# Report SEME for SCALIAN

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# Report SEME for SCALIAN

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In this report we present the procedure that we set up to try to solve a mathematical problem proposed by Scalian. We work in the context of reduced order models for wind turbine applications. The objective is to improve a part of the algorithm developed by the company, in order to make the wind turbine blades more reactive.

## The equation and its discretization

The reduced order model under location uncertainty used for this problem gives us stochastic differential equations for resolved modes denoted by $b = (b_i)_{i\in[\![1,n]\!]}$:

$$\begin{aligned} db_i(t) =& F_i(b(t)) + (\alpha_{\cdot i}.dB_t)^T b(t) + (\theta_i.dB_t) \\ =& F_i(b(t)) + \sum_{j=1}^{n}\left(\sum_{k=1}^{n}\alpha_{kij}b_k(t) + \theta_{ij}\right)dB_t^j. \end{aligned} \tag{1}$$

In this equation $F_i$ is a polynomial function of order two, and $B_t = (B_t^j)_{j\in[\![1,n]\!]}$ is a vector composed of Brownian motions. In the algorithm of the company, these stochastic differential equations are solved using a Euler-Maruyama numerical scheme. In order to improve the algorithm, we chose to change the numerical scheme implemented for these equations, and to implement a Milstein scheme. We rewrite the (1) under a more general form:

$$db(t) = F(b(t)) + \sigma(b(t))dB_t, \tag{2}$$

where $F(b) = (F_i(b))_{i\in[\![1,n]\!]}$ and

$$\sigma(b) = \left(\sum_{k=1}^{n}\alpha_{kij}b_k(t) + \theta_{ij}\right)_{i,j\in[\![1,n]\!]} = (\sigma_{ij})_{i,j\in[\![1,n]\!]}.$$

We discretize $t$ in the interval $[0,T]$ and consider $N \in \mathbb{N}$, and $\Delta t = \frac{T}{N}$. Let $(\gamma_k)_{k\in[\![0,N]\!]}$ be a sequence of vector $(\gamma_k^j)_{j\in[\![1,n]\!]}$ such that all the $\gamma_k^j$ are independent identically distributed standard Gaussian random variables. The Milstein numerical scheme is the following:

$$\begin{aligned} b(t_{k+1}) =& b(t_k) + F(b(t_k))\Delta t + \sigma(b(t_k))\gamma_k\sqrt{\Delta t} \\ &+ \frac{1}{2}(\sigma,\partial\sigma)(b(t_k))\gamma_k^2\Delta t - \frac{1}{2}(\sigma,\partial\sigma)(b(t_k))\mathbb{1}\Delta t. \end{aligned} \tag{3}$$

In this scheme, $\mathbb{1}$ denotes a vector with 1 as coefficients, and $(\sigma,\partial\sigma)(b(t_k))$ denotes a matrix such that

$$(\sigma,\partial\sigma)(b(t_k))_{ij} = \sum_{k=1}^{n} \sigma_{ik}\frac{\partial\sigma_{ij}}{\partial\sigma_{ik}}.$$

# Procedure and results

The aim is to compare the efficiency of the scheme (3) with the classical Euler-Maruyama scheme. First, we need to be sure that the Milstein scheme approximates the solution of (2), so we compare it with a reference solution, obtained using the Euler-Maruyama scheme with $\Delta t$ very small and a very large number of simulations. We denote this reference solution $b^{ref}$. The idea is to compare the strong error between $b^{ref}$ and the Euler-Maruyama scheme for larger values of $\Delta t$

$$\mathbb{E}\left[\sup_{t_k} \|b^{ref}(t_k) - b^{EM}(t_k)\|_2\right]$$

to the strong error between $b^{ref}$ and the Milstein scheme for the same values of $\Delta t$

$$\mathbb{E}\left[\sup_{t_k} \|b^{ref}(t_k) - b^{Mil}(t_k)\|_2\right].$$

This comparison allows us to find the time-step size where the Milstein scheme becomes closer to the reference solution than the Euler-Maruyama scheme. Then we compare the execution time for the two schemes, for several values of $\Delta t$. We compute the strong errors with 100 simulations for each scheme, and Figure 1 shows that the Milstein scheme is slower than the Euler-Maruyama scheme. It makes sense since this scheme is more complex than the intuitive Euler-Maruyama scheme. The strategy is to think that we will need a larger $\Delta t$ for the Milstein scheme than for the Euler-Maruyama scheme, and that the execution time of the Milstein scheme for this larger $\Delta t$ will be smaller than the execution time of the Euler-Maruyama scheme with the smaller $\Delta t$.

The strong error is represented by Figure 2. On this figure we can not observe any visible difference between the two schemes. Hence it does not prove our theory, but it can be due to the number of simulations which is only 100, and it could be good to increase it or to run other tests on these schemes. It could also be interesting to try other schemes like the Runge-Kutta or the multi-step

scheme. The other possibility is that the regime that we studied in Figure 2 is already a regime where Euler-Maruyama and Milstein are close, so il could be interesting to compute the strong error with $N \in [\![0, 50]\!]$. Finally, we do not control the influence of the variance, which creates a Monte-Carlo error in addition to the discretization error. This could be a problem and one can try to use variance reduction methods to improve the algorithm.

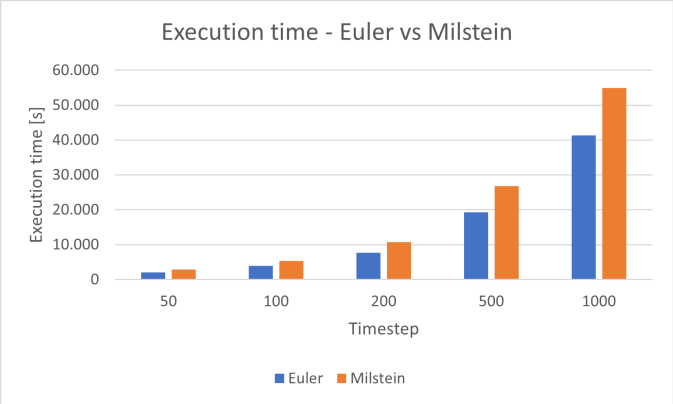


Figure 1: Execution time for the Milstein and Euler-Maruyama schemes, for 100 simulations and several values of $\Delta t$.

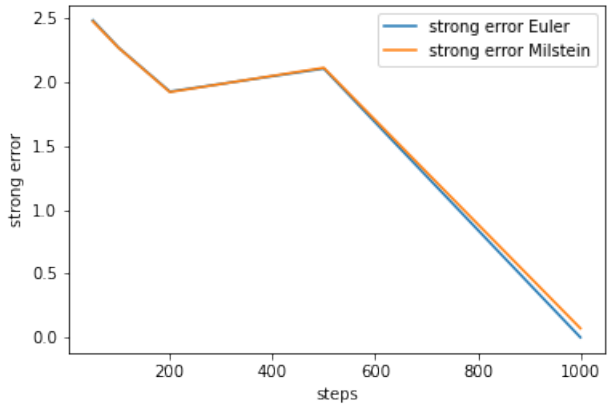


Figure 2: Strong error for the Milstein and Euler-Maruyama schemes, for 100 simulations and several values of $\Delta t$.

For variance reduction techniques one can refer to [3] [2] [7]

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