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CHANGES IN THE TEACHING OF NUMBERS AND RATIOS IN THE PRIMARY CURRICULUM

Christine Chambris

Laboratoire de didactique André Revuz, Université de Cergy-Pontoise, France

Abstract

This paper tackles the issue of changes in the teaching of numbers and ratios in the primary curriculum in France, especially the crucial changes that occurred during the New Math. It investigates specifically the role of mathematical reference knowledge. It shows how these changes still impact the teaching of numbers and ratios.

Keywords: Numbers, ratios, New Math, reference knowledge, didactic transposition.

INTRODUCTION

In the 1960s and 1970s, in many countries throughout the world, the New Math Reform aimed at renewing the teaching of mathematics from the primary school up to the university, and at increasing mathematics proficiency in the general public in the context of the cold war (Furinghetti, Menghini, Arzarello, & Giacardi, 2008). One of the major concerns of the New Math was the teaching of some “new” math. This enables to make the assumption that this period showed major changes in the mathematical foundations of basic arithmetic. The case of the teaching of set theory in primary schools in these years is a famous example. Another concern of the reform was to take into account some psychological features related to the learning or to the child development. The famous psychologists Piaget and Bruner contributed at least indirectly but significantly to the implementation of this reform. It has often been said that New Math failed, however “In no country did school mathematics return to where it had been before the new math movement began: The pendulum is not a suitable metaphor for curriculum change.” (Kilpatrick, 2012, p.569).

The aim of this paper is thus to identify some of the changes promoted by the New Math, and their long term effects. It will focus on the teaching of numbers, operations, and ratio in the primary curriculum in the French curriculum¹. Hence, I aim at identifying possible changes in mathematical foundations for numbers and ratios surrounding the New Math, and possible effects of these changes on the curriculum up to nowadays.

¹ In this note I indicate some specific uses that may be helpful to better understand some of the data presented after. In France, at present time, there is no use of comma to write great numbers, a space is required. It seems a century ago, at least some people were using a point. Moreover, for writing decimal numbers –with the positional notation- a comma is required to indicate the unit. Hence, 30 000,4 means thirty thousand and four tenth.

With regard to multiplication, the sign \times is used between two numerals: 13×15 . In algebra, “ \times ”, “ \cdot ” or no sign can be used to indicate multiplication between two letters or between a numeral and a letter.

METHOD

Theoretical frame

The Theory of Didactic Transposition (TDT) (Chevallard, 1985) considers school mathematics as a reconstruction by the educational institutions from the mathematical knowledge produced by academic scholars. Four steps are distinguished (fig. 1): scholarly knowledge, syllabi and textbooks, teachers, then students. The Anthropological Theory of Didactics (ATD) (e.g. Bosch and Gascon, 2006 for a reference in English) extends the TDT. It postulates that practicing mathematics, as any human practice, can be described with the model of *praxeology*. It is constituted by four components: a *type of tasks* -a set of similar problems-, a *technique* -a “way of doing” for all the tasks of the type-, a *technology* justifies the technique, and a *theory* legitimates the technology. In this model, the reference knowledge is the scholarly knowledge.

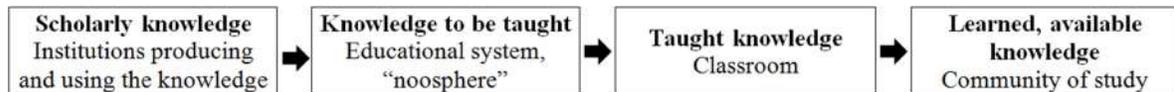


Fig.1: The process of didactic transposition (*op. cite*)

The ATD introduces the “ecology of knowledge” metaphor to describe the evolution of teaching objects. For a given object at a given moment, it defines: its habitats (where it lives) and ecological niches (its roles in each of its habitats). It also considers “trophic chains”: A needs (to eat) B to live. Thus, the ATD asserts the following paradox: To be eaten is a reason to live in the teaching system.

Academic knowledge

In the history of mathematics, the development of numbers has been closely connected to the measurement of quantities, especially that of continuous ones (e.g. Book V by Euclid). But, since 1870 approximately, in academic mathematics, numbers have no longer been elaborated from quantities but from whole numbers, then from sets (Bourbaki, 1984). Did such a change impact the primary curriculum? To what extent has present academic knowledge impacted the primary curriculum during the New Math, and still does?

Background

Bronner (1997) highlights five main periods in the teaching of numbers in the French system: the stable classical period 1870-1950, the New Math highly turbulent period 1970-1980, the continuously evolving contemporary period 1995-2010, and two transitional ones before and after the New Math. Treatises by Bezout (1779), then Reynaud (1821) appear as reference books for primary teacher education, and textbooks by 1940.

Research questions

What are the habitat and niche of quantities in the different praxeologies during the different periods? Are there traces of changes of “reference knowledge” for numbers, operations, ratios and quantities during the different periods, especially around the New Math?

Data

The studied period is huge. In order to get an overview of the curricula, data have been first restricted to the national French syllabi (1882, 1923, 1945, 1970, 1977-1980, 1985, 1995, 2002, 2008) for grades 1 to 5 and to previous works studying teaching of arithmetic throughout the 19th or 20th century. Several 2nd- and-3rd-grade textbooks of have been included after. Mathematical treatises and books for teacher education inform us about scholarly or reference knowledge.

Methodology

First, an extensive analysis of syllabi is conducted in order to identify each occurrence of discrete and continuous quantity. This enables to locate their habitats, and then the role of quantity is interpreted in term of praxeology (niche). Similar analysis is conducted with treatises, textbooks analysis enables to refine, infirm, or confirm the previous analyses on specific topics (addition and multiplication). According to the collected data –i.e. syllabi and textbooks-, only the two first steps of the didactic transposition (Fig. 1) is considered.

FINDINGS

Before the New Math (1870-1970)

The syllabi from 2nd grade (1923) even 1st grade (1945) gather, in the same paragraphs, discrete and continuous quantities: indeed both types of measurement are involved for whole numbers, operations, and place value - which is clearly connected with the metric system-. Moreover, in the 4th grade, in 1945, fractions are fractions of quantities and decimals are introduced using the metric system. Before 1940, the teaching of arithmetic is much inspired by the treatises by Bezout (1779) then Reynaud (1821). This influence remains up to 1970. Both treatises present an arithmetical theory relying on measurement of quantities for numbers and ratios.

In order to highlight the didactic transposition, I present excerpts of the beginning of Reynaud’s treatise and a page of a textbook (fig. 2) of the beginning of the 2nd grade.

First chapter. Numeration, Addition, Subtraction, Multiplication, and Division of whole numbers.

1. Whatever is capable of increase or diminution is called *quantity*. When one thinks about the nature of quantities, one feels it would be impossible to have an exact idea of magnitude of quantities of the same kind without considering one of

these magnitudes as a term of comparison; this magnitude is called *unit*; combining several units of the same magnitude forms a number. The manner of forming, expressing, and writing numbers is the object of *numeration*; and the science which teaches how to perform operations upon numbers is called *Arithmetic*.

2. Numeration (...)

3. A number is *abstract* or *concrete* when no particular denomination is mentioned to which its units belong or not. Thus, 3 and 5 times are *abstract numbers*; 3 toises and 5 leagues are *concrete numbers* (...).

Within whole Arithmetic, as the nature of the units is known in advance, one only has to find their number; this leads to operate on abstract numbers. (Reynaud 1821, p.1 and p.6), my translation.

ADDITION

1. Ajouter des collections.



Il y a 3 poires sur une assiette et 2 poires dans la main de Paul. Paul **met** ses 2 poires avec les 3 poires de l'assiette : il **réunit** les deux collections de poires.

Paul **compte** les poires : il y en a 5 *en tout*. En **ajoutant** 2 poires à 3 poires, on obtient 5 poires.

2. Ajouter des longueurs. — Louis avait deux morceaux de canne à pêche :
l'un de 3 mètres l'autre de 2 mètres.



Il **ajoute** ces morceaux l'un à l'autre en les mettant **bout à bout**.



Il **mesure** la longueur de la canne. Il **compte** les mètres. Il y en a 5 en tout. En **ajoutant** 2 mètres à 3 mètres, on obtient 5 mètres.

3. Addition de nombres. — Qu'il s'agisse de poires ou de mètres, *trois et deux font cinq*.
Compter que 3 et 2 font 5, c'est faire une **addition**.
Au lieu de 3 et 2 font 5
 ↓ ↓
on écrit : 3 + 2 = 5
ce qui se lit : 3 **plus** 2 égale 5.

3. Addition of numbers. Be they pears or meters, *three and two are five*. Counting 3 and 2 are 5: this is doing an *addition*. Instead of 3 and 2 are 5, one writes: $3 + 2 = 5$, what one reads: 3 **plus** 2 equals 5.

Fig. 2: (Marijon *et al.*, grade 2, lesson 4, 1947, p.14), my translation.

In the treatise, numbers emerge from the measurement of quantities. Within the textbook's page, the words "concrete" and "abstract" are not used. Whatever, three cases are taken for talking about addition. Putting together firstly discrete quantities (1. collections of pears), secondly continuous quantities (2. adding length through two pieces of a fishing rod) provide two cases with concrete numbers. A third case using abstract numbers is elaborated from the two first cases (fig. 2).

On figure 3 -a page of the beginning of the 3rd-grade textbook, published in 1932-, the themes of the word problems may seem old-fashioned. I don't know whether they were familiar or not to the children of the 1930s. The structure of the lesson is common for the period: the rule is established using "solved problems", and then it is stated. Exercises –not shown- are proposed after: simple and complex word problems involving multiplication the rule. This excerpt reveals several clues related to the underlying reference knowledge. They may not be easily visible for an unaware reader.

Multiplying by several tens or hundreds

Multiplying an amount of tens. Problem

A box of quills is 60 g heavy. How heavy are 72 boxes?

The weight is 72 times 6 tens of g.;

6 tens \times 72 = 432 tens; 432 tens = **4.320 g.**

Solution. The total weight of 72 boxes is:

60 g \times 72 = **4.320 g.**

Multiplying by an amount of tens. Problem

How much do **60 barrels** of **72 l.** each contain?

A ten of barrels contains: 72 l. \times 10 = 720 l.

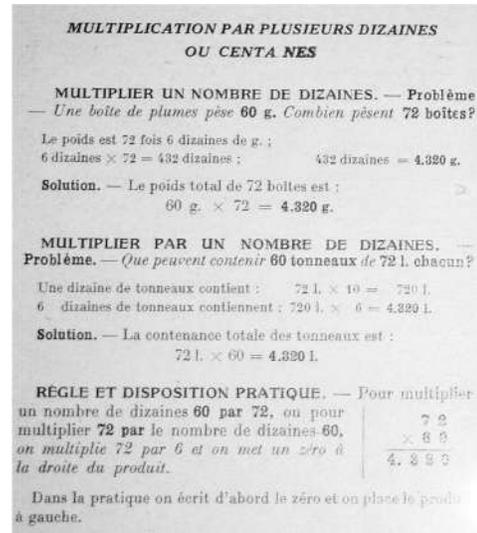
6 tens of barrels contains: 720 l. \times 6 = **4.320 l.**

Solution. The total capacity of the barrels is: 72 l. \times 60 = 4.320 l.

Rule and practical disposition. To multiply an amount of tens **60 by 72**, or to multiply **72 by** the amount of tens **60**, one multiplies 72 by 6 and one put a zero at the right side of the product. [column computation]

Practically, one writes first the zero, then the product one the left.

Fig. 3 : (Châtelet *et al.* 1932, p.160), my translation.



First, I go back to the treatise:

6. The purpose of the **multiplication** is to calculate a number which is called **product**. The latter is formed with a known number called **multiplicand**, similarly as a given number -called **multiplier**- is formed with the unit. So, to obtain the product, one just has to operate on the multiplicand the same operations one would have operated on the unit to form the multiplier. Multiplicand and multiplier are the factors of the product. For instance, for multiplying 5 by 3, one can observe that the multiplier is formed with three times the unit. So the product must be formed with three times the multiplicand 5; this product is thus 5 plus 5 plus 5 or 15. Generally, when the multiplier is a whole number, multiplication is simply repeating the multiplicand, as many times as there are units in the multiplier.

The *multiplier is always an abstract number* because it indicates how many times the multiplicand has to be taken.

The *product is of the same nature as the multiplicand*; indeed it expresses the addition of several numbers that are equal to the multiplicand. (...) (Reynaud 1821, p.10, my translation)

The definition given by Reynaud may seem very complex. Actually, the formulation allows to gather in a same approach multiplication by whole numbers, and by fractions. With whole numbers, it goes back to repeated addition of the multiplicand. Bezout (1779) gives a shorter definition, though in the same vein: “Multiplying a number by another; this means: taking the first one as many times as there are units in the other one. Multiplying 4 by 3; this means: taking three times the number 4” (p.28, my translation). The reference

knowledge is thus related to the external multiplication of quantities by a number. For instance, this implies that multiplication is not commutative. Within the textbook's page, a major consequence of the definition is the distinction between multiplicand and multiplier. It has several implications: e.g. 1) $60 \text{ g.} \times 72$ is read 72 times 60 g or 60 g multiplied by 72; 2) multiplier is written on the right, multiplicand on the left; 3) two cases are needed for elaborating the rule: multiplying tens and being multiplied par tens. Some textbooks elaborate the possibility of changing the order of abstract numbers within a multiplication before, so they refer to it and develop only one case; 4) actually this definition constraints but also allows to refer to quantities for elaborating the rule. Last, another point is the use properties of numeration, especially linked with units (Chambris 2015). This hides the use of associativity.

The New Math reform curriculum (1970)

The introduction of the reformed syllabus asserts the curriculum is not a new one but a “different writing” of the previous one from 1945, a huge ecological reorganization I add. The main visible change is the creation of a new domain: “measurement: practical exercises”. Previously, there were only two domains: arithmetic and geometry. Now, they are three: numeric, geometry, measurement. An acute look at the text shows that: 1) The new measurement domain gathers the study of continuous quantities which was previously in the arithmetical part -measurement and computation with length, capacity, mass, time and corresponding metric units-, and that of the geometrical quantities which was previously in the geometrical part -computation of area and volume, and the corresponding metric units-. 2) The “numeric” domain is devoted to measurement (i.e. counting) and computation of discrete quantities, with many quotations of the set theory. 3) The only exception to this is that of scales problems which moved from geometry to “numeric”, and joined there the old “rule of three” problems under the new “proportionality” label. Last, computation involving units' names are forbidden: “Sentences like “8 apples + 7 apples = 15 apples” do not belong to mathematical language neither to ordinary language” (Instructions for syllabus 1970, my translation).

Decimals and fractions are studied in both domains: numeric and measurement with the same techniques and technologies. It is noteworthy that they do not involve fractioning of quantities. I interpret this as the transposition of the academic construction of the real numbers with whole numbers. The “numeric” domain contains many tables of numbers with operators. They support the new way to study ratios: with linear properties and coefficient, that is a trace of the transposition of the theory of linear application. These are traces of new reference knowledge. All computations are computations between numbers; and proportionality is no longer a relation between two quantities: it became a linear numerical relation.

To sum up: The reorganization of the New Math syllabus is the sign of the implementation of new reference knowledge for numbers and ratios, of academic knowledge. Paradoxically, the birth of the measurement domain is the visible side of the disappearance of quantities as the roots of arithmetic.

From the 1980s

From 1970, the measurement domain has remained with several changes in each new syllabus, but more and more continuous quantities get into the “numeric” domain (fractioning of quantities for fractions and decimals from 1980, number line for whole numbers and fractions from 1980). This can come from the influence of several academic works in mathematics education: e.g. (Perrin-Glorian, 1992). Yet, despite the assertion that fractions are measurement of quantities, there are no fractions of quantities -such as: $\frac{3}{4}$ of a length or of 300 g- in the syllabus. Moreover, most of mathematical definitions of quantities (e.g. Euclid Book I, Rouche 1992, Griesel 2007) involve a basic relation between order and addition: $a < b$ iff it exists c , $a+c=b$; yet, addition is never linked with order relation in the curriculum. Related to proportionality, in 2002, the syllabus indicates examples of reasoning (technologies) with quantities in the form of computation with units:

One needs 400 g of fruit with 80 g of sugar to prepare a fruit salad. How much sugar is required with 1000 g of fruit? Reasoning can be of the following types:

- For 800 g of fruit (twice more than 400), 160 g of sugar (twice more than 80) are required [...]. For 1000 g (800g + 200 g) of fruit, 200 g (160 g + 40 g) of sugar are required;

- The mass of sugar required is five times less than the mass of fruit; 200 g of sugar ($1\ 000 : 5 = 200$) are required. (Instruction for syllabus 2002, 3rd-to-5th grade, my translation)

Looking closely at these rationales, a phenomenon becomes visible: operations with units are used in discursive or in uncomplete arithmetical ways (the only equal sign is used between numbers). Operations with units seem not to belong to mathematics, but seem to fulfil pedagogical needs. My interpretation is that quantities come back for didactical needs, but the underlying mathematical reference knowledge has not changed from the reform. This brings to the fore the need for new reference knowledge: an adequate theorization of quantities for the teaching of numbers and ratios.

Figure 4 displays an excerpt of a present 3rd-grade textbook: the topic is roughly the same as in 1932. According to the teacher’s guide, through the “discovery” activities, the students attempt first to compute 8×40 then 60×30 by their own means, then the focus –within the bubble- is on the implicit use of the associativity, mixed with commutativity of multiplication. It is supposed to be drawn on students’ attempts –and, if need be, on a “solved problem”: Leila’s method-. There is no unit, no quantities. I specify that in the previous lesson -

multiplying by ten-, no link had been established between multiplying tens and converting ones into tens (numeration). Last, current curricula do not propose a specific way for writing the numbers within multiplication (which number on the right?). Some examples of commutativity for multiplication are often given at the very beginning of the curricula. Then 3×5 or 5×3 are used indistinctly. There seems to be very little further investigation about this property.

Multiplying by multiples of 10, of 100, of 1000

Goals: understanding the rule of multiplication by multiples of 10, of 100, of 1000.

Discovery:

1. Compute: 8×40 then 60×30
2. See how Leila (a character of the book) has computed 8×40 then 60×30 : [...]

Compare with your own computation.

Within the bubble: In a product, one can choose the order in which multiplications are performed. Example: $6 \times 10 \times 3 \times 10$ or $6 \times 3 \times 10 \times 10$.

3. Compute: 7×50 and 30×20 using Leila's method.

Exercises:

1. Compute [...]
2. Complete [...]
3. Copy down the correct answer.
 - a. One CD costs 12 €, 10 CD cost: 1 200 € 22 € 120 € 1 210 €
 - b. One bike costs: 100 €, 5 bikes cost: 150 € 500 € 105 € 5 000 €
4. Copy down the table and complete it. [...]

DECOUVERTE

1 Calcule : 8×40 puis 60×30 .

2 Voici comment Leila calcule 8×40 et 60×30 :

8×40
 $8 \times 40 = 8 \times 4 \times 10$
 $8 \times 40 = 32 \times 10$
 $8 \times 40 = 320$

60×30
 $60 \times 30 = 6 \times 10 \times 3 \times 10$
 $60 \times 30 = 6 \times 3 \times 10 \times 10$
 $60 \times 30 = 18 \times 100$
 $60 \times 30 = 1 800$

Dans un produit, on peut choisir l'ordre dans lequel on effectue les multiplications. Exemple : $8 \times 10 \times 3 \times 10 = 6 \times 3 \times 10 \times 10$.

Compare avec ton calcul.

3 Calcule : 7×50 et 30×20 avec la méthode de Leila.

EXERCICES

1 Calcule.

a. 50×6 d. 3×600 g. 40×20
 b. 3×70 e. $8 \times 2 000$ h. 300×60
 c. 400×5 f. $9 000 \times 2$ i. 90×30

2 Complète.

a. ... $\times 50 = 200$ d. ... $\times 75 = 1 500$
 b. $20 \times \dots = 1 000$ e. ... $\times 60 = 180$
 c. $25 \times \dots = 1 000$ f. $50 \times \dots = 3 000$

3 Recopie la bonne réponse.

a. Un CD coûte 12 €, 10 CD coûtent :
 1 200 € 22 € 120 € 1 210 €

b. Un vélo coûte 100 €, 5 vélos coûtent :
 150 € 500 € 105 € 5 000 €

4 Recopie et complète la table.

\times	5	10	20	40
20			180	
		300		
60				

Fig. 4: (Peltier *et al.* 2010 p.89), 3rd grade, middle of the year. My translation

A “learning” theory rules the design of the activity: building on students’ knowledge. The mathematical reference knowledge seems to be that of operations within the set of whole numbers $-N-$. Yet, within the ancient texts, quantities provide rationales for the multiplicative properties. It is not the case within the present excerpt. The rationale seems rather to be a matter of writing. Does this involve learning difficulties? Whatever, depending on the topic and on teachers’ choices, quantities can be solicited or not: e.g. some textbooks refer to “word problems” for this given topic.

This fosters my previous interpretation: quantities have come back for didactical needs, especially when continuous quantities or ratio are involved. This come back is less clear for whole numbers.

DISCUSSION

To begin with, the grain of the current study is rough and further investigation is needed to better characterize even the written curriculum. However, it is clear that the measurement domain cut many old trophic chains in the 1970s. Some new trophic chains were planned –e.g. the reorganization of proportionality- but it is not sure they “compensate” the broken links with quantities. Indeed, more and more continuous quantities have been added in the “numeric” domain after the reform, as if it were a pedagogical need. There is still a measurement domain which role is poorly defined, especially its contribution to the understanding of operations on quantities –e.g. addition-, involved for instance in the learning of fractions.

The New Math is an international phenomenon (Kilpatrick 2012) that impacted several western countries in the 1960s and 1970s, among them the US and Germany. Ma (2013) is very critic on the current written math curricula of the US, and connects its “disorganization” to the New Math and to the disappearance of a classical theory of arithmetic. Singaporean and Chinese curricula are built on such theory. Griesel (2007) identifies different “New reference knowledge” for fractions in West and East Germany during the New Math: with quantities, with sets or whole numbers.

CONCLUSION

This work helps to better understand some effects, even long term effects of the New Math in France. Quantities disappeared in reference knowledge for numbers and ratios surrounding the New Math. Yet, they seem to have partly come back -more and more in recent years- for pedagogical needs. It shows the lack of adequate reference scholar knowledge for numbers and ratios, connected to quantities despite relevant scholar works (e.g. Griesel, 2007; Rouche, 1992). Such theory notably enables to formulate basic arithmetical rules in basic terms. The increasing role of learning theory based on students’ activity that has been promoted from the New Math may hide –or compensate- this lack. Present study leads to ask whether the results could be similar to other countries that were close (in term of curriculum) or not to France. Before the New Math, whole numbers, place value; and operations were taught with both discrete and continuous quantities. The creation of the “measurement” domain is the visible side of the transposition of the set theory and the elimination of continuous quantities in the reference knowledge for numbers, operations and ratios. Yet, continuous quantities seem to be a key input for conceptualization of numbers and ratios (Barrett *et al.*, 2011); and the epistemology (Artigue, 1991) of numbers also fosters the desirable approaches on continuous quantities for numbers and ratios. Another question is: to which extent the roles of quantities in basic arithmetic impact teachers’ perceptions of Mathematics, especially the idea of rigor, and that of the relation between math and everyday life.

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