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ARTICLE TEMPLATE

Global Dissipativity of Fuzzy Genetic Regulatory Networks with Mixed Delays

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ABSTRACT
The synthetic Genetic Regulatory Networks (GRNs) have proven to be a powerful tool in studying gene regulation processes in living organisms. In this article, the global dissipativity and corresponding attractive set for the Fuzzy Genetic Regulatory Networks (FGRNs) with mixed delays are investigated. By utilizing the Lyapunov functional method and the Linear Matrix Inequalities (LMIs) techniques, new sufficient conditions ensuring the global dissipativity and the global exponential dissipativity of the suggested system are given. Moreover, the global attractive set and global exponential attractive set are obtained. The derived criteria are of the form of LMI, and hence they can be verified easily by the numerical software. Lastly, two numerical examples with its simulations are given to illustrate the effectiveness of the obtained results.

KEYWORDS
Fuzzy genetic regulatory networks, global dissipativity, global exponential dissipativity, mixed delays.

1. Introduction

Recently, the developments of biological networks have been quite significant in the field of science and technology. Biologically based networks such as neural networks and GRNs have enabled new applications in integrated circuits like neurochips, as well as in biological and biomedical science Plahte, Gjuvsland,& Omholt (2013); Wang, Qian,& Dougherty (2010). In living cells, a large amount of genes and proteins are interacting with one another by activation and repression. The mathematical modeling of GRNs can provide insight into the complicated biological and chemical processes linked by gene regulation Aouiti, & Dridi (2020, 2021); He, & Cao (2008); Sakthivel, Mathiyalagan,Lakshmanan,& Park (2013); Wang,Liao, Mao, & Liu (2009); Zhang, Fang, & Tang (2011).

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A genetic regulatory network is a collection of DNA segments that control the expression of genes in cells. Its genetic regulation generates the behavior of cells allows cell specialization in multicellular organisms by "turning on" and "turning off" certain parts of the genome. In life sciences, especially in biology, regulatory networks are the basis of models in full development, allowing a better understanding of the functioning of living organisms. Errors in interactions between DNAs, mRNAs, and proteins at the molecular level of corresponding cell lines cause many diseases such as cancer Noor, Serpedin,Nounou,& Nounou (2012); Plahte et al. (2013); Wang et al. (2010); Weber, Defterli,Gk, & Kropat (2011); Zhang, Fan, & Wu (2017).

Mathematically speaking, there are two categories of genetic network models: differential equation model which is a continuous model Cao,& Ren (2008); Li,& Lam (2010), and Boolean model which is a discrete model Ding, Li, Li, & Sun (2018); Vasic, Ravanmehr,& Krishnan (2011). One can easily see that in the progression of the genome sequence, effects of time delay exist due to the limited speed in the slow transcription, translation and transmission process. Time delays are inevitable in modeling the gene regulation process (see papers Hu, Liang,& Cao (2015); Zhang, Wu, & Cui (2014); Zhang, Wu,& Zou (2015)).

In mathematical modeling of real-world problems, uncertainty or vagueness is unavoidable. In order to take into account fuzziness, the fuzzy logic methodology is considered to capture these inconveniences and manipulate data that was rather fuzzy Gao, Xiao,Liu, & Wang (2018); Raza (2019); Su, Xia, Liu, & Wu (2018); Zhao, Wang, Yan, & Shen (2019). Their dynamical behaviors have attracted growing attention from many scholars. For example: In article Ali, Gunasekaran, Ahn, & Shi (2016), Ali, M. S. *et al.,* investigated the sampled-data stabilization problem for Takagi-Sugeno (T-S) FGRNs subject to leakage delays. In paper Poblete,Parra,Gomez,Saldias,Garrido,& Vargas (2009), Poblete, C. M. *et al.,* presented the fuzzy logic in GRN Models. In article Ram, Chetty,& Dix (2006), Ram, R. *et al.,* proposed the fuzzy model for GRN to search microarray datasets for activator/repressor regulatory relationship. In Ratnavelu,Kalpana,& Balasubramaniam (2018), Ratnavelu, K. *et al.,* investigated the existence, uniqueness, and asymptotic stability analysis of the equilibrium point of FGRNs with mixed delays.

The concentrations of mRNAs and proteins are not always approach the equilibrium points, or the points will not be stable. In some cases, they will reach and stay a bounded region far from the equilibrium points. This dynamic characteristic is called dissipativity proposed by Belgian scientist I.Prigogine in the 1970s and has found applications in many complex systems such as chemical reactions, fluid electronic circuits, biological system. Up to now, a huge number of results on the global dissipativity of dynamical systems have been reported in the literature. For instance: In Wang, Dong, Xie,& Cao (2020), Wang, L. *et al.,* discussed the global dissipativity for stochastic GRNs in the presence of time-delays. In Ma, Zhang, & Li (2017), the authors studied the dissipative analysis for comprehensive GRN models with fractional Brownian motion, diffusion-reaction processes, Markovian jump and time-varying delay. In article Shen,Huo, Yan,Park, & Sreeram (2019), Shen, H. *et al.,* dealt with the distributed dissipative state estimation issue of Markov jump GRNs with round-robin scheduling. In Yu, Liu, Zeng, & Wu (2019), Yu, T. *et al.,* investigated the problem of dissipativity-based filtering for switched GRNs in the presence of stochastic perturbation and time-varying delays. In Wang, Zhang, & Han (2015), Wang, J. *et al.,* studied the event-triggered generalized dissipativity filtering for a class of neural network with time-varying delay. In Zhang & Han (2015), Zhang, X. M. and Han, Q. L. dealt with the problem of a decentralized event-triggered dissipative control for systems with the entries of the system outputs having different physical properties. We are aware that the global dissipativity problems of delayed systems have recently received an increasing attention. To the authors' knowledge, the problem of dissipativity for FGRNs have not been researched, it remains essential and challenging. The solution of this problem is of great significance to the study of dynamic performance and control analysis for the system of mRNAs and proteins. The objective of this work is to alternatively study global dissipativity problem of GRNs with fuzzy logics and mixed delays. So, in this article we have the following two parts:

- The global dissipativity of the fuzzy genetic regulatory networks with mixed delays is obtained via Lyapunov functionals and linear matrix inequalities.
- The global exponential dissipativity for the corresponding model is successfully proposed by using Lyapunov functionals.

The highlights and major contributions of this article are reflected in the following key aspects:

✓ The global dissipativity problem of FGRNs with time delays and distributed delays is provided.
✓ New set of sufficient conditions in terms of LMIs is derived to ensure the global dissipativity and global exponential dissipativity results.
✓ The estimations of the positive invariant set, globally attractive set and globally exponential attractive set are computed.
✓ In this work, we proposed a new framework which can cope with the fuzzy logic and mixed delays at the same time.

The main structures of the article includes the following aspects: In Section 2, we will present some notations, basic definitions and helpful lemmas. In Section 3, we will establish our main results. In Section 4, two numerical examples will be presented. In Section 5, we will draw with conclusions.

Notations: In this paper, $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^n$ denotes the set of all n-dimensional real vectors (real numbers). For a symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M > 0$, $M \geq 0$ means that M is positive defined and positive semi-defined matrix respectively, M^{-1} and M^T denotes the matrix inverse and the matrix transpose respectively. The symbol $\star$ means the symmetric term in a symmetric matrix.

2. Preliminaries

In this manuscript, we focus our attention on the study of the global dissipativity of the following model:

$$\begin{cases} \dot{\zeta}_i(t) &= -b_i\zeta_i(t) + \sum_{j=1}^{n} c_{ij} f_j(\xi_j(t-\tau(t))) + \bigwedge_{j=1}^{n} \gamma_{ij} f_j(\xi_j((t-\eta(t))) \\ &+ \bigvee_{j=1}^{n} \delta_{ij} f_j(\xi_j(t-\eta(t))) + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds \\ &+ \bigvee_{j=1}^{n} \beta_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds + \pi_i(t), \\ \dot{\xi}_i(t) &= -a_i\xi_i(t) + e_i\zeta_i(t-\mu(t)) + \bigwedge_{j=1}^{n} \bar{\gamma}_{ij}\zeta_j(t-\eta(t)) \\ &+ \bigvee_{j=1}^{n} \bar{\delta}_{ij}\zeta_j(t-\eta(t)) + \bigwedge_{j=1}^{n} \bar{\alpha}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds \\ &+ \bigvee_{j=1}^{n} \bar{\beta}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds. \end{cases} \tag{1}$$

The description of model 1 is as follows:

* $\zeta_i(\cdot),\ \xi_i(\cdot)$ are the concentrations of mRNA and protein of the i^{th} node.
* $B = diag\{b_1, \cdots, b_n\} > 0,\ A = diag\{a_1, \cdots, a_n\} > 0$ are the decay rates of mRNA and protein, respectively.
* The function $f(\cdot)$ is generally a non-linear function with a form of monotony, denotes the feedback regulation for the proteins during the transcription, which is usually chosen as the Hill form

$$f_j(\zeta_j) = \frac{\left(\frac{\zeta_j}{\chi_j}\right)^{H_j}}{\left(1 + (\frac{\zeta_j}{\chi_j})^{H_j}\right)}$$

where H_j stands for the Hill coefficients and χ_j is a positive constant.
* $\tau(\cdot),\ \eta(\cdot),\ \mu(\cdot)$ and $\sigma(\cdot),\ \rho(\cdot)$ are the time-varying delays and distributed time-varying delays respectively.
* $E = diag\{e_1, \cdots, e_n\}$ represents the translation rate.
* $\pi = diag\{\pi_1, \cdots, \pi_n\}$ is the base transcriptional rate of the repressor of gene i with $|\pi(t)| \leq \tilde{\pi}$.
* $\gamma = (\gamma_{ij})_{n\times n},\ \alpha = (\alpha_{ij})_{n\times n},\ \bar{\gamma} = (\bar{\gamma}_{ij})_{n\times n},\ \bar{\alpha} = (\bar{\alpha}_{ij})_{n\times n}$ and $\delta = (\delta_{ij})_{n\times n},\ \beta = (\beta_{ij})_{n\times n},\ \bar{\delta} = (\bar{\delta}_{ij})_{n\times n},\ \bar{\beta} = (\bar{\beta}_{ij})_{n\times n}$ denotes elements of the fuzzy feedback MIN template and MAX template, respectively.
* $\bigwedge$ and $\bigvee$ denote the fuzzy AND operation and OR operation, respectively.
* $C = (c_{ij})_{n\times n}$ is defined as follows:

$$c_{ij} = \begin{cases} \bar{c}_{ij} & \text{if transcription factor j is an activator of gene i,} \\ 0 & \text{if there is no link from node j to i,} \\ -\bar{c}_{ij}, & \text{if transcription factor j is a repressor of gene i.} \end{cases}$$

Now, we will adapt the following notations:
$\tau^* = \sup_{t\in\mathbb{R}}\{\tau(t)\}$, $\eta^* = \sup_{t\in\mathbb{R}}\{\eta(t)\}$, $\sigma^* = \sup_{t\in\mathbb{R}}\{\sigma(t)\}$, $\mu^* = \sup_{t\in\mathbb{R}}\{\mu(t)\}$, $\rho^* = \sup_{t\in\mathbb{R}}\{\rho(t)\}$, and $\nu = \max\{\tau^*,\ \eta^*,\ \sigma^*,\ \mu^*,\ \rho^*\}$.
The initial values of model (1) are

$$\begin{cases} \zeta_i(t) &= \varphi_i(t), \quad t \in [-\nu,\ 0], \\ \xi_i(t) &= \psi_i(t), \quad t \in [-\nu,\ 0], \quad i = 1,\ 2, \cdots, n, \end{cases} \tag{2}$$

where $\varphi_i(\cdot),\ \psi_i(\cdot) \in \mathcal{C}([-\nu,\ 0], \mathbb{R}^n)$ which $\mathcal{C}([-\nu,\ 0], \mathbb{R}^n)$ represents the Banach space of all continuous functions mapping from $[-\nu,\ 0]$ into $\mathbb{R}^n$.

Throughout this work, the following assumptions are used:
Assumption 1 There exist constants $l_j^-,\ l_j^+ \geq 0$, such that for all $s_1, s_2 \in \mathbb{R},\ s_1 \neq s_2,\ f_j(\cdot)$ satisfy:

$$l_j^- \leq \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \leq l_j^+,\ j = 1, 2, \cdots, n.$$

with $l_j^- = \min_{s\geq 0} \dot{f}_j(s) = 0$ and $l_j^+ = \max_{s\geq 0} \dot{f}_j(s) = \dfrac{(H_j - 1)^{(H_j-1)/H_j}(H_j + 1)^{(H_j+1)/H_j}}{4\chi_j H_j}$.
In the following, we denote

$$F^+ = diag\{l_1^+, l_2^+, \cdots, l_n^+\}.$$

Assumption 2 The time-varying delays and distributed time-varying delays $\tau(\cdot)$, $\eta(\cdot)$, $\sigma(\cdot)$, $\mu(\cdot)$ and $\rho(\cdot)$ are differentiable and satisfy:
$0 < \dot{\tau}(t) < \tilde{\tau} < 1,\ 0 < \dot{\eta}(t) < \tilde{\eta} < 1,\ 0 < \dot{\sigma}(t) < \tilde{\sigma} < 1,\ 0 < \dot{\mu}(t) < \tilde{\mu} < 1$ and $0 < \dot{\rho}(t) < \tilde{\rho} < 1$.

Definition 2.1. Aouiti,Sakthivel, & Touati (2020) The FGRNs (1) is said to be a globally dissipative system, if there exists a compact set $\Upsilon_1 \subset \mathbb{R}^{2n}$ such that for any initial value $(\varphi^T(s),\ \psi^T(s))^T \in \mathbb{R}^{2n}\backslash\Upsilon_1,\ s \in [-\nu,\ 0]$, there exists $\mathcal{T} > 0$ when $t \geq \mathcal{T}$, $(\zeta^T(t,\ \varphi),\ \xi^T(t,\ \psi))^T \subseteq \Upsilon_1$, where $(\zeta^T(t,\ \varphi),\ \xi^T(t,\ \psi))^T$ denotes the solution of system (1) from initial state $(\varphi^T(s),\ \psi^T(s))^T$. In this case, Υ_1 is called a globally attractive set. A set Υ_1 is called positive invariant, if for all $(\varphi^T(s),\ \psi^T(s))^T \in \Upsilon_1$ implies $(\zeta^T(t,\ \varphi),\ \xi^T(t,\ \psi))^T \subseteq \Upsilon_1$ for all $t \geq 0$.

Definition 2.2. Aouiti et al. (2020) Let Υ_2 be a globally attractive set of the FGRNs (1). The FGRNs (1) is said to be a globally exponentially dissipative system, if there exists a compact set $\Upsilon_{2\varepsilon} \supset \Upsilon_2$ in $\mathbb{R}^{2n}$, such that for any initial value $(\varphi^T(s),\ \psi^T(s))^T \in \mathbb{R}^{2n} \setminus \Upsilon_2,\ s \in [-\nu,\ 0]$ there exists $N(\varpi) > 0$ and $\lambda > 0$ such that:

$$\inf_{(\zeta^T(t),\xi^T(t))^T\in\mathbb{R}^{2n}\backslash\Upsilon_{2\varepsilon}} \left\{ \left\| \begin{pmatrix} \zeta(t,\ \varphi) \\ \xi(t,\ \psi) \end{pmatrix} - \begin{pmatrix} \tilde{\zeta} \\ \tilde{\xi} \end{pmatrix} \right\| \ \middle|\ \begin{pmatrix} \tilde{\zeta} \\ \tilde{\xi} \end{pmatrix} \in \Upsilon_{2\varepsilon} \right\} \leq N(\varpi)\exp(-\lambda t),$$

where $\varpi = (\varphi^T(s),\ \psi^T(s))^T$. The set $\Upsilon_{2\varepsilon}$ is called as a globally exponentially attractive set, where $(\zeta^T(t), \xi^T(t))^T \in \mathbb{R}^{2n}\backslash\Upsilon_{2\varepsilon}$ means $(\zeta^T(t), \xi^T(t))^T \in \mathbb{R}^{2n}$ but $(\zeta^T(t), \xi^T(t))^T \notin \Upsilon_{2\varepsilon}$.

Lemma 2.3. *Ratnaveluet al. (2018)Let* $u_j,\ v_j,\ \gamma_{ij},\ \delta_{ij} \in \mathbb{R}$*,* $r_j : \mathbb{R} \to \mathbb{R}$ *be continuous functions, and* $i, j = 1, 2, \cdots, n$*, then the following inequalities hold:*

$$
\begin{aligned}
&\Big|\bigwedge_{j=1}^{n} \gamma_{ij} r_j(u_j) - \bigwedge_{j=1}^{n} \gamma_{ij} r_j(v_j)\Big| \le \sum_{j=1}^{n} |\gamma_{ij}||r_j(u_j) - r_j(v_j)|,\\
&\Big|\bigvee_{j=1}^{n} \delta_{ij} r_j(u_j) - \bigvee_{j=1}^{n} \delta_{ij} r_j(v_j)\Big| \le \sum_{j=1}^{n} |\delta_{ij}||r_j(u_j) - r_j(v_j)|.
\end{aligned}
$$

Lemma 2.4. *Aouiti et al. (2020) Let* $x,\ y \in \mathbb{R}^n$*,* M *be a appropriate dimensions positive definite matrix, then we have:*

$$2x^T y \le x^T M^{-1} x + y^T M y.$$

3. Main results

In this section, we will introduce some criteria to guarantee the global dissipativity and global exponential dissipativity for our FGRNs model.

Theorem 3.1. *Suppose that Assumptions 1-2 hold. If there exists positive definite matrices* $R_1 = (r^1_{ij})_{n\times n}$*,* $R_2 = (r^2_{ij})_{n\times n}$*,* $W = (w_{ij})_{n\times n}$*,* $M_1 = (m^1_{ij})_{n\times n}$*,* $M_2 = (m^2_{ij})_{n\times n}$*,* $M_3 = (m^3_{ij})_{n\times n}$ *and positive diagonal matrices* $J_1 = diag\{J^1_1, \cdots, J^1_n\}$*,* $J_2 = diag\{J^2_1, \cdots, J^2_n\}$*,* $P = diag\{p_1, \cdots, p_n\}$*,* $Q = diag\{q_1, \cdots, q_n\}$*,*
$\chi = diag\{\chi_1, \cdots, \chi_n\}$ *such that the following LMIs holds:*

$$
\Lambda = \begin{pmatrix}
\Xi_{1,1} & 0 & 0 & PC & \Xi_{1,5} & 0 & 0 & \Xi_{1,8} & 0\\
\star & \Xi_{2,2} & 0 & 0 & 0 & QE & \Xi_{2,7} & 0 & \Xi_{2,9}\\
\star & \star & \Xi_{3,3} & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \Xi_{4,4} & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \Xi_{5,5} & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \Xi_{6,6} & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \Xi_{7,7} & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -W & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & -M_1
\end{pmatrix} < 0, \qquad (3)
$$

where
$\Xi_{1,1} = -2PB + P + J_1,\ \Xi_{1,5} = P|\gamma| + P|\delta|,\ \Xi_{1,8} = P|\alpha| + P|\beta|,$
$\Xi_{2,2} = -2QA + \rho^* M_1 + M_2 + M_3 + F^+\chi F^+ + J_2,\ \Xi_{2,7} = Q|\bar{\gamma}| + Q|\bar{\delta}|,$
$\Xi_{2,9} = Q|\bar{\alpha}| + Q|\bar{\beta}|,\ ,\ \Xi_{3,3} = R_1 + R_2 + \sigma^* W - \chi,\ \Xi_{4,4} = -(1-\tilde{\tau})R_1,$
$\Xi_{5,5} = -(1-\tilde{\eta})R_2,\ \Xi_{6,6} = -(1-\tilde{\mu})M_2,\ \Xi_{7,7} = -(1-\tilde{\eta})M_3.$
Then system (1) is globally dissipative, and

$$\Upsilon_1 \;=\; \big\{(\zeta,\ \xi) \in \mathbb{R}^{2n} | \zeta^T(t) J_1 \zeta(t) \le \theta_1 \tilde{\pi}^T P \tilde{\pi},\ \xi^T(t) J_2 \xi(t) \le \theta_2 \tilde{\pi}^T P \tilde{\pi}\big\}$$

is a positive invariant and globally attractive set of (1).

Proof. Consider the following Lyapunov functional:

$$V(t)=\sum_{i=1}^{4}V_i(t), \tag{4}$$

where

$$\begin{aligned}
V_1(t) &= \sum_{i=1}^{n}\zeta_i(t)p_i\zeta_i(t)+\sum_{i=1}^{n}\xi_i(t)q_i\xi_i(t),\\
V_2(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}r_{ij}^{1}\int_{t-\tau(t)}^{t}f_j^2\big(\xi_j(s)\big)ds+\sum_{i=1}^{n}\sum_{j=1}^{n}r_{ij}^{2}\int_{t-\eta(t)}^{t}f_j^2\big(\xi_j(s)\big)ds,\\
V_3(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\sigma(t)}^{t}\int_{s}^{t}f_j\big(\xi_j(s_1)\big)w_{ij}f_j\big(\xi_j(s_1)\big)ds_1ds\\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}^{1}\int_{t-\rho(t)}^{t}\int_{s}^{t}\zeta_j^2(s_1)ds_1ds,\\
V_4(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}^{2}\int_{t-\mu(t)}^{t}\zeta_i^2(s)ds+\sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}^{3}\int_{t-\eta(t)}^{t}\zeta_j^2(s)ds.
\end{aligned}$$

The time-derivative of $V_1(\cdot)$ along the solutions of system (1) is given by

$$\begin{aligned}
\dot{V}_1(t) &= 2\sum_{i=1}^{n}\zeta_i(t)p_i\big[-b_i\zeta_i(t)+\sum_{j=1}^{n}c_{ij}f_j(\xi_j(t-\tau(t)))+\bigwedge_{j=1}^{n}\gamma_{ij}f_j(\xi_j(t-\eta(t)))\\
&+ \bigvee_{j=1}^{n}\delta_{ij}f_j(\xi_j(t-\eta(t)))+\bigwedge_{j=1}^{n}\alpha_{ij}\int_{t-\sigma(t)}^{t}f_j(\xi_j(s))ds\\
&+ \bigvee_{j=1}^{n}\beta_{ij}\int_{t-\sigma(t)}^{t}f_j(\xi_j(s))ds+\pi_i(t)\big]\\
&+ 2\sum_{i=1}^{n}\xi_i(t)q_i\big[-a_i\xi_i(t)+e_i\zeta_i(t-\mu(t))+\bigwedge_{j=1}^{n}\bar{\gamma}_{ij}\zeta_j(t-\eta(t))\\
&+ \bigvee_{j=1}^{n}\bar{\delta}_{ij}\zeta_i(t-\eta(t))+\bigwedge_{j=1}^{n}\bar{\alpha}_{ij}\int_{t-\rho(t)}^{t}\zeta_j(s)ds+\bigvee_{j=1}^{n}\bar{\beta}_{ij}\int_{t-\rho(t)}^{t}\zeta_j(s)ds]
\end{aligned}$$

and by using Lemma 2.3 it yields

$$
\begin{aligned}
\dot{V}_1(t) &\leq -2\sum_{i=1}^{n}\zeta_i(t)p_ib_i\zeta_i(t)+2\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_i(t)p_ic_{ij}f_j(\xi_j(t-\tau(t))) \\
&+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_i(t)p_i|\gamma_{ij}||f_j(\xi_j(t-\eta(t)))|+2\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_i(t)p_i|\delta_{ij}||f_j(\xi_j(t-\eta(t)))| \\
&+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_i(t)p_i|\alpha_{ij}|\int_{t-\sigma(t)}^{t}|f_j(\xi_j(s))|ds \\
&+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_i(t)p_i|\beta_{ij}|\int_{t-\sigma(t)}^{t}|f_j(\xi_j(s))|ds+2\sum_{i=1}^{n}\zeta_i(t)p_i\pi_i(t) \\
&- 2\sum_{i=1}^{n}\xi_i(t)q_ia_i\xi_i(t)+2\sum_{i=1}^{n}\xi_i(t)q_ie_i\zeta_i(t-\mu(t)) \\
&+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_i(t)q_i|\bar{\gamma}_{ij}||\zeta_j(t-\eta(t))|+2\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_i(t)q_i|\bar{\delta}_{ij}||\zeta_j(t-\eta(t))| \\
&+ 2\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_i(t)q_i|\bar{\alpha}_{ij}|\int_{t-\rho(t)}^{t}|\zeta_j(s)|ds+2\sum_{i=1}^{n}\sum_{j=1}^{n}\xi_i(t)q_i|\bar{\beta}_{ij}|\int_{t-\rho(t)}^{t}|\zeta_j(s)|ds.
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
\dot{V}_1(t) &\leq -2\zeta^T(t)PB\zeta(t)+2\zeta^T(t)PCf(\xi(t-\tau(t)))+2\zeta^T(t)P|\gamma||f(\xi(t-\eta(t)))| \\
&+ 2\zeta^T(t)P|\delta||f(\xi(t-\eta(t)))|+2\zeta^T(t)P|\alpha|\int_{t-\sigma(t)}^{t}|f(\xi(s))|ds \\
&+ 2\zeta^T(t)P|\beta|\int_{t-\sigma(t)}^{t}|f(\xi(s))|ds+2\zeta^T(t)P\pi(t)-2\xi^T(t)QA\xi(t) \\
&+ 2\xi^T(t)QE\zeta(t-\mu(t))+2\xi^T(t)Q|\bar{\gamma}||\zeta(t-\eta(t))| \\
&+ 2\xi^T(t)Q|\bar{\delta}||\zeta(t-\eta(t))|+2\xi^T(t)Q|\bar{\alpha}|\int_{t-\rho(t)}^{t}|\zeta(s)|ds \\
&+ 2\xi^T(t)Q|\bar{\beta}|\int_{t-\rho(t)}^{t}|\zeta(s)|ds.
\end{aligned}
$$

The time-derivative of $V_2(\cdot)$ along the solutions of system (1) is given by

$$
\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}r_{ij}^1f_j^2\big(\xi_j(t)\big)-\sum_{i=1}^{n}\sum_{j=1}^{n}(1-\dot{\tau}(t))r_{ij}^1f_j^2\big(\xi_j(t-\tau(t))\big) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}r_{ij}^2f_j^2\big(\xi_j(t)\big)-\sum_{i=1}^{n}\sum_{j=1}^{n}(1-\dot{\eta}(t))r_{ij}^2f_j^2\big(\xi_j(t-\eta(t))\big),
\end{aligned}
$$

and then we obtain

$$\begin{aligned}
\dot{V}_2(t) &\leq \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^1 f_j^2\big(\xi_j(t)\big) - \sum_{i=1}^{n}\sum_{j=1}^{n}(1-\tilde{\tau}) r_{ij}^1 f_j^2\big(\xi_j(t-\tau(t))\big) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^2 f_j^2\big(\xi_j(t)\big) - \sum_{i=1}^{n}\sum_{j=1}^{n}(1-\tilde{\eta}) r_{ij}^2 f_j^2\big(\xi_j(t-\eta(t))\big) \\
&= f^T\big(\xi(t)\big) R_1 f\big(\xi(t)\big) - (1-\tilde{\tau}) f^T\big(\xi(t-\tau(t))\big) R_1 f\big(\xi(t-\tau(t))\big) \\
&+ f^T\big(\xi(t)\big) R_2 f\big(\xi(t)\big) - (1-\tilde{\eta}) f^T\big(\xi(t-\eta(t))\big) R_2 f\big(\xi(t-\eta(t))\big).
\end{aligned}$$

The time-derivative of $V_3(\cdot)$ along the solutions of system (1) is given by

$$\begin{aligned}
\dot{V}_3(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma(t) f_j\big(\xi_j(t)\big) w_{ij} f_j\big(\xi_j(t)\big) - \sum_{i=1}^{n}\sum_{j=1}^{n} \int_{t-\sigma(t)}^{t} f_j\big(\xi_j(s)\big) w_{ij} f_j\big(\xi_j(s)\big) ds \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^1 \rho(t) \zeta_j^2(t) - \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^1 \int_{t-\rho(t)}^{t} \zeta_j^2(s) ds,
\end{aligned}$$

and then we obtain

$$\begin{aligned}
\dot{V}_3(t) &\leq \sigma^* f^T\big(\xi(t)\big) W f\big(\xi(t)\big) - \bigg(\int_{t-\sigma(t)}^{t} f\big(\xi(s)\big) ds\bigg)^T W \bigg(\int_{t-\sigma(t)}^{t} f\big(\xi(s)\big) ds\bigg) \\
&+ \rho^* \zeta^T(t) M_1 \zeta(t) - \bigg(\int_{t-\rho(t)}^{t} \zeta(s) ds\bigg)^T M_1 \bigg(\int_{t-\rho(t)}^{t} \zeta(s) ds\bigg).
\end{aligned}$$

The time-derivative of $V_4(\cdot)$ along the solutions of system (1) is given by

$$\begin{aligned}
\dot{V}_4(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^2 \zeta_i^2(t) - \sum_{i=1}^{n}\sum_{j=1}^{n} (1-\dot{\mu}(t)) m_{ij}^2 \zeta_i^2(t-\mu(t)) + \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^3 \zeta_j^2(t) \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^3 (1-\dot{\eta}(t)) \zeta_j^2(t-\eta(t)),
\end{aligned}$$

and then we obtain

$$\begin{aligned}
\dot{V}_4(t) &\leq \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^2 \zeta_i^2(t) - \sum_{i=1}^{n}\sum_{j=1}^{n} (1-\tilde{\mu}) m_{ij}^2 \zeta_i^2(t-\mu(t)) + \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^3 \zeta_j^2(t) \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}^3 (1-\tilde{\eta}) \zeta_j^2(t-\eta(t)) \\
&= \zeta^T(t) M_2 \zeta(t) - (1-\tilde{\mu}) \zeta^T(t-\mu(t)) M_2 \zeta(t-\mu(t)) + \zeta^T(t) M_3 \zeta(t) \\
&- (1-\tilde{\eta}) \zeta^T(t-\eta(t)) M_3 \zeta(t-\eta(t)).
\end{aligned}$$

By employing the Lemma (2.4), we get

$$2\zeta^T(t) P \pi(t) \leq \zeta^T(t) P \zeta(t) + \pi^T(t) P \pi(t).$$

Under the Assumption 1. the following inequality holds

$$-f^T(\xi(t))\chi f(\xi(t)) + \xi^T(t)F^+\chi F^+\xi(t) \geq 0,$$

for any positive diagonal matrix $\chi = \text{diag}\{\chi_1,\ \chi_2, \cdots, \chi_n\}$. It implies that

$$\begin{aligned}
\dot{V}(t) \;\; &\leq \;\; \zeta^T(t)[-2PB + P + J_1]\zeta(t) + 2\zeta^T(t)PCf(\xi(t-\tau(t))) \\
&+ \;\; 2\zeta^T(t)[P|\gamma| + P|\delta|]|f(\xi(t-\eta(t)))| + 2\zeta^T(t)[P|\alpha| + P|\beta|]\int_{t-\sigma(t)}^{t}|f(\xi(s))|ds \\
&+ \;\; \xi^T(t)[-2QA + \rho^* M_1 + M_2 + M_3 + F^+\chi F^+ + J_2]\xi(t) \\
&+ \;\; 2\xi^T(t)QE\zeta(t-\mu(t)) + 2\xi^T(t)[Q|\bar{\gamma}| + Q|\bar{\delta}|]|\zeta(t-\eta(t))| \\
&+ \;\; 2\xi^T(t)[Q|\bar{\alpha}| + Q|\bar{\beta}|]\int_{t-\rho(t)}^{t}|\zeta(s)|ds + f^T\big(\xi(t)\big)[R_1 + R_2 + \sigma^* W - \chi]f\big(\xi(t)\big) \\
&- \;\; (1-\tilde{\tau})f^T\big(\xi(t-\tau(t))\big)R_1 f\big(\xi(t-\tau(t))\big) \\
&- \;\; (1-\tilde{\eta})f^T\big(\xi(t-\eta(t))\big)R_2 f\big(\xi(t-\eta(t))\big) \\
&- \;\; \left(\int_{t-\sigma(t)}^{t} f\big(\xi(s)\big)ds\right)^T W\left(\int_{t-\sigma(t)}^{t} f\big(\xi(s)\big)ds\right) \\
&- \;\; \left(\int_{t-\rho(t)}^{t} \zeta(s)ds\right)^T M_1\left(\int_{t-\rho(t)}^{t} \zeta(s)ds\right) \\
&- \;\; (1-\tilde{\mu})\zeta^T(t-\mu(t))M_2\zeta(t-\mu(t)) - (1-\tilde{\eta})\zeta^T(t-\eta(t))M_3\zeta(t-\eta(t)) \\
&+ \;\; \pi^T(t)P\pi(t) - \zeta^T(t)J_1\zeta(t) - \xi^T(t)J_2\xi(t),
\end{aligned}$$

and finally we obtain

$$\dot{V}(t) \;\; \leq \;\; \triangle^T(t)\Lambda\triangle(t) - \zeta^T(t)J_1\zeta(t) - \xi^T(t)J_2\xi(t) + \tilde{\pi}^T P\tilde{\pi},$$

where

$$\triangle^T(t) = \Bigg[\zeta^T(t),\ \xi^T(t),\ f^T\big(\xi(t)\big),\ f^T\big(\xi(t-\tau(t))\big),\ |f\big(\xi(t-\eta(t))\big)|^T,$$

$$\zeta^T(t-\mu(t)),\ |\zeta(t-\eta(t))|^T,\ \left(\int_{t-\sigma(t)}^{t}|f\big(\xi(s)\big)|ds\right)^T,\ \left(\int_{t-\rho(t)}^{t}|\zeta(s)|ds\right)^T\Bigg]^T.$$

We choose given positive constants $\theta_1,\ \theta_2$ such that $\theta_1 + \theta_2 = 1,\ \theta_1 \neq 0,\ \theta_2 \neq 0$. Then, by $\Lambda < 0$, one obtains:

$$\dot{V}(t) \;\; \leq \;\; -\zeta^T(t)J_1\zeta(t) - \xi^T(t)J_2\xi(t) + (\theta_1 + \theta_2)\tilde{\pi}^T P\tilde{\pi}, \tag{5}$$

when, $(\zeta^T(t),\ \ \xi^T(t))^T \in \mathbb{R}^{2n}\diagdown\Upsilon_1$, i.e. $(\zeta^T(t),\ \ \xi^T(t))^T \notin \Upsilon_1$. Accordingly, if $(\varphi^T(s),\ \phi^T(s))^T \in \Upsilon_1$, then $(\zeta^T(t), \xi^T(t))^T \subseteq \Upsilon_1,\ t \geq 0$, which results Υ_1 is a positive invariant set of (1). If $(\varphi^T(s),\ \phi^T(s))^T \notin \Upsilon_1$, there exist some $\mathcal{T} > 0$ such that $(\zeta^T(t),\ \xi^T(t))^T \subseteq \Upsilon_1,\ t \geq \mathcal{T}$, which results the system (1) is globally dissipative and Υ_1 is a positive invariant and globally attractive set of (1). $\square$

Remark 1. In Theorem 3.1, we construct the best Lyapunov functional and estimating their derivative by improved inequality technique is the main challenges in the field of delayed systems. The method used in Theorem 3.1 attempts to do some efforts to estimate the fuzzy terms, and the distributed delays to obtain a linear matrix inequality can be easily checked.

Remark 2. The conditions of Theorem 3.1 are expressed in terms of solution of several LMIs. Therefore, the global dissipativity of the FGRNs (1) can be easily verified by using the numerically effect Matlab LMI toolbox.

Theorem 3.2. *Suppose that Assumptions 1-2 hold. If there exists constants $\theta_1,\ \theta_2 \in (0,\ 1)$ and $\lambda > 0$ such that:*

$$\lambda - (1-\theta_1)b_i + \frac{|e_i|e^{\lambda\mu^*}}{1-\tilde{\mu}} + \sum_{j=1}^{n} \frac{(|\bar{\delta}_{ij}| + |\bar{\gamma}_{ij}|)e^{\lambda\eta^*}}{1-\tilde{\eta}} + \sum_{j=1}^{n} \frac{(|\bar{\alpha}_{ij}| + |\bar{\beta}_{ij}|)\rho^* e^{\lambda\rho^*}}{1-\tilde{\rho}} < 0, \quad (6)$$

$$\begin{aligned} &\lambda - (1-\theta_2)a_i + \sum_{j=1}^{n} \frac{|c_{ij}|l_j^+}{1-\tilde{\tau}} e^{\lambda\tau^*} + \sum_{j=1}^{n} \frac{(|\delta_{ij}| + |\gamma_{ij}|)l_j^+}{1-\tilde{\eta}} e^{\lambda\eta^*} \\ &+ \sum_{j=1}^{n} \frac{(|\alpha_{ij}| + |\beta_{ij}|)\sigma^* e^{\lambda\sigma^*} l_j^+}{1-\tilde{\sigma}} < 0,\ i = 1,\ 2, \cdots, n. \end{aligned} \quad (7)$$

Then system (1) is globally exponentially dissipative, and

$$\Upsilon_2 \;=\; \left\{ (\zeta^T,\ \xi^T)^T \in \mathbb{R}^{2n} |\ \sum_{i=1}^{n} |\zeta_i| \leq \frac{\vartheta_1 \tilde{\pi}}{\theta_1 b_i},\ \sum_{i=1}^{n} |\xi_i| \leq \frac{\vartheta_2 \tilde{\pi}}{\theta_2 a_i},\ i = 1,\ 2, \cdots, n \right\}$$

is a positive invariant and globally exponentially attractive set of (1).

Proof. Consider the following Lyapunov functional:

$$V(t) = \sum_{i=1}^{5} V_i(t), \quad (8)$$

where

$$
\begin{aligned}
V_1(t) &= e^{\lambda t}\sum_{i=1}^{n}|\zeta_i(t)| + e^{\lambda t}\sum_{i=1}^{n}|\xi_i(t)|,\\
V_2(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|c_{ij}|}{1-\tilde{\tau}}\int_{t-\tau(t)}^{t}|f_j(\xi_j(s))|e^{\lambda(s+\tau^*)}ds\\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\delta_{ij}|+|\gamma_{ij}|)}{1-\tilde{\eta}}\int_{t-\eta(t)}^{t}|f_j(\xi_j(s))|e^{\lambda(s+\eta^*)}ds,\\
V_3(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)}{1-\tilde{\sigma}}\int_{-\sigma(t)}^{0}\int_{t+v}^{t}e^{\lambda(s+\sigma^*)}|f_j(\xi_j(s))|dsdv,\\
V_4(t) &= \sum_{i=1}^{n}\frac{|e_i|}{1-\tilde{\mu}}\int_{t-\mu(t)}^{t}e^{\lambda(s+\mu^*)}|\zeta_i(s)|ds\\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)}{1-\tilde{\eta}}\int_{t-\eta(t)}^{t}e^{\lambda(s+\eta^*)}|\zeta_j(s))|ds,\\
V_5(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)}{1-\tilde{\rho}}\int_{-\rho(t)}^{0}\int_{t+v}^{t}e^{\lambda(s+\rho^*)}|\zeta_j(s)|dsdv.
\end{aligned}
$$

Calculating the upper-right Dini derivative of $V(\cdot)$ along the solution of system (1) we have:

$$
\begin{aligned}
D^+V_1(t) &\leq e^{\lambda t}\sum_{i=1}^{n}\Big[(\lambda-b_i)|\zeta_i(t)| + \sum_{j=1}^{n}|c_{ij}||f_j(\xi_j(t-\tau(t)))|\\
&+ \sum_{j=1}^{n}|\gamma_{ij}||f_j\xi_j((t-\eta(t)))| + \sum_{j=1}^{n}|\delta_{ij}||f_j(\xi_j(t-\eta(t)))|\\
&+ \sum_{j=1}^{n}|\alpha_{ij}|\int_{t-\sigma(t)}^{t}|f_j(\xi_j(s))|ds + \sum_{j=1}^{n}|\beta_{ij}|\int_{t-\sigma(t)}^{t}|f_j(\xi_j(s))|ds + |\pi_i(t)|\Big]\\
&+ e^{\lambda t}\sum_{i=1}^{n}\Big[(\lambda-a_i)|\xi_i(t)| + |e_i||\zeta_i(t-\mu(t))| + \sum_{j=1}^{n}|(\bar{\gamma}_{ij}|+|\bar{\delta}_{ij}|)|\zeta_j(t-\eta(t))|\\
&+ \sum_{j=1}^{n}|(\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\int_{t-\rho(t)}^{t}|\zeta_j(s)|ds\Big],
\end{aligned}
$$

$$
\begin{aligned}
D^+V_2(t) \quad \leq \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|c_{ij}|}{1-\tilde{\tau}}|f_j(\xi_j(t))|e^{\lambda(t+\tau^*)} \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|c_{ij}|(1-\dot{\tau}(t))}{1-\tilde{\tau}}|f_j(\xi_j(t-\tau(t)))|e^{\lambda(t-\tau(t)+\tau^*)} \\
+ \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\delta_{ij}|+|\gamma_{ij}|)}{1-\tilde{\eta}}|f_j(\xi_j(t))|e^{\lambda(t+\eta^*)} \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\delta_{ij}|+|\gamma_{ij}|)(1-\dot{\eta}(t))}{1-\tilde{\eta}}|f_j(\xi_j(t-\eta(t)))|e^{\lambda(t-\eta(t)+\eta^*)} \\
\leq \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{|c_{ij}|e^{\lambda\tau^*}l_j^+}{1-\tilde{\tau}}|\xi_j(t)|e^{\lambda t}-\sum_{i=1}^{n}\sum_{j=1}^{n}|c_{ij}||f_j(\xi_j(t-\tau(t)))|e^{\lambda t} \\
+ \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\delta_{ij}|+|\gamma_{ij}|)e^{\lambda\eta^*}l_j^+}{1-\tilde{\eta}}|\xi_j(t)|e^{\lambda t} \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}(|\delta_{ij}|+|\gamma_{ij}|)|f_j(\xi_j(t-\eta(t)))|e^{\lambda t},
\end{aligned}
$$

$$
\begin{aligned}
D^+V_3(t) \quad = \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\dot{\sigma}(t)}{1-\tilde{\sigma}}\int_{t-\sigma(t)}^{t}e^{\lambda(s+\sigma^*)}|f_j(\xi_j(s))|ds \\
+ \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\sigma(t)}{1-\tilde{\sigma}}e^{\lambda(t+\sigma^*)}|f_j(\xi_j(t))| \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)}{1-\tilde{\sigma}}\int_{-\sigma(t)}^{0}e^{\lambda(t+\sigma^*+v)}|f_j(\xi_j(t+v))|dv \\
= \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\sigma(t)}{1-\tilde{\sigma}}e^{\lambda(t+\sigma^*)}|f_j(\xi_j(t))| \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)(1-\dot{\sigma}(t))}{1-\tilde{\sigma}}\int_{t-\sigma(t)}^{t}e^{\lambda(s+\sigma^*)}|f_j(\xi_j(s))|ds \\
\leq \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\sigma^*}{1-\tilde{\sigma}}e^{\lambda(t+\sigma^*)}|f_j(\xi_j(t))| \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\int_{t-\sigma(t)}^{t}e^{\lambda(s+\sigma^*)}|f_j(\xi_j(s))|ds \\
\leq \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\sigma^*e^{\lambda\sigma^*}l_j^+}{1-\tilde{\sigma}}e^{\lambda t}|\xi_j(t)| \\
- \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)e^{\lambda t}\int_{t-\sigma(t)}^{t}|f_j(\xi_j(s))|ds,
\end{aligned}
$$

$$
\begin{aligned}
D^+V_4(t) &= \sum_{i=1}^{n}\frac{|e_i|}{1-\tilde{\mu}}e^{\lambda(t+\mu^*)}|\zeta_i(t)| \\
&- \sum_{i=1}^{n}\frac{|e_i|(1-\dot{\mu}(t))}{1-\tilde{\mu}}e^{\lambda(t-\mu(t)+\mu^*)}|\zeta_i(t-\mu(t))| \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)}{1-\tilde{\eta}}e^{\lambda(t+\eta^*)}|\zeta_j(t))| \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)(1-\dot{\eta}(t))}{1-\tilde{\eta}}e^{\lambda(t-\eta(t)+\eta^*)}|\zeta_j(t-\eta(t))| \\
&\leq \sum_{i=1}^{n}\frac{|e_i|e^{\lambda\mu^*}}{1-\tilde{\mu}}e^{\lambda t}|\zeta_i(t)| - \sum_{i=1}^{n}|e_i|e^{\lambda t}|\zeta_i(t-\mu(t))| \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)e^{\lambda\eta^*}}{1-\tilde{\eta}}e^{\lambda t}|\zeta_j(t))| - \sum_{i=1}^{n}\sum_{j=1}^{n}(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)e^{\lambda t}|\zeta_j(t-\eta(t))|,
\end{aligned}
$$

$$
\begin{aligned}
D^+V_5(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\dot{\rho}(t)}{1-\tilde{\rho}}\int_{t-\rho(t)}^{t}e^{\lambda(s+\rho^*)}|\zeta_j(s)|ds \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\rho_j(t)}{1-\tilde{\rho}}e^{\lambda(t+\rho^*)}|\zeta_j(t)| \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)}{1-\tilde{\rho}}\int_{-\sigma(t)}^{0}e^{\lambda(t+\rho^*+v)}|\zeta_j(t+v)|dv \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\rho(t)}{1-\tilde{\rho}}e^{\lambda(t+\rho^*)}|\zeta_j(t)| \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)(1-\dot{\rho}(t))}{1-\tilde{\rho}}\int_{t-\rho(t)}^{t}e^{\lambda(s+\rho^*)}|\zeta_j(s)|ds \\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\rho^*}{1-\tilde{\rho}}e^{\lambda(t+\rho^*)}|\zeta_j(t)| \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\int_{t-\rho(t)}^{t}e^{\lambda(s+\rho^*)}|\zeta_j(s)|ds \\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\rho^*e^{\lambda\rho^*}}{1-\tilde{\rho}}e^{\lambda t}|\zeta_j(t)| \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)e^{\lambda t}\int_{t-\rho(t)}^{t}|\zeta_j(s)|ds,
\end{aligned}
$$

then, we can write

$$
\begin{aligned}
D^+V(t) &\leq e^{\lambda t}\sum_{i=1}^{n}\left[\lambda-(1-\theta_1)b_i+\frac{|e_i|e^{\lambda\mu^*}}{1-\tilde{\mu}}+\sum_{j=1}^{n}\frac{(|\bar{\delta}_{ij}|+|\bar{\gamma}_{ij}|)e^{\lambda\eta^*}}{1-\tilde{\eta}}\right.\\
&+\left.\sum_{j=1}^{n}\frac{(|\bar{\alpha}_{ij}|+|\bar{\beta}_{ij}|)\rho^*e^{\lambda\rho^*}}{1-\tilde{\rho}}\right]|\zeta_i(t)|\\
&+e^{\lambda t}\sum_{i=1}^{n}\left[\lambda-(1-\theta_2)a_i+\sum_{j=1}^{n}\frac{|c_{ij}|l_j^+}{1-\tilde{\tau}}e^{\lambda\tau^*}+\sum_{j=1}^{n}\frac{(|\delta_{ij}|+|\gamma_{ij}|)l_j^+}{1-\tilde{\eta}}e^{\lambda\eta^*}\right.\\
&+\left.\sum_{j=1}^{n}\frac{(|\alpha_{ij}|+|\beta_{ij}|)\sigma^*e^{\lambda\sigma^*}F^+}{1-\tilde{\sigma}}\right]|\xi_i(t)|\\
&-e^{\lambda t}\sum_{i=1}^{n}\big(\theta_1b_i|\zeta_i(t)|+\theta_2a_i|\xi_i(t)|-\tilde{\pi}\big).
\end{aligned}
\tag{9}
$$

Combining (6),(7) with (9) and we choose given positive constants $\vartheta_1,\ \vartheta_2$ such that $\vartheta_1+\vartheta_2=1,\ \vartheta_1\neq 0,\ \vartheta_2\neq 0$. Then, the following inequality can be reached:

$$
D^+V(t)\leq -e^{\lambda t}\sum_{i=1}^{n}\big[\big(\theta_1b_i|\zeta_i(t)|-\vartheta_1\tilde{\pi}\big)+\big(\theta_2a_i|\xi_i(t)|-\vartheta_2\tilde{\pi}\big)\big],
\tag{10}
$$

when $(\zeta^T(t),\xi^T(t))\in\mathbb{R}^{2n}\backslash\Upsilon_2$, i.e. $(\zeta^T(t),\xi^T(t))\notin\Upsilon_2$. Based on the proof of Theorem 3.1, we can say that the FGRNs system (1) is globally dissipative and Υ_2 is a positive invariant and globally attractive set of (1). Now, we have to manifest that system (1) is globally exponentially dissipative, and Υ_2 is globally exponentially attractive set of (1). Immediately thereafter, integrating (10) between 0 to t ($t>0$), we obtain $V(t)\leq V(0)$, which yields

$$
e^{\lambda t}\sum_{i=1}^{n}|\zeta_i(t)|+e^{\lambda t}\sum_{i=1}^{n}|\xi_i(t)|\leq V(t)\leq V(0),
$$

and

$$
\sum_{i=1}^{n}|\zeta_i(t)|+\sum_{i=1}^{n}|\xi_i(t)|\leq \sup_{-\nu\leq s\leq 0}V(s)e^{-\lambda t}.
$$

Let $N=\sup\limits_{-\nu\leq s\leq 0}V(s)$, we can write

$$
\begin{aligned}
\inf_{(\zeta^T(t),\xi^T(t))^T\in\mathbb{R}^{2n}\backslash\Upsilon_{2\varepsilon}}\left\{\left\|\begin{pmatrix}\zeta(t)\\ \xi(t)\end{pmatrix}-\begin{pmatrix}\tilde{\zeta}\\ \tilde{\xi}\end{pmatrix}\right\|\ \middle|\ \begin{pmatrix}\tilde{\zeta}\\ \tilde{\xi}\end{pmatrix}\in\Upsilon_{2\varepsilon}\right\} &\leq \left\|\begin{pmatrix}\zeta(t)-0\\ \xi(t)-0\end{pmatrix}\right\|\\
&\leq Ne^{-\lambda t},
\end{aligned}
$$

where $\Upsilon_{2\varepsilon}\supset\Upsilon_2\in\mathbb{R}^{2n}$ is a compact set.
Depending on definition 2.2, we can conclude that system (1) is globally exponentially dissipative and Υ_2 is globally exponentially attractive set. □

Remark 3. In article Aouiti, & Dridi (2020), the authors discussed the existence, uniqueness and global exponential stability of the weighted pseudo almost automorphic solution for a class of delayed fuzzy genetic regulatory networks with Stepanov-like weighted pseudo almost automorphic coefficients by using Banach fixed point theorem and novel analysis techniques. In article Xue,Zhang, & Zhang (2020), Xue, Y. *et al.,* investigated the reachable set estimation problem for genetic regulatory networks with time-varying delays and bounded disturbances and based on spectral properties of Metzler matrices they established the global exponential stability criteria. In Liu, Wang,& Xue (2020), the authors studied the problem of exponential stability analysis of discrete-time genetic regulatory networks with time-varying discrete delays and unbounded distributed delays without any auxiliary function or Lyapunov Krasovskii functional. In Duan, Di,& Wang (2020), Duan, L. *et al.,* derived the existence and global exponential stability of almost positive periodic solutions for a class of genetic regulatory networks with time varying delays by using the theory of dichotomy and contraction mapping principle. However, our approach is based on Lyapunov functional which is simple and easy to calculate their derivative and the linear matrix inequality which is easily to check by MATLAB LMI toolbox. Therefore, the advantages of the proposed method can be found better dissipativity performance it is faster than classical ones and more precisely. Therefore, it can be obtained less conservative results.

Remark 4. In this article, we propose the fuzzy logic model to predict changes in expression values and infer causal relationship between genes. Our manuscript offers a theoretical basis for the design of the fuzzy genetic regulatory networks with mixed delays more effective in the resolution of many problem thanks to the template input and/or output besides the sum of product operation. Hence, the obtained results can enrich the study on dynamical characteristics of FGRNs and generalized many previous works such as Liu et al. (2020); Manivannan,Cao,& Chong (2020); Qiao, Yan, Duan, & Miao (2020); Zhang,Zhang,Xue, & Zhang (2020).

Remark 5. In Aouiti,Sakthivel, & Touati (2021), the authors investigated the global dissipativity of fuzzy bidirectional associative memory neural networks with proportional delays. In Chen, Lin, & Lan (2020), Chen, X. *et al.,* studied the global dissipativity of delayed discrete-time inertial neural networks. In Duan, Jian,& Wang, B (2020), Duan, L. *et al.,* discussed the problem of the global exponential dissipativity of neutral-type BAM inertial neural networks with mixed time-varying delays. In Liu,& Jian (2019), Liu, J. and Jian, J. analyzed the global dissipativity of a class of quaternion-valued BAM neural networks with time delay. In Zhang (2021), Zhou, L. dealt with the problem of the global exponential dissipativity of impulsive recurrent neural networks with multi-proportional delays. However, in this article we studied the global dissipativity of FGRNs for two different concentrations variables of mRNA and proteins which is different from model of neural networks. To the best of our knowledge no results from the global dissipativity of fuzzy genetic regulatory networks with mixed delays have appear in the literature. So, we attempted this goal successfully and we obtained a sufficient condition to find a solution of this problem.

Remark 6. The method used in this article is based on the framework of Lyapunov-Krasovskii functional (LKF) and linear matrix inequality (LMI). The LKF-based method can be used to handle all time delays mentioned before and it is available for not only global dissipativity but also many other problems, like stability, state estimation, passivity analysis, and so on Chen, Zhou, &Zhang (2014); Sakthivel et al.

(2013); Yu, Liu, Zeng, Zhang, Zeng, & Wu (2017). Meanwhile, the LMI-based criteria can be easily checked through MATLAB/LMI toolbox for determining the global dissipativity of the adressed system. .

4. Numerical Examples

In this section, numerical examples are presented to show the effectiveness of the obtained theoretical analysis. Example 4.1 is provided to illustrate theorem 3.1, and example 4.2 aims to verify theorem 3.2.

Example 4.1. For $i = 1, 2$ considering the following FGRNs:

$$\begin{cases} \dot{\zeta}_i(t) &= -b_i\zeta_i(t) + \sum_{j=1}^{2} c_{ij} f_j(\xi_j(t-\tau(t))) + \bigwedge_{j=1}^{2} \gamma_{ij} f_j \xi_j((t-\eta(t))) \\ &+ \bigvee_{j=1}^{2} \delta_{ij} f_j(\xi_j(t-\eta(t))) + \bigwedge_{j=1}^{2} \alpha_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds \\ &+ \bigvee_{j=1}^{2} \beta_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds + \pi_i(t), \\ \dot{\xi}_i(t) &= -a_i\xi_i(t) + e_i\zeta_i(t-\mu(t)) + \bigwedge_{j=1}^{2} \bar{\gamma}_{ij}\zeta_j(t-\eta(t)) \\ &+ \bigvee_{j=1}^{2} \bar{\delta}_{ij}\zeta_j(t-\eta(t)) + \bigwedge_{j=1}^{2} \bar{\alpha}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds \\ &+ \bigvee_{j=1}^{2} \bar{\beta}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds. \end{cases} \tag{11}$$

The regulatory function is taken as

$$f_j(\xi_j) = \frac{\xi_j^2}{1+\xi_j^2}, \quad j = 1, 2,$$

i.e., the Hill coefficient is 2, so, it can be easily seen that: $F^+ = \mathrm{diag}\{0.65,\ 0.65\}$. Let

$$\begin{aligned} B &= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix},\ C = \begin{pmatrix} -1.1 & 1.3 \\ -1.4 & 1.5 \end{pmatrix},\ \gamma = \begin{pmatrix} -0.7 & 1.2 \\ 1.3 & -0.5 \end{pmatrix}, \\ \delta &= \begin{pmatrix} 1 & 0.1 \\ 0.4 & 0.6 \end{pmatrix},\ \alpha = \begin{pmatrix} 0.4 & 0.3 \\ 0.6 & 0.4 \end{pmatrix},\ \beta = \begin{pmatrix} 0.1 & -0.2 \\ -0.4 & 0.15 \end{pmatrix}, \\ A &= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix},\ E = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.6 \end{pmatrix},\ \bar{\gamma} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \\ \bar{\delta} &= \begin{pmatrix} 0.01 & 0.02 \\ 0.01 & 0.02 \end{pmatrix},\ \bar{\alpha} = \begin{pmatrix} 0.01 & -0.01 \\ -0.02 & 0.02 \end{pmatrix},\ \bar{\beta} = \begin{pmatrix} -0.04 & 0.06 \\ 0.03 & -0.05 \end{pmatrix}, \end{aligned}$$

$\tau(t) = 1 + 0.5\sin(t)$, $\eta(t) = 0.5(1+\cos(t))$, $\sigma(t) = 0.2 + 0.1\sin(t)$, $\mu(t) = 0.3 + 0.1\cos(t)$, $\rho(t) = 0.1(1+\sin(t))$, for $i = 1,\ 2$, $u_1(t) = 2\cos(t)$, $u_2(t) = 2.5\sin(t)$, and

we choose $\theta_1 = 0.6,\ \theta_2 = 0.4$.
By solving (3) in Theorem 3.1, using Matlab LMI toolbox, we can obtain the feasible solutions:

$$
\begin{aligned}
P &= \begin{pmatrix} 64.2344 & 0 \\ 0 & 55.2398 \end{pmatrix},\ Q = \begin{pmatrix} 264.2962 & 0 \\ 0 & 258.9128 \end{pmatrix},\\
R_1 &= \begin{pmatrix} 264.9188 & -47.3078 \\ -47.3078 & 292.2987 \end{pmatrix},\ R_2 = \begin{pmatrix} 261.4040 & 34.5731 \\ 34.5731 & 254.1741 \end{pmatrix},\\
W &= \begin{pmatrix} 252.1110 & 8.3404 \\ 8.3404 & 257.4912 \end{pmatrix},\ M_1 = \begin{pmatrix} 260.5774 & -45.8304 \\ -45.8304 & 271.3998 \end{pmatrix},\\
M_2 &= \begin{pmatrix} 354.3491 & -93.7484 \\ -93.7484 & 313.7145 \end{pmatrix},\ M_3 = \begin{pmatrix} 542.3147 & 28.4778 \\ 28.4778 & 581.5108 \end{pmatrix},\\
J_1 &= \begin{pmatrix} 201.6960 & 0 \\ 0 & 156.7445 \end{pmatrix},\ J_2 = \begin{pmatrix} 105.2631 & 0 \\ 0 & 126.2111 \end{pmatrix},\\
\chi &= \begin{pmatrix} 729.6485 & 0 \\ 0 & 761.8957 \end{pmatrix}.
\end{aligned}
$$

All the conditions in Theorem 3.1 are satisfied. Therefore, system (11) is globally dissipative, and

$$
\Upsilon_1 = \left\{ \begin{pmatrix} \zeta \\ \xi \end{pmatrix} \in \mathbb{R}^4 \middle|\ \zeta_1^2 + \zeta_2^2 \leq 2.0859,\ \xi_1^2 + \xi_2^2 \leq 2.0706 \right\}
$$

is a positive invariant and globally attractive set of (11). Using MATLAB/Simulink, the figures 1, 2, 3 and 4 represent the trajectories of concentrations of mRNA $\zeta(t)$, the trajectories of protein concentrations $\xi(t)$, the transients responses of mRNA concentrations $\zeta(t)$ and the transients responses of protein concentrations $\xi(t)$, respectively in the FGRNs equation (11). Through the simulation, the protein concentrations and mRNA enter and stay inside the set Υ_1.

Example 4.2. For $i = 1, 2$, considering the following FGRNs:

$$
\left\{
\begin{aligned}
\dot{\zeta}_i(t) &= -b_i\zeta_i(t) + \sum_{j=1}^{2} c_{ij} f_j(\xi_j(t-\tau(t))) + \bigwedge_{j=1}^{2} \gamma_{ij} f_j \xi_j((t-\eta(t)))\\
&+ \bigvee_{j=1}^{2} \delta_{ij} f_j(\xi_j(t-\eta(t))) + \bigwedge_{j=1}^{2} \alpha_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds\\
&+ \bigvee_{j=1}^{2} \beta_{ij} \int_{t-\sigma(t)}^{t} f_j(\xi_j(s))ds + \pi_i(t),\\
\dot{\xi}_i(t) &= -a_i\xi_i(t) + e_i\zeta_i(t-\mu(t)) + \bigwedge_{j=1}^{2} \bar{\gamma}_{ij}\zeta_j(t-\eta(t))\\
&+ \bigvee_{j=1}^{2} \bar{\delta}_{ij}\zeta_j(t-\eta(t)) + \bigwedge_{j=1}^{2} \bar{\alpha}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds\\
&+ \bigvee_{j=1}^{2} \bar{\beta}_{ij} \int_{t-\rho(t)}^{t} \zeta_j(s)ds.
\end{aligned}
\right. \tag{12}
$$

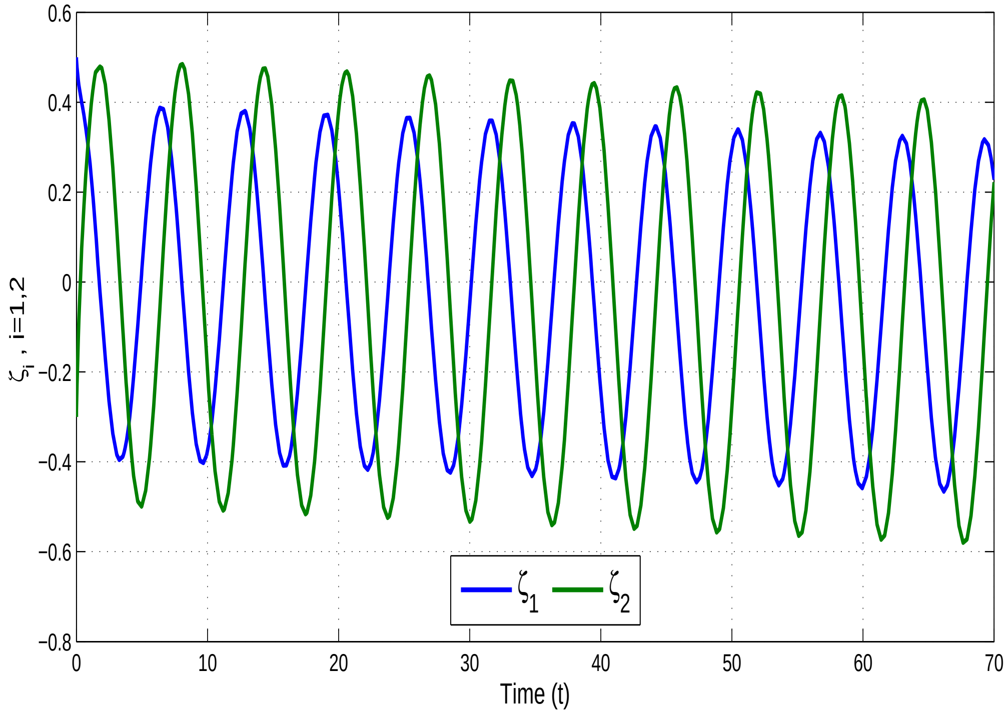

Figure 1. The trajectories of concentrations of mRNA $\zeta(t)$ in system (11)

The regulatory function is taken as

$$f_j(\xi_j) = \frac{\xi_j^2}{1+\xi_j^2}, \quad j = 1, 2,$$

i.e., the Hill coefficient is 2, so, it can be easily seen that: $F^+ = \text{diag}\{0.65,\ 0.65\}$. Let

$$\begin{aligned}
&B = \begin{pmatrix} 7 & 0 \\ 0 & 5 \end{pmatrix}, \ C = \begin{pmatrix} -0.7 & 0.5 \\ -0.9 & 0.4 \end{pmatrix}, \ \gamma = \begin{pmatrix} -0.4 & 0.9 \\ 0.8 & -0.3 \end{pmatrix}, \\
&\delta = \begin{pmatrix} 0.7 & 0.3 \\ 0.2 & 0.3 \end{pmatrix}, \ \alpha = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{pmatrix}, \ \beta = \begin{pmatrix} 0.3 & -0.1 \\ -0.5 & 0.3 \end{pmatrix}, \\
&A = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}, \ E = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.3 \end{pmatrix}, \ \bar{\gamma} = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.6 \end{pmatrix}, \\
&\bar{\delta} = \begin{pmatrix} 0.04 & 0.05 \\ 0.03 & 0.06 \end{pmatrix}, \ \bar{\alpha} = \begin{pmatrix} 0.05 & -0.04 \\ -0.05 & 0.07 \end{pmatrix}, \ \bar{\beta} = \begin{pmatrix} -0.09 & 0.08 \\ 0.06 & -0.03 \end{pmatrix}.
\end{aligned}$$

$\tau(t) = \eta(t) = 0.1 + 0.1\sin(t),\ \sigma(t) = \mu(t) = 0.2 + 0.1\cos(t),\ \rho(t) = 0.2(1+\sin(t))$, for $i = 1,\ 2$, $u_1(t) = 1.5e^{-0.1t}\cos(t),\ u_2(t) = 1.3e^{-0.1t}\sin(t)$, and we choose $\theta_1 = 0.1,\ \theta_2 = 0.15,\ \lambda = 1.2,\ \vartheta_1 = 0.3,\ \vartheta_2 = 0.7$.
All the conditions in Theorem 3.2 are satisfied. Therefore, system (12) is globally

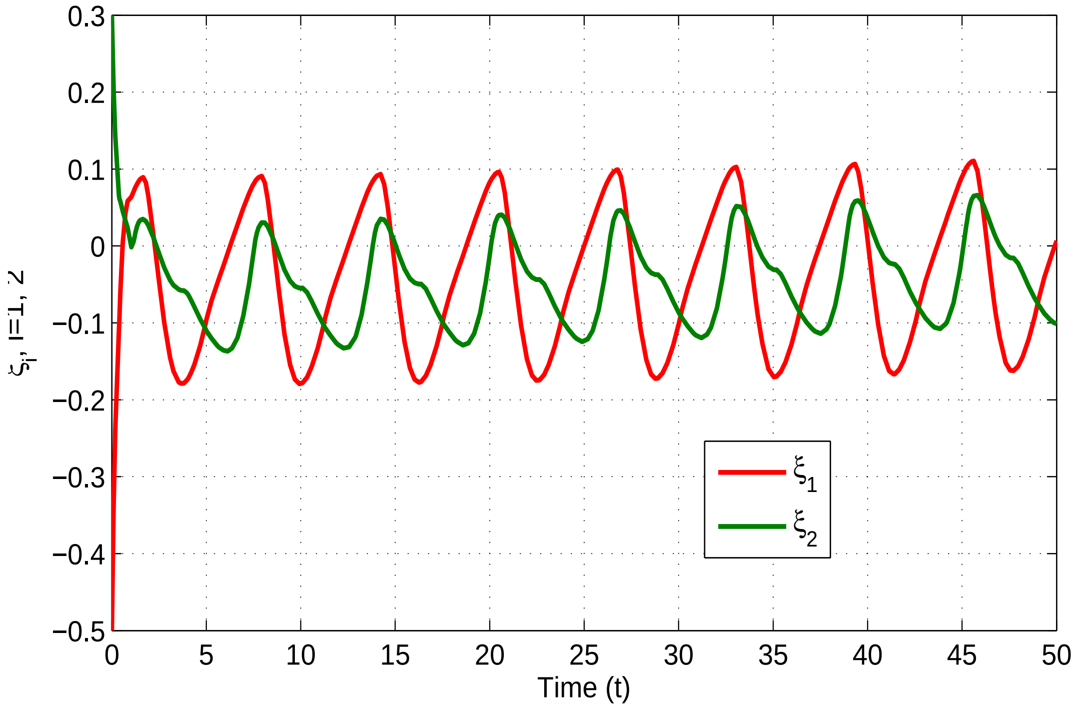

Figure 2. The trajectories of protein concentrations $\xi(t)$ in system (11)

exponentially dissipative, and

$$\Upsilon_2 = \left\{ \left(\begin{array}{c} \zeta \\ \xi \end{array} \right) \in \mathbb{R}^4 \middle| \; |\zeta_1| + |\zeta_2| \leq 1.6333, \; |\xi_1| + |\xi_2| \leq 1.4229 \right\}$$

is a positive invariant and globally exponentially attractive set of (12). Using MATLAB/Simulink, the figures 5, 6 and 7 represent the trajectories of concentrations of mRNA and protein, the transients responses of mRNA concentrations $\zeta(t)$ and protein concentrations $\xi(t)$, respectively in the FGRNs equation (12). Through the simulation, the protein concentrations and mRNA enter and stay inside the set Υ_2.

5. Conclusions

Fuzzy genetic regulatory networks is a hot research topic in biology and biomedicine. In this manuscript, we studied the global dissipativity problem for a class of FGRNs with mixed delays. By using the Lyapunov functionals and the LMIs approach, we obtained new sufficient conditions to guarantee the global dissipativity and global exponential dissipativity of our proposed network model. At last, two numerical examples with their simulations are presented to prove the applicability of our theoretical results. To the best of our knowledge, there have been no results on the global dissipativity of delayed fuzzy genetic regulatory networks until now. Hence, the obtained results are essentially new and can enrich the corresponding ones known in the literature. Our future work will focus on the investigation of

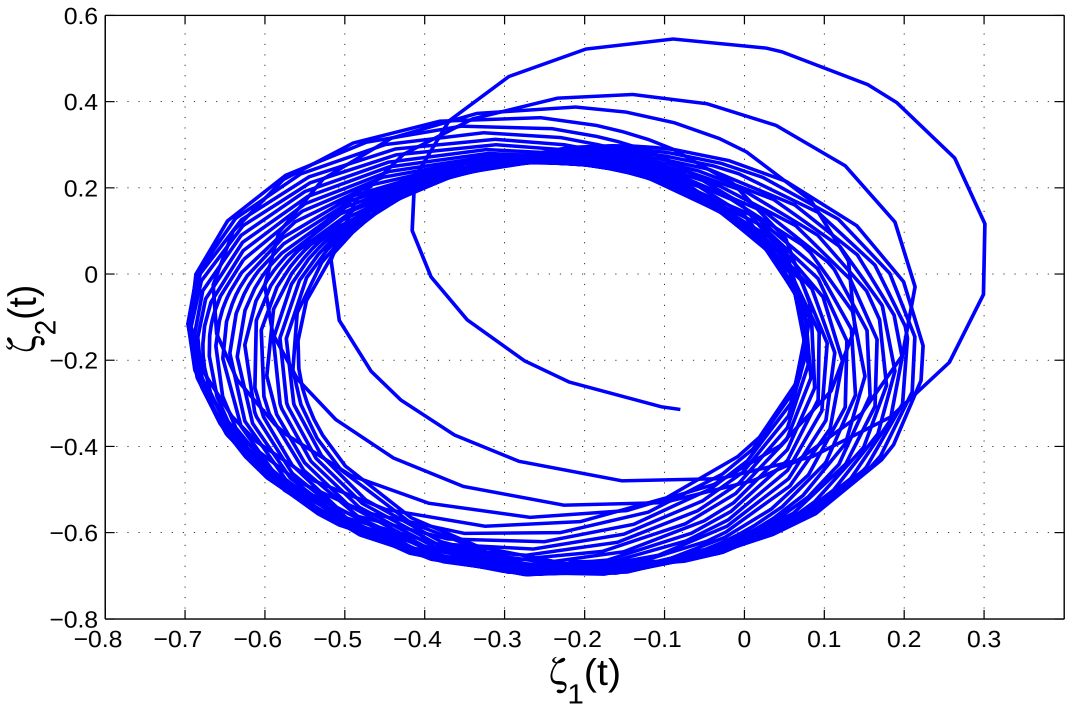

Figure 3. Transient response of mRNA concentrations $\zeta(t)$ in system (11)

the global dissipativity of fuzzy inertial genetic regulatory networks with mixed delays.

Disclosure statement

No potential conflict of interest was reported by the authors.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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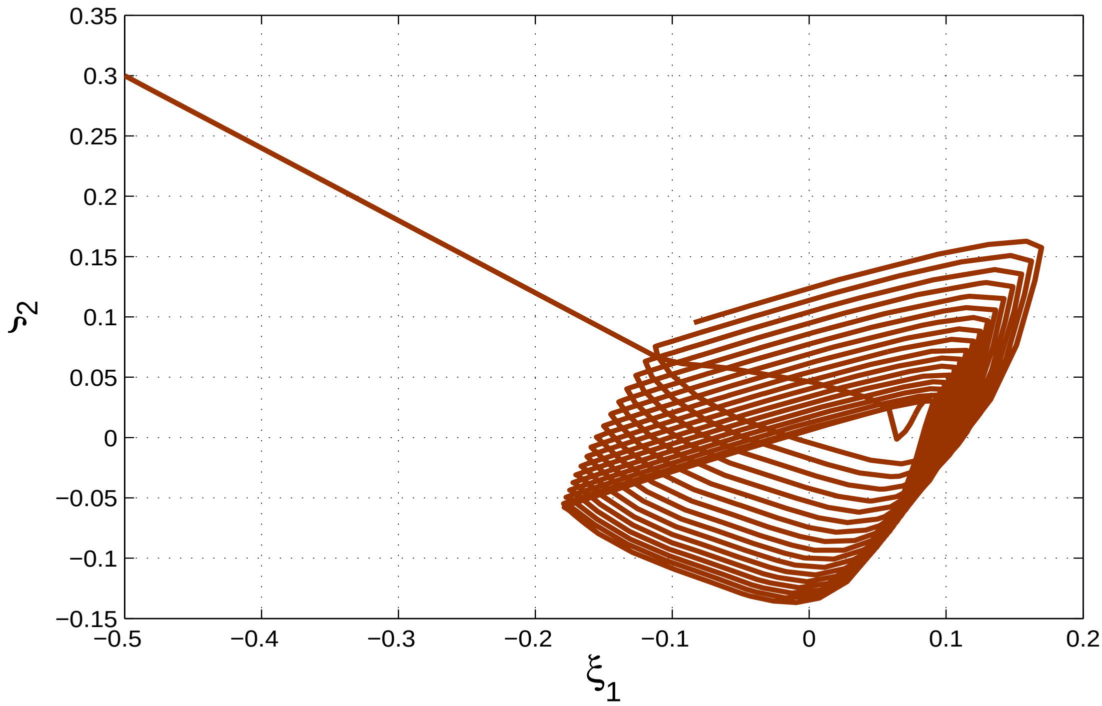

Figure 4. Transient response of protein concentrations $\xi(t)$ in system (11)

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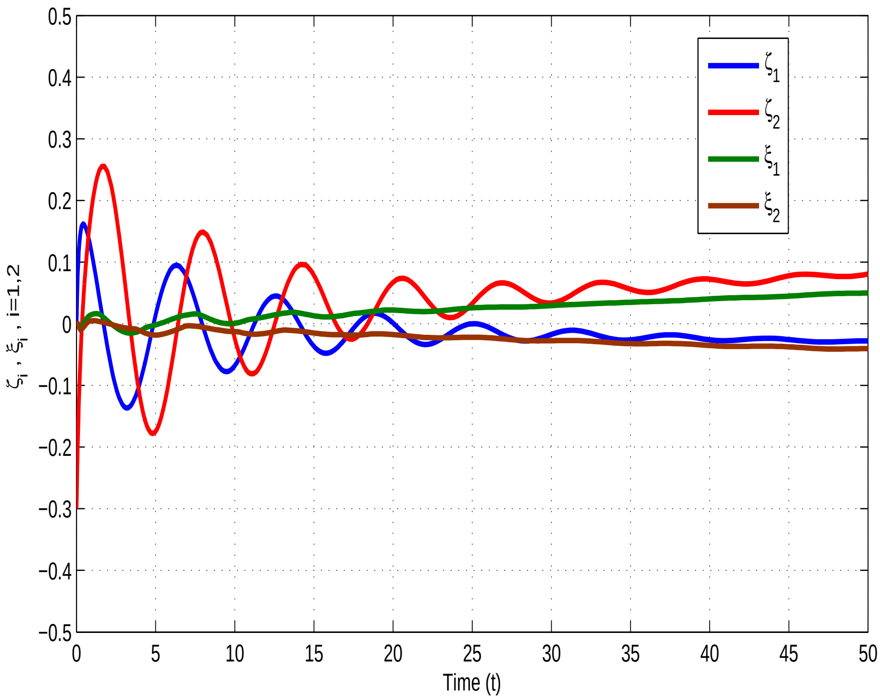

Figure 5. The trajectories of concentrations of mRNA and protein in system (12)

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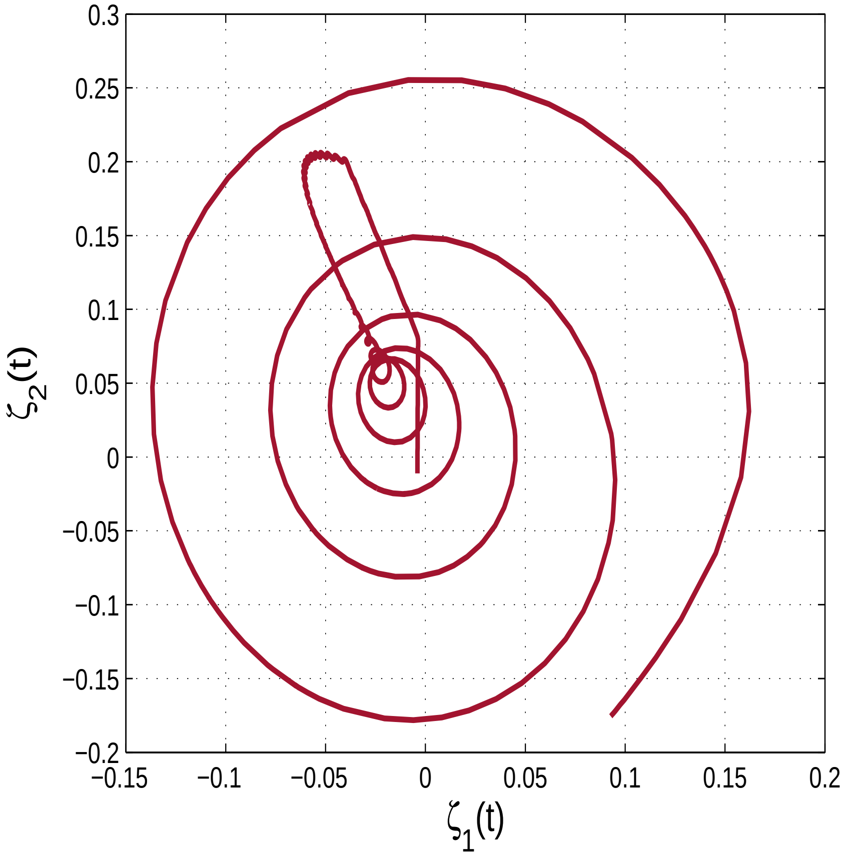

Figure 6. Transient response of mRNA concentrations $\zeta(t)$ in system (12)

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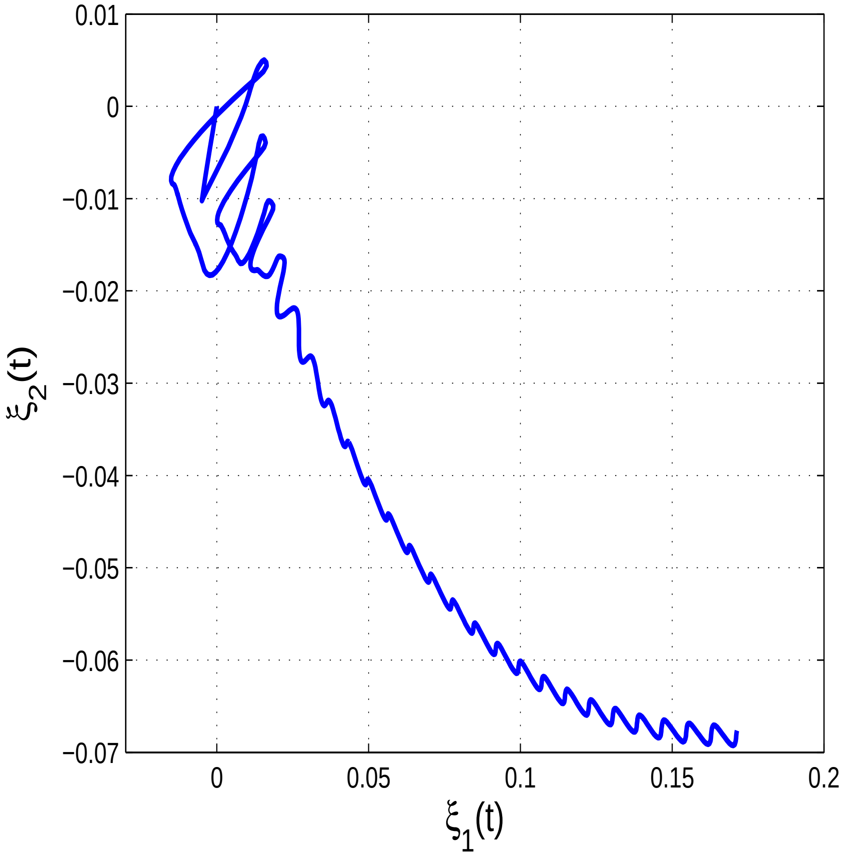

Figure 7. Transient response of protein concentrations $\xi(t)$ in system (12)

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