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1 **A BRAESS PARADOX EXPLOITATION FRAMEWORK FOR OPTIMAL TRAFFIC**
2 **BANNING STRATEGY**

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1 ABSTRACT

2 The transportation operators seek for control and management strategies to push the system to-
3 wards the system optimum in spite of selfish drivers who make decisions in a user optimal manner.
4 Motivated by the Braess paradox, we propose a framework for developing optimal traffic banning
5 strategies to improve the total travel time (TTT) of a network. More specifically, our goal is to
6 identify the links whose closure cause the Braess paradox. This problem belongs to the family of
7 discrete network design problem (DNDP), often formulated as a bi-level mixed-integer program.
8 While the literature offers several algorithms to solve the mathematical problem, additional efforts
9 are required to provide a solution method that guarantees global optimality in a reasonable com-
10 putation time for real-world urban networks. We propose a heuristic optimization method based
11 on link filtering to reduce the solution space for an exact method and evaluate its effectiveness
12 on two sub-areas of the Chicago-Sketch network. We have designed two experiments to validate
13 our approach and to evaluate the trade-off between computation time and solution optimality. Our
14 results show that the proposed framework converges to the same solution as the exact method in
15 most cases. Also, under a fixed computation time budget, the exact method could analyze only a
16 tiny fraction of all feasible solutions, while our method not only converged significantly fast but
17 also provided the same solution as the exact method. Lastly, we show that by implementing an
18 optimal traffic banning strategy, the system performance can be improved up to 40%.
19
20 *Keywords:* Network design problem, Braess paradox, User equilibrium, System optimum, Optimal
21 banning.

1 INTRODUCTION

2 The transportation network design problem (NDP) concerns with optimal decision-making on
3 modifying a network under cost constraints. The objective function of the NDP is to optimize
4 a system performance measure, e.g., minimizing the total travel time (TTT), while accounting for
5 the route choice behavior of drivers. The route choice behavior by itself is an optimization problem
6 known as traffic assignment problem (TAP), which is solved to optimize one or more objectives.
7 In road networks, fully rational drivers choose their path in a way to minimize their own travel
8 cost. This principle is called user equilibrium (UE) (Wardrop, 1). The network under the UE is
9 a situation in which no driver can change her path to improve her travel cost. By definition, the
10 UE does not tend to optimize the network performance compared to a situation wherein drivers
11 cooperate with each other in order to reach a social optimum condition (also known as the system
12 optimum (SO) principle). The ratio of TTT between these two principles is known as the "price
13 of anarchy" (PoA) (Koutsoupias and Papadimitriou, 2). Clearly, the ultimate goal of transporta-
14 tion operators is to keep the value of PoA close to 1. This fact has motivated many studies to
15 propose traffic management measures and strategies to ensure the selfish behavior of drivers re-
16 sults in the SO, i.e., decreasing the PoA, while the drivers follow the UE principle (Aashtiani and
17 Poorzahedy, Ma et al., Yao et al., 3–5).

18 To achieve this goal, in this paper, we employ a rather counter-intuitive but well-known
19 phenomenon in transportation referred to as the Braess paradox (Braess, 6). In his remarkable
20 work, Braess demonstrated that adding a new link to a network degrades its performance and in-
21 creases the TTT. There is a large number of theoretical and empirical implications of this paradox,
22 see e.g. (Aashtiani and Poorzahedy, Yao et al., Pas and Principio, Roughgarden, Gisches and
23 Rapoport, Ameli et al., Colini-Baldeschi et al., Bagloee et al., 3, 5, 7–12)). In this study, we aim to
24 answer the following question: in a given network with a given Origin-Destination (OD) matrix,
25 which roads should be closed to obtain the minimum TTT under the UE principle. In other words,
26 the goal is to find links whose closure cause the Braess paradox. Within the scope of this study, we
27 rely on static traffic assignment models. Despite their limitations in representing traffic dynamics,
28 they have proved useful particularly for design and management of road networks with the merit
29 of simplicity and computational efficiency Wang et al. (13).

30 In our problem, the decision variable is binary, i.e., closing a link or not. This problem
31 can be categorized as discrete NDP (DNDP). It is a mixed-integer problem by taking into account
32 the route choice behavior of drivers under the UE principle. This problem is often formulated via
33 bi-level programming (Yang and Bell, 14). Due to their intrinsic complexity, bi-level models (par-
34 ticularly when a discrete decision variable exists in either the upper or the lower level problem) are
35 considered as one of the most challenging global optimality problems in transportation (Ben-ayed
36 and Blair, Farvaresh and Sepehri, 15, 16). Nevertheless, several methods have been proposed to
37 solve it that can be categorized as exact or heuristics. The exact methods aim at finding the global
38 optimum. For example, Leblanc (17) proposed a branch and bound algorithm, which he solved
39 by a sequence of shortest path problems to improve computation time. Farvaresh and Sepehri (16)
40 revised the branch and bound algorithm and added a Steiner minimal tree inequality to reduce the
41 computation time. In a more recent study, Bagloee et al. (18) employed a generalized Benders' de-
42 composition to speed up the branch and bound algorithm. Using the Karush-Kuhn-Tucker (KKT)
43 conditions to solve the lower-level problem is another approach that has been used extensively,
44 see for example (Farvaresh and Sepehri, Fontaine and Minner, 16, 19). The application of exact
45 methods to real-world networks is restricted (Rey, 20). Therefore, many researchers have devel-

1 oped heuristic methods to find nearly optimal solutions for medium-size networks in a reasonable
2 duration of time by relaxing some of the properties of the problem. For instance, [Friesz et al.](#)
3 [\(21\)](#) proposed simulated annealing and [Poorzahedy and Abulghasemi, Poorzahedy and Rouhani](#)
4 [\(22, 23\)](#) developed a meta-heuristic approach based on ant colony systems. The main challenge
5 these algorithms face is that they may converge to a local optimum solution that is only not worse
6 than a solution in which the Braess paradox occurs. [Rey \(20\)](#) has conducted a comprehensive
7 comparison of the computation time for some of these methods. He argues that DNDP is NP-hard
8 and even medium-size instances of this problem are computationally challenging. Thus, there is a
9 need for developing stable and fast exact methodologies. In this paper, we propose a framework to
10 address this need.

11 Traditionally, the NDP in transportation involves selecting the best solution from a limited
12 number of possible expansion projects for an existing network under budget constraints. In our
13 case, we are interested in the inverse problem in which some links must be closed. In the litera-
14 ture, only very few studies have formulated the NDP in this way. [Bagloee et al. \(12\)](#) proposed a
15 surrogate-based algorithm for identifying the links that need to be closed in a network. The authors
16 have used supervised machine learning to train a multivariate linear regression model that treats
17 the original objective function, TTT, as a black box. While such an approach may provide optimal
18 results, it lacks in-depth explanatory power to be accepted as a generic solution. [Ma et al. \(4\)](#) have
19 suggested link restrictions as a traffic management measure to cope with the Braess paradox under
20 various demand scenarios. They have adapted the SO-relaxation method of [Wang et al. \(24\)](#) to find
21 links that cause Braess Paradox. However, their method cannot be applied to real-world networks
22 because of the extensive computation time required by the SO-relaxation method. Moreover, they
23 have used the interdiction constraint from the original method, which imposes a minimum number
24 of two changes between the possible network designs, whereas the intended solutions should seek
25 for only one change. Later in section 3 of this paper, we put this issue under scrutiny. [Wu and Zhu](#)
26 [\(25\)](#) have also looked into the features of the links that may cause the Braess paradox. By running
27 series of experiments, they have concluded that the minimum saturation rate is a notable common
28 feature among such links. [Sun et al. \(26\)](#) have come to a very similar conclusion by studying the
29 network of San Francisco and show that low value of volume over capacity is an indicator for
30 a link to be inefficient in the network, and its removal could have positive impacts on the TTT.
31 [Ameli et al. \(10\)](#) have developed a simulation-based framework to compare OD path flows under
32 different principles, including UE and SO. Furthermore, by introducing the concept of demand
33 breakpoints at which routing decisions are significantly changed, the authors propose a method
34 that could detect the Braess paradox and close these roads to improve the network performance.

35 Our main goal in this paper is to extract the best subnetwork out of a given network that
36 has the minimum TTT under the UE principle compared to the original network. Mathematically
37 speaking, this problem can also be formulated as a 0-1 Knapsack optimization problem. The family
38 of Knapsack problems considers a set of items, each having an associated profit and weight. The
39 problem is then to select a subset of items such that the profit is maximized without exceeding the
40 capacity of the knapsack [Pisinger et al. \(27\)](#). In 0-1 Knapsack problems, each item may be selected
41 at most once. Here, items are the set of all built traffic network roads (links), and the Knapsack
42 is a subset of links to be banned. The Knapsack has no capacity limit. The optimal sub-network
43 obtained from banning the links belong to the most profitable Knapsack should serve the demand
44 (i.e., keep all ODs connected), while providing minimum TTT under the UE conditions. Note
45 that the number of candidates (binary decision variables) in this problem is equal to the number of

1 links in the original network. Thus, regardless of the complexity of the constraints, the problem
2 becomes computationally expensive by increasing the number of links. To overcome this curse of
3 dimensionality, we propose a framework to identify a set of candidate links, which has the most
4 potential to be banned and contribute to the problem. In other words, we aim to decrease the
5 number of items in the 0-1 Knapsack problem in order to solve it faster.

6 In this study, we propose a filtering methodology to reduce the cardinality of the set of links
7 whose closure cause the Braess paradox. This enables us to solve the optimal banning strategy
8 problem within a reasonable computation time for even medium-size real-world networks. To the
9 best of our knowledge, such an approach to solving the NDP has not been proposed in the literature.
10 First, we carry out a feature selection procedure to identify relevant link features. Second, we
11 build a framework to analyze links features evolution under the UE and SO principles to highlight
12 and extract the potential candidates. Third, we solve the problem with an exact method for the
13 potential candidates. The application of the proposed framework to a real network shows that
14 by considering a small subset of links (less than 5% of the total number of links) as potential
15 candidates, the computation time decreases dramatically. Finally, we compare our solution with
16 the global optimum solution obtained by the exact method without any filtering. The results show
17 that the impact of the roads that are not detected by our method on the system performance is
18 neglectable in comparison to the global optimum solution found by the exact method. Besides the
19 proposition of a new methodology, we review the most advanced exact method in the literature and
20 rectify an error in the solution method, which is used in some studies.

21 The remainder of this paper is structured as follows: in the next section we first review
22 the fundamentals of the DNDP and its bi-level programming formulation and then present our
23 solution. In section case study, we introduce our study network in which we evaluate the efficiency
24 of our methodology and compare it with an exact solution approach. In section results, we present
25 our results from the case study and discuss our findings, followed by some concluding remarks in
26 the last section.

27 **METHODOLOGICAL FRAMEWORK**

28 In this section, we first formulate the original problem as a leader-follower game. This game can
29 be formulated as a bi-level programming model. The upper-level problem is to find the best sub-
30 network in which the difference between the sub-network TTT under the UE assignment and the
31 original network TTT under the SO assignment is minimized. We formulate this problem as a 0-1
32 Knapsack problem. The lower-level problem deals with solving the TAP under the UE. Before we
33 continue with the problem formulation, we provide an overview of the notations used in this paper
34 in table 1.

35 **Problem formulation**

36 Let us denote a given transportation network with a directed graph $G = (V, E)$ where V and E are
37 the sets of nodes and links, respectively. We define the 0-1 Knapsack problem as follows: for
38 each link $(i, j) \in E$ we define an associated weight and profit. The objective is to choose a subset
39 of links from the set of all links E to maximize the profit without exceeding the capacity i.e. the
40 number of links in the network $|E|$.

41 For the sake of simplicity, in this paper, we assume the weight of all links are equal to one.
42 The difficulty of the solution lies in the calculation of the value of the profit for each link. One main
43 difference of our 0-1 Knapsack problem with the classic one is that the value of profit of each item

TABLE 1: Nomenclature

E	Set of all links in the original network.
\bar{E}	Set of all open links in the solution subnetwork.
Λ	Set of candidate links to be considered for solution algorithm; $\Lambda \subseteq E$.
V	Set of all nodes in the network.
D	Set of all destination nodes; $D \subseteq V$.
(i, j)	Link from node i to node j ; $i, j \in V$; $(i, j) \in E$.
rs	OD pair of a trip from node r to node s ; $r \in V$; $s \in D$.
u_{ij}	Binary decision variable for the link from node i to j to indicate whether it is opened or not.
\bar{u}_{ij}	Binary decision variable for the link from node i to j to indicate whether it is banned or not.
t_{ij}	Travel time function of link (i, j) .
t_{ij}^0	Free flow travel time of link (i, j) .
q_{rs}	Traffic demand between origin r and destination s .
Q	The OD matrix, $Q = [q_{rs}]$, $\forall r, s \in V$.
x_{ij}	Traffic flow on link (i, j) .
x_{ij}^{SO}	Traffic flow on link (i, j) under the SO principle.
x_{ij}^{UE}	Traffic flow on link (i, j) under the UE principle.
$x_{ij,s}$	Traffic flow on link (i, j) travel to destination node s .
Ω_u	Set of all feasible network designs.
$\bar{\Omega}_u$	Set of all explored network designs.
Ω_x	Set of all feasible traffic flows.
Ω_f	Set of all feasible path flows.
M	An arbitrarily large number.
λ_{ij}^θ	Value of feature θ for link $(i, j) \in \Lambda$.
ξ_{ij}	Normalized difference between traffic flow on link (i, j) under the UE and SO principle.
Θ	Vector of all considered link features.
$Z(x)$	Objective function of the original problem.
$Z_{RP}(x)$	Objective function of the relaxed problem.
f_k^{rs}	Traffic flow on path k between OD pair (r, s) .
$\delta_{ij,k}^{rs}$	Binary variable, 1 if link (i, j) is part of path k between r and s , 0 otherwise.
$solution_{RP}$	Set of tuple of banned links and links flows obtained form the solution of the relaxed problem.

1 varies depending on the other selected items. More specifically, because of the interdependencies
2 among the links of a network, the value of profit for banning a link depends on which other links
3 in the network are also banned. In our problem, the profit is defined as how much the TTT in the
4 solution subnetwork under the UE gets closer to the TTT under the SO of the original network. We
5 can obtain this profit by solving the lower-level problem which is a classic UE traffic assignment
6 problem.

7 Let D be the set of destination nodes and Q the OD matrix for this network with elements
8 q_{rs} that show the demand travelling from node $r \in V$ to destination $s \in D \subseteq V$. Let x_{ij} and t_{ij}
9 represent the traffic flow and travel time on link $(i, j) \in E$ where $t_{ij}(\cdot)$ represents the volume-delay
10 function, which is usually modelled as a strictly increasing and convex function of x_{ij} to make sure
11 the traffic flow on each link under UE has a unique solution (Ameli et al., 28). In this paper, we

1 use the well-known BPR function:

$$2 \quad t_{ij}(x_{ij}) = t_{ij}^0 \left(1 + \beta_{ij} \left(\frac{x_{ij}}{c_{ij}}\right)^{\eta_{ij}}\right) \quad (1)$$

3 where t_{ij}^0 and c_{ij} are the free-flow travel time and capacity of link (i, j) , respectively. β and η
4 are the model parameters. Let us denote a feasible solution subnetwork with $\bar{G} = (V, \bar{E})$. We can
5 then rewrite the classical profit maximization Knapsack problem as the following minimization
6 problem:

$$\min Z(x) = \sum_{(i,j) \in \bar{E}} x_{ij} t_{ij}(x_{ij}) - \sum_{(i,j) \in E} \tilde{x}_{ij} t_{ij}(\tilde{x}_{ij}) \quad (2)$$

7 where x_{ij} is obtained from the UE assignment on the designed subnetwork and \tilde{x}_{ij} from the SO
8 assignment on the original network. $\bar{E} \subseteq E$ is the set of all links that remain open in the designed
9 subnetwork i.e. $\bar{E} := \{(i, j) \mid u_{ij} = 1; \forall (i, j) \in E\}$. u_{ij} is the binary decision variable indicating if
10 link (i, j) is open or not.

11 Since demand is given and fixed, the second term in the objective function (2) is positive
12 and constant. Thus, the objective function becomes simply to minimize the first term. We can then
13 re-write the upper-level program as follows:

$$\arg \min_x \sum_{(i,j) \in \bar{E}} x_{ij} t_{ij}(x_{ij}) \quad (3)$$

14 subject to:

$$t_{ij}(x_{ij}) = t_{ij}^0 \left(1 + \beta_{ij} \left(\frac{x_{ij}}{c_{ij}}\right)^{\eta_{ij}}\right) \quad (4)$$

$$u_{ij} \in \{0, 1\}; \quad \forall (i, j) \in E \quad (5)$$

15 where x_{ij} is the solution of the lower-level (UE) problem (Beckmann et al., 29):

$$\arg \min_x \sum_{(i,j) \in \bar{E}} \int_0^{x_{ij}} t_{ij}(\omega) d\omega \quad (6)$$

16 subject to:

$$\sum_{j \in V: (i,j) \in E} x_{ij,s} - \sum_{j \in V: (j,i) \in E} x_{ji,s} = q_{is} \quad \forall i \in V, \forall s \in D \quad (7)$$

$$\sum_{s \in D} x_{ij,s} = x_{ij}; \quad \forall (i, j) \in E \quad (8)$$

$$q_{ss} = - \sum_{i \in V} q_{is}, \forall s \in D \quad (9)$$

$$x_{ij,s} \geq 0; \quad \forall (i, j) \in E \quad (10)$$

17 where constraints (7) and (9) assure flow conservation and that the demands of all destinations
18 will be served. Besides, they also balance the inflow and the outflow in each node. Constraint (8)

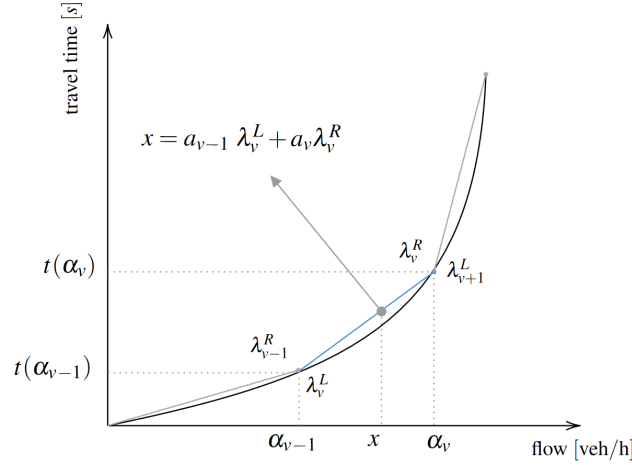


FIGURE 1: Piecewise linear approximation of BPR. function

1 ensures that the total flow on each link is the sum of the flows of all destinations on that link. In
 2 order to take advantage of the state-of-the-art mixed-integer linear programming (MILP) solvers,
 3 we must use a piecewise linear approximation of the original BPR function 1. For that, we cut the
 4 link flow x_{ij} into m_{ij} segments between values $\alpha_{ij,n-1}$ and $\alpha_{ij,n}$ ($n \in [1, m_{ij}]$). The goal is to find
 5 $\alpha_{ij,n}^L$ and $\alpha_{ij,n}^R$ such that:

$$x_{ij} = \sum_{n=1}^{m_{ij}} \alpha_{ij,n} \lambda^L + \alpha_{ij,n} \lambda^R \quad (11)$$

6 Figure 1 provides an illustration of the linear approximation of the BPR function. Moreover, we
 7 need to introduce an arbitrary large number M in the function to ensure the travel time on closed
 8 links is high enough that no traffic will be assigned to them. As a result, we rewrite the travel time
 9 function $t_{ij}(\cdot)$ as follows:

$$t_{ij}(x_{ij}) = t_{ij}^0 \left[1 + \frac{\beta_{ij}}{\alpha_{ij}} \sum_{n=1}^{m_{ij}} (\alpha_{ij,n} \lambda^L + \alpha_{ij,n} \lambda^R) \right] + (1 - u_{ij}) M \quad (12)$$

10 **The exact solution method: SO-relaxation approach**

11 Solving the lower-level program is relatively straightforward because the link travel times and the
 12 constraints are linear and convex. Therefore, the solution procedure is to enumerate the finite set
 13 of all feasible subnetworks Ω_u to find the optimal solution u^* . However, practically, this approach
 14 is computationally prohibitive as the cardinality of Ω_u increases exponentially. In order to rectify
 15 this problem, the SO-relaxation approach was proposed by Wang et al. (24). The idea is to use
 16 the SO principle as a good approximate of the optimal solution for the UE principle and sort all
 17 the solutions successively in the order of increasing TTT. Optimal solution is guaranteed when the
 18 TTT for the unexplored solutions under the SO is not smaller than an already explored solution
 19 under the UE principle. The authors have used this methodology for adding new lanes and links to
 20 a network under budget constraints. We adapt their exact method and use it as the reference solution
 21 for our problem to develop an optima traffic banning strategy. The principle of the SO-relaxation
 22 method is described below.

1 In the first step, a relaxed problem (RP) is defined instead of the original problem in which
 2 the UE assignment constraint is replaced with the SO assignment. This single-level optimization
 3 problem is much easier to solve and yields a lower bound on the optimum of the original problem.
 4 Such a lower bound can serve as a starting point of an iterative process. More specifically, all
 5 solutions $u \in \Omega_u$ are sorted in the order of increasing based on their SO travel time. Then, from
 6 the top of that list, the corresponding UE traffic assignment is solved. The travel time under the
 7 UE assignment of each iteration serves as an upper bound for the next possible solution. This
 8 iterative process is repeated until the resulting SO travel time of a solution (lower bound) is larger
 9 than the UE travel time of the previous solution (upper bound). Wang et al. (24) proved that the
 10 final solution derived from this procedure is the global optimum. In other words, there is no need
 11 to explore any of the remaining feasible solutions because the UE-based travel time of all the
 12 remaining unexplored designs cannot be less than that SO-based travel time. We formulate the
 13 objective function of the relaxed problem referred to as Z_{RP} as follows:

$$\arg \min_{u \in \Omega_u, x \in \Omega_x} Z_{RP} = \sum_{(i,j) \in E} x_{ij} t_{ij}(x_{ij}) \quad (13)$$

14 where Ω_x and Ω_f denote the set of link and path flows, defined by:

$$\Omega_x := \{x \mid x_{ij} = \sum_r \sum_s \sum_k f_k^{rs} \delta_{ij,k}^{rs}; \quad \forall (i,j) \in E, \quad \forall f_k^{rs} \in \Omega_f\} \quad (14)$$

$$\Omega_f := \{f \mid \sum_k f_k^{rs} = q_{rs}; \quad \forall r, s \in V\}. \quad (15)$$

15 Note that the constraints of the UE assignment are already incorporated in the domain $x \in \Omega_x$
 16 and $u \in \Omega_u$. While iterating this process, an interdiction constraint is imposed in order to avoid
 17 exploring an already tested solution in the process:

$$\sum_{(i,j) \in E} [(1 - u_{ij}) \bar{u}_{ij} + (1 - \bar{u}_{ij}) u_{ij}] \geq 1, \forall \bar{u} \in \bar{\Omega}_u \quad (16)$$

18 where $\bar{\Omega}_u$ is the set of already explored solutions. The pseudo-code of this iterative process is
 19 presented in Algorithm 1.

20 It is worth to mention that in Wang et al. (24) the right side of the inequality in the constraint
 21 (16) is set to 2 instead of 1. This implies that the current network that is being explored must have
 22 at least two differences with all of the already explored networks, which is not intended by the
 23 proposed algorithm. Apparently, this error roots in the numerical example provided wherein a
 24 hypothetical scenario, the addition of two potential links to the network is being investigated and
 25 the constraint of the example is used for the general scenario as well. Some of the existing literature
 26 such as (Ma et al., Rey, 4, 20) have used the same constraint which implies that their solutions are
 27 not guaranteed to be global optimum .

28 Proposed solution method: identifying candidate links

29 Despite the significant reduction in computation time, the SO-relaxation method is still impractical
 30 for large-scale networks. Therefore, we propose a heuristic approach to further reduce the size of
 31 the solution space that the SO-relaxation algorithm must explore. We postulate that the optimal
 32 or nearly optimal solution is found by considering only a small fraction of the links in a network.

Algorithm 1: Pseudo-code for the SO-relaxation solution method

```

 $\bar{\Omega}_u \leftarrow []$ ;
 $u^*, u_{temp}^* \leftarrow []$ ;
 $UB \leftarrow +\infty$ ;
 $LB \leftarrow 0$ ;
 $Z_{RP} \leftarrow +\infty$ ;
converged  $\leftarrow False$ ;
while converged == False do
  solutionRP = solve((13), s.t. inequality (14)-(16));
  if |solutionRP|  $\geq 0$  then
    return  $u^*$ ;
    break;
  else
     $(u_{temp}^*, \tilde{x}_{ij}) \leftarrow$  solutionRP;
     $Z_{RP} \leftarrow \sum_{(i,j) \in E} \tilde{x}_{ij} t_{ij}(\tilde{x}_{ij})$ ;
     $x_{ij} =$  solve((6), s.t. (7)-(10));
     $LB \leftarrow \sum_{(i,j) \in E} x_{ij} t_{ij}(x_{ij})$ ;
    if  $LB \leq UB$  then
       $UB \leftarrow LB$ ;
       $u^* \leftarrow u_{temp}^*$ ;
       $\bar{\Omega}_u = \bar{\Omega}_u \cup u_{temp}^*$ 
    else
       $\bar{\Omega}_u = \bar{\Omega}_u \cup u_{temp}^*$ 
    end
    if  $Z_{RP} \geq UB$  then
      converged == True;
      return( $u^*$ );
      break;
    end
  end
end

```

1 In our approach, the problem formulation and the solution algorithm remain the same as before.
2 The main difference is that we provide a list of candidate links that are more likely to exhibit the
3 Braess paradox and reduce the solution space to these candidate links only. In order to identify
4 such candidate links, we use some of the key features of the links in the network. The goal is to
5 significantly decrease the computation time to make the solution method practical, while the final
6 solution remains very close to that of the global optimum.

7 In order to identify which features are important for our problem, like any other learning
8 process, we need to collect a reasonable number of Braess occurrences. For a given network and
9 demand profile, we do this in an iterative process as depicted in the flowchart of Figure(2). We
10 scale the given demand by a global factor at each iteration step and solve the SO and UE traffic
11 assignments. Afterwards, we run algorithm 1 to identify the set of links that cause the Braess
12 paradox, i.e., $E - \bar{E}$. At this point, we can calculate the value of each feature. In this paper, we

1 refer to each feature by θ and denote its value for link (i, j) with λ_{ij}^θ . We define λ_{ij}^θ as the relative
 2 ranking position of link (i, j) after sorting in the order of increasing all links in the network based
 3 on the value of θ . The value of each feature for link (i, j) as $\lambda_{ij}^\theta, \forall \theta \in \Theta$. We define the value of
 4 θ as the relative ranking position of link (i, j) after sorting in the order of increasing all links in
 5 the network based on the value of θ . In order to decide which features are important, we compare
 6 the summation of λ^θ with a predefined threshold f . Only features that are above this threshold are
 7 considered for the link filtering, which is the last step in our procedure. Here, we set a cut-off score,
 8 i.e., p^{th} percentile for each of the important features and filter links that are only above this score.
 9 Consequently, in our method, the solution space, i.e., E in objective function (13), is replaced with
 10 Λ defined as:

$$\Lambda := \{(i, j) \mid \lambda_{ij}^\theta \geq P_{10\%} \quad \forall \theta \in \Theta\} \quad (17)$$

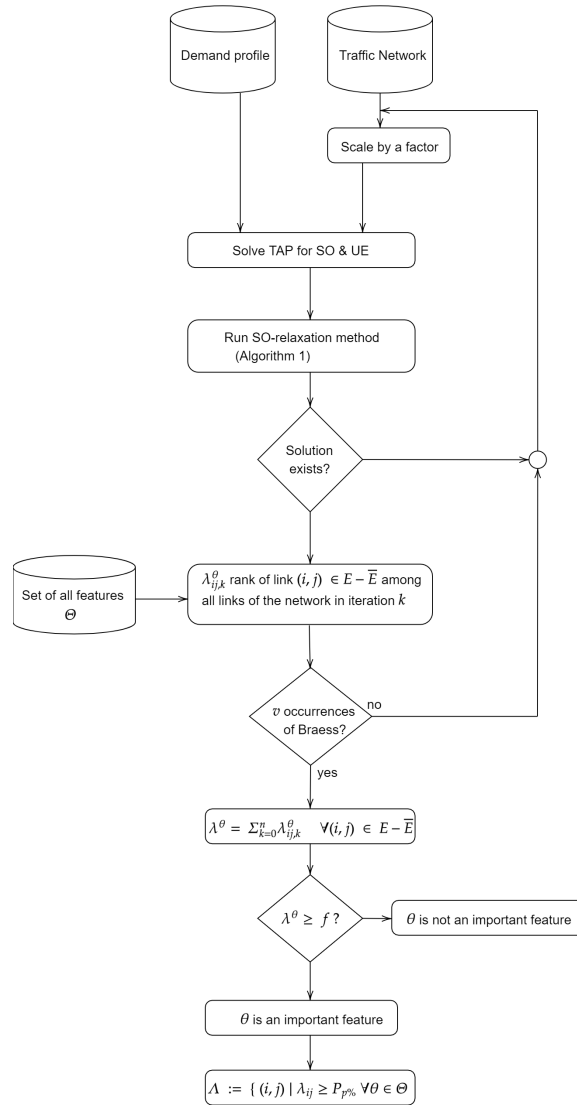


FIGURE 2: The flow chart of the proposed feature selection procedure and filtering process

1 CASE STUDY

2 In this section, we present the toy networks that we considered in order to evaluate the efficiency of
 3 our method. There are several transportation networks with demand provided by [Stabler et al. \(30\)](#)
 4 that have been extensively used in transportation studies [Amini et al., Tilg et al. \(31, 32\)](#). From
 5 this repository, we consider two medium-size areas in the network of the Chicago-Sketch that
 6 allow us to conduct our benchmark experiments. Because, as mentioned before, calculation of the
 7 SO-relaxation is still very expensive on these networks but feasible by normal computer resource.
 8 Therefore, we can have a reference point (the global optimum solution) in order to evaluate the
 9 quality of our solution. The two considered areas of the Chicago-Sketch are depicted in Figure
 10 3 highlighted by the red (hereafter called network 1) and the green (hereafter called network 2)
 11 rectangles. Some general information regarding the size of each of these areas is provided in table
 12 2. We perform two types of experiments on these two networks. But before explaining the details
 13 of these experiments, we first need to discuss the settings of our feature selection and link filtering
 14 procedure.

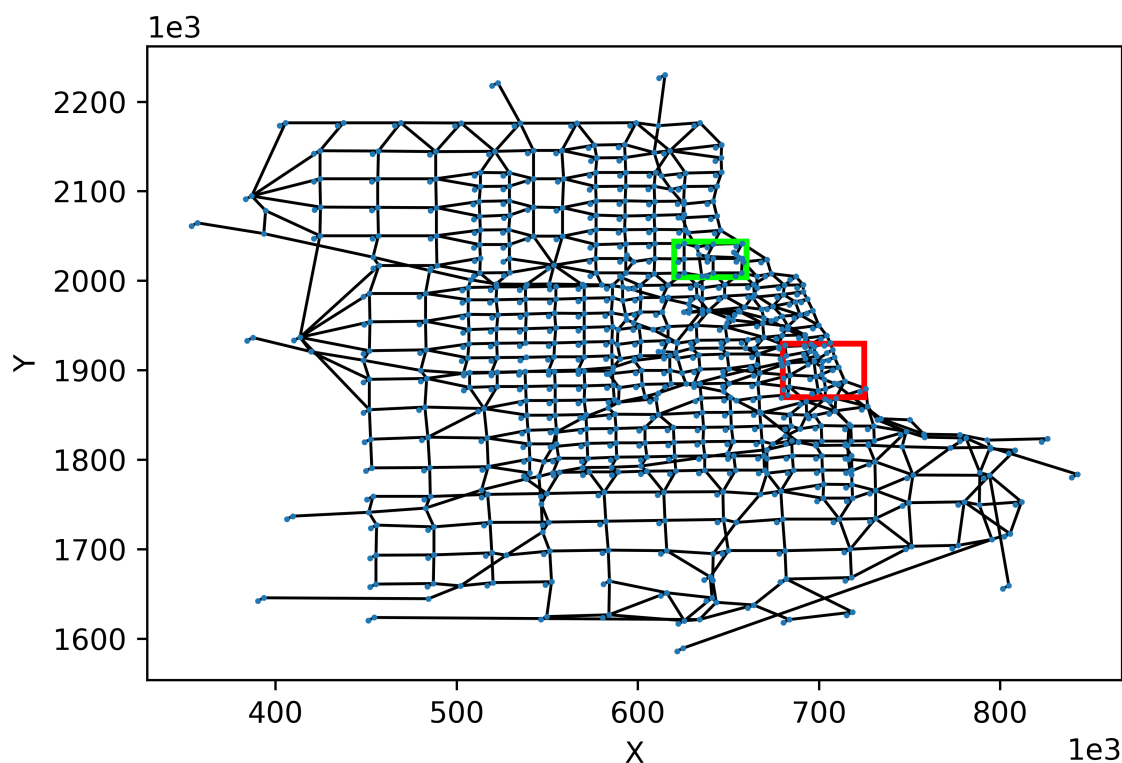


FIGURE 3: The targeted areas from the network of Chicago-Sketch ([Stabler et al., 30](#)): Network 1 boundaries are highlighted by red, and the green rectangle defines the boundaries of Network 2.

TABLE 2: Test networks characteristics

Network	No. nodes	No. links	No. OD pairs	Scenarios [Demand factors]
1	48	136	210	[2.8, 3.0, 3.5, 4.0, 4.2]
2	26	66	72	[8.9, 9.0, 9.1]

1 Settings of the feature selection procedure

2 As mentioned in the methodology, we design a feature selection process intending to find the
 3 most relevant link features to our problem. After identifying the feature(s), we apply the filtering
 4 procedure and pre-select a set of links. These selected links make the set of candidate links that
 5 we believe it contains most likely links belongs to the final optimum solution based on the learned
 6 feature(s). In this part, we specify the details of that general procedure for the current case study.
 7 First, we select network 1 and the provided demand as the inputs. The next step is to define the
 8 initial set of features Θ that include free flow travel time (fftt), capacity, flow under the UE (x_{ij}^{UE})
 9 and the SO assignment (x_{ij}^{SO}) and their difference as well as an indicator that reflects the centrality
 10 of a link - by counting the number of paths that a certain link is part of them. Please note that
 11 this set can be extended by considering different aspects. Then, we basically run the algorithm of
 12 flowchart (2) until we collect 50 occurrences of Braess paradox. Please note that, 50 refers to the
 13 number of links and not the number of iterations. Lastly, we set the feature value threshold and
 14 the cut-off score, which depending on computation resources and network size can be adjusted, to
 15 75% and 10th percentile, respectively.

16 Experiments setup

17 We conduct two different experiments on both networks. In the first experiment, the exact method
 18 is executed to obtain the global optimum solution, as the reference, in a number of different demand
 19 scenarios. These demand scenarios are derived by scaling up the demand by a global factor as
 20 provided in table 2. The number of demand scenarios and the factor for which we observed the
 21 Braess paradox are different for each network. This is due to the fact that the Braess paradox occurs
 22 only when demand lies within an intermediate range of values (Pas and Principio, 7). Once the
 23 results from the exact method are ready, we re-run the SO-relaxation method on the set of candidate
 24 links, obtained from the previous step. Finally, we compare the optimality of the obtained solution
 25 with the reference solution. In this experiment, we investigate the effectiveness of our filtering
 26 method in terms of providing a solution with an objective value, TTT, closed to the exact method.

27 Basically, the exact method has to explore a large number of feasible solutions (maximum
 28 the total number of possibilities ($2^{|E|}$) to evaluate different traffic banning strategies for each sce-
 29 nario). Note that the number of evaluations cannot be predicted until the algorithm converges to the
 30 optimum solution; however, typically, on large real-world networks, this could take up to weeks or
 31 months if a solution exists. For the second experiment, the performance of the proposed solution
 32 method is evaluated in front of the the exact method. We impose a computation time limit of three
 33 hours and compare the solution found by each method. The motivation for this experiment is to
 34 account for some reasonable computation time that the transportation planner may consider while
 35 making a decision which algorithm to implement. With this experiment, we aim to investigate the
 36 trade-of between computation time and the accuracy of the solution methods.

37 RESULTS

38 In this section, we first present the results of our feature selection procedure that is used to identify
 39 the relevant features for link filtering. Then, we provide the results of the first experiment in which
 40 we show the validation of the proposed methodology. After that, we discuss the second experiment
 41 and evaluate the performance of both solution methods under computation time budget. Lastly, we
 42 discuss the effectiveness of traffic banning strategy as a traffic management measure to improve
 43 the overall system performance.

1 Analysis of the link features

2 After a careful analysis of the 50 (link) occurrences of the Braess paradox, we found only one
 3 feature to be relevant among all the selected features. In Figure 4a, we present the value of each
 4 feature for every link in the solution of the exact method i.e. links that exhibited the Braess paradox.
 5 We can easily see, in the figure, that only two features seem to be relevant. By calculating the λ^θ
 6 for these two features it becomes clear, as shown in Figure 4b, that only one feature namely the
 7 flow difference between the UE and SO ($x_{ij}^{UE} - x_{ij}^{SO}$) should be considered for link filtering. Note
 8 that this feature is significantly correlated with PoA; however, we are going to investigate whether
 9 all links in the optimal banning solution always belong to the top candidate links ranked by this
 10 feature or not. Note that, according to the characteristics of a network, banning a link with a high
 11 value of $x_{ij}^{UE} - x_{ij}^{SO}$ does not necessarily improve the system. It can even provide an infeasible
 12 sub-network, e.g., disconnecting some OD pairs. Therefore, we consider this feature for further
 13 investigation and validation in the next subsections through the filtering method. Regarding the
 14 second-best feature, intuitively, travel time on a link depends on the link travel time function $t_{ij}(\cdot)$
 15 in which the ffft is only one parameter. In our study, other parameters of this function are identical
 16 for all links (η and β in BPR function) which can be another reason that why the ffft is not a
 17 representative feature for the filtering procedure.

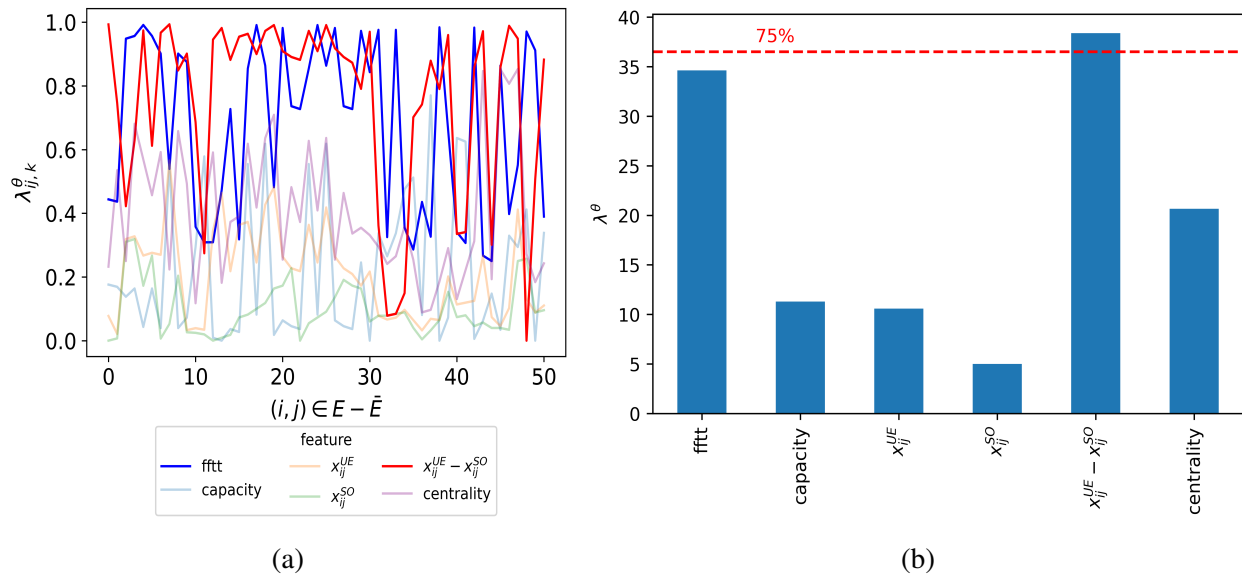


FIGURE 4: (a) Ranking position of the feature with respect to the rest of the links in the corresponding network and (b) Number of times a feature appears in the top 10%.

18 In order to be able to compare the value of $x_{ij}^{UE} - x_{ij}^{SO}$ in different scenarios and links with
 19 different levels of congestion, we normalize its value by dividing it by the x_{ij}^{UE} . Consequently, we
 20 consider a single feature $\xi_{ij} = \frac{x_{ij}^{UE} - x_{ij}^{SO}}{x_{ij}^{UE}}$ in our link filtering procedure. In fact, ξ_{ij} shows the degree
 21 to which removal of a link brings the system closer to the SO principle. $\xi_{ij} = 1$ implies that link
 22 (i, j) is used under the UE principle but not under the SO.

23 Moreover, we plot the value of λ for each network-demand scenario as presented in table
 24 2 in both networks. In Figure 5, the links in the global optimum solution are mostly at the top of

1 the boxplot (the red dots in the scatter plot), which confirms our previous conclusion on the feature
 2 selection process. In Figure 5b we observe some red dots i.e. links that are in the solution of the
 3 exact method in the bottom of the boxplot which means they have a very low value for the selected
 4 feature. This motivates us to evaluate the contribution of each link to the objective value of the
 5 optimal solution. In other words, how much these links further reduce the TTT compared to our
 6 solution.

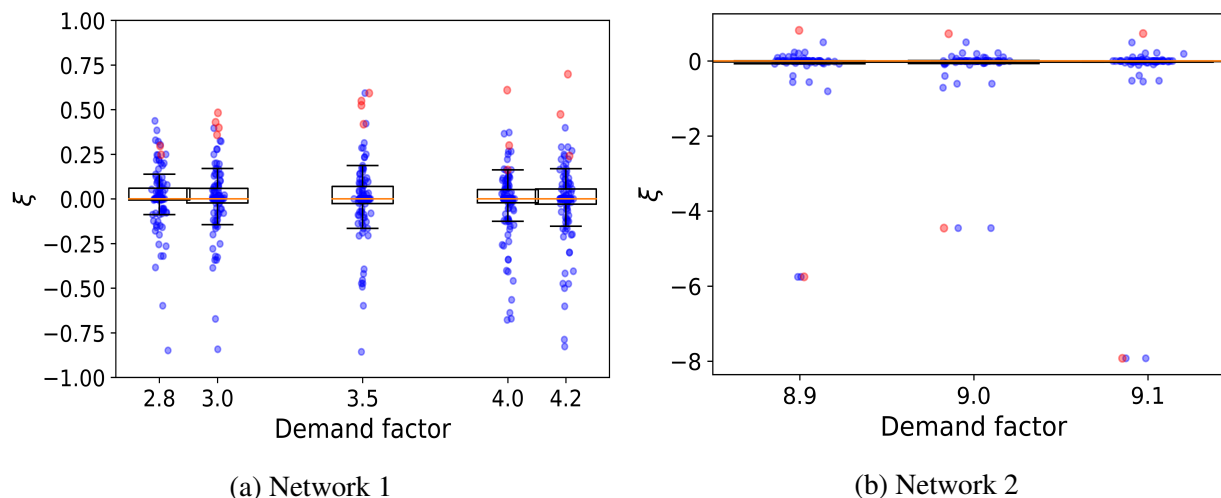


FIGURE 5: The value of the selected feature ξ for different scenarios. The red dots indicate the links in the global optimum solution whereas the blue dots are all other links in the network.

7 The first experiment: Validation

8 In order to prove the effectiveness of the selected feature, we compare the results of the obtained
 9 solution by both methods in the first experiment. Each solution method will return a set of links
 10 that must be closed to the traffic. While the exact method requires several days to converge, our
 11 method needs only a few minutes to deliver the results. Figure 6 show the results for network 1
 12 and network 2 respectively. The red links are the ones found by both algorithms, while the yellow
 13 links are only found by the exact method. The blue links are the set of candidate links obtained
 14 from our filtering process. Clearly, the results are very similar. Regarding network 2, our method
 15 finds only one of the two links that must be closed. After a quick check, it turned out that the
 16 second link (the yellow link in Figures 6f-6h) does not carry any traffic. This problem arises due
 17 to the fact that in the solution algorithm 1 when two solutions have the same objective value, the
 18 last one is returned as the optimal solution. Regardless of that, the calculated TTT in both methods
 19 for all three demand scenarios of this network are identical. Figure 7 depicts the differences in the
 20 calculated TTT with both solution methods.

21 Network 1 is twice as big as network 2, and therefore, it is more challenging to find a
 22 solution for it. As expected, the offset between the solutions found by the algorithms will increase
 23 as the size of the network grows. We can see in Figures 6a-6e that for lower demand scenarios in
 24 this network, our method finds the same solution as in the exact method. However, for the higher
 25 demand scenarios, one link with non-zero traffic flow is not found by our algorithm. The main
 26 reason is that these links are not in the set of candidate links. We argue that these links are trivial

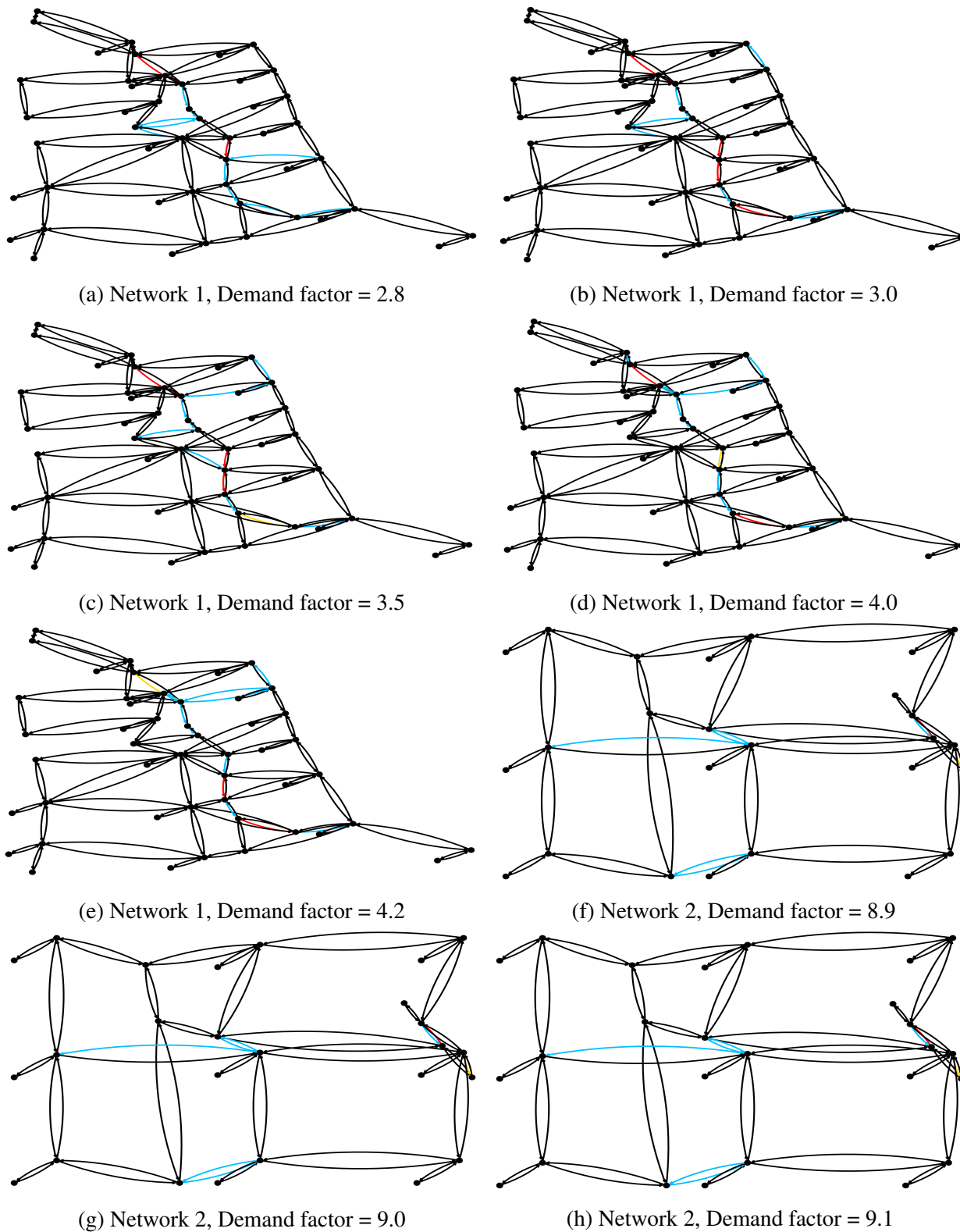


FIGURE 6: Traffic banning strategy in each scenario for network 1 and network 2: Red links are in both solutions, yellow links are only in the exact method solution, and blue links are the candidate links considered by our method but eventually remain open.

1 to the traffic banning strategy. We can clearly see this fact in Figure 7 that by adding the yellow
 2 links, only 0.04% improvements in the TTT can be achieved.

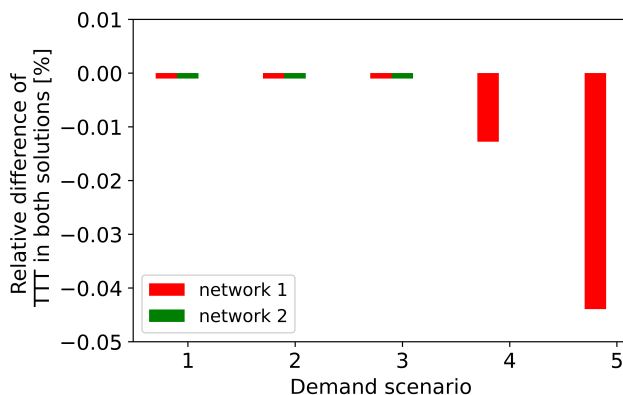


FIGURE 7: Difference of the TTT between the solution obtained by the exact solution method and the solution obtained by the proposed filtering approach.

3 The second experiment: Performance evaluation

4 The results of the second experiment are also very promising. After three hours, which was set
 5 as a reasonable computation time limit, the exact method managed to analyze only 583 possible
 6 solutions, that is only a fraction of maximum number of feasible solutions i.e. 2^{136} it must explore.
 7 Interestingly, our method converged to the same solution in only 12 minutes, which implies that
 8 the exact method had found the near-global optimum solution but still had to test thousands of
 9 feasible solutions. Even though we cannot guarantee our solution is the global optimum solution,
 10 but we can be up to a certain level confident that this solution is very close to the global optimum
 11 solution, and it is not worth it to spend more time to find those trivial links that may reduce the
 12 TTT only by a negligible fraction. Figure 8 illustrates the solutions from both methods on the
 13 toy network. Similar to the previous results, red links represent the solution of the algorithm. We
 14 can see that these links are identical in both solutions. The blue links in Figure 8b are the set of
 15 candidate links.

16 Remarks on the value of the traffic banning strategy

17 Another crucial observation we can make is that the solution is relatively robust to small changes in
 18 demand. This is crucial to the real-world implementation of such an idea using traffic management
 19 measures. A sudden change of the location of the optimal links with slight changes in demand
 20 would have required a bang-bang controller, which is not efficient for such complex problems. Our
 21 results on the effectiveness of traffic banning as a management measure are also very encouraging.
 22 Figure 9a and 9b illustrate the TTT under both the UE and SO principle and Figure 9c depicts
 23 the improvements in the PoA after applying the traffic banning strategy for all network-demand
 24 scenarios. We looked into the PoA in the original network and the network obtained after banning
 25 the links. It turns out that implementing the proposed traffic banning strategy can reduce the PoA
 26 on average about 20% in network 1 and up to 40% in small and less congested scenarios in network
 27 2. This is a very motivating observation that should be indeed further investigated in more realistic
 28 settings.

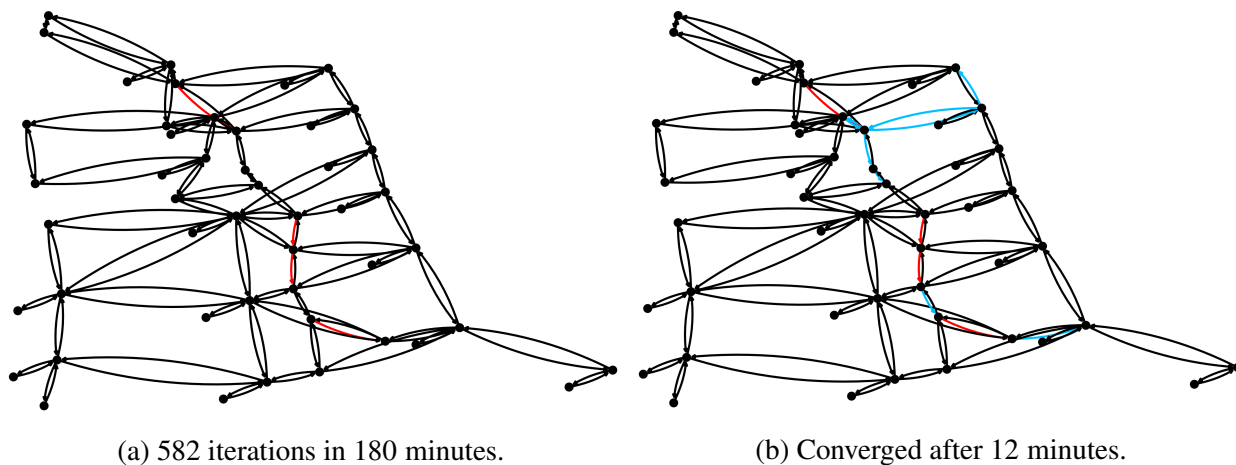
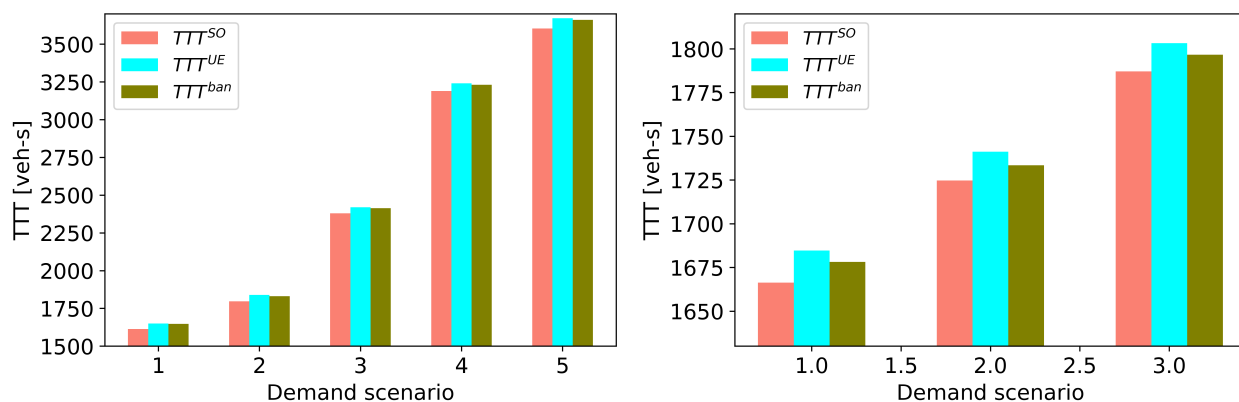
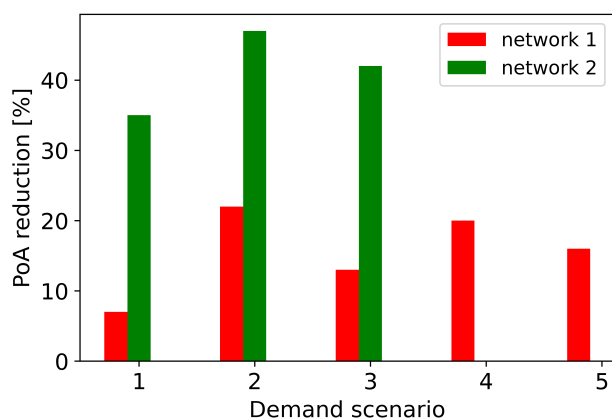


FIGURE 8: Identical traffic banning found by both solution methods.



(a) TTT in network 1 in different scenarios.

(b) TTT in network 2 in different scenarios.



(c) The PoA reduction for both networks.

FIGURE 9: The effectiveness of traffic banning strategy on reducing the PoA.

1 CONCLUSION

2 In this paper, we proposed a heuristic optimization framework based on network link features in
 3 order to find the links that cause the Braess paradox. The framework improves the network per-
 4 formance by imposing a traffic banning strategy. This improvement by banning link(s) is occurred
 5 because of the difference in link cost and flow distribution of the network status at the users op-
 6 timum (i.e., UE) and system optimum (SO). The methodology consists of three main steps: (i)
 7 Identify the feature of candidate links by selecting the link features on the UE and the SO prin-
 8 ciples for different demand profiles on a traffic network, (ii) Filter the candidate links which are
 9 the most potential link(s) to ban, and (iii) Find the optimal set of links to be banned by an exact
 10 solution method. The last step is essentially based on the SO-relaxation method first for discrete
 11 network design problem (DNDP). First, we performed a complete analysis on this solution method
 12 and fixed the original interdiction cut constraint in order to account for one change between the
 13 network designs when seeking the optimum solution. This correction guarantees the global opti-
 14 mum solution for DNDP. Second, we adapt the solution method to find the optimal network design
 15 from an existing network under the UE principle. Third, we added a link filtering procedure based
 16 on the first step of our methodology to reduce the solution space. The filtering procedure is based
 17 on a single feature ξ , which was found to be very effective. This feature, in essence, reflects the
 18 importance of a link during the UE in comparison to the SO assignment. The value of 1 basically
 19 means the link is only used in the UE and not SO, which is a sign for the Braess paradox.

20 In order to examine the effectiveness of the proposed solution, we conducted a case study
 21 on the network of Chicago-Sketch. We designed two experiments, one without a computation time
 22 limit in order to let the exact method converge, and the second experiment in which we imposed
 23 a reasonable computation time limit to account for the trade-off between computation time and
 24 accuracy of the solution. The results of the first experiment confirmed our initial postulation re-
 25 garding the proposed feature for link filtering. $\xi_{ij} = \frac{x_{ij}^{UE} - x_{ij}^{SO}}{x_{ij}^{UE}}$ was the most influential feature to
 26 distinguish a link that may exhibit the Braess paradox among other features we investigated. Addi-
 27 tionally, the first experiment results proved that for low-demand scenarios, our method converges
 28 to the same solution as in the exact method. We also showed that for higher levels of demand our
 29 method returns a near-global solution. In other words, the difference between the global optimum
 30 and our solution is trivial, and we conclude that it is not worth the computation time required to
 31 find the global optimum solution. We used the second experiment to test this statement. After three
 32 hours, the exact method could only evaluate a fraction of the solution space, whereas our method
 33 converged in 12 minutes. The solutions were identical by both methods, which implies that the
 34 exact method found a good solution but had to explore the rest of the feasible solutions, which for
 35 real-world networks can take up days or weeks.

36 Despite the promising results of the proposed method, we suggest some improvements
 37 for future work. First and foremost, it is essential to perform a sensitivity analysis on different
 38 demand levels in other realistic network settings. In this paper, we only scaled the demand by a
 39 factor on two networks. It would be interesting to see how stable the solution is in different OD
 40 relations, such as peak hours versus non-peak hours. This is very similar to the idea of defining
 41 demand breakpoints as discussed in (Ameli et al., 10). Another possibility to enrich our study is
 42 to add operational budget constraints. While closing a link may not need high monetary costs in
 43 comparison with building a new road, but there are other aspects in terms of easiness to implement
 44 a link closure, e.g., if the banning infrastructure exists.

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4 **AUTHOR CONTRIBUTION STATEMENT**

5 All the authors have contributed to all aspects of this study, ranging from the conception and design
6 of the methodology, analysis and interpretation of the results and discussion, and the manuscript
7 preparation.

8 **CONFLICT OF INTEREST**

9 The authors declare no potential conflict of interests.

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