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GERAD

GROUP FOR RESEARCH
IN DECISION ANALYSIS

Fonds de recherche
Nature et
technologies

Québec 



CRSNG
NSERC

2022 Optimization Days

Escaping unknown discontinuous regions in blackbox optimization

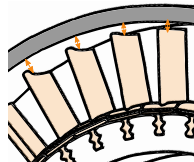
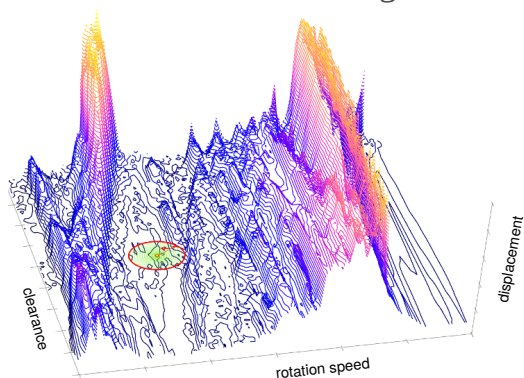
Solène KOJTYCH

Supervisors : Charles AUDET, Alain BATAILLY

May, 18th 2022

An example of discontinuous problem

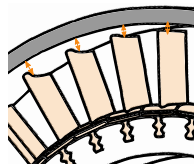
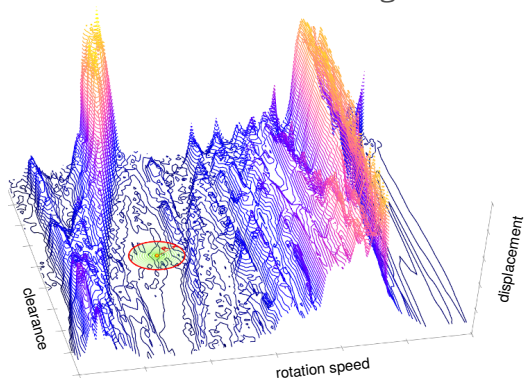
- ▶ vibrations of aircraft engine blades



Source (right picture): Source: Harvey, Ramsden 2000 - ASME Turbo Expo Robert 2011 - Brevet US20110085896 A1

An example of discontinuous problem

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Problem characterization

$$\min_{x \in X} f(x)$$

$$\text{s.t. } c(x) \leq 0$$

$$d(x) \leq 0 \implies x \text{ "far away" from discontinuities of } f \text{ or } c$$

- ▶ f and c : given by a **blackbox**, possibly **discontinuous functions**

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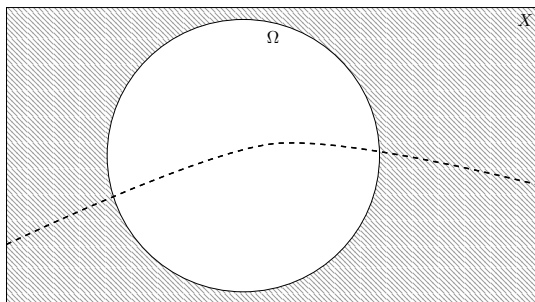
- ▶ f and c : given by a **blackbox**, possibly **discontinuous functions**
- ▶ d : **infinite constraint**

Objective : treat constraint d in a **blackbox optimization context with the Mesh Adaptive Direct Search**

Outline

- 1 Modeling
- 2 Basics on direct search methods
- 3 DiscoMads
- 4 Numerical results
- 5 Conclusion

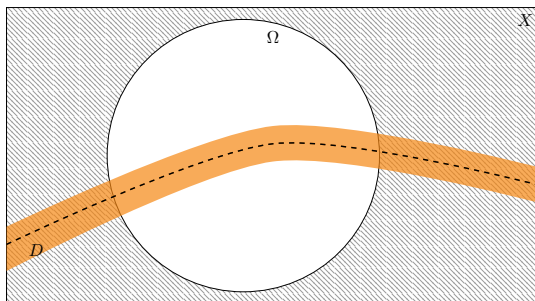
Problem modeling



$$\Omega = \{x \in X : c(x) \leq 0\}$$

Problem modeling

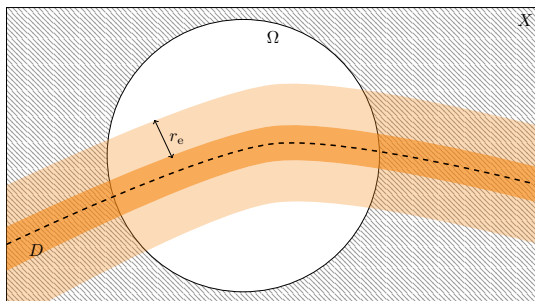
- ▶ region of weak discontinuities D (■): slope of f or c_j between two points at distance at most r_d exceeds a limit slope τ



$$D = \{y \in X : \exists j \in J, \exists z \in X \cap B_{r_d}(y), |c_j(y) - c_j(z)| > \tau \|y - z\|\}$$

Problem modeling

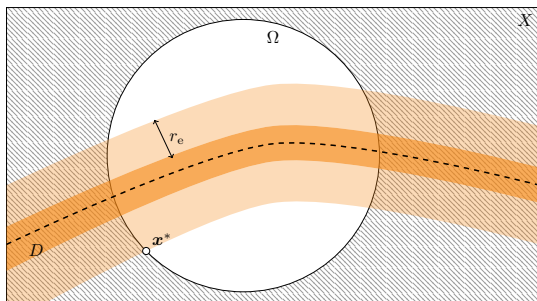
- ▶ region of weak discontinuities D (■): slope of f or c_j between two points at distance at most r_d exceeds a limit slope τ
- ▶ safety margin M (■ \cup ■): region of radius r_e around D



$$M = X \cap (\cup_{x \in D} B_{r_e}(x))$$

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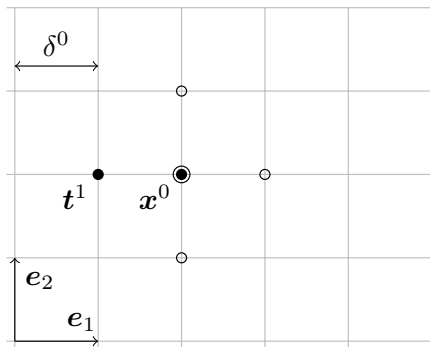
$$x \notin M \iff d(x) \leq 0$$

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Coordinate search¹

- ▶ example: $\min_{x \in \mathbb{R}^2} f(x)$

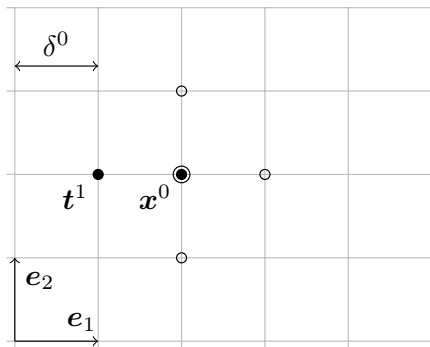


- ▶ poll at iteration 0

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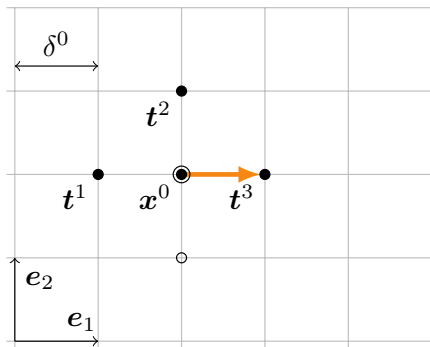
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- $f(x^0) \leq f(t^1)$

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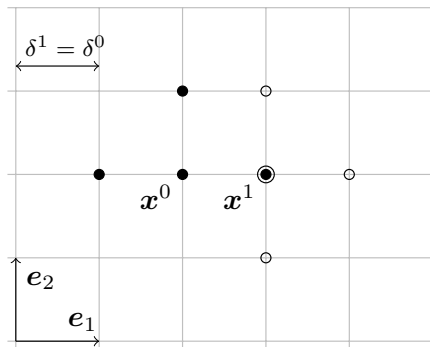
- $f(x^0) \leq f(t^1)$
- $f(x^0) \leq f(t^2)$
- $f(x^0) > f(t^3)$

⇒ successful

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Coordinate search¹

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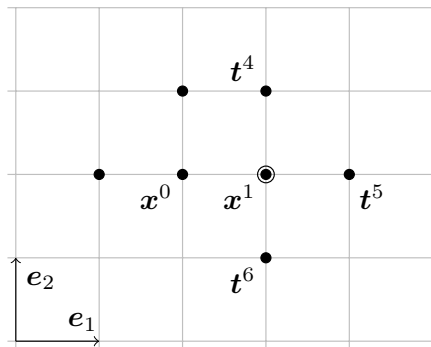


- ▶ poll at iteration 1

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► poll at iteration 1

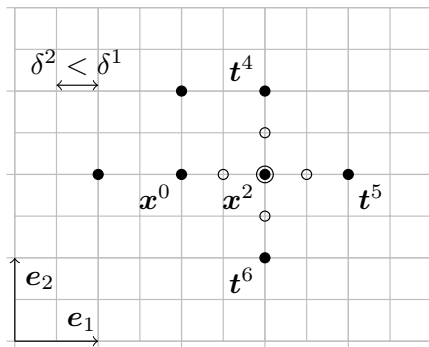
- $f(x^1) \leq f(t^4)$
- $f(x^1) \leq f(t^5)$
- $f(x^1) \leq f(t^6)$

⇒ **unsuccessful**

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Coordinate search¹

► example: $\min_{x \in \mathbb{R}^2} f(x)$



► poll at iteration 2 ...

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Mesh Adaptive Direct Search (Mads)²

- ▶ a Mads iteration

²C. Audet and J.E. Dennis, Jr. "Mesh Adaptive Direct Search Algorithms for Constrained Optimization". *SIAM Journal on Optimization* 17.1 (2006), pages 188–217. URL: <http://dx.doi.org/doi:10.1137/040603371>.

Mesh Adaptive Direct Search (Mads)²

- ▶ a Mads iteration
 - ① search step : user-defined points on the mesh

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stronger convergence results

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 - progressive barrier approach

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 - constraint violation function

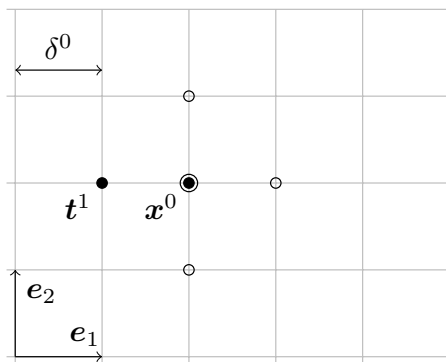
$$h(x) = \begin{cases} \sum_{j=1}^m (\max(c_j(x), 0))^2 & \text{if } x \in X, \\ \infty & \text{otherwise.} \end{cases}$$

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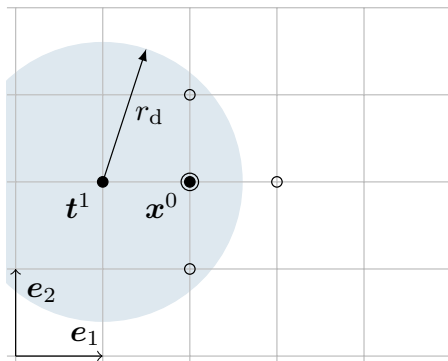
Key algorithmic mechanisms : example on Cs



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Key algorithmic mechanisms : example on Cs

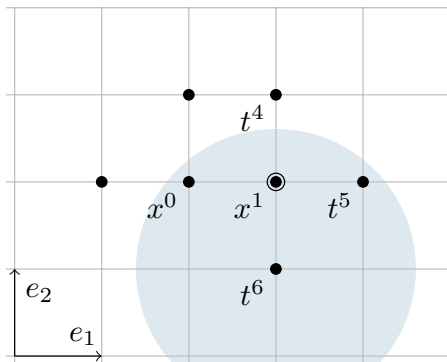
- 1 revelation (radius r_d and limit slope τ)



$$|f(t^1) - f(x^0)| \leq \tau \|t^1 - x^0\|$$

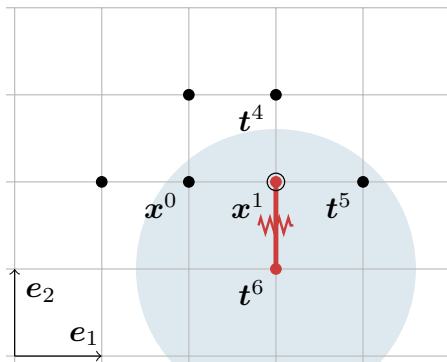
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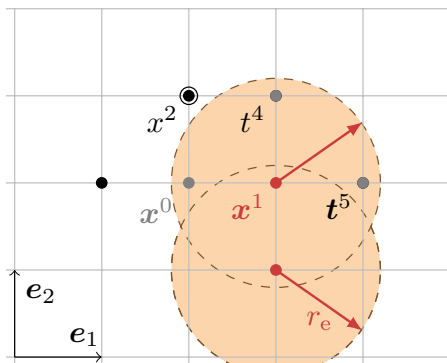
① revelation (radius r_d and limit slope τ)



$$|f(t^6) - f(x^1)| > \tau \|t^6 - x^1\| \implies t^6 \text{ and } x^1 \text{ are revealing points } \in D$$

Key algorithmic mechanisms : example on Cs

- 1 revelation (radius r_d and limit slope τ)
- 2 exclusion (radius r_e)



t^6 and x^1 are revealing points \Rightarrow exclusion constraint

Exclusion mechanism

- ▶ set of revealing points D^k at iteration k

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- ▶ exclusion constraint d^k at iteration k

$$d^k(x) = \begin{cases} 1 - \frac{\text{dist}(x, D^k)}{r_e} & \text{if } D^k \cap B_{r_e}(x) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

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- ▶ problem solved at iteration k :

$$\begin{aligned} \min_{x \in X} \quad & f(x) \\ \text{s.t.} \quad & c(x) \leq 0 \\ \text{and} \quad & d^k(x) \leq 0 \end{aligned}$$

DiscoMads algorithm

▶ Mads iteration :

① search step

② poll step

③ parameters update depending on iteration type

DiscoMads algorithm

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DiscoMads algorithm

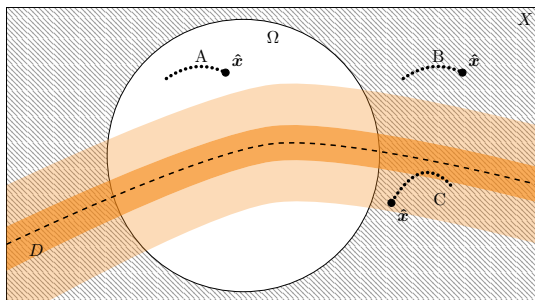
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 - revealing iteration
 - otherwise usual iteration types from Mads

Convergence in brief

- 1 mesh gets infinitely fine when $k \rightarrow \infty$

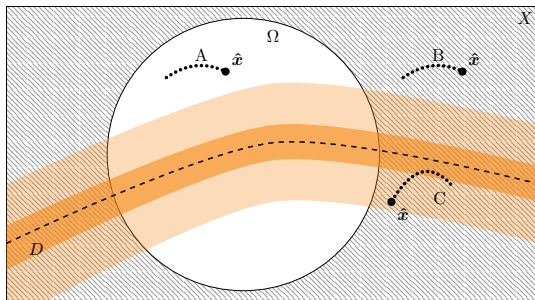
Convergence in brief

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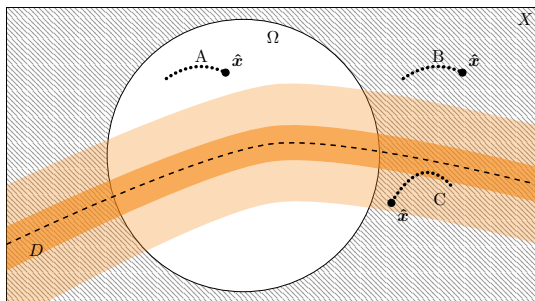
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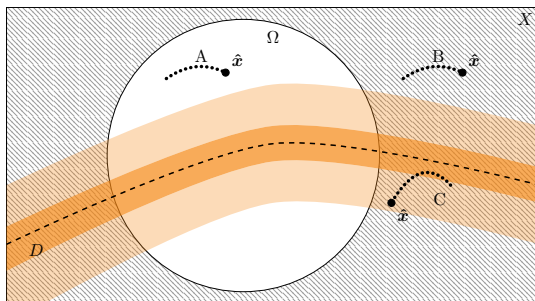
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Convergence in brief

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- 2 refining point analysis
 - case A $\implies f^\circ(\hat{x}, p) \geq 0$ (from Mads)
 - case B $\implies \hat{h}^\circ(\hat{x}, p) \geq 0$ (from Mads)
 - case C $\implies \hat{x}$ is a local minimizer of \hat{h} on X
from revealing poll + piecewise continuity assumption

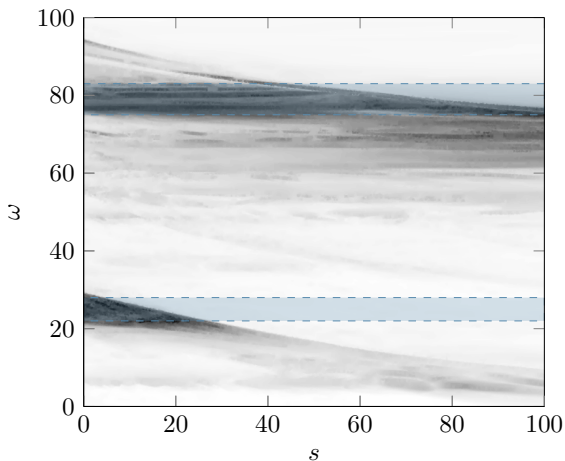


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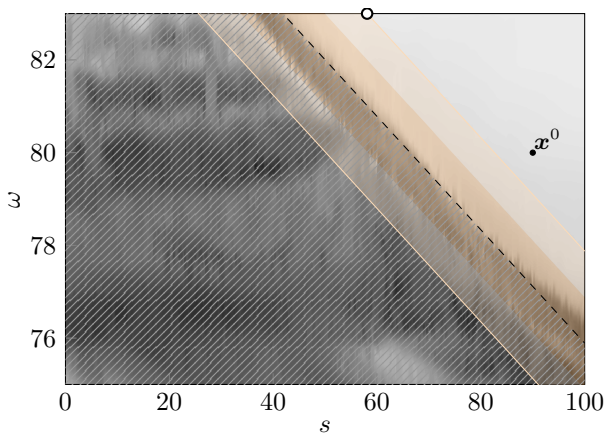
Vibration of aircraft engine blades

- ▶ magnitude of vibration $u(s, \omega)$



Vibration of aircraft engine blades

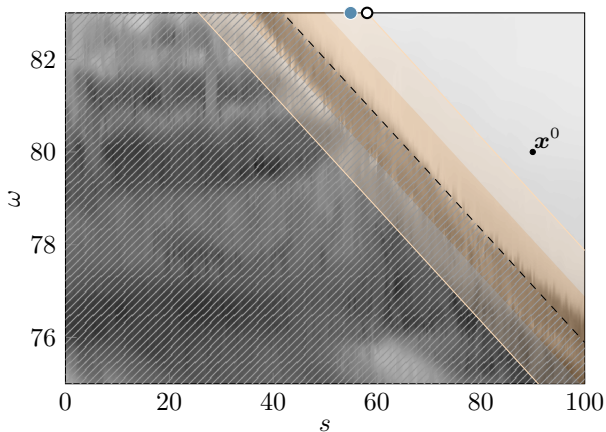
- $\min_{(s,\omega)} s$ s.t. $u(s,\omega) - u_{max} \leq 0$ and $d(s,\omega) \leq 0$
- $r_d = 1, \tau = 0.4, r_e = 1$



$$x^0 \in \hat{\Omega}$$

Vibration of aircraft engine blades

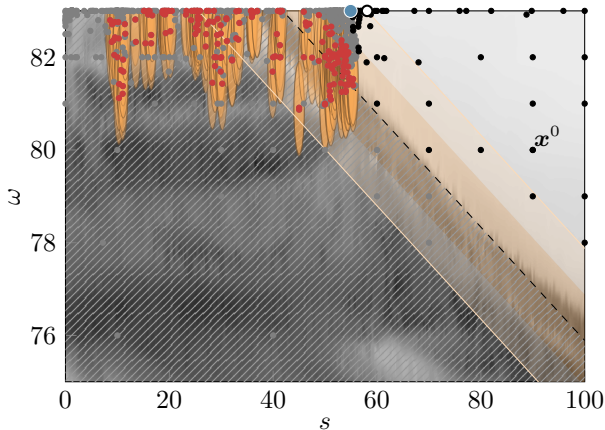
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Escaping regions of hidden constraints

- ▶ styrene production problem³ \implies STYRENE blackbox⁴
 - 8 variables (between 0 and 100), 11 constraints
 - simulation fails for some $x \implies$ hidden constraints

³C. Audet, V. Béchar, and S. Le Digabel. "Nonsmooth optimization through Mesh Adaptive Direct Search and Variable Neighborhood Search". *J. Global Optim.* 41.2 (2008), pages 299–318.

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 - failures $\implies f = 10^{20} \implies$ weak discontinuities of f
- ▶ how do we know if d is satisfied at x ?
 - evaluation of severity of hidden constraints around x :

$$H(x, \sigma) := \#\{y^i \in B_\sigma(x), i \in \{1, \dots, 1000\}, \text{evaluation at } y \text{ fails}\}$$

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Escaping regions of hidden constraints

▶ problem

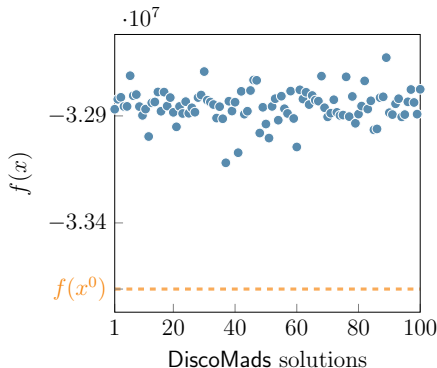
- x^0 : one of the best known solutions $f(x^0) = -33\,709\,000$
- $H(x^0, 15) = 435 \implies x^0$ in hidden constraints regions
- $r_d = 5, \tau = 10^{15}, r_e = 10$

Escaping regions of hidden constraints

▶ problem

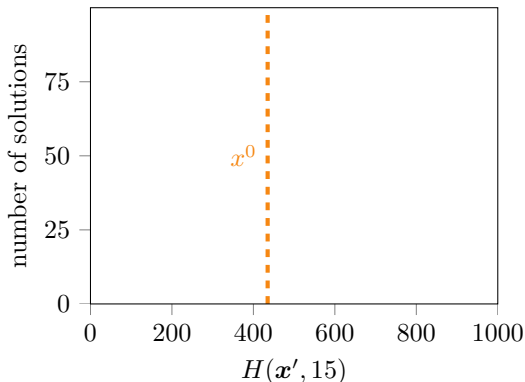
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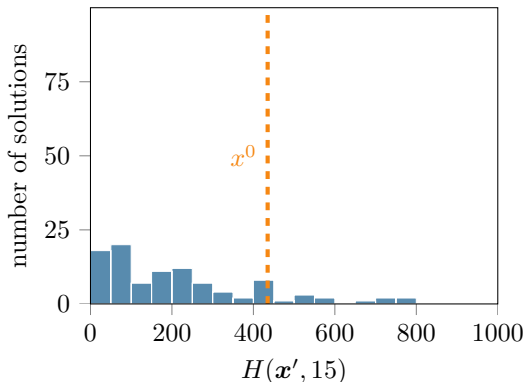


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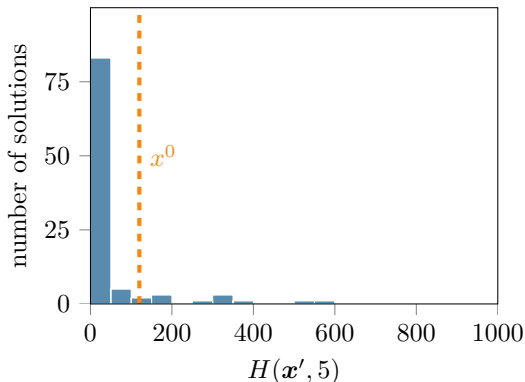
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$$\sigma = r_d + r_e$$

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$$\sigma = r_d$$

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Conclusion

- ▶ DiscoMads⁵ to escape unknown discontinuous regions
 - relies on Mads \Rightarrow preserves convergence properties
 - validated on low-dimensional numerical problems

⁵C. Audet, A. Batailly, and S. Kojtych. "Escaping unknown discontinuous regions in blackbox optimization". *SIAM Journal on Optimization* (2022). recently accepted.

Conclusion

- ▶ DiscoMads⁵ to escape unknown discontinuous regions
 - relies on Mads \Rightarrow preserves convergence properties
 - validated on low-dimensional numerical problems

- ▶ perspectives
 - improvements \Rightarrow revealing poll, scaling of variables, surrogates...
 - other problems \Rightarrow complex blade design, crash simulations...
 - other infinite constraints?

⁵C. Audet, A. Batailly, and S. Kojtych. "Escaping unknown discontinuous regions in blackbox optimization". *SIAM Journal on Optimization* (2022). recently accepted.

Thank you for attention !