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THE PHILOSOPHICAL SIGNIFICANCE OF CARROLL'S REGRESS

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Introduction

Lewis Carroll's "What the Tortoise said to Achilles" (Carroll 1895) is as enigmatic as it is famous. The puzzle stated in it, which results in a certain kind of regress, is often called the "paradox of inference" and "Carroll's regress" although it is unclear that it states a genuine paradox and what kind of regress is in question¹. It raises many kinds of issues for logic, philosophy, psychology, cognitive science, and in general for all disciplines concerned with the nature of rationality and reasoning, although there is large room for dispute about what kind of issues are involved and how they can be settled. The story is supposed to have a moral, perhaps several, but after more than one century of comments, neither the exact lessons nor the full implications of it have reached consensus. In what follows, I am not concerned here with how Carroll himself might have conceived his tale. Neither shall I try to propose my own treatment of the puzzle². I limit myself here to giving a sort of map of some of the philosophical problems which have figured most prominently in the literature and to suggesting some of the contemporary answers, without attempting to draw a full catalogue of their implications. I shall focus here on four main problems: (a) the nature of logical inference, (b) the nature of our understanding of logical rules and logical knowledge, (c) the justification of logical rules, (d) the nature of normative guidance in both theoretical and practical reasoning. I shall finally indicate some pathways for dealing with these issues.

1. The nature of logical inference

Achilles presents to the Tortoise three propositions:

- (A) Things that are equal to the same are equal to each other
- (B) The two sides of this triangle are things that are equal to the same
- (Z) The two sides of this triangle are equal to each other

¹ See e.g. Clark 2002. It is not clear, however, that this puzzle is a genuine paradox, in the ordinary sense of a set of acceptable premises leading to an unacceptable conclusion through apparently acceptable rules of inference, for in the case at point precisely nothing is inferred from the premises.

² The present essay draws in part from my previous publications (Engel 1989, 1998, 2005, 2005 a, 2006, 2007, 2010)

The Tortoise accepts A and B, but does not accept Z, although he accepts the proposition which Achilles presents to her:

(C) If A and B are true then Z must be true

As Achilles insists, the Tortoise is also prepared to accept that if A and B and C are true, then Z must be true, but he still refuses to accept Z, and the regress never ends.

There has been a lot of discussion about regresses, and on whether they are always vicious. But in the case at hand, everyone agrees that there is a regress, and that it is harmful, although the problem is to understand what kind of harm is here done.³

Two things are *prima facie* noteworthy. The first is that the Tortoise agrees that (Z) “follows logically “from (A) and (B), and that “the sequence [(C)] is a valid one” (Carroll 1895: 691). The second is that the Tortoise makes his acceptance conditional on C being entered as a further premise. The puzzle is: how, given these facts, can he fail to accept Z?

The most usual moral of the story focuses on the second point: the fact that the Tortoise needs to add the conditional (C) as a supplementary premise shows that he fails to make a distinction between the propositions A and B on the one hand and the logical truth (C) on the other, and more generally between a premise and a rule of inference⁴. This is Carroll’s own diagnosis when he explains his article to the Editor of *Mind*: “My paradox turns on the fact that in a Hypothetical, the truth of the Protasis, the truth of the Apodosis, and the *validity of the sequence* are three distinct propositions.”⁵ In other terms, we can neither treat a rule of inference, such as *Modus ponens* or a true conditional, as a premise. Once we respect this distinction, it seems that Carroll’s regress cannot start. It is the lesson that we get from most logic manuals today⁶. The point looks simple and straightforward, especially now that we have a clear grasp of the difference between a proposition, a tautology, and a rule of inference. Lewis Carroll’s logic was traditional syllogistic, and although he was acquainted with the logic of Boole and Jevons and had a sophisticated theory of logical diagrams, he did not have the kind of distinctions that we have today between rules of natural

³ See Wisdom 1974, Gratton 2010, 38-43

⁴ Among the reactions to Carroll’s paradox along these lines, see Peirce 1902 (*Collected Papers* , Harvard , Harvard University Press, 2.27), Russell 1903, p. 35, note : “The principles of inference which we have accepted lead to the proposition that, if *p* and *q* be propositions, then *p* together with “*p* implies *q* ” implies *q*. At first sight, it might be thought that this would enable us to assert *q* , provided *p* is true and implies *q*. But the puzzle in question shows that this is not the case, and that, until we have some new principle, we shall only be led into an endless regress of more and more complicated implications, without ever arriving at the assertion of *q*. We need, in fact, the notion of *therefore*, which is quite different from the notion of *implies*, and holds between different entities. See also Brown 1954 ,Geach 1965, Thomson 1960

⁵ Dogson 1977, p.472, quoted by Smiley 1995. According to Smiley (p.727) Carroll lacked the distinction between a deduction and assertion of a hypothetical proposition.

⁶ see e.g Goldstein , Brennan, Deutsch and Lau, 2005, p. 41-42.

deduction and axioms⁷. Still, the fact that the 1995 article is about an inference in *modus ponens* form is not indifferent. In 1894 Carroll published, also in *Mind*, a problem about “hypotheticals”, known as “the Barbershop paradox”, which deals with the paradoxes of material implication, and a number of his reflections at this time show that he was interested in hypotheticals, which we would call today conditionals.⁸ Nevertheless, Carroll’s problem is formulated in terms of a perfectly classical syllogistic form, and the 1895 *Mind* article, unlike the 1894 one, does not discuss any paradoxical form of hypotheticals. One clue, however, is given to us by A.N. Prior when he remarks that just before the publication of Carroll’s piece, the logician Alfred Sidgwick (not to be confused with his cousin Henry Sidgwick) published in *Mind* a paper arguing that *If p then q* always means that *one can deduce q from p*, and that Carroll may have wanted to give a counterexample to that claim. The article by Sidgwick may well be one of the origins of Carroll’s piece, which, on that hypothesis, would have been written precisely in order to distinguish a hypothetical in the “predicative sense” (i.e. propositional) and not in the “rule of inference” sense.⁹

Another relevant logical issue (which is present in Sidgwick’s 1895 article) is the nature of assertion, and of what Geach (1965) has called “the Frege point”: “A proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition”. Carroll might well have seen the point, since the sentences (A) and (B) are asserted as premises of the target inference, although they are not asserted as antecedent of the conditional in (Z). Given the importance that the Frege-Geach point has taken in discussions of the logic of moral discourse and imperatives¹⁰, Carroll’s story reinforces it.

But even if we take Carroll’s point to consist in making the distinction between a rule of inference and a premise, the problem that he raised would still have been with us. For what the story tells us is that *even* when the proposition (C) is explicitly laid out (written down in the Tortoise’s notebook), that is even when the difference between a mere premise in a proof and a principle of inference is put *explicitly* in front of him, the stubborn animal refuses to draw the conclusion. Why? Indeed the whole story turns upon how we answer that question, but Carroll gives us a hint when he explicitly presents the problem as one about the force of the logical “must”. The puzzle does not

⁷ See Moktefi 2008 on Lewis Carroll’s logic

⁸ See Moktefi 2007, pp. 490-496

⁹ Prior 1963 p. 19, with reference to Sidgwick 1895, p. 43. A. Sidgwick (1850-1943) was a logician interested in what we would call today argumentation theory. Two years before Carroll’s 1895 article, he published a logic book on arguments (Sidgwick 1893) and he was one of the reviewers of Carroll’s *Game of logic* in 1887 (see Moktefi 2006, who quotes his discussion of Carroll 1894 and another piece on hypotheticals in 1895, but he does not quote the article “Context and Meaning”, which may well have been the proximal source of Carroll’s problem.

¹⁰ The point is meant to be a problem for any anti-factualist or expressivist conception of moral discourse. For instance, if one says “It’s wrong to lie” and “if it wrong to lie you must not induce your little brother to lie” one is entitled to conclude “you must not induce your little brother to lie”. If the premises are true, the conclusion is true as well. But if moral statements are neither true nor false, it’s hard to take this inference as valid. There is huge literature on this issue.

consist in the fact that the Tortoise fails to make the difference between a proposition and a rule of inference or a principle of logic. For he recognizes that Z follows from A and B by “logical necessity”. He agrees that “any one who accepts A and B as true, *must* accept Z as true”, and that “The youngest child in High School...will grant that.” So he has a clear grasp of the difference in modal status between the premise and the rule the former may be a contingent proposition, whereas the latter is necessary and forces us to infer. For all that, he still refuses to accept Z, and challenges Achilles “to force [him], logically, to accept Z as true.” So the issue is not simply one of distinguishing rules from propositions, but of understanding the nature of logical rules and the kind of *force* they have¹¹.

2. *Knowledge and understanding of logical rules*

The second moral that one can draw from Carroll’s story has to do with the *epistemology of understanding* and with our *knowledge of logical rules*. It would seem that the very first condition for a logical rule to have a force in moving us to infer is that we know the logical rule in question. The Carroll puzzle can be formulated thus: how can someone who understands the rule of *modus ponens* (MP) fail to apply it? The Tortoise seems to know the rule, since he grants that from (A) and (B) and (C) one can infer (Z). But does the fact that he nevertheless fail to infer shows that he does not actually understand the rule? It seems that “understanding and accepting premises of the form P, and if P then Q, but not accepting the corresponding proposition Q describe nothing at all” (Black 1970: 21)¹². Either the person in question does not understand the meaning of these sentences or of the words which they contain (in particular the logical word *if*), or her acceptance of the premises is not sincere or faked. The same point can be put in terms of the notion of *knowledge*: “A man knows that if P, then Q if, when he knows that P, he is able to see that, consequently, Q: If a

¹¹ Mathieu Marion, in his contribution to this volume, suggests in a note (48) that I went wrong in some of my previous essays on Carroll’s paradox when I raised the question ‘How can logic move the mind?’ or ‘How can a rule of inference make one act according to it?’. According to Marion, “This is only a relevant question if one confuses again rules of inference with their corresponding logical truths: one believes sentences such as logical truths, but only does not believe rules, one acts according to them or not, and one does not have to ‘internalize’ rules for that reason ; one might argue further that rules of inference as such are merely making explicit features of a prior practice.” But I do not see where I made the confusion in question. Marion seems to believe that once we have a rule rather than a logical truth, the problem of how logic moves the mind disappears, because one only “acts” on rules. In other words he seems to believe that the issue about how one can understand rules and of how they can guide us in our inferential behavior arises only of one takes logic to be a set of logical truths. Perhaps, when he tells us that rules are implicit in our practice, he means to remind us the famous Wittgensteinian saying : “I follow a rule *blindly*”. I have never understood how this Wittgensteinian move could dispense us with any epistemology of understanding rules. The question of how a rule can move the mind is just as pressing as the question of how a logical truth can move the mind, unless one thinks that rules act like magic just because we are engaged in “practices”.

¹² Quoted by Stroud 1969: this is also, basically, the moral that Stroud derives from the paradox.

man knows that P, but cannot see that Q, this is just what shows him not to know that if P, then Q.” (Brown 1954: 175). The Tortoise seems to be a person of this kind. Hence he does not know what “If P, then Q” means, or does not understand it. This raises the question of what it takes to understand a logical rule.

More specifically we can ask what it is to *know* a logical rule. For it seems that on one conception of logical rules, Carroll’s puzzle is still with us. It is the conception according to which knowing a logical rule consists in knowing a certain *proposition*, and a certain kind of *fact*. It is this conception which seems responsible for the regress. For suppose that one knows the rule of *modus ponens*. Suppose also that one knows that A and that B in the story. Now if knowing (MP) consists in knowing the fact that Q follows from P and If P then Q that could not explain how we can perform the inference from these premises to Q, since it would be just knowing a further fact. And to know a further fact cannot move us to infer. Alternatively, if we talk of propositions instead of facts, we can say that knowing the proposition C, and writing it down into one’s notebook cannot move us either. Ryle takes Carroll’s tale to refute the hypothesis that ‘knowing how to reason’ is ‘analysable into the knowledge or supposal of some propositions, namely, (1) the [particular] premises, (2) the conclusion, plus (3) some extra propositions about the implication of the conclusion by the premises, etc., etc., *ad infinitum*’ (Ryle 1945, 227). This line of thought leads to the view that the kind of knowledge involved in this case cannot be expressed by a *proposition* like C. Our knowledge of the rule is not a form of knowledge *that* or propositional knowledge but a form of knowledge *how*, or of practical knowledge. This was indeed Ryle’s reaction to the paradox and to his distinction between knowing how and knowing that.¹³ The foregoing reasoning about one’s knowledge of logical rules seems to show that this kind of knowledge cannot be knowledge of a logical fact, or knowledge of a proposition. Since it’s not a form of knowledge that, the Rylean line suggests, it must be a kind of knowledge *how*. Knowledge *how*, Ryle argues, is fundamentally different from knowledge that. It implies the existence of certain dispositions and abilities, in which inferring according to a rule consists, which do not presuppose the grasp of propositions, but certain habits of inference. If one removes the presupposition that our knowledge of logical rules is a kind of intellectual knowledge, and if one accepts instead that it is a form of practical knowledge, the Tortoise’s regress should not arise, according to Ryle, for the Tortoise does not *need*, when given the propositions (A) and (B), to consider the proposition (Z). If he has the relevant inference-habit or aptitude, he ought to move to (Z) *directly*, as any schoolboy who knows the rules of addition does when given a simple arithmetical operation to perform. On Ryle’s reading, a subject who knows the

¹³ Ryle 1945-46, Ryle 1949

modus ponens rule has a disposition, or perhaps several dispositions, to infer according to that rule. The disposition need not issue in conscious or reflective thought, or at least it has to manifest itself in some way if the rule is supposed to be known. The regress is engendered by *considering reflectively* the rule in propositional form (Z). If we let, so to say, the habit act by itself, the regress does not arise. Ryle was a behaviourist, even though he was not a reductionist behaviourist (he held that a concept could be defined by a set, possibly open, of interrelated dispositions). For him there was no question that once the dispositions of the subject were fixed, he ought – in the sense of there being a behavioural regularity issuing in a normal behaviour – to follow the rule. Hence on the *knowing how* interpretation of logical knowledge, what is missing in the Tortoise is simply the relevant disposition or set of dispositions.

But can dispositions to infer really account for whether a given rule is being followed? Famously Wittgenstein (1951, § 201), at least according to Kripke's reading (1981), held that neither interpreting a rule by giving an explicit proposition which is supposed to state what the rule is nor a set of dispositions can tell us *what* rule is followed. According to Kripke, Wittgenstein is a "sceptic" about rules: no fact, mental, physical or dispositional, *a fortiori* no Platonic fact about what a rule consists in, can fix whether a given rule is followed. This "sceptical paradox" about rule-following bears some similarity with the Carroll paradox. What the Tortoise's refusal to accept the conclusion seems to suggest, on this reading, is that there is no principled reason to interpret the sequence A-B-Z as an ordinary instance of *modus ponens* rather than as an instance of a deviant rule of *schmodus ponens* (say: "from P and if P and Q infer Q, unless P and Q are propositions about geometry, in which case do not accept Q"). So the Tortoise would not infer because she does not accept that the "correct" interpretation of the rule to follow in inferring from (A) and (B) to (C) can be (Z). Any other rule could go, and it is why the Tortoise is stuck. Of course it is hard to suppose that Carroll could have anticipated Wittgenstein or Kripke, but there are clear similarities with Wittgenstein's problem. Wittgenstein points out that following a rule is not an interpretation. Adding successive, possibly indefinite, interpretations of the rule as conditionals cannot fix its meaning. According to a number of other Wittgenstein interpreters, he held that following a rule is a kind of practice, whereby it is by definition settled which rule one follows: one obeys the rule "blindly". Such a reading, however, leaves us with no account about what knowledge of a logical rule consists in. It vaguely hints at the notion of a practice, which is itself quite indeterminate. Could Achilles have retorted to the Tortoise: "But don't you know what the practice of inferring according to Modus Ponens is?" Perhaps Achilles could have bitten him with a stick until he inferred, as the

Wittgensteinian line suggests. But it's likely that the Tortoise would still have resisted, probably by re-entering his head under its shell.

3. *The justification of logical laws and the epistemology of logic*

Understanding and knowing what a logical rule have a clear potential relevance to the Tortoise failure to infer, since these questions are closely related to the psychological explanation of her strange behaviour. But one can draw an even more abstract lesson from the story, which is clearly not the one intended by Carroll, but which he might have been pointing at. One clear lesson that we can draw is that it is not necessary to *justify* a logical rule or to establish the validity of an inference in order to make the inference, for otherwise we would be embarked in the regress. But we could also read the story as making a point about the justification of logical rules as part of the general problem of giving the correct epistemology for logic. What is it to justify a logical rule? This question is closely linked to a problem which was emphasised strongly by John Stuart Mill in his *System of Logic* (II, 3). Mill remarked that there is a tension within deductive inference of the syllogistic kind: the conclusion of a syllogism is valid only if, in some sense, the conclusion is already contained in the premises, but a syllogism is useful or learns us something only if the conclusion is, in some sense, new to us. Carroll was certainly familiar with Mill's view that deductive inference is a *petitio principii* and hence useless. The proposition (C) is indeed one that we already know. So inferring (Z) does not learn us anything. Thus the Tortoise's refusal to draw the conclusion might be interpreted as an acknowledgement of the tension between the usefulness and the legitimacy of deductions, and the regress as an illustration of the question begging character of deductive inference.¹⁴

Traditionally there are three kinds of answers to the problem of the justification of deduction: either logical rules are based upon fundamental propositions or axioms (foundationalism), or they are based upon certain conventions (conventionalism)¹⁵, or they are justified by other rules (holism). The first solution is a form of logical platonism, which was accepted for instance by Frege. It seems to correspond actually to Achilles's position. For Achilles the truths of logic are necessary and they are based upon nothing else. But this leaves us with a mystery: why are they primitive? The second solution, conventionalism about logic rules, has been defended in the XXth century by Carnap (1935). But this view has been subjected to a famous criticism by Quine in "Truth by Convention" (1936).

¹⁴ See Dummett 1973 Haack 1976, Marton 2004, Engel 1991, Engel 2007

¹⁵ For a discussion of these classical answers, see Dummett 1973, 1991, Engel 1991

Suppose that logical rules could be justified by the fact that we have accepted general conventions. Suppose, in particular that such conventions took the form: “For any x , y , and z , if x and y stand in the MP relation to z , then x and y imply z .” Now suppose that the particular sentences ‘ P ’ and ‘If P then Q ’ stand in the MP relation to ‘ Q ’. Do ‘ P ’ and ‘if P then Q ’ imply ‘ Q ’? If we reason thus:

- (a) ‘ P ’ and ‘if P then Q ’ stand in the MP relation to ‘ Q ’
- (b) If ‘ P ’ and ‘If P then Q ’ stand in the MP relation to ‘ Q ’, then ‘ P ’ and ‘If P then Q ’ imply ‘ Q ’
- (c) Hence ‘ P ’ and ‘if P then Q ’ imply ‘ Q ’

But the question whether (c) can be inferred from (a) and (b) turns out just to be the question whether it is true that for any x , y , and z , if x and y stand in the MP relation to z , then x and y imply z . In other words, if we want to explain the nature of logical truths or rules from the existence of conventions, we must presuppose these logical truths and rules to derive them from conventions, and our derivation is thus circular. Quine explicitly refers to Lewis Carroll’s regress when he mounts his argument, and the latter could serve to illustrate this general difficulty¹⁶.

The third alternative consists in granting that the justification of deduction is in some sense circular: we can justify the basic principles of logic without using these principles. So logical laws and rules support each other, without any one of them being properly basic. Thus the fundamental metatheorems of logic, such as the completeness and consistency proofs, can hardly be formulated without using the *modus ponens* and other primitive rules. If one holds, in addition, that there is no principled distinction between analytic truths – of which the truths of logic are part – and synthetic or empirical truths, or between *a priori* and *a posteriori* truths, logic becomes part of the general web of belief. This holistic conception of the justification of deduction has been defended by Quine and Goodman (1957): rules of logic depend upon other rules, and on the network of our beliefs, where no proposition can be really isolated as primitive.

4. The normativity of logic: how can logic move the mind?

Carroll’s story can be interpreted in a fourth way, as emphasising the problem about the normative force of logical rules mentioned above. Remember that the Tortoise *accepts* explicitly the

¹⁶ Quine 1936, in (1976), p.104, note 21. The presentation above is from Harman 1996 p.395

truth of C, which is the semantic equivalent of the *modus ponens*. Given this, why does he not accept Z? How can someone who accepts a logical truth or a logical rule *as such* fail to infer accordingly? If we answer that he actually does not really accept the logical truth C because he does not really understand the meaning of C we are brought us back to interpretation (2). Another answer would be that the Tortoise accepts C, fully understands what it means, but still *cannot*, or possibly *refuses*, to draw the appropriate conclusion. He would thus intend to convey another form of scepticism, distinct from the Wittgensteinian one, which bears upon about *normative force* of the logical rules. On this reading the Tortoise is fully aware that C is a logical truth or a valid rule of inference. But he fails to draw the conclusion because he does not take the rule or law as binding and as being able to *move* him to the appropriate conclusion. How is that possible? One interpretation of this strange behaviour is that the tortoise explicitly questions the normative power of the *logical must*. He recognizes that Z follows from A and B by “logical necessity”, but takes logic to be *non normative at all*. This is why his acknowledgement of the authority of logic is somewhat ironic: “Whatever *Logic* is good enough to tell me is worth *writing down*.”. Similarly Hume question the power of reason in the practical domain: reason, or belief, as such, cannot move us to act, only passions or desires can do so.¹⁷ In parallel fashion, the Tortoise would be a sceptic about the power of logical reasons to force us to believe any sort of conclusion. She would be a sceptic about *the force of logical reasons*. The question is: “How can logic move the mind?”¹⁸

In order to be clear on the scope of this question, we have to distinguish three things:

- (1) Do the two premises A and B imply the conclusion Z?
- (2) Given that one has good reasons to believe that A and B are true, is it reasonable to believe that Z is true?
- (3) Supposing it is reasonable for me to believe Z on the basis of A and B, what is supposed to move our mind to believe that Z ? ¹⁹

(1) is not in question, unless we revise logic strongly (see below). Neither is (2), if we are equipped with an appropriate justification of our rules of deduction. The point of scepticism about the force of logical reasons is that neither a positive answer to (1) nor a positive answer to (2) can yield a positive answer to (3). However willing to recognize the validity of the inference, and however well equipped with a full justification of deduction in general, this kind of sceptic will not be moved.

¹⁷ See Korsgaard 1986

¹⁸ Blackburn 1995, Engel 1998.

¹⁹ see Schueler 1995

It is interesting to compare Carroll's puzzle with a parallel kind of challenge for practical reasoning. Simon Blackburn (1995), in his essay for the centenary of Carroll's piece in *Mind*, shows that a similar problem to Carroll's arises when we consider an instance of practical reasoning:

- (P) I prefer lettuce to souvlaki
- (B') The moment of decision is at hand
- (Z') Let me choose to eat lettuce rather than souvlaki

How many premises of the form

- (P*) It is right (good, desirable, rational, etc) to prefer lettuce to souvlaki

are added, the Tortoise still does not act. Blackburn argues for a "Humean conclusion" to the problem of scepticism about practical reasons: "There is always something else, something that is not under the control of fact and reason, which has to be given as a brute extra, if deliberation is ever to end by determining the will." (1995: 695).

There are, of course, strong differences between practical reasoning on the one hand, and logical reasoning and theoretical reasoning on the other hand: the former goes from beliefs and desires to intentions or actions, whereas the latter goes from beliefs to beliefs. But the form of the sceptic's challenge about the force of logical reason is the same. For suppose that the Tortoise had accepted not only C but the apparently stronger:

- (C*) It is a norm of correct deductive reasoning that if one accepts that MP kind of inferences are valid, one should in the presence of a particular instance of MP, accept the conclusion

Suppose also that the Tortoise had recognised that

- (I) The propositions A, B and Z stand in the MP relation

Wouldn't she be bound to accept Z? No. (I) here plays the same role as the minor premise of the practical syllogism (P)-(Z') above. But even though the Tortoise sees its truth, and accepts (C*), she is not be moved. Neither would she if we reinforced again (C*) into :

- (C**) There are absolute rational justifications for the norm of correct deductive reasoning that

if one accepts that MP kinds inferences are valid, one should in the presence of an instance of MP, accept the conclusion

and so on. Whether we say that our norms of reasoning are justified because they are metaphysically necessary, or because they correspond to what ideal agents would believe in ideal circumstances, they would not be able to *force* us to reason according to them.

It is important to distinguish this scepticism raised by the Tortoise's resistance to accept the obvious conclusion from scepticism about justification. The sceptic about the force of logical reasons grants that we do have good reasons and justifications to infer according to a given rule of inference. Following the terminology which is common in the literature on practical reasoning, let us call these reasons *normative reasons*.²⁰ Now to say that they are normative is to say that, at least *prima facie*, they are such that a subject who recognises them as reasons would be ready to act, or in the case at hand, to infer, according to them. A subject can, however, have good reasons to do an action, or to have a certain belief, but still not act upon this reason, and act upon other reasons, which might not be good ones. Such reasons will explain our action, and motivate it, but they will not necessarily coincide with the normative reasons that we have. They can be called *motivating reasons*. Of course the distinction between normative and motivating reasons does not imply that they cannot be identical. In the normal case, when, so to say, everything goes right, the reasons *for* acting or believing in a certain way just are the reason *why* the agent acted or believed in that way. But the two can diverge. *Akrasia* in the practical domain consists in acting against one's own best judgement. The tortoise illustrates a kind of *logical akrasia*: he sees what he ought to infer, but fails to comply. In *akrasia* normative reasons fail to be motivating.

The *modus ponens* rule is a proposition of logic. As such it is worth being written down. But he does not tell you in itself anything about your beliefs. It tells you that if P, and if P then Q imply Q, but it does not tell you that if you *believe* that P, and if you believe that if P then Q, then you *believe* that Q.²¹ Similarly when we open a logic book, we just see a number of propositions written. The fact that they are in the book does not in itself move us to reason logically. We might nevertheless want to formulate the MP rule as a normative requirement of reasoning and belief. Then we would formulate it as a normative reason for belief, using the deontic term "ought":

(O1) You ought (to believe Q, if you believe that P, and that if P then Q)

²⁰ See for instance Dancy 2000. For a more elaborate parallel between the practical case and the theoretical case, and a discussion of logical akrasia, see Engel 2005

²¹ Harman 1986 makes this point.

This is just a reformulation of (C*) above. But the “ought” which figures in this proposition has a wide scope: it governs a conditional. (O1) does not allow you to infer or detach that

(O2) You ought to believe that Q ²²

for the reason that we have just noted. Your normative belief O1 does not tell you what particular belief you have to adopt relatively to Q, it just tells you that you recognise O1 as a normative reason. On the present interpretation of the paradox, the Tortoise intends to remind us of this simple, but very important, point.

If we take this lesson of Carroll’s tale seriously, the important question becomes: normative reasons, which are supposed to be objective and independent of the agent’s specific mental states, be able to move our minds in a particular way? ²³

5 *What Achilles could have said to the Tortoise*

I have suggested four kinds of verdicts about what is at stake in Carroll’s tale. There are others, with which I cannot deal here.²⁴ The lessons of the puzzle which I have outlined here are distinct, but they are obviously connected: if a logical rule is known by a subject to be a rule, and if it is justified, it ought to move the mind. The Tortoise’s refusal to comply can come from denying either one of the antecedents of this conditional, or the consequent. So there must be some connection between these three elements: our being justified in accepting a rule of logic, the normative status of that rule, and our being psychologically guided by it. But to understand the connections between our knowledge of a rule or principle, its justification, and its normative power is one of the main tasks of a philosophy of logic as well as of philosophical psychology if one attends to the parallel between logical normativity and ethical normativity. In this last section, I would like to indicate some of the possible contemporary developments of these issues.

Let us start with the problem of the nature of logical inference. Since Gentzen 1934, and with the work of Prawitz (1965), Dummett (1991) and many other logicians, we have known a better understanding of the nature of basic logical inferences, and of the nature of logical constants.

²² Broome 2000, 2002 emphasises this point.

²³ Simchen 2001 formulates well, this form of scepticism, but his analysis of the paradox focuses on the question whether a rule can be action guiding, especially when it is quoted. I cannot deal here with this specific interpretation of the paradox.

²⁴ For instance, W. Bartley III 1962 confronts the puzzle with some claims about scientific explanation. D. Hofstadter introduced the puzzle into AI and cognitive research. Dennett 1978 and S. Pinker 1997 mention the regress in the examination of the problem of the *homunculus* in cognitive science.

These issues are indeed too wide to claim that they could be illustrated by Carroll's tale, but at least two of them are relevant. Wittgenstein held a quasi-conventionalist conception of logic, according to which logic is a matter of rules, which he opposed to necessary truth of logic, rejecting the idea that there could be a special "hardness of the logical *must*". Ryle took logical rules to be "inference tickets" on a par with « rules of grammar, chess, etiquette and military funerals, are performance-rules". In a famous article, A.N. Prior (1960) reacting to such conventionalist views, by introducing the inconsistent "logical" connective *tonk*, which obeys the following introduction and elimination rules: $A / A \text{ tonk } B$, $A \text{ tonk } B / B$, from which one can derive anything from anything. The idea is that if one only stipulates a rule, without caring for its coherence, one will end up with such things as *tonk*. But what kind of conditions must we impose on logical rules? A vast literature is devoted to this and to the nature of logical constants (Martin Löf 1985, Prawitz 1985, 2006, Dummett 1991, Tennant 1986). These discussions deal largely with the question, which was completely out of reach for Carroll, of whether one should abandon classical logic and adopt an alternative logic, such as intuitionist and constructivist logics. In so far as this large trend in contemporary logic deals with the most abstract possible characterisation of logical inference, this work in a sense descends from Carroll's paradox. But it descends as well from Bolzano's pioneering definition of a logically valid inference as that which remains constant whatever variations are made in the contents of its non logical terms. This definition was taken up by Tarski. Now a question which Bolzano raised was this: *If an inference from a sentence A to a sentence B is valid, is it the case that a person who is justified in holding A true is also justified in holding B true?* And he showed that if one expected to justify the inferring of B from A through the consideration of the validity of the inference, then we would get into a regress similar to the one that Carroll formulated.²⁵ The point is that the validity of the sequence, as Carroll would say, does not entitle us into drawing the conclusion. According to Prawitz and other constructivist logicians, this point vindicates the Tortoise's attitude, and calls for a notion of a proof which justifies directly the steps of the inference. The very fact that an inference is valid in the Bolzano-Tarski sense is not enough, and we have to change our notion of validity.

One can also object to the *modus ponens* as a logical rule. In a much discussed paper, Van Mc Gee (1985) proposed a counterexample to this familiar rule, showing that when conditionals are embedded, our intuitions about the validity of the rule vacillate. Certainly Carroll's story at no point suggests that he intended to put into doubt the rule of *modus ponens*. His point was not, like

²⁵ I owe this reference to Bolzano and the point from Dag Prawitz, who kindly allowed me to see his 2006 a, where he draws the parallel. See also his earlier article Prawitz 1978 on Carroll's regress.

relevant logicians for instance, to contest basic rules such as disjunctive syllogism, although, as we saw above, he was thinking about the logic of conditionals.

The justification issue, however, as, we saw, is not the only moral. Carroll was not clearly discussing the foundations of deductive reasoning. But he was clearly asking what could be our *reasons* for our inferences. In so far as epistemology deals with our reasons for believing, and as these reasons are in part inferential ones, the tale is very relevant. The notion of a reason, for actions and for belief, as been prominent in recent philosophy and Carroll regress plays an important role for revealing various positions²⁶. There are reasons to act, reasons, to believe, reasons to infer. In the vocabulary adopted above (§ 4), Carroll tale does not bear only upon our motivating reasons to believe and to infer, but also about our normative reasons to do so. It can actually be construed as asking what kind of relation there can be between our normative reasons to infer and our motivating ones. The options on the table are not that many.

At one extreme there is a form of Platonism about logical reasons. It was Frege's view: once we grasp that a certain proposition is a truth of logic, a "law of Being-True", we cannot but infer accordingly. Logical laws *as such* are normative, both in the sense that they are absolutely correct, but also in the sense that they motivate us to infer in conformity with them. Being eternal laws of truth, the logical laws do not tell us anything about our psychology or about mental representations. But we have nevertheless a certain kind of relationship with them, which Frege, as most Platonists, conceives as a kind of intuitive grasp.²⁷ So it is the direct, intuitive apprehension of the validity of *modus ponens* that is sufficient to move our mind to infer accordingly. In the field of practical rationality, this kind of view has been revived by writers who construe normative reasons as certain objective *facts*, the intuitive recognition of which is supposed to move directly the agent into doing the appropriate action. The view can be transposed to logical reasons.²⁸ The obvious problem with this view is precisely the one that the Tortoise, on our present interpretation of the story, is meant to remind to Achilles (who, for that matter, seems to be a straight Platonist): a Fregean Thought or a Platonic fact does not have any causal impact upon our minds, and intuition of Platonic fact seems even more mysterious than the idea of a causal contact with Laws of being true and our minds.

At the opposite extreme lies the view that only a *psychological* state can play the *causal* role of moving the mind. This is the Humean solution that Blackburn suggests for the practical case. On

²⁶ See Scanlon 1996, Railton 1997, 2003, Skorupski 2011.

²⁷ In the Preface to the *Grundgesetze der Arithmetik*, Frege tells us that we should distinguish two senses of "law" when we talk about the laws of thought: the normative and the causal. Psychologism makes a confusion between these two senses. For Frege, the laws of thought are normative in so far as they are descriptive of a Third Realm.

²⁸ See Engel 2005. The Platonist view in the practical case is defended by Dancy 2000.

such a view the existence of objective reasons for inference will never “move the mind”. The mind, when it comes to ending a deliberation by an action, or a reasoning by the assertion of a conclusion, is always moved by some “brute extra”. In the case of action, it must be, for a Humean, a desire, which is the only psychological state which can motivate one to act. In the case of reasoning from beliefs to beliefs, it has to be also some psychological state. But it cannot be a belief, for beliefs, at least on the official Humean doctrine, do not in themselves cause us to act. They cannot be motivating reasons. Hence the “brute extra” must consist in something else. The most plausible view would be that it is a *habit*. We could understand this in the way Hume gives a “sceptical solution” to his sceptical doubts about induction, by pointing out the force of habit and custom, or in a Wittgensteinian fashion, by saying that at some point, reasons must come to an end, and that drawing a conclusion from premises according to a rule of inference is “simply what we do”. It is a brute fact of our practices that we reason *that way*.²⁹

The problem is that it gives us a plausible answer to the causal question about the normative force of reasons at the expense of the justificatory nature of reasons. We lose sight, on this view, of why logical reasons are good reasons, and of how they can be objective. But to be told that we have the practice or habit of inferring in a certain way, and that *modus ponens* kinds of inferences belong to these practices, does not tell us whether we should maintain this practice if we believed the theory according to which the force of reasons consists in nothing but the existence of this practice.³⁰ A Wittgenstein says, “We obey the rule *blindly*”. But this leaves us with a mystery: why is it that we find the reasons for our beliefs, and especially our inferential reasons, satisfactory and even compelling? We ought not to rest content with saying, of any feature of our linguistic practice, “That is simply what we do”. Not only does this view deprive logical reasons of any normativity, but also it does not tell us what they *do*.

There are, however, intermediate answers between these extremes. To answer the worry raised by the Wittgensteinian position, we would have to insist upon the requirement that the subject who infers a conclusion from certain premises has to be in some sense conscious of what he is doing. But we cannot say that this involves having a *belief* about the validity of the rule, for it is precisely the view which is the target of Carroll’s paradox. Let us call this *reflective internalism*.³¹ According to

²⁹ This would in some sense harmonise with what Kripke (1981) calls Wittgenstein’s “sceptical solution” to his “sceptical paradox” about rules. Dummett hints at this “brute fact” when he describes Wittgenstein’s position: “We do, by and large, agree on what the consequences are, what follows from what, what is a valid proof and what is not... On Wittgenstein’s view, it is a *brute fact*: nothing explains it.” See also Railton 1999

³⁰ Dummett “Wittgenstein on necessity: some questions”, in Dummett 1993 : 446

³¹ The view described is quite close to what Boghossian (2003) describes as “simple inferential internalism” (2003, see also his 2002 article, where adopts slightly different formulations). I use here only *partly* Boghossian’s formulation for

it, the subject would be reasoning by entertaining beliefs to the effect that a given rule is valid, and that he is presented with a particular instance of this rule, by reasoning in the following way:

- (i) Any inference of the form MP is valid
- (ii) This particular inference (from A and B to Z) is of MP form
- (iii) Hence this particular inference (from A and B to Z) is valid

Not only such beliefs would be reflective, in the sense that they would be beliefs about beliefs, but they would also be inferential, since (i)-(iii) is an inference. But the point of Carroll's story is that this kind of reflective belief would lead us to the familiar regress. Reflective internalism is the psychological counterpart of the Carroll story. Reflective beliefs about the validity of our inference rules can no more move our minds than the fact that further premises are added to the initial ones in the tale.

Alternatively one might defend a form of *externalism* about logical reasons. If internalism leads to the regress or to the postulation of some mysterious intuitive power, and if neither the Platonist nor the Humean view can give us a plausible account of the normative force of logical rules, why not accept the idea that we are justified in our rules, and guided by them, because (i) they are valid and thus transmit warrant from the premises to the conclusions, and (ii) there exist causal processes which not only justify our logical rules, but to which we do not have necessarily access? After all, knowledge of logical rule is a kind of psychological competence that humans have acquired, and which may well be partly innate through mastery of language. Much of the psychological literature on reasoning shows that we are endowed with such rules as part of our natural equipment. The rules could be in some sense tacit, and operative in our inferential psychological processes. What would be, on such a view, the problem with the Tortoise? Given that, by definition, upon the externalist view, the modus ponens rule is valid and truth transmitting, no issue about its justification arises. No issue either about how the rule can motivate a subject to infer a conclusion arises, since, on the externalist view, there is no need to have access to the rule to infer correctly. It seems clear that, on this view, the only thing that we can register is the success or the failure of the agent in drawing the appropriate conclusion. Some brain lesion could impede the subject from drawing the conclusion, and we could conclude from this, on neuropsychological grounds, that the agent does not "have the rule". Alternatively a given subject might perform perfectly in drawing a certain kind of inference,

he is concerned, in his paper, with the question of the *justification* of the rules of reasoning, and not with the question of their normative force.

without having the slightest idea he does so.³² But that cannot be a satisfactory solution to Carroll's problem, unless the inferential externalist simply wants to bite the bullet and to claim that the problem does not arise. We have the resilient internalist intuition that if an argument is valid, a subject must be in position to know that it is, and if he is to infer according to a rule, he must in some sense be able to justify – in the sense of giving reasons – for his inferring. But how could we satisfy this requirement without being in some sense hostage to Carroll's paradox? As Boghossian says: "At some point it must be possible to use a rule in reasoning in order to arrive at a *justified* conclusion, without this use needing to be supported by some knowledge about the rule one is relying on. It must be possible simply to move between the thoughts in a way that generates justified belief, without this movement being grounded in the thinker's justified belief about the rule of the reasoning (Boghossian 2002:37)

This suggests a fourth kind of view. Reflective internalism says that our belief in the validity of the rule is the product of an inference. But we might still accept the idea that we must have in some sense an access to the thought that the inference is valid without this access being either *inferential* or *reflective*. We could say that this access is a direct access, which allows us to go from premises to conclusion through some insight or intuition, rather than through an act of reflection and an inference.³³ On this view, rational insight *is not a belief* and depends neither on any further inferential process, or on any reflective apprehension of the rule. The mental state involved in the recognition of a certain pattern as instantiating a valid rule is neither reflective nor inferential: if it is not, then no regress like the Carrollian one arises. We may grant this point, however, without finding this kind of description of what happens when one makes simple inferences less problematic than the Platonist's appeal to a faculty of intuition. But it is not clear that non reflective internalism needs a form of mysterious insight or intuition into the rule. There is room for the idea that our recognition of the rule is not inferential without it being based on a faculty of insight. Neither need we go into a regress by supposing that the justification of the rules has to go through an inference. In some way we are entitled to the conclusion, in the same way as we can be entitled to take at face value what perception delivers to us Entitlement to a belief, in this sense, is a weaker relationship than knowledge, closer to the idea of being authorised or permitted to infer a conclusion.³⁴

³² For notorious examples of this sort against externalism, see Bonjour 19

³³ See e.g. Bonjour 1998: 105-106

³⁴ Boghossian 2003: 235-236. Crispin Wright suggests something like non reflective internalism sort in his commentary of Boghossian 2002 (Wright 2002: 83). He calls it « simple internalism ». For similar views of this kind, see Brewer 1995. Leite 2005 has a close view, but he calls it, in my view misleadingly "reflective internalism". My own approach in Engel 2010 is an attempt along these lines. In Engel 2005 I was closer to an externalist view.

A number of writers have suggested, along these lines, that the conditions placed upon our understanding of logical rules and concepts could solve both the justification problem and the normative problem. In particular if one requires that the rules of logical inference can be in some sense constitutive of the meaning of our logical concepts and if we place specific conditions on them so that spurious concepts such as Prior's *tonk* cannot arise, any thinker would be endowed with such concepts would grant the inferences governed by these concepts as valid and would comply by the rules.³⁵ Two problems arise for such views. First how can we guarantee that our rules are meaning constitutive in the intended sense? In what sense is, for instance, Modus ponens constitutive of the meaning of the logical constant *if.. then*? Second, in what sense is the idea that we can be entitled - or committed - by our concepts to draw various kinds of inference give us an account of the fact that Achilles intended to remind the Tortoise: that he was *forced* to conclude. Non classical logicians and philosophers inspired by the Wittgenstein view according to which logic is a matter of rules, might find logical *must* less hard than it seems, but anyone favouring realist and classical account of logic will have to admit that the logical *must* is hard.

Another resilient intuition is that the Tortoise *knows* the rule of *modus ponens*, and that if he knows that A and B, he knows that Z is true. This is different from saying that he believes, even rationally believes that A and B and that he ought to believe that Z. Knowledge is factive, and subject to the much discussed principle of closure: if one knows that P, and knows that P entails Q, then one knows that Q. The Carroll case would not simply be, in the current terminology a case of transmission of warranted belief, but a case of transmission of knowledge. If one attends to this principle, the Tortoise's refusal to draw the conclusion (Z) becomes even more problematic, since the closure principle seems so compelling, especially if we transpose it into the context of discussions of scepticism about knowledge in general. We can actually transpose Carroll's story into the context of a discussion of knowledge in general. Suppose that P is an empirical knowledge such as knowledge that one has hands, and that Q is a consequence of the "heavyweight" kind, such as that there is an external world, which the sceptics are prone to doubt and which common sense philosophers are prone to claim to know. And suppose also that the Tortoise knows that P entails Q and he able competently to deduce the second from the first. Now suppose that Achilles asks him

Whether he agrees that P. He asserts that he does: "Yes," he says. Achilles then ask S whether he realises that Q follows from P. "Yes," he says. Achilles then asks him whether he agrees that Q.

³⁵ See Peacocke 1992, 2004, Boghossian 2003, 2004, 2005. This kind of view has much in common with those of Dummett and Prawitz, although, unlike these they accept a form of rule circular kind of justification for logical rules. Enich and Schechter 2006 advance a "pragmatic account" of our justification of logical rules. Phillie 2007 takes general requirements on rationality to provide the appropriate notion of normative assessment. My proposal in Engel 2005 relies on the notion of a rational disposition.

"I'm not agreeing to that," he says. I ask him whether he now wishes to retract his earlier claims. "Oh no," he says. "I'm sticking by my claim that P and my claim that P entails Q. I'm just not willing to claim that Q." The dialogue then becomes exactly Carroll's³⁶. It is applied to knowledge in general, but we can also restrict it to the case of local knowledge of logical rules.

If we understand the story in terms of knowing logical rules, rather than having justified beliefs about our inferences, the question arises again about the nature of this knowledge. If we understand it in the internalist way, as knowledge of propositions to which we have actual or potential access, then the regress associated with reflective internalism is likely to prop up again. If we understand it in the externalist way, then the difficulties raised above about our need to be able to give reasons for our inferences, will also prop up again. And if we conceive of this knowledge in terms of *know how* or in practical terms, in the Rylean or Wittgensteinian fashion, then we shall also have meet the internalist challenge. If we understand it as a kind of tacit knowledge, we shall have also to understand in what it can consists. We shall also have to ask the relevant questions about logical knowledge: is it *a priori*, of a conceptual nature? Is it innate or acquired?

The regress seems to show that logical knowledge cannot be propositional, hence that it has to involve either logical concepts conceived as dispositions or some dispositional knowledge. This line of thought, which has long been dominant under the influence of Ryle's arguments for the difference between *knowing how* and *knowing that* has been the subject of criticism. It has been argued that *knowing how* is much closer to a form of *knowing that* than the dispositional analysis allows, and that the former can actually be reduced to the latter. Transposed to the logical case, the idea would be that there can be knowledge *how* of logical rules, which is in fact a form of propositional knowledge. This view seems more in line with the (Chomskyan and cognitive science inspired) idea that our logical knowledge is a form of tacit propositional knowledge of rules. If this were correct, the "intellectual legend" that Ryle meant to eradicate, might still be with us. But this kind of proposal would still have to tell us how it can account for the normative force of logical norms, and how it can steer between the Charybdis of our internalist intuitions (without which the Carroll puzzle would not arise) and the Scylla of full blown externalism.³⁷

6. Conclusion

³⁶ The remark is borrowed from Hawthorne 2005. The "knowledge first epistemology" was present in Lewis Carroll's Oxford, with the work of Cook Wilson, with whom he was acquainted, and its contemporary descendent is Williamson 2000.

³⁷ Stanley and Williamson 2000, Stanley 2011, and for a critical perspective, see Rumfitt 2011. Besson 2010 give an interesting critique of dispositional accounts such as Boghossian, but does not clearly propose a positive view or a diagnosis on Carroll's paradox. For criticisms, see Martinez 2011.

It seems clear that the Carroll puzzle not only is a mirror in which the epistemology of logic reflects itself, but also a mirror for epistemology in general, and perhaps also for moral epistemology. One's notion of inference, of proof, of logical validity bear upon it, but also our notion of an argument, both in the formal and the informal sense. It illustrates some of the difficulties of notion of rule-following, of knowledge of logical rules, but also of the idea that these can constitute normative reasons for our inferential beliefs. It invites us to draw parallels between logic and ethics, and between epistemology and meta-ethics. It is a litmus test for many of the most interesting issues of contemporary philosophy. At the end of Carroll's dialogue, the Tortoise asked Achilles: "Would you mind, as a personal favour, considering what a lot of instruction this colloquy of ours will provide for the Logicians of the Nineteenth Century?" He could have added easily: "For the logicians and philosophers Of the Twentieth and the Twenty-first centuries."³⁸

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³⁸ I thank Dag Prawitz and Concha Martinez for having let me see the drafts of their papers 2006a and 2011. This paper was written while I was visiting the University of Lund. Thanks to Erik Olsson, Frank Zenker, and the member of the *Filosofiska Föreningen*. It has been also realised with the help of the FNS project "Knowledge, evidence and practice".

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