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ABSTRACT
Separating activated sludge from the liquid phase is the last operation in a wastewater treatment plant. Our goal is to present a knowledge-based dynamic model with both dynamic mass and momentum balances and that can be used for decision-making. The numerical scheme used for simulation is a method of lines with a spatial discretization based on a finite volume method using the Rusanov approximation. It is well-suited to such nonlinear hyperbolic systems exhibiting discontinuities or shock waves. Such a model takes into account the various behaviours of the mixture inside the clarifier and not only the nominal operation and is more general than the other models existing in the literature. The simulation results are confronted with experimental results from measurements of the sludge blanket during a dynamic event.

KEYWORDS
1-D dynamic hyperbolic PDE model; urban sludge continuous-flow settling process; Comparison of simulated results with experimental data

1. INTRODUCTION

A proper understanding of organic urban sludge settling in wastewater treatment plants is very important a) to ensure that the clarified water discharged into the environment meets the required quality and environmental standards and b) to help decide which closed-loop control strategies should be used in order to have lower operating costs (energy, ...) or better effluent quality. During settling operation, the solid particles of the activated sludge settle inside the suspension and three zones form: the upper clarification zone containing liquid only; the intermediate zone in which solid particles settle freely with no interaction; and the compression zone at the bottom where solid particles form a high concentration porous bed where the solid particle mass concentration exceeds a given threshold above which interaction forces between particles begin to act.

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These three zones appear to be separated by two mobile interfaces: the sludge blanket separating the clarification and intermediate zones and an interface defined by a concentration threshold separating the intermediate and the compression zones, [1], [2], [3], [4].

Two situations are mainly addressed in the literature devoted to dynamic settling modeling. The batch settler that allows investigating sludge settling properties and the continuous settler (clarifier) that is operated in urban wastewater treatment plants. To date, 1-D dynamic models represent the best compromise for capturing the complexity and significance of the studied phenomena, [5]. In these vertical settlers, the solid phase moves at variable velocity in the liquid phase. Therefore, the model must include a momentum balance (static or dynamic) or a constitutive equation to represent velocity variation in space and time. Four families of 1-D models are described in the literature:

1. Models based on a dynamic solid particle mass balance coupled with a constitutive equation that gives the solid particle velocity, the so-called batch or hindered settling velocity, [1], [6], [3], [7], [8], [9]. I. Takács initiated this approach;
2. Models based on a dynamic solid particle mass balance and a static momentum balance which defines the flux density function, [10], [4];
3. Models based on both a dynamic solid particle mass balance and a dynamic momentum balance, [2], [11]. This approach is more recent and is more general due to its two dynamic balances;
4. Models based on both the dynamic solid particle mass and momentum balances and the sludge blanket location as an additional variable whose variation is governed by a dynamic equation (issued from a mass balance), [12], [13]. This approach is more specific to batch settling for the moment.

A very interesting review on clarifier 1-D dynamic models of families 1. and 2. can be found in [14]. In the last three families, constitutive equations are required in order to express the forces acting on the solid particles: pressure, effective interparticles stress, gravity and drag. The interest of models with a dynamic momentum balance as family 3. is that they do not require an additional constitutive equation to define the velocity of solid particles in the mixture. These models are more general and represent a wider range of settling processes (primary, secondary, in mines, cities, ...). Only the values of the model parameters have to be adapted to the context. The first three models are nonlinear hyperbolic partial differential equation (PDE) systems and the fourth one is an ODE-PDE system.

In [13], a two PDEs system obtained from the solid particles mass and momentum balances and an ODE representing the variation in the location of the sludge blanket mobile interface is used. Simulating this model using a centered finite difference numerical scheme gives a sludge blanket descent dynamics that is in good agreement with measurements. However, with this numerical scheme, only the change in the average concentration under the sludge blanket is obtained, because it is unable to detect spatial discontinuities or shock waves in some state variables as the solid particles velocity. Therefore, it cannot predict the change in sludge concentration at the bottom over time nor the lowest interface location between the intermediate zone and the compression zone.

In this paper is presented a 1-D dynamic model based on particles mass and momentum balances of family 3. so that it can be used for decision-making and for a further design of a closed-loop controller to regulate the water quality at the top of the clarifier. Such a model takes into account the various behaviours of the mixture inside the clarifier and not only the nominal operation. A Rusanov finite volume numerical scheme,

[15], is implemented to take into account the specificity of this nonlinear hyperbolic system with source terms. This type of scheme naturally detects the two mobile interfaces between the three zones that form when the sludge settles. The velocity of the fastest of these two shocks needs to be calculated in order to define the time and space steps in the numerical scheme in order to ensure its stability.

This paper is organized as follows: section 2. deals with a one dimension functional description of the secondary clarifier and of a dynamic model of the behaviour of sludge based on both dynamic mass and momentum balances. It is a hyperbolic PDE system with non linear source terms associated with specific constraints, constitutive equations and boundary conditions. This model is more general than the other models presented in the literature and more simple than the model of the same family, 3. presented in [2] in the field of sedimentation in a river estuary. A numerical scheme well-suited to this nonlinear hyperbolic systems exhibiting discontinuities or shock waves is presented in section 3. In section 4., a dynamic event scenario is described and simulated results obtained with our model are compared with experimental data. Finally, section 5. gives some conclusions and perspectives of this promising work.

2. A 1-D dynamic physical model

2.1. Clarifier and settling description

The clarifier (settling tank) is represented schematically Fig. 1 in 1D view. The open-air settler content can be divided into 2 interfaces moving in space and time separating the three above-mentioned zones:

- The upper interface is the sludge blanket and is located at depth $z_v(t)$. It separates the clarification zone (which no longer contains any solid particles) and the intermediate zone,
- The lower interface is defined by the intermediate/compression threshold and is located at depth $z_c(t)$ where a change in behavior occurs as the solid particle concentration $C_s(z,t)$ exceeds the threshold C_c above which an interparticle stress comes into effect. Under $z_c(t)$, the liquid phase flows through the porous network, [16].

The clarifier is connected to the wastewater treatment process through three flows:

- one inlet where activated sludge flows inside the clarifier by gravity from the upstream biological aeration tank with volume flow rate $Q_f(t)$ and solid particles concentration $C_f(t)$. Practically, the activated sludge feeding takes place inside a skirt at depth around $z = z_f$,
- one top outlet of clarified water at $z = 0$, with volume flow rate $Q_e(t)$ and solid particles concentration $C_e(t)$,
- one bottom outlet of compressed sludge at $z = z_b$, with volume flow rate $Q_u(t)$ and solid particles concentration $C_u(t)$. Some of the compressed sludge is recirculated back to the aeration tank with volume flow rate $Q_{ur}(t)$ and some can be extracted from the clarifier with volume flow rate $Q_{ue}(t)$ such that $Q_u(t) = Q_{ur}(t) + Q_{ue}(t)$.

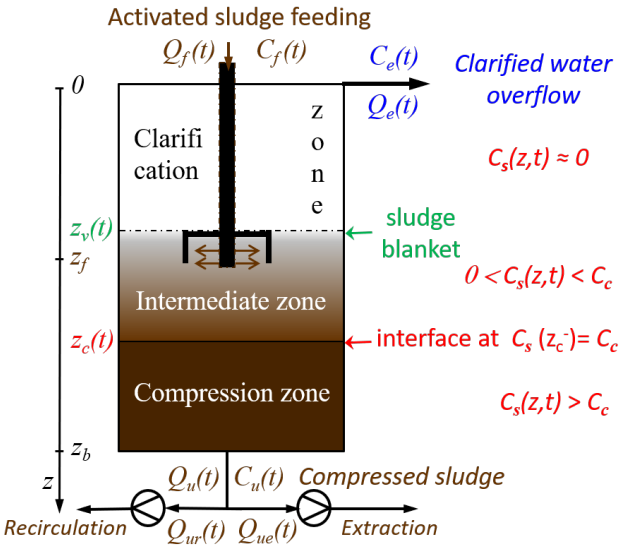

Figure 1. One-dimensional schematic view of a sludge clarifier.

2.2. 1-D dynamic mass and momentum balances

The objective of the model is to calculate the time evolution of the solid particles concentration profile in the clarifier as well as the outlet concentrations $C_e(t)$ and $C_u(t)$ and the sludge blanket position $z_v(t)$, knowing the activated sludge feeding flow rate and concentration $Q_f(t)$ and $C_f(t)$ and the compressed sludge outlet flow rate $Q_u(t)$. The dynamic model describing the behavior of the sludge suspension in the clarifier is based on the dynamic mass and momentum balances that can be written for the two phases under the following commonly-used simplifying assumptions, [14]:

1. The liquid and solid phases completely fill the clarifier volume that is constant.
2. There is no biological activity during the settling operation and the suspension is fully flocculated before sedimentation, [10].
3. The solid particles have the same size and shape, [6], [3], [4].
4. There is a uniform particle concentration at a given depth, [6], [3].
5. The vessel wall friction is negligible.
6. The solid particles are small with respect to the containing vessel and have the same density, [4].
7. The solid particle and fluid mass densities, ρ_s (kg/m^3) and ρ_l (kg/m^3), are constant, [2], [3] and there is no mass transfer between them, [6], [4].
8. The open-air clarifier has a constant cross-sectional area.

Let $\varepsilon_s(z,t)$ denote the solid particle volume fraction with z the depth from the top of the clarifier, and t the time. $\varepsilon_l(z,t)$ denotes the liquid volume fraction. The solid particle mass concentration is then $C_s(z,t) = \rho_s \varepsilon_s(z,t)$ (kg/m^3). Let $v_s(z,t)$ (m/s) and $v_l(z,t)$ (m/s) denote the solid and liquid phase Eulerian average velocity respectively.

Remark: for sake of clarity, the notations have been simplified by omitting (z,t) *from the following equations wherever the variables for all* z *and all* t *are considered. A subscript notation is used for the partial derivatives:* $\frac{\partial f}{\partial t} = \partial_t f$ *and* $\frac{\partial f}{\partial z} = \partial_z f$.

The feeding is located over an interval of height Δz_f and centered at z_f to take into account the different types of skirts that drive the flow of sludge downwards. This area is located between $z_2 = z_f + \Delta z_f/2$ and $z_1 = z_f - \Delta z_f/2$. The following gate (Top-Hat) function, $\Pi(z, z_f, \Delta z_f)$, can be defined in order to represent the source terms due to the sludge feeding in the balance equations. For simplicity, it is denoted $\Pi_f(z)$:

$$\forall z, z_1, z_2 \in \Re^+, \Pi_f(z) = \begin{cases} 1 \text{ if } z_1 \leq z \leq z_2 \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

The dynamic mass balances for the solid phase and the liquid phase are then the following:

Solid phase mass balance:

$$\partial_t(\rho_s \varepsilon_s) + \partial_z(\rho_s \varepsilon_s v_s) = f_{1s}(Q_f, C_f)\Pi_f(z) \tag{2}$$

Liquid phase mass balance:

$$\partial_t(\rho_l \varepsilon_l) + \partial_z(\rho_l \varepsilon_l v_l) = f_{1l}(Q_f, C_f)\Pi_f(z) \tag{3}$$

with $f_{1s}\Pi_f$ and $f_{1l}\Pi_f$ the source terms representing the activated sludge inlet feeding in the solid and liquid mass variation respectively. They both depend on Q_f, the total volume flow rate of the feed and on C_f, the mass concentration of solid particles at the feed and A, the cylindric clarifier section.

As well, the dynamic momentum balance equations can be written for the solid phase and the liquid phase, [2], [17], [18]:

Solid phase momentum balance:

$$\begin{aligned} \partial_t(\rho_s \varepsilon_s v_s) &= -\partial_z(\rho_s \varepsilon_s v_s^2) + \varepsilon_s \rho_s g - \varepsilon_s \partial_z P - \partial_z \sigma_e(\varepsilon_s) + r(\varepsilon_s)(v_l - v_s) \\ &\quad + f_{2s}(Q_f, C_f)\Pi_f(z) \end{aligned} \tag{4}$$

with:

$\varepsilon_s \rho_s g$	volumetric gravitational force (body force)
$\partial_z P(z,t)$	gradient of the pore pressure (hydrodynamic pressure)
$\partial_z \sigma_e(\varepsilon_s)$	gradient of the interparticle stress between the solid particles, [10].
$r(\varepsilon_s)(v_l - v_s)$	Stokes like drag force i.e. liquid/solid dynamic interaction force standing for viscous friction between the two phases. $r(\varepsilon_s)$ is the resistance coefficient.

Liquid phase momentum balance:

$$\begin{aligned} \partial_t(\rho_l \varepsilon_l v_l) &= -\partial_z(\rho_l \varepsilon_l v_l^2) + \varepsilon_l \rho_l g - \varepsilon_l \partial_z P - r(\varepsilon_s)(v_l - v_s) \\ &\quad + f_{2l}(Q_f, C_f)\Pi_f(z) \end{aligned} \tag{5}$$

with $f_{2s}\Pi_f$ and $f_{2l}\Pi_f$ the source terms representing the activated sludge inlet feeding in the solid and liquid momentum variation respectively. They depend on Q_f, C_f and A.

In the next two sections, specific constraints inherent in a two-phase suspension with non-constant velocity and constitutive relations of $\sigma_e(\varepsilon_s)$ and $r(\varepsilon_s)$ will be presented.

2.3. Specific constraints

As the sludge is a two-phase (liquid and solid) suspension, the sum of the solid particle volume fraction and the liquid volume fraction is:

$$\varepsilon_l + \varepsilon_s = 1 \tag{6}$$

As solid particles and fluid are incompressible, the total volume flux of the suspension (or mixture average velocity), i.e. the sum of the volume flux of the two phases is denoted $v_m(z,t)$ and can be calculated by the following algebraic equation:

$$v_m = \varepsilon_l v_l + \varepsilon_s v_s \tag{7}$$

Moreover by using equation (6), the sum of the two mass balances (2) and (3), each divided by the respective phase density, gives:

$$\partial_z v_m = \left[\frac{f_{1s}}{\rho_s} + \frac{f_{1l}}{\rho_l}\right] \Pi_f(z) \tag{8}$$

It means that v_m gradient is null everywhere except in the feeding zone, thus v_m is constant on both sides of the feeding zone with a linear change inside the zone and can be calculated by (9).

$$v_m(z,t) = \begin{cases} -Q_e/A & \text{if } z < z_1 \\ \frac{1}{A\Delta z_f}\left[(Q_e+Q_u)z - z_1 Q_u - z_2 Q_e\right] & \text{if } z_1 \leq z \leq z_2 \\ Q_u/A & \text{if } z > z_2 \end{cases} \tag{9}$$

Therefore, $\varepsilon_l(z,t)$ can be calculated from (2) and (6), and $v_l(z,t)$ can be deduced from (2), (4), (6) and (9) as follows:

$$v_l = \frac{v_m - \varepsilon_s v_s}{(1-\varepsilon_s)} \tag{10}$$

Note that this equation is well defined because there is liquid everywhere in the clarifier in the interstices between the solid particles, then $\varepsilon_s \neq 1$.

2.4. Constitutive equations

Some quantities, such as the interparticle stress, σ_e, and some parameters, such as the drag force resistance coefficient, r, depend on the solid particle volume fraction ε_s. These are characterized by constitutive expressions that are usually derived from experimental data. They are empirical in nature and depend on the characteristics of the sludge. Different constitutive equations have been proposed by various authors for a range of contexts (wastewater from cities, mines, estuary or coastal zones etc). [14] presented most of the proposed approaches. $\sigma_e(\varepsilon_s)$ and $r(\varepsilon_s)$ were chosen from those adapted to organic urban sludge. The constitutive equations presented by [4] for $\sigma_e(\varepsilon_s, \alpha)$ and by

[2] for $r(\varepsilon_s)$ are well suited to the urban sludge:

$$\sigma_e(\varepsilon_s, \alpha) = \alpha(\varepsilon_s)\sigma_0 \frac{\varepsilon_s^{n_s} - \varepsilon_c^{n_s}}{\varepsilon_c^{n_s}} \tag{11}$$

$$r(\varepsilon_s) = \frac{\rho_l g}{K(\varepsilon_s)} \text{ with } K(\varepsilon_s) = \frac{A_k}{\varepsilon_s^{2/(3-n_r)}} \tag{12}$$

with σ_0, n_s, A_k and n_r being the constant parameters that characterize the sludge which permeability is K and $\alpha(\varepsilon_s)$, a Boolean parameter such that:

$$\alpha(\varepsilon_s) = \begin{cases} 0 & \text{for } \varepsilon_s \leq \varepsilon_c \\ 1 & \text{for } \varepsilon_s > \varepsilon_c \end{cases} \tag{13}$$

with ε_c, the solid volume fraction intermediate/compression threshold. $\alpha(\varepsilon_s)$ is equal to zero in the intermediate zone where the particles are relatively distant from one another due to the low concentration, and is equal to one in the compression zone where an interparticle stress between the solid particles comes into effect due to their proximity to each other.

Thus, the constitutive equation for $\sigma_e(\varepsilon_s, \alpha)$ depends on the zones in the clarifier. The formulation ensures that $\sigma_e(\varepsilon_s, \alpha)$ is a continuous function at $\varepsilon_s = \varepsilon_c$. The need for continuous and Boolean variables in the settler dynamic model makes it hybrid, [19].

2.5. PDE 1-D sludge continuous settling dynamic model

After all these considerations, the four dynamic balance equations given in section 2.2 can be expressed only in terms of the solid particle volume fraction, ε_s, and the solid particle volume velocity, v_s, after having performed all the simplifications implied by the assumptions 1. to 8. and the algebraic equations specific to this system established in section 2.3.

Moreover, a simplified expression of the pore pressure gradient can be considered for this system because the settling is very slow and the suspension is at a low concentration. This involves a pressure profile identical to the static gradient due to Archimedes' buoyancy force, $\partial_z P = \rho_l g$, and this throughout the duration of the operation, [12], which is an additionnal 9th assumption:

9. $\forall z \in [0, z_b], \partial_z P = \rho_l g$.

The solid particle volume fraction ε_s and the velocity variations v_s can be written in conservative form, after some manipulation of the equations (2), (4) and (10):

$$\partial_t \varepsilon_s + \partial_z(\varepsilon_s v_s) = \frac{f_{1s}}{\rho_s}\Pi_f(z) \tag{14}$$

$$\begin{aligned} \partial_t v_s + \partial_z(\frac{v_s^2}{2}) &= (1 - \frac{\rho_l}{\rho_s})g - \frac{1}{\rho_s \varepsilon_s}\partial_z \sigma_e - \frac{r(\varepsilon_s)(v_m - v_s)}{\rho_s \varepsilon_s (1 - \varepsilon_s)} \\ &\quad + \frac{f_{21s}}{\rho_s}\Pi_f(z) \end{aligned} \tag{15}$$

with:

$$f_{1s}(Q_f, C_f) = \frac{C_f Q_f}{A\Delta z_f} \tag{16}$$

$$f_{21s}(Q_f, C_f) = \frac{f_{2s} - v_s f_{1s}}{\varepsilon_s} = \frac{C_f Q_f}{\varepsilon_s A \Delta z_f}\left[\frac{Q_f}{S_f} - v_s\right] \tag{17}$$

S_f being the sludge feeding surface.

Equations (14) and (15) constitute a nonlinear hyperbolic system with two first order PDEs, two continuous nonlinear source terms (gradient of the interparticle stress and Stokes like drag force), two constant source terms (volumetric gravitational and Archimedes' buoyancy forces) representing the forces applied to solid particles and two discontinuous source terms representing the contributions of the activated sludge feeding. Here it is expressed in matrix form, with x the state variable vector:

$$x = \begin{pmatrix} \varepsilon_s \\ v_s \end{pmatrix}$$

And $f_s(x)$ the flux:

$$f_s(x) = \begin{pmatrix} \varepsilon_s v_s \\ \frac{v_s^2}{2} \end{pmatrix}$$

Then, the dynamic model of the urban sludge settling is:

$$\partial_t x + \partial_z f_s(x) = \mathcal{S}_1(x) + \mathcal{S}_2 \tag{18}$$

with $\mathcal{S}_1(x)$ equal to the three source terms depending on x and $\mathcal{S}_2$ equal to the three others not depending on x.

2.6. Boundary conditions

Two boundary conditions are required for these two first order PDEs. They come from a continuity of solid particles flow at the top and at the bottom of the clarifier, assuming a continuity/equality on the solid particles volume fraction, ε_s.

At the top, at $z = 0$:

$$\varepsilon_s(0,t) = \varepsilon_s^e \tag{19}$$

$$\varepsilon_s(0,t) v_s(0,t) = -\varepsilon_s^e \frac{Q_e(t)}{A} \tag{20}$$

with ε_s^e the solid particles volume fraction at the very beginning of the overflow.

At the bottom, at $z = z_b$:

$$\varepsilon_s(z_b, t) = \varepsilon_s^u \tag{21}$$

$$\varepsilon_s(z_b, t) v_s(z_b, t) = \varepsilon_s^u \frac{Q_u(t)}{A} \tag{22}$$

with ε_s^u the solid particles volume fraction at the entry to the outlet pipe.

During the operation of the clarifier, the activated sludge feed volume flow rate, $Q_f(t)$, and concentration, $C_f(t)$, are known as well as the withdrawal (recirculated and/or removed) volume flow rate at the bottom, $Q_u(t)$. It follows that the top overflow rate, $Q_e(t)$, is also known since $Q_f(t) = Q_u(t) + Q_e(t)$, the volume of sludge in the clarifier being constant.

Thus, the two boundary conditions are:

$$v_s(0, t) = -\frac{Q_e(t)}{A} \tag{23}$$

$$v_s(z_b, t) = \frac{Q_u(t)}{A} \tag{24}$$

Then, the state-space representation of urban sludge continuous settling in a clarifier is given by the nonlinear hyperbolic system (18) with $\sigma_e(\varepsilon_s, \alpha)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ constitutive equations in (11), (12) and (13), v_m algebraic equation in (9), contributions of sludge feeding in (16) and (17) and the boundary conditions in (23) and (24). The initial conditions are given in the simulation part.

The simulation is based on a numerical scheme that is well-suited to nonlinear hyperbolic systems exhibiting discontinuities or shock waves.

3. Numerical scheme

A very interesting discussion of various numerical schemes for this kind of non linear hyperbolic system is provided in the review [14]. Our simulations are carried out using semi-discretization (also called method of lines) leading to a system of ODEs. The Finite Volume method is used for spatial discretization. The time integration is performed using ode45 of Matlab®. This method is the most efficient and simple we implemented so far for non linear hyperbolic systems with source terms. It is based on the integral form of the balance laws and is well suited for simulation of fluid mechanics, as well as heat and mass transfer. One of this main feature is that it locally preserves the local balances with respect to fluxes, [15].

The state space vector is spatially discretized as a uniform mesh of N_z volumes of thickness Δz and constant cross-sectional area, A. Each volume i is between an upstream frontier boundary, indexed $i - \frac{1}{2}$, and a downstream frontier boundary, indexed $i + \frac{1}{2}$.

Equation (18) can be written in integral form :

$$\int_{i-1/2}^{i+1/2} \frac{dx(z,t)}{dt} dz = f_{i-\frac{1}{2}}(t) - f_{i+\frac{1}{2}}(t) + \int_{i-1/2}^{i+1/2} (\mathcal{S}_1(x) + \mathcal{S}_2)\, dz \tag{25}$$

where $f_{i\pm\frac{1}{2}}(t) = f_s(x(z,t))|_{z=(i\pm1/2)\Delta z}$

Considering that the state variables are uniform in each mesh and equal to some average value, $\bar{x}_i(t)$:

$$\bar{x}_i(t) = \frac{1}{\Delta z} \int_{i-1/2}^{i+1/2} x(z,t) dz$$

the previous equation can be approximated in integral form by :

$$\frac{d\bar{x}_i(t)}{dt} = \frac{1}{\Delta z} \left(f_{i-\frac{1}{2}}(t) - f_{i+\frac{1}{2}}(t) \right) + \mathcal{S}_1(\bar{x}_i) + \mathcal{S}_2 \tag{26}$$

where we assume that $\mathcal{S}_1(\bar{x}_i)$ is a good approximation of $\frac{1}{\Delta z} \int_{i-1/2}^{i+1/2} \mathcal{S}_1(x) dz$.
Let $F_{i\pm\frac{1}{2}}(t)$ represent an approximation of the fluxes $f_{i\pm\frac{1}{2}}(t)$ as a function of $\bar{x}_i$.
We can then write:

$$\frac{d\bar{x}_i(t)}{dt} = \frac{1}{\Delta z} \left(F_{i-\frac{1}{2}}(t) - F_{i+\frac{1}{2}}(t) \right) + \mathcal{S}_1(\bar{x}_i) + \mathcal{S}_2 \tag{27}$$

For hyperbolic systems a special care has to be taken, for the choice of these approximations, [20], [15]. The Rusanov approximation is chosen since it allows capturing the shock waves of the model:

$$F_{i-\frac{1}{2}} = \frac{1}{2} \left(f_s(\bar{x}_{i-1}) + f_s(\bar{x}_i) \right) - \frac{w_s}{2} \left(\bar{x}_i - \bar{x}_{i-1} \right) \tag{28}$$

where $w_s(t)$ is the propagation velocity of the fastest wave in the hyperbolic system. To ensure the stability of this method, the mesh size must respect the necessary CFL (Courant-Friedrichs-Lewy) condition i.e. $|w_s \frac{\Delta t}{\Delta z}| < 1$. Δt is chosen as the maximum of the time discretization step used in ode45. This propagation velocity, w_s, can be found as follows.

Let us express the nonlinear hyperbolic system (18) with the Jacobian matrix $J(x)$:

$$\begin{aligned} \partial_t x + \partial_x f_s(x) \partial_z x &= \\ \partial_t x + J(x) \partial_z x &= \mathcal{S}_1(x) + \mathcal{S}_2 \end{aligned} \tag{29}$$

with:

$$J(x) = \begin{bmatrix} v_s & \varepsilon_s \\ 0 & v_s \end{bmatrix} \tag{30}$$

The shock velocity, $w_s(t)$, is calculated as a function of the eigenvalues, λ_J, of the Jacobian matrix, $J(x)$, [20]. Thus, the double real eigenvalues is: $\lambda_J = v_s$ which means

that this system is not strictly hyperbolic (weakly hyperbolic) and therefore: $w_s(t) = \max_{z_i} \mid \lambda_J \mid = \max_{z_i} \mid v_s(z_i, t) \mid$.

Remark: This Rusanov numerical scheme works also for the weakly hyperbolic system (29), [15]

4. simulation of a dynamic event

A dynamic event was experimentally applied to a full-scale clarifier located at the wastewater treatment plant SYSTEPUR (Vienne, France). Inlet flow was artificially increased by a magnitude of approximately 2. Flow rates $Q_f(t)$, $Q_{ur}(t)$ and $Q_{ue}(t)$, depth of sludge blanket $z_v(t)$, and particle concentrations $C_f(t)$ and $C_{ur}(t)$ were continuously monitored. The sludge blanket level was measured with an ultrasound Royce device located on the rotating deck on top. In this section, simulated results obtained with our model are compared with experimental data specifically collected to test the model. First let us describe how the plant has handle the event.

4.1. Wastewater treatment plant description

The urban sludge treatment plant, [21] includes a primary settling tank, two biological aeration tanks in parallel and two secondary clarifiers in parallel.

Each secondary clarifier is equiped with two ON/OFF pumps for the sludge recirculation and two ON/OFF pumps for sludge extraction. According to the control strategy, one or two recirculation pumps are in operation as well as zero, one or two extraction pumps.

Our experimental study consisted in investigating the dynamic evolution of the separation of liquid and sludge particles in a clarifier.

Scenario description: a dynamic event was applied by routing all activated sludge from biological aeration tanks to one clarifier during approximately $8h$ (beginning at $t = 1.40am$; end at $t = 9.45am$). It has provoked a sudden increase of the flow rate of activated sludge Q_f at the inlet of the secondary clarifier.

During this scenario, the sludge recirculation flow rate was $Q_{ur} = 180m^3/h$ until $9.45am$, which corresponds to one recirculation pump running, and $Q_{ur} = 360m^3/h$ thereafter, which corresponds to two recirculation pumps running. The sludge extraction flow rate, $Q_{ue}(t)$ is null all the time. The experimental time profile of $Q_f(t)$ as well as $Q_f(t)$ measurement average are given in Fig. 2 with the other flow rates values.

4.2. Dynamic Simulations of the dynamic event

A uniform 28-node spatial mesh, a variable time step with a maximum of $\Delta t = 1\ s$ and the parameters values given table 1, chosen to fit the measurements, are used to run the simulations. The flow rate $Q_f(t)$ and the solid particles concentration $C_f(t)$ of activated sludge at the inlet of the clarifier measurements used in the simulation are averaged (respectively red curve in Fig. 2 and value in table 1).

Some parameters values come from the characteristics of the clarifier or the numerical discretization. The other parameter values are determined from the measurements and the ranges proposed in [4] and [2] adapted from mineral to organic sludges. Indeed,

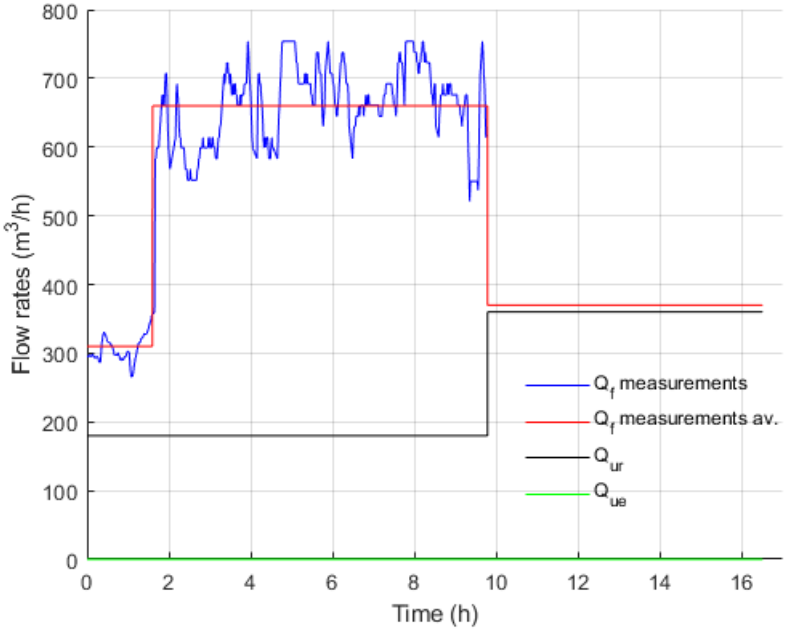

Figure 2. Measured activated sludge feeding flow rate and average value, $Q_f(t)$, recirculation flow rate, $Q_{ur}(t)$ and extraction flow rate, $Q_{ue}(t)$.

Table 1. Model parameters values.
**G: determined from fitting measurements with [4] constitutive equations*
**C: determined from fitting measurements with [2]constitutive equations*
**E: determined from fitting measurements with our model*

A	1175 m^2	clarifier section
A_k	9.81 10^{-4} m/s	$*C$
C_c	4.18 kg/m^3	$*E$
C_f	2.83 kg/m^3	average feeding concentration
Δt	1 s	numerical time discretization
Δz	10 cm	numerical spatial discretization
Δz_f	10 cm	feeding zone height
ε_c	4.1 10^{-3}	$*E$
n_r	2	$*C$
n_s	11	$*G$
ρ_s	1030 kg/m^3	$*E$
ρ_l	1000 kg/m^3	liquid density
S	1.44 m^2	sludge feeding zone surface
σ_0	0.5 kg/ms^2	$*G$
z_b	2.8 m	clarifier height
z_f	1.8 m	feeding zone average location

mineral particulate system density is about twice the density of the organic sludge density which changes the settling dynamics through the magnitude of the forces applied to the solid particles.

The initial stationary profile of the sludge concentration in the clarifier is given Fig. 4: profile indexed with t_{start}. It corresponds to the stationary profile of our model corre-

sponding to operating conditions before the dynamic event. The activated sludge feeding takes place at $z = z_f$.
Fig. 3 presents simulated and measured values of the sludge blanket level which are very close. It suggests that the model properly represents the main settling phenomena involved inside the clarifier.

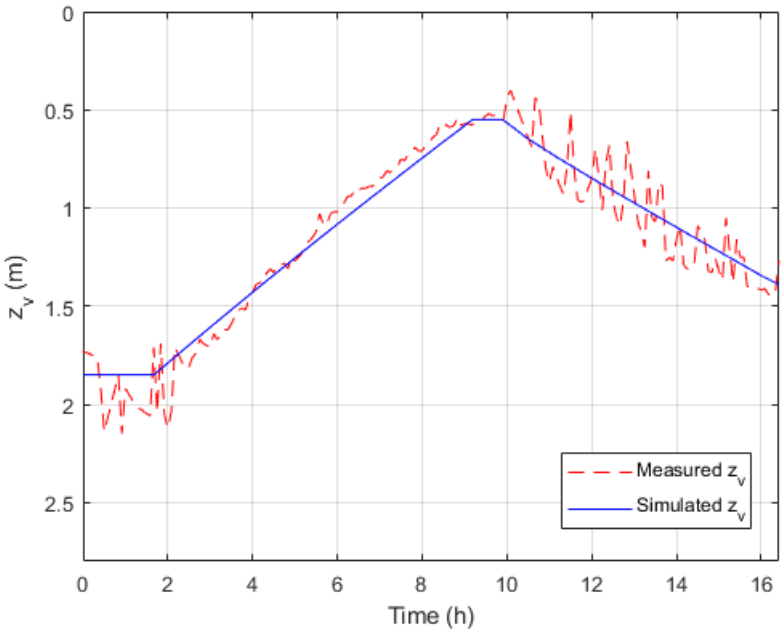

Figure 3. Comparison of simulated and measured sludge blanket level, $z_v(t)$.

Seven simulated solid particle concentration spatial profiles at different levels of progress of the simulated dynamic event are shown in Fig. 4. It can be observed in Fig. 4 that, as soon as the dynamic event was applied at the inlet of the clarifier, it leaves the stationary state and the sludge blanket level, $z_v(t)$, rises (Blue curves) up to depth $0.55m$ at $9.48am$. At this level, a second recirculation pump starts which causes the sludge blanket level to move down until the end of the measurements (cyan curves). The sludge blanket level corresponds to the spatial position of the maximum concentration gradient of solid particles. It also corresponds to the position of the shock wave (discontinuity) in solid particles concentration. As in practice, a change in behaviour appears below the lower interface when the compression threshold, C_c (red dashed line) is exceeded. There is a higher concentration of solid particles in this low compression zone.

Fig. 5 shows the time and space evolution of the solid particles concentration as a general overview of how the scenario progresses.

Currently, the execution time of a simulation on a workstation with Intel Xeon at $3.8GHz$ is $43min$. It can be explained by the fact that the settling phenomenon is very slow whereas the numerical scheme must allow the detection of the sludge blanket position variation which is a concentration discontinuity.

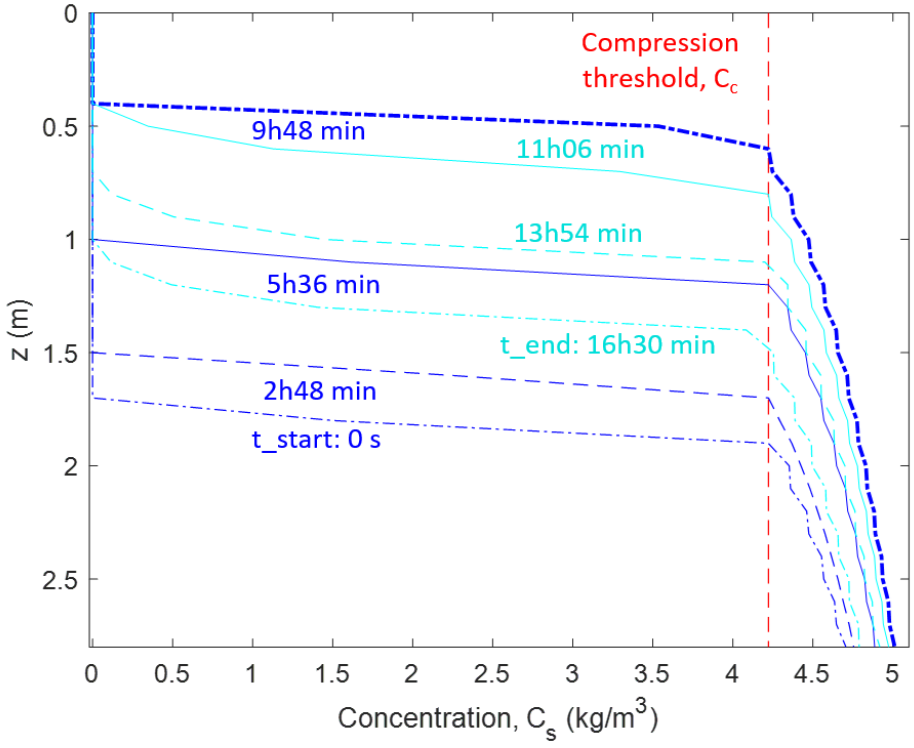

Figure 4. Simulated solid particle concentration C_s vertical spatial profiles at different instants. (Blue curves: the sludge blanket moves up, cyan curves: the sludge blanket moves down)

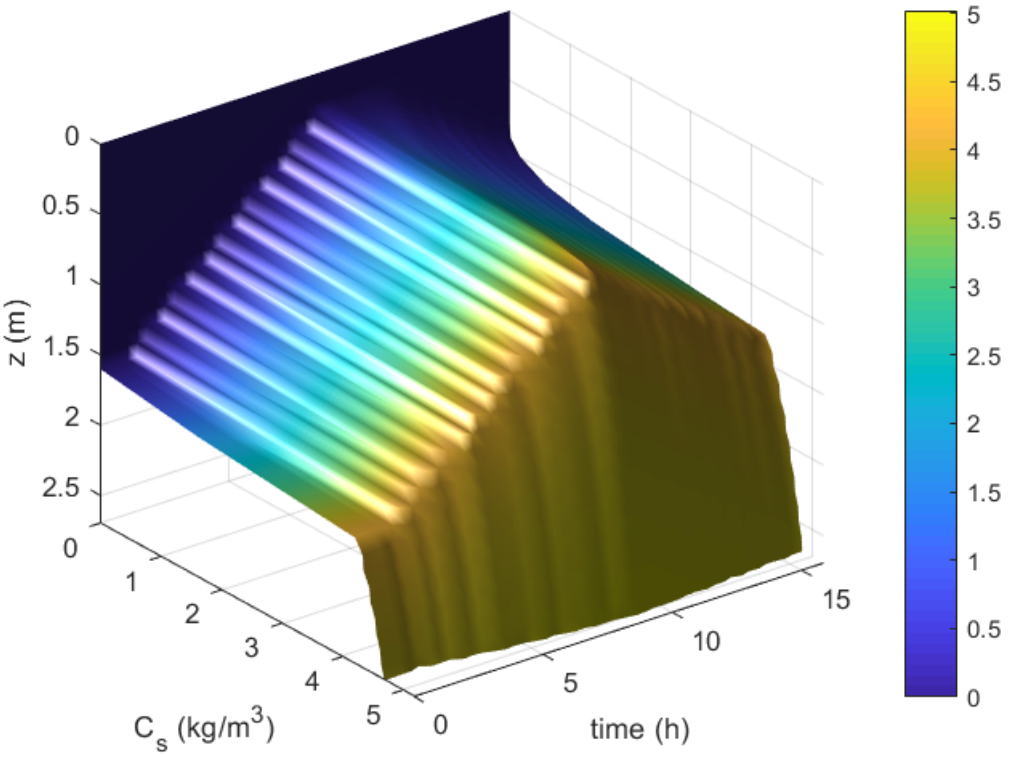

Figure 5. Solid phase volume fraction over time and space.

5. Conclusions and perspectives

In this paper, a 1-D dynamic knowledge-based model of urban sludge settling in a continuous secondary clarifier has been presented. Its originality is in its constitution of two dynamic mass and momentum balances, which makes it more general and representative of a wider range of settling processes (primary, secondary, in mines, cities, ...). Only the values of the model parameters have to be adapted to the context and there is no need for an additional equation to define the hindered settling velocity as in family 1. models. To the best of our knowledge, the model and the numerical scheme presented in this paper are the only ones of family 3. that are well adapted to urban sludge. They calculate everything that enables operating decisions to be made for the clarifier: the time evolution of the solid particles concentration profile in the clarifier as well as the outlet concentrations $C_e(t)$ and $C_u(t)$ and the sludge blanket position $z_v(t)$, knowing the activated sludge feeding flow rate and concentration $Q_f(t)$ and $C_f(t)$ and the compressed sludge outlet flow rate $Q_u(t)$. The dynamic model is a partial differential non linear hyperbolic system including source terms. It will also be very useful for a further design of a closed-loop controller to regulate the water quality at the top of the clarifier, through an automatic control of the sludge blanket depth by acting on recirculation and/or extraction fluxes. The simulation results were confronted with experimental results from measurements of the sludge blanket during a dynamic event. The numerical scheme used for simulation is a method of lines with a spatial discretization based on a finite volume method using the so called Rusanov approximation. This model is able to correctly simulate the behaviour of the clarifier.

It can also be noted that the estimation of the parameters of the dynamic model to match the measurements is an indirect method to determine the intermediate/compression threshold C_c.

One of our perspectives is to improve the numerical scheme or the way to implement it in Matlab® in order to reduce the execution time. The method of lines was implemented in [6] too without specifying how the flux were approximated and with a single hyperbolic equation corresponding to the mass balance which makes the system strongly hyperbolic and not weakly hyperbolic as in this paper. The CPU time was much shorter.
If a real time exploitation of these methods is not presently possible, they can be the heart of a decision support tool to help operators in wastewater treatment plants in their decision making.

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7. Notations

Index i stands for liquid phase (l) or solid phase (particles) (s).

$\alpha(\varepsilon_s)$	boolean intermediate / compression zone flag
A	cylindric clarifier section (m^2)
A_e	overflow surface (m^2)
A_u	outlet pipe section (m^2)
$C_i(z,t)$	solid (liquid) phase mass concentration (kg/m^3) $C_i(z,t) = \rho_i \varepsilon_i(z,t)$
$C_f(t)$	mass concentration of solid at the activated sludge feed (kg/m^3)
$\varepsilon_i(z,t)$	solid (liquid) phase volume fraction
ε_c	solid volume fraction intermediate/compression zone threshold
$f_s(z,t)$	system average volumetric flux (m/s)
$P(z,t)$	excess pore pressure (Pa)
$Q_f(t)$	volume flow rate of the activated sludge feed (m^3/s)
$Q_e(t)$	volume flow rate of clarified water released at the top of the clarifier (m^3/s)
$Q_u(t)$	volume flow rate of compressed sludge which is pumped at the bottom of the clarifier (m^3/s)
$r(\varepsilon_s)$	resistance coefficient of the drag force proposed by Darcy and Gersevanov in a two-phase model ($kg.m^{-3}.s^{-1}$)
ρ_i	solid (liquid) phase density (kg/m^3)
$\sigma_e(\varepsilon_s)$	effective solid stress function (Pa)
S_f	Sludge feeding surface
$v_i(z,t)$	solid (liquid) phase average velocity (m/s)
$v_m(z,t)$	volume average velocity (total volume flux of the suspension) (m/s)
z_b	cylindric clarifier height (m)
$z_c(t)$	intermediate/compression interface location (m)
z_f	average location of the activated sludge feeding zone (m)
$z_v(t)$	sludge blanket location (m)

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