8 - 119844225322340843520 t^9 y^{10} + 135080198664722841600 t^7 y^{12} + \\
& 238817356029225861120 t^5 y^{14} + 133101424080888791040 t^3 y^{16} + 25011866430508892160 t y^{18} - 293924896282855342080 t^{16} y^2 - \\
& 1295858906269168435200 t^{14} y^4 - 2962576234447204515840 t^{12} y^6 - 347756602278509281280 t^{10} y^8 - \\
& 1298582398498536161280 t^8 y^{10} + 1429890330823156039680 t^6 y^{12} + 1750429548293809766400 t^4 y^{14} + \\
& 608771264880039690240 t^2 y^{16} - 4875233003784319795200 t^{15} y^2 - 18898835776356954931200 t^{13} y^4 - \\
& 37108313617168513105920 t^{11} y^6 - 35822107135945211904000 t^9 y^8 - 8989199474238601297920 t^7 y^{10} + \\
& 11512041773954236416000 t^5 y^{12} + 8787106933576512307200 t^3 y^{14} + 1566822013610565304320 t y^{16} - \\
& 96402615118848 t^{20} y^2 - 666487215636480 t^{18} y^4 - 2538998916710400 t^{16} y^6 - 5416531022315520 t^{14} y^8 - \\
& 4493714625921024 t^{12} y^{10} + 7988826001637376 t^{10} y^{12} + 30433622863380480 t^8 y^{14} + 4508684868489600 t^6 y^{16} + \\
& 37405385576939520 t^4 y^{18} + 17099604835172352 t^2 y^{20} - 9426033478287360 t^{19} y^2 - 58650874976010240 t^{17} y^4 - \\
& 198606137484902400 t^{15} y^6 - 370731456638484480 t^{13} y^8 - 263631258054033408 t^{11} y^{10} + 390564826746716160 t^9 y^{12} + \\
& 1190292805323325440 t^7 y^{14} + 1322547561470361600 t^5 y^{16} + 731483095726817280 t^3 y^{18} + 167196136166129664 t y^{20} - \\
& 445467359936839680 t^{18} y^2 - 2485059295279841280 t^{16} y^4 - 7441912400141352960 t^{14} y^6 - 12063127262472437760 t^{12} y^8 - \\
& 7218348116146126848 t^{10} y^{10} + 9121069489276846080 t^8 y^{12} + 21657718249047982080 t^6 y^{14} + 17541344626747637760 t^4 y^{16} + \\
& 6117858618806108160 t^2 y^{18}
\end{aligned}$$

and

$$\begin{aligned}
d_3(x, y, t) = & 16777216 x^{12} + (-738197504 - 150994944 t) x^{11} + (6090129408 t + 15435038720 + 622854144 t^2 + \\
& 100663296 y^2) x^{10} + (-754974720 t y^2 - 1557135360 t^3 - 200991047680 - 3690987520 y^2 - 22837985280 t^2 - \\
& 115762790400 t) x^9 + (251658240 y^4 + 64760053760 y^2 + 51385466880 t^3 + 2627665920 t^4 + 1799188643840 + \\
& 1356689571840 t + 390699417600 t^2 + 2548039680 t^2 y^2 + 24914165760 t y^2) x^8 + (-7381975040 y^4 - \\
& 10795131863040 t - 4070068715520 t^2 - 1509949440 t y^4 - 708669603840 y^2 - 11563863900160 - 781398835200 t^3 - \\
& 388560322560 t y^2 - 5096079360 t^3 y^2 - 77078200320 t^4 - 3153199104 t^5 - 74742497280 t^2 y^2) x^7 + \\
& (38755368960 t y^4 + 335544320 y^6 + 1019970846720 t^2 y^2 + 103347650560 y^4 + 28337221140480 t^2 + \\
& 66886044160 t^4 y^2 + 60710285475840 t + 130799370240 t^3 y^2 + 3963617280 t^2 y^4 + 80932110336 t^5 + \\
& 2759049216 t^6 + 1025585971200 t^4 + 54293755330560 + 7122620252160 t^3 + 5308311142400 y^2 + 3720515420160 t y^2) x^6 + \\
& (-8371159695360 t^2 y^2 - 1529956270080 t^3 y^2 - 6019743744 t^5 y^2 - 1509949440 t y^6 - 1773674496 t^7 - \\
& 23887400140800 t y^2 - 186336485048320 - 244321898987520 t - 136598142320640 t^2 - 42505831710720 t^3 - \\
& 28245114224640 y^2 - 87199580160 t^2 y^4 - 890534625280 y^4 - 5945425920 t^3 y^4 - 923027374080 t^5 - \\
& 147149291520 t^4 y^2 - 60699082752 t^6 - 8012947783680 t^4 - 465064427520 t y^4 - 7381975040 y^6) x^5 + \\
& (458103560601600 t^2 + 5573836800 t^4 y^4 + 108999475200 t^3 y^4 + 27682406400 t y^6 + 110361968640 t^5 y^2 + \\
& 2831155200 t^2 y^6 + 3762339840 t^6 y^2 + 107786901913600 y^2 + 105919178342400 t y^2 + 44788875264000 t^2 y^2 +
\end{aligned}$$

1434334003200  $t^4 y^2 + 871995801600 t^2 y^4 + 10463949619200 t^3 y^2 + 3339504844800 t y^4 + 698761818931200 t +$   
 5106984550400  $y^4 + 39849217228800 t^4 + 170747677900800 t^3 + 75833016320 y^6 + 576892108800 t^6 +$   
 6009710837760  $t^5 + 831409920 t^8 + 32517365760 t^7 + 251658240 y^8 + 460685574144000)x^4 + (-1048142728396800 t^2 -$   
 81749606400  $t^4 y^4 - 3344302080 t^5 y^4 - 1612431360 t^7 y^2 - 2831155200 t^3 y^6 - 754974720 t y^8 - 871995801600 t^3 y^4 -$   
 227499048960  $t y^6 - 860600401920 t^5 y^2 - 41523609600 t^2 y^6 - 55180984320 t^6 y^2 - 277136640 t^9 -$   
 292272524492800  $y^2 - 323360705740800 t y^2 - 158878767513600 t^2 y^2 - 7847962214400 t^4 y^2 - 5009257267200 t^2 y^4 -$   
 44788875264000  $t^3 y^2 - 15320953651200 t y^4 - 1382056722432000 t - 19704840192000 y^4 - 128060758425600 t^4 -$   
 458103560601600  $t^3 - 459024629760 y^6 - 3004855418880 t^6 - 23909530337280 t^5 - 12194012160 t^8 -$   
 247239475200  $t^7 - 3690987520 y^8 - 792542262067200)x^3 + (1554813812736000 t^2 + 490497638400 t^4 y^4 +$   
 1592524800  $t^4 y^6 + 849346560 t^2 y^8 + 1254113280 t^6 y^4 + 453496320 t^8 y^2 + 36787322880 t^5 y^4 + 17736744960 t^7 y^2 +$   
 31142707200  $t^3 y^6 + 8304721920 t y^8 + 3756942950400 t^3 y^4 + 1032805416960 t y^6 + 3531582996480 t^5 y^2 +$   
 255936430080  $t^2 y^6 + 322725150720 t^6 y^2 + 3048503040 t^9 + 544467635404800 y^2 + 657613180108800 t y^2 +$   
 363780793958400  $t^2 y^2 + 25193742336000 t^4 y^2 + 17236072857600 t^2 y^4 + 119159075635200 t^3 y^2 + 44335890432000 t y^4 +$   
 1783220089651200  $t + 49143821107200 y^4 + 257683252838400 t^4 + 786107046297600 t^3 + 1727382159360 y^6 +$   
 8966073876480  $t^6 + 57627341291520 t^5 + 62355744 t^{10} + 69536102400 t^8 + 965846384640 t^7 + 23488102400 y^8 +$   
 100663296  $y^{10} + 891610044825600)x^2 + (-1337415067238400 t^2 - 1408853606400 t^4 y^4 - 11678515200 t^4 y^6 -$   
 6228541440  $t^2 y^8 - 9196830720 t^6 y^4 - 3325639680 t^8 y^2 - 147149291520 t^5 y^4 - 69155389440 t^7 y^2 -$   
 127968215040  $t^3 y^6 - 35232153600 t y^8 - 8618036428800 t^3 y^4 - 2591073239040 t y^6 - 7558122700800 t^5 y^2 -$   
 774604062720  $t^2 y^6 - 882895749120 t^6 y^2 - 8503056 t^{11} - 11589350400 t^9 - 634836431667200 y^2 -$   
 816701453107200  $t y^2 - 493209885081600 t^2 y^2 - 44684653363200 t^4 y^2 - 33251917824000 t^2 y^4 - 181890396979200 t^3 y^2 -$   
 73715731660800  $t y^4 - 1337415067238400 t - 71259547238400 y^4 - 294790142361600 t^4 - 777406906368000 t^3 -$   
 3871845908480  $y^6 - 14406835322880 t^6 - 77304975851520 t^5 - 457275456 t^{10} - 181096197120 t^8 -$   
 1921301544960  $t^7 - 76168560640 y^8 - 738197504 y^{10} - 75582720 t^9 y^2 - 268738560 t^7 y^4 - 477757440 t^5 y^6 -$   
 424673280  $t^3 y^8 - 150994944 t y^{10} - 594406696550400)x + 8312979456000 t^3 y^4 + 57126420480 t y^8 +$   
 944765337600  $t^6 y^2 + 531441 t^{12} + 193651015680 t^3 y^6 + 16777216 y^{12} + 31177872 t^{11} + 15091349760 t^9 +$   
 356725555200000  $y^2 + 476127323750400 t y^2 + 306263044915200 t^2 y^2 + 34104449433600 t^4 y^2 + 2903884431360 t y^6 +$   
 6702698004480  $t^5 y^2 + 971652464640 t^2 y^6 + 445805022412800 t + 46939294924800 y^4 + 145763794944000 t^4 +$   
 334353766809600  $t^3 + 4160078479360 y^6 + 9663121981440 t^6 + 44218521354240 t^5 + 869201280 t^{10} +$   
 180122019840  $t^8 + 1543589498880 t^7 + 27643399372800 t^2 y^4 + 123302471270400 t^3 y^2 + 53444660428800 t y^4 +$   
 501530650214400  $t^2 + 1615881830400 t^4 y^4 + 23994040320 t^4 y^6 + 13212057600 t^2 y^8 + 18393661440 t^6 y^4 +$   
 6483317760  $t^8 y^2 + 211328040960 t^5 y^4 + 94595973120 t^7 y^2 + 108045271040 y^8 + 1677721600 y^{10} +$   
 5668704  $t^{10} y^2 + 25194240 t^8 y^4 + 59719680 t^6 y^6 + 79626240 t^4 y^8 + 56623104 t^2 y^{10} + 277136640 t^9 y^2 +$   
 985374720  $t^7 y^4 + 175177280 t^5 y^6 + 1557135360 t^3 y^8 + 553648128 t y^{10} + 198135565516800.$