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Testing the accuracy of WIM systems: application to a B-WIM case

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Abstract

The assessment of the accuracy of a weigh-in-motion (WIM) system is described in this paper. It is based on a statistical composite hypothesis test for which the Type I error is the supplier risk for the WIM system. This statistical test is built in the asymptotical setting, namely for large samples of measurements, with an implicit estimator of the tolerance. Considering an observation sample of independent and identically distributed relative errors, this procedure is used in the classical WIM assessment method described in the COST 323 action and compared to the classical estimator of the tolerance. The test also gives formulæ to practitioners implying the observation sample size comparing to the usual estimator for which the observation sample size is tabulated. An application on a bridge-WIM system is also done.

Keywords: Accuracy estimation, bridge weigh-in-motion, hypothesis test, metrology, WIM specification.

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1 Introduction

Heavy traffic loads are growing, both in intensity and frequency, while road infrastructures are aging and the available resources need to be allocated in an optimal manner. Therefore, it is necessary to collect reliable and accurate enough traffic data, above all traffic loads on the road network.

To obtain this information, a weigh-in-motion system (WIM) is generally used, where sensors in, on or in the surroundings of the road make it possible to infer the loads and dimensions of the vehicles passing at that location. These sensors can be of various natures and technologies, see [8] for example of descriptions.

When using or selling a weighing system, one important information is the relative error, meaning the difference between the value of the load assessed by the WIM station and the real load relatively to the real load.

Currently, several accuracy assessment methods exist for WIM systems: in legal metrology, OIML [17] makes it possible to characterize a weighing systems by considering the maximum absolute relative error (see also BIPM [3]). In other words, all measured absolute relative errors must be lower than a fixed accuracy in order to obtain the certification.

For non-legal applications, other statistical methodologies were proposed by COST323 [9] (see also [10, 11] for the statistical framework) and ASTM1318 [1]. They mainly specify a minimum proportion of measurements within a given tolerance. This tolerance characterizes the accuracy class. Both standards have been compared in a recent study [16]. From the COST 323 action [9, 12] and the WAVE project [18] to more recent analysis (see [5] and the reference therein), several WIM technologies have been investigated and methodologies for assessing the accuracy and performances of WIM systems have been proposed in order to provide extensive results of large-scale tests.

In this paper, we propose to define the supplier risk for the assessment of the accuracy of WIM systems as a Type I error of a test of hypothesis. The model of observation sample composed of independent and identically distributed relative errors and the statistical composite hypothesis test of the tolerance of the system are described in Section 2. It is built in the asymptotical setting,

namely for large samples of measurements, in Section 3. In the assessment of the accuracy described in the COST 323 action, our procedure is compared numerically to the classical estimator of the tolerance. The procedure can be applied to all WIM systems and we finally illustrate it to assess the accuracy of a real bridge-WIM (B-WIM) in Section 4. A conclusion section ends the paper.

2 Assessment of the accuracy and classification of the WIM systems

2.1 Relation between the tolerance and the confidence level

Let X_i be the relative error (with respect to the reference value) of the i -th predicted (computed) weight (for instance gross weight, single axle weight or group axle weight) in the load test sample defined by

$$X_i = \frac{W_i^{pred} - W_i^{ref}}{W_i^{ref}}, \quad i = 1, \dots, n,$$

where W_i^{ref} is the reference weight of the i -th truck in the load test and W_i^{pred} is the estimated weight by the WIM system. Speed, inter-axle distance and number of axles can also be tested.

In this setting, the observation sample (X_1, X_2, \dots, X_n) is supposed to be composed of independent and identically distributed (i.i.d.) random variables. Then, the notation of the WIM system usually reduces to the estimation of the quantiles q_1 and q_2 such that

$$P(q_1 \leq X_1 \leq q_2) = \pi_0 \tag{1}$$

or the tolerance $\delta > 0$ defined by

$$P(|X_1| < \delta) = \pi_0 \tag{2}$$

for a confidence level π_0 , given the observation sample (X_1, X_2, \dots, X_n) . In particular, for the COST 323, the sample is supposed to be composed of i.i.d. Gaussian random variables of unknown mean μ and unknown variance σ^2 . The confidence level π_0 corresponds to the required confidence

level π_0 for a sample of size $n = \infty$ in the COST 323 [10].

It is worth mentioning that the i.i.d. setting considered here does not take into account the possible change in the distribution of the relative errors for several WIM systems, due to the temperature for instance (see [4] and the conclusion section 5).

2.2 Assessment of the supplier risk by a test of hypothesis

In the COST 323 procedure, the accuracy of the WIM system is characterized by the tolerance $\delta > 0$ defined by (2) for a fixed confidence level π_0 . The tolerance is to be estimated given the observation sample (X_1, X_2, \dots, X_n) composed of i.i.d. Gaussian random variables of unknown mean μ and unknown variance σ^2 . This definition is used for gross weight, single axle weight or group axle weight with no distinction on the statistical experiment. The values of π_0 are provided according to the environmental and test conditions (repeatability and reproducibility) based on experimental experiments [7]. The tolerance δ characterizes the accuracy class to be assigned. For instance, the accuracy classes defined in the COST 323 [9] are tabulated (see Table 1). When the COST was created, these classes cover all the WIM systems for non-legal applications: heavy traffic monitoring (from A to D), bridge damage assessment (from A to C) and preselection of overloaded vehicles (A or B).

Type of measurement	δ (in %) for different accuracy classes					
	A (5)	B+ (7)	B (10)	C (15)	D (25)	E
1. Gross weight	5	7	10	15	25	> 25
2. Group of axles	7	10	13	18	28	> 28
3. Single axle	8	11	15	20	30	> 30
4. Axle of a group	10	15	20	25	35	> 35
Speed	2	3	4	6	10	> 10
Inter-axle distance	2	3	4	6	10	> 10
Axle/vehicle count	1	1	1	3	5	> 5

Table 1: The COST 323 classes of accuracy with respect to the tolerance δ

The accuracy parameter δ is unknown and has to be estimated based on an observation sample of relative errors (X_1, X_2, \dots, X_n) .

We propose a test of hypothesis to assess the class of accuracy of the WIM system by controlling

the supplier risk (see also [13] for general consideration on hypothesis testing). Indeed, the supplier risk α is a Type I error of a composite hypothesis test in this paper (see Equation (4)) whereas it is a confidence interval error in the classical COST 323 procedure. We also propose in the Section 3 an (implicit) estimator of the tolerance δ which will take into account the statistical errors of the bias and the standard deviation (which is simplified to the bias only in the classical COST 323 estimation procedure).

The customer can control the risk by selecting an appropriate value for π_0 in Equation (2). The supplier risk is assessing the WIM system to be in a class of accuracy (from A to E in Table 1) lower than appropriate. This supplier risk, denoted α , can therefore be defined as the error of the first kind in a statistical composite hypothesis test

$$H_0 : \delta > \delta_0 \quad \text{against} \quad H_1 : \delta \leq \delta_0. \quad (3)$$

$$P_{H_0}(\text{ assess } H_1) \leq \alpha. \quad (4)$$

Here δ_0 is the tolerance of the proposed class of accuracy. In assessment of WIM systems, the supplier risk is generally $\alpha = 0.05$.

Finally, let us remark that Equation (2) can be formulated as

$$\pi_0 = \Phi\left(\frac{\delta - \mu}{\sigma}\right) - \Phi\left(\frac{-\delta - \mu}{\sigma}\right) \quad (5)$$

where Φ is the cumulative distribution function of the standard Gaussian random variable.

The tolerance δ of the measurement system is consequently defined as the unique implicit root of the function

$$f(d, \mu, \sigma^2) = \pi_0 - \left(\Phi\left(\frac{d - \mu}{\sqrt{\sigma^2}}\right) - \Phi\left(\frac{-d - \mu}{\sqrt{\sigma^2}}\right) \right) \quad (6)$$

for a fixed level of confidence π_0 . It is also denoted $\delta(\mu, \sigma^2)$ and

$$f(\delta(\mu, \sigma^2), \mu, \sigma^2) = 0. \quad (7)$$

In the following, we detail the corresponding statistical hypothesis test in the asymptotic setting as the sample size n tends to infinity.

3 Construction of the test in the asymptotical setting

3.1 Construction of the test

Let us denote the unknown parameter $\vartheta = (\mu, \sigma^2)$ of the Gaussian random variable X_1 . We consider the estimator $\widehat{\vartheta}_n = (\overline{X}_n, S_n^2)$ composed of the empirical mean and the estimated variance defined by

$$\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \quad \text{and} \quad S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X}_n)^2. \quad (8)$$

As n tends to infinity, the central limit theorem holds and

$$\mathcal{L} \left(\sqrt{n} \left(\widehat{\vartheta}_n - \vartheta \right) \right) \longrightarrow \mathcal{N}(0, \mathcal{I}(\vartheta)^{-1}) \quad (9)$$

where

$$\mathcal{I}(\vartheta) = -\mathbf{E}_{\vartheta} \left(\nabla^2 \log \left(\frac{1}{\sigma} \varphi \left(\frac{X_1 - \mu}{\sigma} \right) \right) \right) = \begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{pmatrix}.$$

Here φ is the probability distribution function of the standard Gaussian distribution.

Let us denote the implicitly defined random variable $\widehat{\delta}_n(\overline{X}_n, S_n^2)$ defined as the unique root of the function

$$f(d, \overline{X}_n, S_n^2) = \pi_0 - \left(\Phi \left(\frac{d - \overline{X}_n}{\sqrt{S_n^2}} \right) - \Phi \left(\frac{-d - \overline{X}_n}{\sqrt{S_n^2}} \right) \right), \quad (10)$$

namely

$$f(\widehat{\delta}_n(\overline{X}_n, S_n^2), \overline{X}_n, S_n^2) = 0. \quad (11)$$

The computation of $\widehat{\delta}_n$ generally needs the use of a root-finding algorithm.

The asymptotic properties of the estimator $\widehat{\delta}_n$ of the tolerance δ is considered in the following.

Asymptotically, it is possible to use the delta method for implicitly defined random variables as in [2]. Direct computations lead to

$$\sqrt{n}(\widehat{\delta}_n - \delta) \sim \mathcal{N}(0, \Sigma^2(\mu, \sigma^2)) \quad (12)$$

where

$$\Sigma^2(\mu, \sigma^2) = \frac{\sigma^2 f_\mu^2(\delta(\mu, \sigma^2), \mu, \sigma^2) + 2\sigma^4 f_{\sigma^2}^2(\delta(\mu, \sigma^2), \mu, \sigma^2)}{f_d^2(\delta(\mu, \sigma^2), \mu, \sigma^2)} \quad (13)$$

where f_x is the derivative with respect to the variable x of the function f defined in Equation (6). The explicit expressions are given in Appendix A. It is worth mentioning that this result encompasses the statistical error for the bias and the standard deviation simultaneously.

The composite hypothesis test is built with the statistical decision

$$\psi((X_1, \dots, X_n)) = \mathbb{1}_{\{\widehat{\delta}_n - \delta_0 \leq \epsilon\}}.$$

The critical region $[0, \epsilon]$ is defined by

$$\begin{aligned} \sup_{(\mu, \sigma^2), \delta(\mu, \sigma^2) > \delta_0} P_\delta \left(\widehat{\delta}_n - \delta_0 \leq \epsilon \right) &= \sup_{\delta > \delta_0} P_\delta \left(\frac{\sqrt{n}(\widehat{\delta}_n - \delta_0)}{\Sigma(\overline{X}_n, S_n^2)} \leq \frac{\sqrt{n}\epsilon}{\Sigma(\overline{X}_n, S_n^2)} \right) \\ &= P_{\delta_0} \left(\frac{\sqrt{n}(\widehat{\delta}_n - \delta_0)}{\Sigma(\overline{X}_n, S_n^2)} \leq \frac{\sqrt{n}\epsilon}{\Sigma(\overline{X}_n, S_n^2)} \right) \\ &= \alpha \end{aligned}$$

and classical computations lead to

$$P_{\delta_0} \left(\sqrt{n} \frac{\widehat{\delta}_n - \delta_0}{\Sigma(\overline{X}_n, S_n^2)} \leq \sqrt{n} \frac{\epsilon - \delta_0}{\Sigma(\overline{X}_n, S_n^2)} \right) = \alpha$$

and

$$\epsilon = \frac{u_\alpha}{\sqrt{n}} \Sigma(\overline{X}_n, S_n^2) \quad (14)$$

where u_α is the α -quantile of a standard Gaussian random variable defined by $P(Z \leq u_\alpha) = \alpha$,

$Z \sim \mathcal{N}(0, 1)$.

In order to determine the class of accuracy of the measurement system (WIM) in the asymptotical setting, we compare the estimated tolerance $\widehat{\delta}_n$ (numerically computed with Equation (11)) with the border of the critical region of the test $\delta_0 + \epsilon < \delta_0$ (here $\epsilon < 0$) where ϵ is defined in Equation (14) with Σ defined in Equation (13). Namely, if $\widehat{\delta}_n < \delta_0 + \epsilon$ we assess H_1 and accuracy δ_0 for the WIM system.

It is worth mentioning that the Type II error, namely

$$P_{H_1}(\text{assess } H_0) = \sup_{(\mu, \sigma^2), \delta(\mu, \sigma^2) \leq \delta_0} P_\delta \left(\widehat{\delta}_n > \delta_0 + \epsilon \right),$$

reduces to zero as the sample size n tends to infinity (the test is consistent) showing the importance of considering the largest possible dataset of relative errors.

In summary, considering the observation sample (X_1, X_2, \dots, X_n) and a reference tolerance δ_0 , the statistical procedure is:

1. Compute the empirical mean and the estimated variance given by Equation (8);
2. Compute the implicit tolerance $\widehat{\delta}_n$ defined by Equation (11) with a root-finding algorithm;
3. Compute the border of the critical region of the test $\delta_0 + \epsilon$ where ϵ is defined in Equation (14) with Σ defined in Equation (13);
4. Compare $\widehat{\delta}_n$ with $\delta_0 + \epsilon$ and decide accordingly.

This method is now compared to the usual COST 323 estimators for large sample $n \geq 30$.

3.2 Simulations

In order to compare the proposed methodology with the classical estimator in the COST 323 procedure, we consider fixed estimation of the bias μ , with three distinct values $\overline{X}_n = 0$, $\overline{X}_n = 0.02$ and $\overline{X}_n = 0.04$. Then we compare the maximal admissible empirical standard deviation $\widehat{\sigma}_n^{max}$ to be in an accuracy class (defined by fixing the tolerance of the proposed accuracy class δ_0). In our

context, it corresponds to the solution of the equation

$$\delta(\bar{X}_n, (\hat{\sigma}_n^{max})^2) = \delta_0 + \frac{u_\alpha}{\sqrt{n}} \Sigma(\bar{X}_n, (\hat{\sigma}_n^{max})^2). \quad (15)$$

With the usual COST 323 estimators, it corresponds to the root δ^* of

$$s \mapsto \pi_n - \left(\Psi_{n-1} \left(\frac{\delta_0 - \bar{X}_n}{s} - \frac{t_{1-\frac{\alpha}{2}}}{\sqrt{n}} \right) - \Psi_{n-1} \left(\frac{-\delta_0 - \bar{X}_n}{s} + \frac{t_{1-\frac{\alpha}{2}}}{\sqrt{n}} \right) \right) \quad (16)$$

where $t_{1-\frac{\alpha}{2}} = \Psi_{n-1}^{-1} \left(1 - \frac{\alpha}{2} \right)$ and π_n is tabulated (see Table 5 in [10] where full repeatability condition (r1) and environmental repeatability condition (I) are selected in our numerical experiments).

$n / \delta_0(\%)$	A (5)	B+ (7)	B (10)	C (15)	D (25)
30	1.56 - 1.78	2.18 - 2.50	3.11 - 3.56	4.67 - 5.35	7.78 - 8.91
60	1.64 - 1.83	2.29 - 2.56	3.28 - 3.65	4.92 - 5.48	8.20 - 9.13

$n / \delta_0(\%)$	A (5)	B+ (7)	B (10)	C (15)	D (25)
30	1.00 - 1.21	1.67 - 2.00	2.66 - 3.18	4.31 - 5.08	7.54 - 8.74
60	1.06 - 1.22	1.77 - 2.04	2.83 - 3.24	4.57 - 5.18	7.96 - 8.95

$n / \delta_0(\%)$	A (5)	B+ (7)	B (10)	C (15)	D (25)
30	0.33 - 0.40	1.10 - 1.20	2.00 - 2.41	3.66 - 4.40	6.98 - 8.28
60	0.33 - 0.40	1.11 - 1.22	2.12 - 2.44	3.89 - 4.48	7.40 - 9.44

Table 2: Comparison of $\hat{\sigma}_n^{max}$ (in %) between the described methodology in the asymptotic setting (on the left with $\pi_0 = 0.992$) and the COST 323 procedure (on the right for I-r1 conditons) in the unbiased case for gross weight and $\alpha = 0.05$ for $\bar{X}_n = 0$ (on the top), $\bar{X}_n = 0.02$ (in the middle) and $\bar{X}_n = 0.04$ (on the bottom).

For a confidence level $\pi_0 = 0.992$ and a classical supplier risk $\alpha = 0.05$, the simulations in the asymptotical setting are summarized in the Table 2. Our estimations are more stringent than the usual COST 323 estimators for all samples, this for all classes of accuracy. It shows that the supplier risk α is slightly undervaluated by the usual COST 323 estimators. Naturally, all values of the maximal admissible empirical standard deviation $\hat{\sigma}_n^{max}$ to be in a fixed accuracy class (defined by δ_0) are smaller for the biased setting than the unbiased setting.

It is worth mentioning that no testing solution has yet be found in the general case when the observation sample is small. But if the bias where reduced to zero, the same methodology

(composite hypothesis test) could be executed on the variance for any sample size.

4 Application to a real Bridge Weigh-in-Motion system

4.1 Instrumentation

The methodology has been applied to assess the accuracy of a real bridge-WIM (BWIM) system currently operational on a road bridge over a highway in western France. The B-WIM system uses the bridge as the weighing system. It considers the measured deformation of the bridge due to the passage of the trucks. In our application, the bridge has four spans and the B-WIM strain sensors are installed on the two main spans, the length of which is 14.5 m. The bridge deck is of a type which is uncommon for B-WIM applications: ten parallel precast post-tensioned concrete beams support a concrete slab (see Figure (1)). Usually B-WIM solutions are used on short spanned concrete frames or slabs, or on orthotropic steel decks. Thus it was necessary to quantify the accuracy of the system in this unusual configuration. The B-WIM system tested is the “WIM+D” solution by OSMOS using optical strands (see Figure (1)), simultaneously considering WIM application and structural health monitoring [6].



Figure 1: One main span of the bridge instrumented by optical strands

The result for which the accuracy was quantified is the gross weight of the vehicles. To perform the analysis, a load test was carried out on the bridge with three different trucks (19.2, 31.8, 43.9 tons), the static weight of which had been measured on a classical scale. Each one of the trucks performed many runs on the bridge, in both directions, in various transverse locations (in or out of

lanes) and at different speeds.

The total number of recorded runs with B-WIM results was 55. Out of them, 22 were not taken into account as they correspond to extreme configurations for the B-WIM, such as very low speed, and they are not representative for the usual conditions of operation. The 33 remaining results were divided in two sets. One set of 6 runs was used to calibrate the B-WIM through an automatic learning of the actual measured effects of the trucks on the bridge. The remaining 27 runs were used to apply the methodology and to quantify the accuracy of the B-WIM.

4.2 Assessment of the accuracy class

We consider these 27 runs of trucks, their real gross weights, and their measured gross weights (see Figure 2) and compute the relative errors sample (X_1, \dots, X_{27}) . Relative errors range is -0.116 to 0.048. The empirical bias is $\bar{X}_n = 0.002$ and the estimated standard deviation is $S_n = 0.042$.

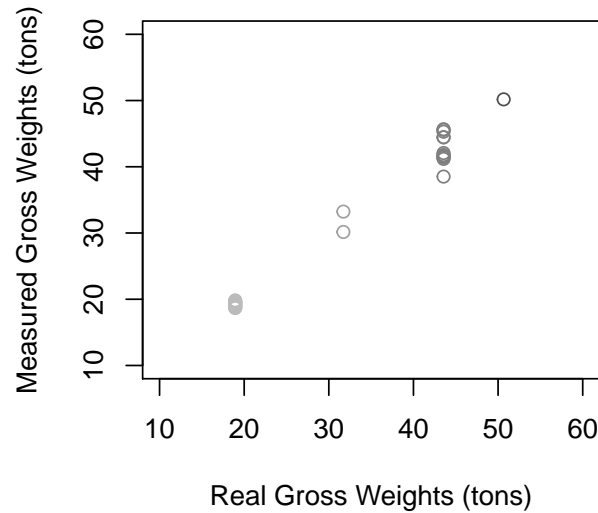


Figure 2: Real gross weights and gross weights measurements (in tons) for 27 runs of trucks.

Under COST 323 I-R1 conditions (environmental repeatability - limited reproducibility) the

usual COST 323 estimators gives $\pi_n = 0.925$. Consecutively, the root in Equation (16) is $\delta_* = 0.0935 < 0.1$ and the system is B(10).

With the alternative estimator and test procedure, considering we are in the asymptotical setting for $n = 27$, we get for $\pi_0 = 0.97$ and $\alpha = 0.05$ that, for $\delta_0 = 0.1$, $\hat{\delta}_n = 0.090 > 0.080 = \delta_0 + \epsilon$ (and that the system is not B(10)) but that for $\delta_0 = 0.115$, that $\hat{\delta}_n = 0.090 < 0.094$ which is close.

The proposed measurement analysis in the COST 323 procedure make it possible to assess the risk for the seller and the risk for the customer. The customer may choose any other level of confidence (the level of confidence is given in the COST 323 procedure based on real experiments) which will lead to another tolerance and accuracy class. The corresponding values may be fixed according to the strategy of the company.

For instance, with $\pi_0 = 0.85$, direct computation of the tolerance is $\hat{\delta}_n = 0.060$. The test of the B-WIM system to be in B(7) is positive for a error of type I fixed to $\alpha = 0.15$. The test becomes negative for $\pi_0 \geq 0.90$ (and a similar error $\alpha = 0.15$). The test just becomes negative for $\alpha \leq 0.01$ (and a similar level of confidence $\pi_0 = 0.85$).

5 Conclusion

We propose in this paper a formulation of the supplier risk of a WIM system in the COST 323 procedure by means of a Type I error of a test of composite hypothesis. This test is built with an alternative (implicit) estimator of the tolerance parameter. It allows to assess the class of accuracy of a WIM system considering the level of confidence, the supplier risk and the size of the observation sample of independent and identically distributed relative errors. In this test, both statistical errors on the bias and the standard deviation are taken into account.

Composite hypothesis tests are built for the asymptotic case using the delta method for implicitly defined random variables.

No explicit solution has been found in the general case the observation sample is small (when the bias is not reduced to zero). This setting could be considered in a further dedicated work. This work could also be extended to the Gaussian regression case (independent but not identically

distributed) in order to consider the effect of an exogenous variable on the sensors of the WIM system (for instance the temperature) and, consecutively, on the distribution of the relative errors.

A Derivatives of f

The closed-form derivatives of the function $f(d, \mu, \sigma^2)$ given in (6) are

$$f_\mu(d, \mu, \sigma^2) = \frac{1}{\sqrt{\sigma^2}} \left(\varphi \left(\frac{d - \mu}{\sqrt{\sigma^2}} \right) - \varphi \left(\frac{-d - \mu}{\sqrt{\sigma^2}} \right) \right),$$

$$f_{\sigma^2}(d, \mu, \sigma^2) = \frac{1}{2(\sigma^2)^{3/2}} \left((d - \mu) \varphi \left(\frac{d - \mu}{\sqrt{\sigma^2}} \right) - (-d - \mu) \varphi \left(\frac{-d - \mu}{\sqrt{\sigma^2}} \right) \right)$$

and

$$f_d(d, \mu, \sigma^2) = -\frac{1}{\sqrt{\sigma^2}} \left(\varphi \left(\frac{d - \mu}{\sqrt{\sigma^2}} \right) + \varphi \left(\frac{-d - \mu}{\sqrt{\sigma^2}} \right) \right)$$

where φ is the probability distribution function of the standard Gaussian distribution.

Heuristically, in order to obtain the Equation (12), we can formally write the following Taylor expansion

$$\begin{aligned} 0 &= f(\widehat{\delta}_n, \overline{X}_n, S_n^2) - f(\delta, \mu, \sigma^2) \\ &= (\widehat{\delta}_n - \delta) f_d(d, \mu, \sigma^2) + (\overline{X}_n - \mu) f_\mu(d, \mu, \sigma^2) + (S_n^2 - \sigma^2) f_{\sigma^2}(d, \mu, \sigma^2) + \text{remainder.} \end{aligned}$$

It can be rewritten as

$$\sqrt{n}(\widehat{\delta}_n - \delta) = -\sqrt{n}(\overline{X}_n - \mu) \cdot \frac{f_\mu(d, \mu, \sigma^2)}{f_d(d, \mu, \sigma^2)} - \sqrt{n}(S_n^2 - \sigma^2) \cdot \frac{f_{\sigma^2}(d, \mu, \sigma^2)}{f_d(d, \mu, \sigma^2)} + \text{remainder.}$$

Due to Equation (9), the right hand term is a sum of asymptotically independent random variables that gives the result.

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