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# Recent developments on the modelling of outdoor sound propagation in complex environments using the parabolic equation.

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## Abstract

In this paper we develop a method which aims to evaluate the propagation of an acoustic wave above a non flat ground and which could include realistic outdoor conditions. The acoustic waves are propagated in a parabolic approximation. The effect of topography is taken into account by rotating the coordinate system. Our model of outdoor sound propagation is validated both with classical numerical benchmark cases and recent experiments done in St-Berthevin.

## 1. Introduction

During the last decade, the propagation of sound above plane and heterogeneous grounds has been extensively studied analytically, numerically and/or experimentally. However in complex environments, the modelling of outdoor noise pollution implies to take into account the mixed influence of the terrain topography and the atmospheric conditions. Significant progress have been made in the modelling of sound propagation over distances ranging from hundreds meters to kilometers, and the agreement between calculated and measured fields have been greatly improved. New developments appear in the parabolic-equation method. The new PE algorithm is based on a wide-angle parabolic equation in which the influence of the wind velocity and wind velocity gradients could be incorporated. In this paper we also present a method which evaluates the propagation of an acoustic wave above non-flat terrain using a PE method and which includes realistic meteorological parameters. In our approach the effects of the topography are modelled using appropriate rotated coordinates systems in order to treat the non-flat ground as a succession of flat domains. Our model of outdoor sound propagation is validated both with classical numerical benchmark cases and recent experiments done in St-Berthevin. Acoustical and meteorological measurements are performed simultaneously.

## 2. Description of the model

The Helmholtz equation in two dimensions is classically used to solved outdoor propagation problems. For an acoustical wave  $\hat{p}(\mathbf{r}, \omega)$  at the pulsation  $\omega$ , this equation is in the frequency domain:

$$[\Delta + k_0^2(1 + \epsilon)] \hat{p}(\mathbf{r}, \omega) = 0 \quad (1)$$

where  $k_0 = \omega/c_0$ ,  $c_0$  is a reference celerity,  $\epsilon = (c_0/c(\mathbf{r}))^2 - 1$  is the variation of the standard refraction indice and  $c(\mathbf{r})$  is the celerity at the point  $\mathbf{r}(x, z)$ .  $\mathbf{x}$  and  $\mathbf{z}$  are respectively the horizontal and the vertical directions. Equation (1) is exact for a homogeneous and motionless medium. It still remains exact when only the celerity  $c(\mathbf{r})$  varies. For numerical simulations of outdoor sound propagation, parabolic equations have been derived using the approximation of the effective sound speed. In this conventional approach the real moving atmosphere is replaced by a hypothetical motionless medium with the effective sound speed  $c_{eff} = c + v_x$ , where  $v_x$  is the wind velocity component along the direction of propagation between the source and the receiver. When the source and the receiver are closed to the ground, the preferred direction of sound propagation is nearly horizontal, and standard parabolic equation can be used to predict sound pressure levels. The previous wave equation (Eq.1) was reduced to a wide-angle parabolic equation [1, 5]. The first step is to write the equation for forward propagation:

$$\left[ \frac{\partial}{\partial x} - ik_0 \sqrt{\mathcal{Q}} \right] \hat{p}(\mathbf{r}, \omega) = 0 \quad (2)$$

where  $\mathcal{Q} = 1 + \epsilon + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}$ . From here, the square of the propagation operator  $\mathcal{Q}$  is simplified using a Padé approximation to yield:

$$\sqrt{\mathcal{Q}} = \frac{1 + p\mathcal{L}}{1 + q\mathcal{L}}$$

where  $\mathcal{L} = \mathcal{Q} - 1$ ,  $p = 3/4$  and  $q = 1/4$ . This procedure leads to the following scheme:

$$[1 + q\mathcal{L}] = \frac{\partial\phi(\mathbf{r})}{\partial x} = ik_0[(p - q)\mathcal{L}]\phi(\mathbf{r}) \quad (3)$$

where  $\phi$  is the envelope of the pressure field  $\hat{p}(\mathbf{r})$ :  $\phi(\mathbf{r}) = \hat{p}(\mathbf{r}) \exp(-ik_0x)$ . To get the numerical scheme, we use a second-order finite difference method. This finite difference method allows to use an impedance boundary condition. We use a gaussian starting field.

The problem of the propagation above a non flat ground has recently been studied by several acoustic researchers. A curved terrain version of the parabolic equation has been adapted for acoustic propagation in the atmosphere over fairly simple terrain profiles which can be reduced to a set of joined circular section pieces [3]. In that approach, a separate conformal coordinate transformation was applied to each circular section piece of atmosphere. Sack and West [4] chose to use a transformation which follows the terrain profile. Their method can be used for any smooth terrain but seems difficult to apply for a parabolic equation including wind terms. We chose to develop another model which can be used above any terrain and with a parabolic equation including wind terms [2]. The non flat ground is treated as a succession of flat domains (see figure 1), the PE is computed until the line l1.

Initial condition

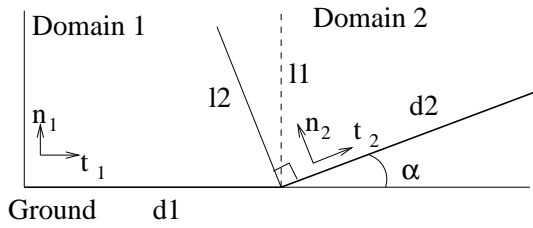


Figure 1: Schematic representation of the rotation of the coordinate systems in the upslope case.

The values of the pressure in the first domain are then used to calculate an initial solution for the second domain along the line l2 (figure 1). If we call  $p_1(t_1, n_1) = \phi_1(t_1, n_1)e^{ik_0t_1}$  and  $p_2(t_2, n_2) = \phi_2(t_2, n_2)e^{ik_0t_2}$  the pressure in the domains 1 and 2, the continuity of the pressure along l2 is written as :

$$\phi_2(0, n_2) = \phi_1(t_1|_{l2}, n_1|_{l2})e^{ik_0t_1|_{l2}} \quad (4)$$

where  $t_1|_{l2} = d1 - n_2 \sin \alpha$  and  $n_1|_{l2} = n_2 \cos \alpha$ . In the case of downslope propagation, we notice that the calculation in the first domain must be done slightly beyond the slope discontinuity so that the rotated PE for the second domain can be initialized. The wide-angle parabolic equation (Eq. 3) is included in the rotated PE method. In order to validate the method, we consider the propagation above a wedge. A reference solution can be established analytically as the sum of a geometrical field and

a diffracted field ([2]). The first benchmark case is the propagation above the wedge drawn on the left in figure 1, with  $d1 = 60$  meters and  $\alpha = 20$  degrees. In figure 2, we draw on the left the pressure amplitude calculated analytically and on the right the pressure amplitude calculated by the rotated PE method. The pressure amplitude is relative to a Dirac source located at 2 meters above the ground. The acoustic frequency is 340 Hz. The qualitative agreement between the solution is good. The patterns of interferences above the slope are well predicted by the rotated PE.

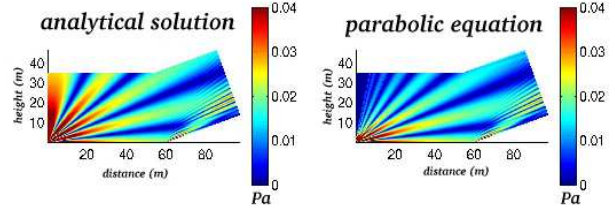


Figure 2: Comparison of the non flat ground model (right) with an analytical solution (left). Parameters:  $h_s = 2$  meters;  $d1 = 60$  meters;  $d2 = 40$  meters.

In order to have a more quantitative validation of the rotated PE method, we evaluate the pressure amplitude on the same benchmark case on a line perpendicular to the slope at 40 meters of the bottom discontinuity. We consider the case of a positive slope and of a negative slope. We chose  $d1=60$  metres,  $d2=40$  meters and  $\alpha = 20$  degrees. The unit point source is located at 2 meters above the ground. The acoustic frequency is 340 Hz. On figures 3 and 4, we compare two analytical solutions and the PE solution. The first analytical solution is the geometrical part of the pressure ; the second is the total analytical pressure field (geometrical plus diffracted). The agreement between the PE solution and the second analytical solution (geometrical plus diffracted field) is excellent in both cases (upslope and downslope). The difference between the two analytical solutions is due to diffracted part of the pressure field. We conclude that the rotated PE method calculates accurately the diffracted part of the field above the wedge. Others numerical comparisons in the cases of the curved surfaces have been done([2]) : the rotated PE method gives accurate results until an angle of 40 degrees and for frequencies between 100 and 3200 Hz.

### 3. Outdoor measurements

Outdoor meteorological and acoustical measurements were carried out in Saint-Berthevin (France) ([2]). The site has complex features: it is non flat and the ground impedance is heterogeneous. Meteorological measurements of wind and temperature at several heights during the acoustical measurements allow us to model the mean profiles of wind and temperature above the ground .On

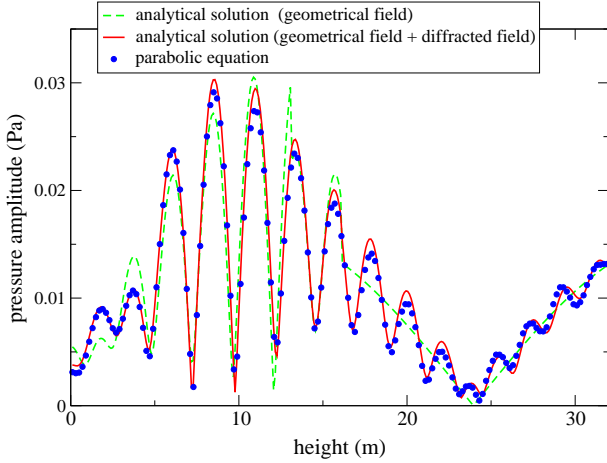


Figure 3: Analytical validation of the model for non flat ground in the cases of a flat ground followed by upslope. Parameters:  $h_s = 2$  meters;  $d1 = 60$  meters;  $d2 = 40$  meters.

the figure 5 we can see a photo of the terrain above which were carried out the experiments. For the calculations, the ground is modelled as a succession of 3 flat domains (see figure 5) : the first is 25 meters long the second is 75 meters long and the third is 25 meters long. Five microphones ( M1 to M5) were used to give measures between 25 and 100 meters of the source . The reference microphone was located at a distance of 10 meters from the source ( $M_{ref}$ ).

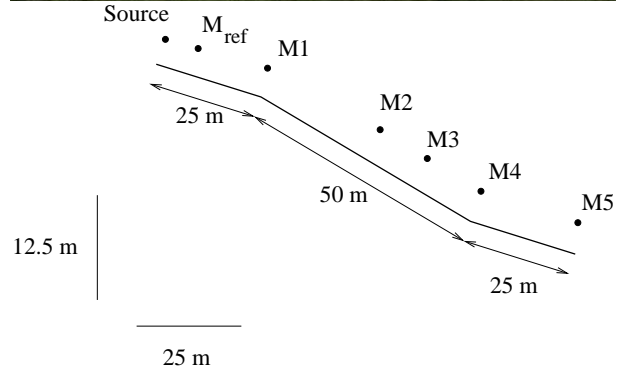


Figure 5: Photography of the site with the source, the microphones and the meteorological tower (top) and representation of the ground as 3 flat domains for the calculations (bottom).

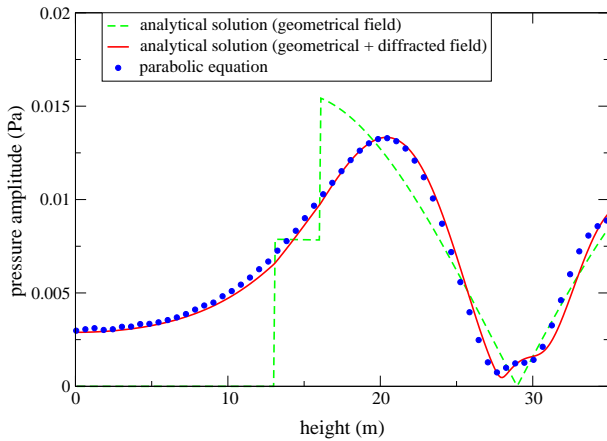


Figure 4: Analytical validation of the model for non flat ground in the cases of a flat ground followed by downslope. Parameters:  $h_s = 2$  meters;  $d1 = 60$  meters;  $d2 = 40$  meters.

The numerical scheme can include an impedance boundary condition. The value of the impedance  $Z$  ( $Nm^{-3}s$ ) is defined by a measurable quantity, the flow resistivity  $\sigma$  ( $Nm^{-4}s$ ), through an impedance model. We use the classical Delany and Bazley model because the parameter  $\sigma$  can be determined experimentally. This

models gives  $Z$  as a function of the frequency  $f$ :

$$Z = \rho_0 c_0 \left[ 1 + 0.0571 \left( \frac{\rho_0 f}{\sigma} \right)^{-0.754} + i0.087 \left( \frac{\rho_0 f}{\sigma} \right)^{-0.732} \right] \quad (5)$$

where  $c_0$  is a reference celerity speed ( $m.s^{-1}$ ) and  $\rho_0$  is the air density ( $kg.m^{-3}$ ). We measured the ground impedance between the source and the microphone M5. The method of ground characterization uses short range attenuation measurements, the measured value of  $\sigma$  are : around the source:  $\sigma = 600.10^3 N.m^{-4}s$  , around M1:  $\sigma = 90.10^3 N.m^{-4}s$  , around M2:  $\sigma = 160.10^3 N.m^{-4}s$  , around M4:  $\sigma = 200.10^3 N.m^{-4}s$ . The first hundreds meters above the ground are characterized by the presence of an atmospheric boundary layer where mean gradients of wind and temperature and turbulent fluctuations exist. When the topography is smooth enough, the leading wind component is the horizontal component. It can be modelled with a logarithmical profile :  $u(z) = a \ln(z/z_0)$  for  $z > z_0$  , where  $z_0$  is the rugosity of the ground (0.1 meter for a field). With the bidimensional model, only the wind component in the direction of propagation is taken into account. If we call  $\theta$  the angle between the horizontal wind direction and the sound propagation direction, the quantity  $U(z) \cos \theta$  must be

introduced in the effective celerity  $c_{eff}$ . The effective celerity used in the propagation equation (2) is defined with the velocity and the temperature field as:

$$c_{eff}(z) = c(0) \sqrt{\frac{T(z) + 273.15}{273.15}} + u(z) \cos(\theta) \quad (6)$$

We use a meteorological tower to evaluate the meteorological parameters used in the calculation of acoustical propagation. The tower is located on the slope, far from the measurement line for the acoustical propagation not to be perturbed. The tower is equipped with thermometers and anemometers to measure the temperature, the direction and intensity of wind. These thermometers and anemometers are located at 1, 3 and 10 meters high. The data are recorded every minutes.

measure	$a$	$\theta$	temp. T
6	0.5	20 degrees	290 K
7	0.65	20 degrees	290 K

Table 1: Meteorological parameters for the model of sound propagation.

Acoustic measurements are done at the positions defined on the figure 5. We use a data recorder with four channels. The source is impulsive and omnidirectional. For a single measure, the signal is averaged over 10 snapshots. The relative sound pressure level is the difference of the spectrum at the microphone M1, M2, M3, M4 or M5 and the spectrum at the reference microphone  $M_{ref}$ . The wind was moderate and the propagation was downwind; the meteorological parameters are summarized in table 1. We use the model with a wind profile defined by the measured parameter  $a$  and without wind profile. We present now the results of the measures 6 and 7 and those of the model (see figure 6). The sound pressure level was recorded at M4 (75 m) (see figure 5). The comparison of the experimental results and those of the model shows that the model gives accurate relative pressure levels. Besides, the comparison of the model with and without wind profile show the influence of the meteorological parameters. Above 2000 Hz, the relative sound pressure level is 10 dB higher with the wind profile.

#### 4. Conclusion

We have developed a model for outdoor sound propagation. The model is based on a wide-angle parabolic equation which can include the profiles of wind and temperature through the definition of an effective sound celerity. The ground impedance can be included in the model as a boundary condition. The model was developed to handle sound propagation over a non flat ground. This specification was fulfilled by using a rotated PE method. The model has been validated with analytical results for

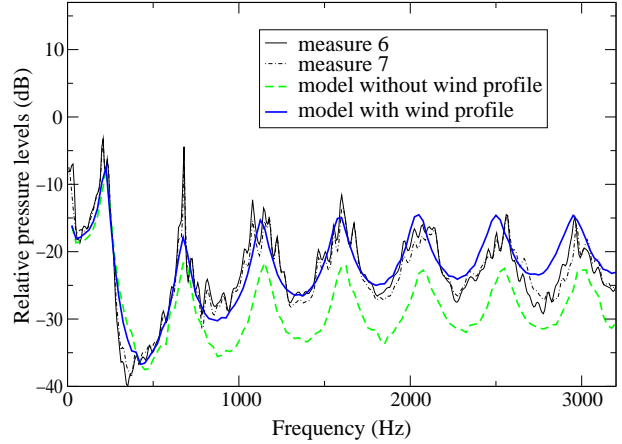


Figure 6: Experimental validation of the model for a non flat ground. Parameters:  $h_s = 2$  meters;  $h_m = 2$  meters; Microphone 4;  $D = 75$  meters. Measures 6 and 7.

the wedge case. Outdoor measurements of acoustic attenuation and meteorological parameters were performed simultaneously on a complex site (mixed terrain, non flat ground). Comparisons between the measurements and the model with and without a moving atmosphere are very good.

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