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Fault detection using Upper Bound Interval Kalman Filter for unmaned aerial vehicle

Carine Jaubert¹ and Soheib Fergani¹ and Emelyne Saulnier¹

Abstract—This paper deals with dynamics estimation and fault detection of unmaned aerial vehicles (uav) using the Upper Bound Interval Kalman Filter (UBIKF). An upper bound for all positive semi-definite matrices included in an interval matrix is calculated. This upper bound is used to generate envelopes for the variables to be estimated which are the dynamics of the UAV. It allows to provide a guaranteed estimation envelope for the considered dynamics.

Then, the fault detection scheme is used based on a χ^2 test. The faults concern sensors and actuators. Simulations on a discrete uncertain UAV model highlight the efficiency of the proposed filter for both UAV dynamics estimation and fault detection.

I. INTRODUCTION

Using Upper Bound Interval Kalman Filter instead of classical Kalman Filter is mainly motivated by the idea that stochastic and set-membership estimation approaches have specific advantages: they complement each other more than they compete. In fact, the experimental conditions about noise and disturbances, in a stochastic estimation framework, are usually properly modeled through appropriate probability distributions. Yet, other sources of uncertainty are not well-suited to stochastic modeling, as for example the parameter uncertainties that generally arise from system design tolerances and from process ageing. These uncertainties are often better represented with bounded uncertainties, as intervals. Thus this modeling is described through the set-membership framework.

Combining bounded and stochastic uncertainties opens hence new perspectives for modeling complex systems more accurately.

Thus, motivated by above-mentioned reasons, the filtering issue for discrete time linear models considering bounded uncertainties on parameters and gaussian noise on measurements is studied in this paper. In [1], the classical Kalman filter [2] has been extended to this type of uncertain systems. This proposed solution is not guaranteed, i.e. the solution set may not include all the classical Kalman filter solutions consistent with the bounded uncertainties represented in the system. In [3], an improved interval Kalman filter has been proposed solving the interval matrix inversion problem with the set inversion algorithm SIVIA (Set Inversion Via Interval Analysis) and constraint satisfaction problems [4]. However, an high computational time is required by the proposed algorithm if the considered system is affected by large uncertainties [5].

Thus the Minimum Upper Bound of Variance Interval Kalman Filter (UBIKF) has been presented in [6] with two main goals: minimizing an upper bound for the estimation error covariance and enclosing the set of possible solutions of the filtering problem for interval linear systems. Since the gain matrix handled by UBIKF is punctual, this approach encloses all the estimates consistent with the parameter uncertainties in a much less conservative manner than the iIKF. This filter is used in the following work.

In the proposed work, the fault detection strategy consists in combining the UBIKF filter with the well-known χ^2 test. The χ^2 test is a very useful statistical test providing information about on the significance of any observed differences (with the predicted behaviour) and also detailed information on which categories account for any differences found. Since the χ^2 square is a significance test, coupling it with an accurate estimation strategy can provide satisfying results. Here, the study case deals with data that are estimated as intervals with no predefined distribution. This choice was also motivated by two main factors: the first is the computation time and the easiness of implementation for both UBIKF filter and the χ^2 test. The second is the compliance of this test with the mixed uncertainties handled by the proposed algorithm. Here, we focus on the efficiency of the filter estimation performances and though the robustness against disturbances is to be developed in on going works.

Interesting cases of study for the proposed strategy are Unmanned Aerial Vehicles (UAVs). Indeed, in the last decade, both industrial and academic communities have been interested by the UAV systems, for civilian and military applications. Due to their high manoeuvrability and flexibility in indoor and outdoor application, a special focus has been given to multirotor UAVs. Therefore, one of the most challenging topics for their autonomous navigation is the vehicle attitude estimation. Also, since most of the localization and control strategies are based on these dynamics estimation, fault detection and isolation on these information is crucial to anticipate system dysfunctions and prevent missions failure. In this work, authors propose an efficient fault detection and estimation strategy based on the Minimum Upper Bound of Variance Interval Kalman Filter. The main objective is estimate UAVs dynamics while supervising its behaviour to detect the eventual actuators and sensors faults.

This paper is organized as follows. The problem formulation is described in Section II. In Section III, the fault detection scheme, based on UBIKF, is described. Then a numerical example has been used to highlight the potential of the proposed approach in Section IV. Some conclusions

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are presented in Section V.

II. PROBLEM FORMULATION

Interval analysis was developed by [7] and is useful to deal with bounded uncertainties. Most of the notions of interval analysis can be found in [4]. In this framework, an interval $[x]$ is a closed and connected subset of \mathbb{R} :

$$[x] = \{x \in \mathbb{R} \mid \underline{x} \leq x \leq \bar{x}\}, \quad (1)$$

where \underline{x} and \bar{x} are respectively the lower and upper bound. The center of $[x]$ is defined by $\text{mid}([x]) = (\bar{x} + \underline{x})/2$ and its radius is $\text{rad}([x]) = (\bar{x} - \underline{x})/2$. The set of all intervals in \mathbb{R} is noted as \mathbb{IR} .

An interval vector (or matrix) is a vector (or matrix) whose elements are considered as intervals. In this paper, an interval vector and an interval matrix are denoted as $[\mathbf{x}]$ and $[M]$. The set of n -dimensional interval vectors (or $m \times n$ interval matrices) is denoted as \mathbb{IR}^n (or $\mathbb{IR}^{m \times n}$). Given $[M] \in \mathbb{IR}^{m \times n}$, the two functions $\text{mid}([M])$ and $\text{rad}([M])$ provide two $m \times n$ real matrices containing the centers and radius of elements of $[M]$.

In the proposed work, the following class of uncertain linear discrete-time stochastic systems is considered:

$$\begin{cases} \mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k, \\ \mathbf{y}_k = C_k \mathbf{x}_k + \mathbf{v}_k, \end{cases} \quad (2)$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u}_k \in \mathbb{R}^{n_u}$ is the input vector, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is the measurement vector and $\mathbf{w}_k, \mathbf{v}_k$ are white Gaussian noise sequences with zero mean and covariance matrices Q and R . The initial state \mathbf{x}_0 is also Gaussian with mean μ_0 and covariance matrix P_0 . In addition, $\mathbf{x}_0, \{\mathbf{w}_1, \dots, \mathbf{w}_k\}$ and $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ are assumed to be mutually independent.

The structure of the classical Kalman filter for system (2) is represented as follows ([2]):

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k (\mathbf{y}_k - C_k \hat{\mathbf{x}}_{k|k-1}), \quad (3)$$

where $\hat{\mathbf{x}}_{k|k}$ is the posterior state estimate, $\hat{\mathbf{x}}_{k|k-1} = A_k \hat{\mathbf{x}}_{k-1|k-1}$ is the prediction of state a priori and K_k is the gain matrix that minimizes the mean square error $E[(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)^T]$. We consider the case where the matrices A_k, B_k, C_k and the covariance matrices Q, R are assumed bounded and are represented by interval matrices, denoted $[A_k], [B_k], [C_k], [Q]$ and $[R]$, containing all possible values of each parameter. Since it is impossible to solve directly the Kalman filtering problem due to parameter uncertainties, our goal is to obtain an upper bound $P_{k|k}^+$ such that:

$$E[(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)(\hat{\mathbf{x}}_{k|k} - \mathbf{x}_k)^T] \preceq P_{k|k}^+, \quad (4)$$

for the set of all models with parameters bounded by the above interval matrices. The envelope enclosing the set of state estimates corresponding to the gain K is then computed. This idea is similar to [3] in which the envelopes of the optimal gains and the state estimates given by the Kalman filtering procedure are determined by using interval analysis. In contrast, the proposed algorithm determines a gain matrix

K which minimizes the trace of the upper bound on the error covariance instead of finding a set of gain in [3]. Since the gain matrix in the UBIKF is punctual, it allows to reduce the conservatism and the computational time of interval operations.

In the following, the UBIKF is combined with a χ^2 -statistics test for fault detection of an unmanned aerial vehicle.

III. FAULT DETECTION SCHEME

As the conventional Kalman filter, the UBIKF can be designed in two steps: prediction and correction. A description of this filter is given in [6] and the main algorithm is recalled below:

A. UBIKF algorithm

The algorithm steps are summarized below:

Algorithm 1 UBIKF Algorithm

Input: $[\hat{\mathbf{x}}_{0|0}], P_{0|0}^+, [A], [B], [C], [Q], [R], \mathbf{y}_k, \mathbf{u}_k, A_k, Q_k, R_k, k = 1, 2, \dots$

Output: $[\hat{\mathbf{x}}_{k|k}], P_{k|k}^+$
 1: **for** $k = 1, 2, \dots$ **do**
 2: **Prediction step:**

$$\begin{aligned} [\hat{\mathbf{x}}_{k|k-1}] &= [A] [\hat{\mathbf{x}}_{k-1|k-1}] + [B] \mathbf{u}_k, \\ P_{k|k-1} &= A_k P_{k-1|k-1}^+ A_k^T + Q_k, \\ P_{k|k-1}^+ &\succeq P_{k|k-1} \end{aligned}$$

3: **Correction step**

$$\begin{aligned} R_k^+ &\succeq R_k \\ K_k &= (n_0 + 1) P_{k|k-1}^+ C_m^T S_k^{-1}, \\ S_k &= (n_0 + 1) \text{mid}([C]) P_{k|k-1}^+ \text{mid}([C])^T \\ &\quad + 2 \sum_{i=1}^{n_y} \sum_{j=1}^{n_x} C_r^{(i,j)} P_{k|k-1}^+ (C_r^{(i,j)})^T \\ &\quad + \frac{1}{2} \sum_{i=1}^{n_y} \sum_{j=1}^{n_x} \sum_{m=1}^{n_y} \sum_{l=1}^{n_x} S^{(m,l)} \\ &\quad + R_k^+. \end{aligned}$$

$$P_{k|k}^+ = (n_0 + 1) (I - K_k \text{mid}([C])) P_{k|k-1}^+.$$

$$[\hat{\mathbf{x}}_{k|k}] = (I - K_k [C_k]) [\hat{\mathbf{x}}_{k|k-1}] + K_k \mathbf{y}_k.$$

4: **end for**

B. Fault detection scheme

In this section, consider the following system:

$$\begin{cases} \mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k + \mathbf{f}_k^a, \\ \mathbf{y}_k = C_k \mathbf{x}_k + \mathbf{v}_k + \mathbf{f}_k^c, \end{cases} \quad (5)$$

where $\mathbf{f}_k^a \in \mathbb{R}^{n_{fa}}$ and $\mathbf{f}_k^c \in \mathbb{R}^{n_{fc}}$ are actuators and sensors faults vectors.

The difference between measurement and model output is defined by:

$$[\mathbf{r}_k] = \mathbf{y}_k - [C] [\hat{\mathbf{x}}_{k|k-1}], \quad (6)$$

and it contains all values of innovation \mathbf{r}_k defined by $\mathbf{r}_k = \mathbf{y}_k - C_k \hat{\mathbf{x}}_{k|k-1}$ whose the covariance matrix is given by:

$$[S] = [C] P_{k|k-1} [C]^T + [R], \quad (7)$$

with $[\hat{\mathbf{x}}_{k|k-1}]$ and $P_{k|k-1}$, the predicted state and the covariance matrix of the prediction error respectively.

The interval $[\beta_k]$ defined by $[\beta_k] = [\mathbf{r}_k]^T S_k^{-1} [\mathbf{r}_k]$

contains all possible values of $\beta_k, \forall \mathbf{r}_k \in [\mathbf{r}_k]$ where β_k is a statistical function detecting a change in the mean of the innovation sequence:

$$\beta_k = \sum_{i=k-W+1}^k \mathbf{r}_i^T S_i^{-1} \mathbf{r}_i, \quad (8)$$

In the literature, using the χ^2 -statistics test for fault detection is a kind of Innovation approach mentioned in [8]. In [9] or [10], this method is applied for fault detection problems in which the β_k statistic is used where W is a window size ($W \leq k$). The authors argue that this statistic is χ^2 -distributed with Wn_y degrees of freedom. A rule for the fault detection test was established: (H_0) $\beta_k \leq \delta$, no error occurred; (H_1) $\beta_k > \delta$, an error occurred, where δ is the threshold determined by $\mathbb{P}(\chi^2(Wn_y) > \delta) = \alpha$ with α a chosen significance level.

Algorithm 2 Fault detection algorithm

Initialization: $[\hat{x}_{0|0}], P_{0|0}, [A], [B], [C], [D], [Q], [R], W, u_k, y_k, \alpha, k = 1, 2, \dots, N.$

for $k = 1, 2, 3, \dots, N$ **do**

Implementation:

Use Algo.1 to get : $[\hat{x}_{k|k}], P_{k|k}^+, [\hat{x}_{k|k-1}], P_{k|k-1}.$

$[r_k] = y_k - [C][\hat{x}_{k|k-1}]$

$[S_k] = [C]P_{k|k-1}[C]^T + [R]$

$U_k = \sup\{\text{abs}([\mathbf{r}_k]^T [\mathbf{r}_k])\}$

Calculate β_k

Find δ s.t.: $\mathbb{P}(\chi^2(Wn_y) > \delta) = \alpha$

Detection signal : $\pi_k = \mathbb{I}(U_k > \delta)$

end for

where $\mathbb{I}(x)$ equal 1 if x holds true and null otherwise.

IV. APPLICATION TO UAV

The drone studied in this work is the Parrot ARDRONE 2 which attracted a lot of attention and many works (see for example [11] or [12]).

The ARDRONE 2, depicted in Fig. 1 is an underactuated system. Indeed, it has 6 degrees of freedom (see Fig. 1), 3 translations (w.r.t axes (x,y,z)) and 3 rotations (w.r.t (ϕ, θ, ψ)). The different movements are obtained by varying the speeds of each one of the 4 motors ([13] [14]).

A. Estimation of the UAV dynamics

In this work, the commonly used 12-DOF (degree of freedom) linearized model of the UAV is considered (some details can be found in [15]). The state vector X is defined as follows:

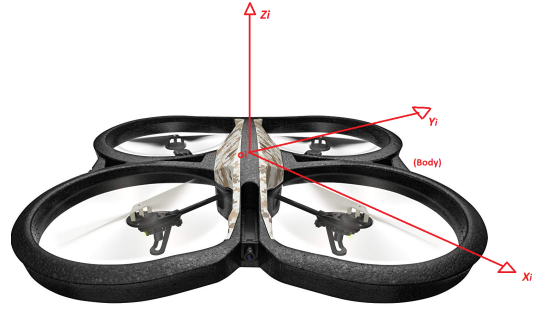


Fig. 1. UAV Parrot ARDRONE 2

$X = (x, y, z, v_x, v_y, v_z, \phi, \theta, \psi, p, q, r)$ where x, y, z represent the relative positions of the UAV center in the world frame; v_x, v_y, v_z correspond to the translation speed of the drone in the world frame; ϕ, θ, ψ correspond to rotation angles of the UAV in the world frame, and p, q, r are the angular velocities in the body frame.

Then, the inputs of the system are:

$u = (u_1, u_2, u_3, u_4)$ where :

$$u_1 = F_1 + F_2 + F_3 + F_4,$$

$$u_2 = (-F_1 + F_2 + F_3 - F_4)l,$$

$$u_3 = (-F_1 - F_2 + F_3 + F_4)l',$$

$$u_4 = F_1 - F_2 + F_3 - F_4$$

where l and l' are the distance between the center of gravity of the quadrotor and the rotors axis. F_i are the vertical forces generated by the rotors.

Thus the system can be described by Equation 2.

In the following, the attitude dynamics of the UAV are under interest. The drone is hovering at a considered altitude, and then its dynamics are excited by varying the input u_4 (the force differential at each time instant k to study, mainly, the roll and pitch behaviours).

Remark 4.1: It is worth noticing that in this work, the study has been focusing on the roll dynamics (considering that the pitch behaviour is the quit the same on the y-axis). Also, the yaw dynamic estimation is not considered, and usually measured using an OptiTrack capture system.

In Fig 2, the UBIKF filter is used to estimate the UAV states using only four output signals $(z, v_x, \dot{v}_y, \dot{v}_z)$. The objective is to efficiency bound the estimated states in the guaranteed intervals.

One can notice that the UBIK filter provides tight estimated intervals that includes the vehicle dynamics which can be very useful for its behaviour monitoring (especially for UAV pitch and roll dynamics) as in Fig.3:

Also, in Fig.4 the trace of the upper bound of the covariance matrix is represented.

Thus after few time steps, the trace of this upper bound is constant.

B. Fault detection

1) *Sensor fault detection:* To test the efficiency of the UBIK filter for estimating the faulty dynamics, the following

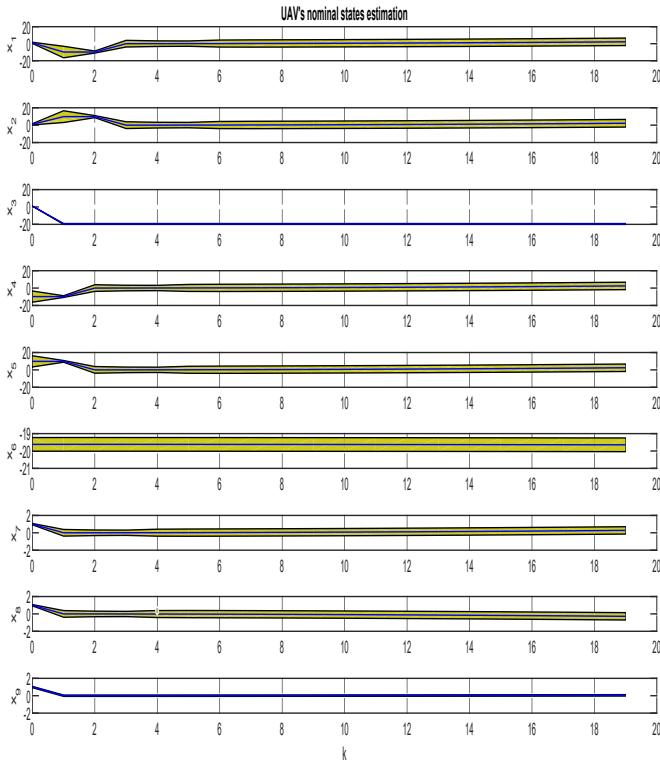


Fig. 2. State estimation

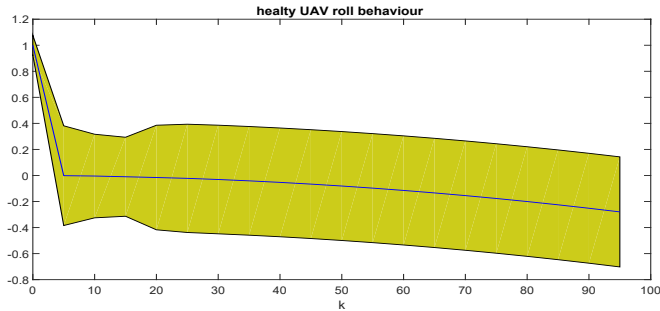


Fig. 3. Roll θ dynamics estimation

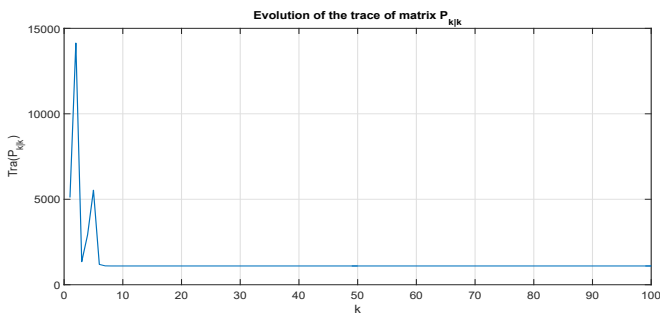


Fig. 4. Trace of the covariance matrix $P_{k|k}$.

scenario is proposed. First, a bias for $k \in [30, 70]$ has been added on the roll dynamics sensor to simulate a sensor fault. For this sake, we add for $k \in [30, 70]$ a constant on one of the components of the output vector of the system (namely for the roll dynamics output). Fig.5 shows the performance of the

proposed filter for the UAV faulty roll dynamics estimation.

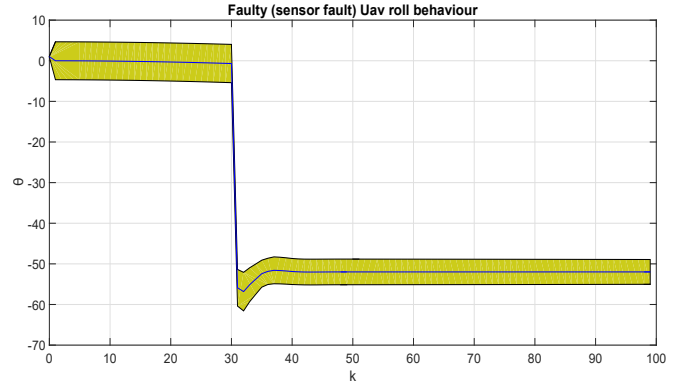


Fig. 5. Faulty roll dynamics estimation

Notice that the estimated intervals are larger (which normal in the faulty case) but the UBIKF filter still manages to provide satisfactory results.

For the fault detection, the χ^2 test is performed with a confidence index $1 - \alpha = 0.99$. The curves obtained are displayed using a sliding window $W = 4$ time steps, the detection threshold is represented by a red dashed line and the curve in blue corresponds to the values of $\inf([\beta_k])$.

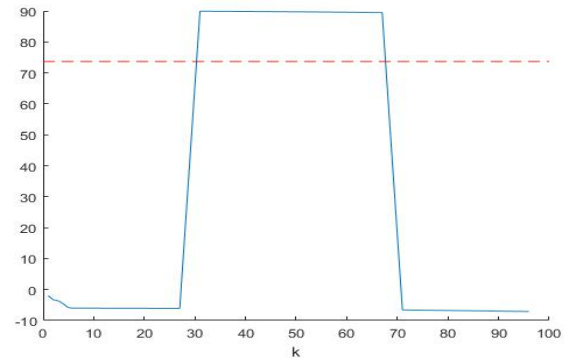


Fig. 6. Detection test of χ^2 law for a sensor fault

The detection threshold is crossed between 30 and 70, which corresponds to the added fault. The χ^2 test detects the bias introduced by the faulty sensor.

2) **Actuator fault detection:** Inspired by [16], the matrices that allow to provide a configuration where one of the 4 rotors of the UAV would fail are used (actuator fault scenario). The inputs of the UAV are scheduled using these matrices to simulate the actuator fault.

The considered scenario is the following: The drone is hovering at a considered altitude, and then the attitude dynamics are excited. Also, by simulating a fault on the first rotor (u_1) at $k = 31$. It can be easily noticed that the curves representing the states are affected by this input deterioration. Indeed, one can observe a jump to $k \approx 32$ on each of the states.

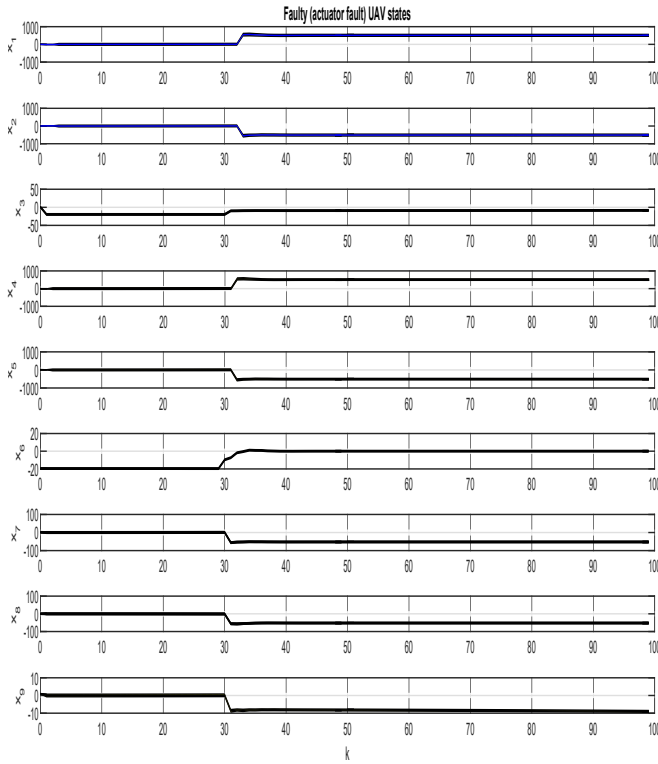


Fig. 7. State estimation in faulty situation

A comparison between Fig. 7 and Fig. 2 shows the degradation of the roll dynamics of the UAV (compared to the healthy state).

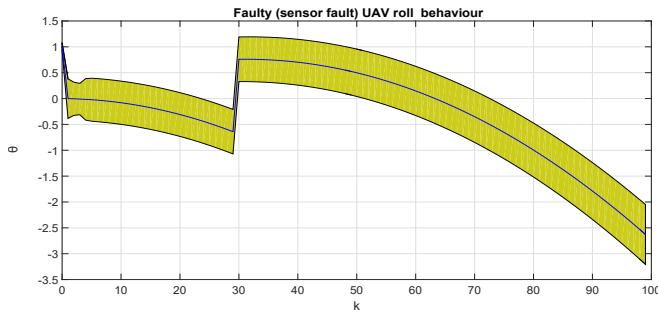


Fig. 8. UAV roll dynamics with an actuator fault

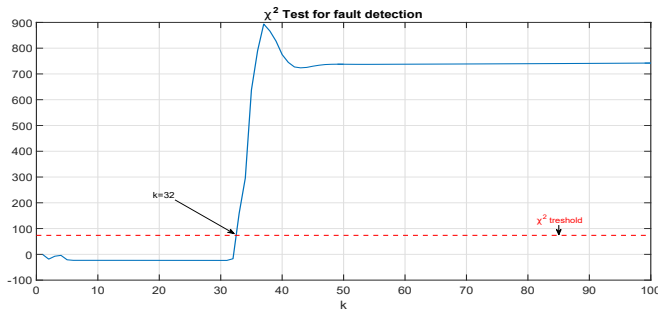


Fig. 9. Detection test of χ^2 law for an actuator fault

Using the χ^2 test, it can be seen in Fig. 9 that at $k \approx 32$ the curve crosses the detection threshold. Thus, the test

detects the fault correctly.

V. CONCLUSION

An approach to fault detection applied to a UAV model is proposed in this paper. It combines UBIKF and a classical χ^2 test. The proposed filter allows to obtain tight guaranteed estimation intervals of the considered dynamics while the χ^2 -test allows to detect some abnormal behaviours. Simulation results highlight the efficiency of the proposed fault detection scheme. Beside being efficient, it is easily implemented and can be very useful in real time implementation. Also, the robustness analysis of the filtering considering external disturbances and comparison with the well-known classical estimation approaches represent challenging issues that have been considered by authors in their ongoing works. Furthermore, this study can inspire performance fault tolerant control strategy by taking advantage of the provided guaranteed interval estimations to handle the systems disfunctions (see Fig.10).

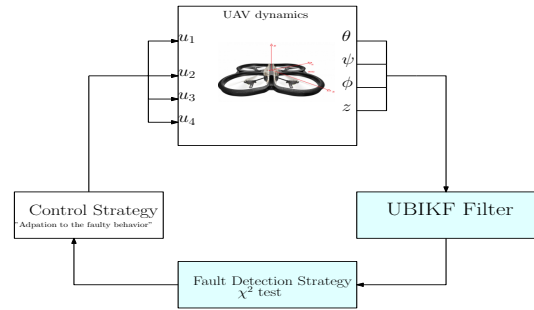


Fig. 10. Fault tolerant control strategy

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