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Horizontal Axis Wind Turbine Systems: Optimization Using Genetic Algorithms

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Key words: genetic algorithms; optimization; Mediterranean site; wind turbine A method for the optimization of a grid-connected wind turbine system is presented. The behaviour of the system components is coupled in a non-linear way, and optimization must take into account technical and economical aspects of the complete system design. The annual electrical energy cost is estimated using a cost model for the wind turbine rotor, nacelle and tower and an energy output model based on the performance envelopes of the power coefficient of the rotor, CP, on the Weibull parameters k and c and on the power law coefficient α of the wind profile. In this study the site is defined with these three parameters and the extreme wind speed V_{max} . The model parameters vary within a range of possible values. Other elements of the project (foundation, grid connection, financing cost, etc.) are taken into account through coefficients. The optimal values of the parameters are determined using genetic algorithms, which appear to be efficient for such a problem. These optimal values were found to be very different for a Mediterranean site and a northern European site using our numerical model. Optimal wind turbines at the Mediterranean sites considered in this article have an excellent profitability compared with reference northern European wind turbines. Most of the existing wind turbines appear to be well designed for northern European sites but not for Mediterranean sites. Copyright © 2001 John Wiley & Sons, Ltd.

Introduction

A wind turbine is a complex system working in a complex environment. It is composed of subsystems working in a tightly coupled way. The design choice of a component may affect the annual energy production of the system and may indirectly affect the other component costs and behaviour. Furthermore, wind turbine systems must be adapted to the specific meteorological and topographical characteristics of each site.

The aim of our study is to develop optimization tools for the design of wind turbines dedicated to a site. The objective function of the optimization is the cost of the generated electrical energy calculated by means of conceptual models, most of which come from Reference 1. These models use a set of parameters, called principal parameters, that define the configuration of a wind turbine system. The optimized values of these variables are determined by minimizing the objective function with an optimization tool well adapted to the nature of this function.

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Most of the classical optimization methods are based on differential or variational calculus. Indeed, in most of the optimization problems the objective function appears to be continuous and derivable, and optimal values of the function may be calculated using iterative methods. These methods are limited to the treatment of optimization problems leading to continuous, derivable and well-defined objective functions. Since the mid-1950s, more difficult problems have been handled in the artificial intelligence field. These optimization problems are solved using knowledge-based systems, fuzzy logic, inductive learning, neural networks and genetic algorithms.²

Owing to their efficiency and robustness in the global optimization of design problems involving non-linear and non-explicit variables, genetic algorithms are used in this study as an optimization technique.^{3,4} The use of genetic algorithms is relevant since the design of wind turbine systems is a combinatorial problem and the number of possible configurations of a system is very substantial. The main objective of this article is to validate the use of genetic algorithms for wind turbine systems. Thus the class of grid-connected horizontal axis wind turbines which is considered in this article is the most widespread. The principal characteristics of the turbines considered in this article are presented in Table I (see Results and Discussion Section).

Wind turbine sites are characterized by the Weibull parameters k and c, by a constant power law coefficient α and by the extreme wind speed $V_{\rm max}$. The parameters k, c and α are used for the estimation of the annual energy output, and $V_{\rm max}$ is involved in the design of several components through the estimation of extreme loads due to the wind. Other characteristics of the wind (direction, turbulence) are not taken into account in our model. However, despite this restrictive hypothesis, those four parameters are sufficient to identify interesting differences between optimal wind turbines for Mediterranean sites and northern European sites.

In this article the global process of our work is presented through the description of the models for the estimation of the objective function, and the principle of genetic algorithms. Then the results that validate the methodology and prove its usefulness are discussed. A sensitivity study is performed showing the relevance of the design parameters used and the benefits deriving from the optimization. Optimal wind turbines are determined for different sites in order to point out interesting classes of wind turbines that may be compared with existing wind turbines. Finally, interesting prospects of our work that are currently being developed are provided.

Modelling

Wind turbines are complex systems that contain multiple components interacting through non-linearly coupled behaviours. A large number of design parameters and technological choices are required to define a wind turbine. The optimized design of a wind turbine well adapted to a specific site requires a complex economical and technical study. This study involves many different models. Our work is based on a single-criterion optimization method for which the objective function is the generated electrical energy cost. A synoptic diagram of the optimization method developed in this article is presented in Figure 1. The objective function is calculated using a wind turbine cost model and an energy annual production model, which have been developed using information from the literature. These models define the principal characteristics of a wind turbine system and include a set of equality constraints (geographical, material, aerodynamic and actualization variables, cut-in and cut-out wind speeds) and inequality constraints (hub height, rotor diameter and rotation speed). Figure 2 illustrates the different elements that constitute the two models.

Wind Turbine Cost Model

A global cost model of a wind turbine has been derived from cost models of all the components of the wind turbine and of some other costs of the project. These models are not presented here because they require a lot of parameters, but most of them can be found in References 1 and 4. For most of the components the models are derived from mechanical design laws and from the calibration of these models. Mechanical laws are used to calculate the weight of components, and calibration is performed by means of statistical data of component

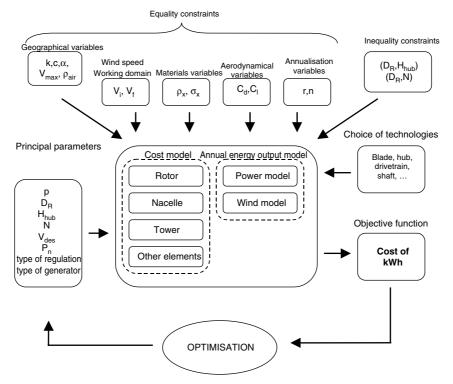


Figure 1. Synoptic diagram of the wind turbine optimization method

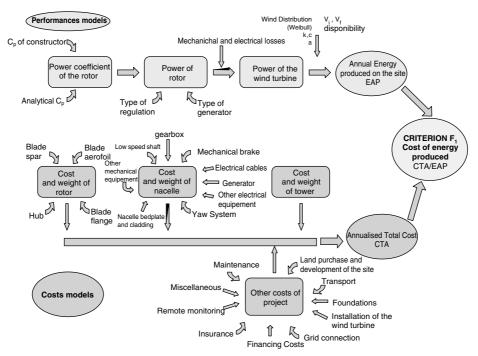


Figure 2. Global schema of the model

costs. For the generator and electrical equipment the price is derived from the value of the nominal power. The total wind turbine cost is the sum of all the costs, and a calibration factor $F_{\rm WT}$ allows us to use real wind turbine costs and take into account some unknown project parameters such as the manufacturer's margins:

$$C_{\text{WT}} = F_{\text{WT}} \times \sum_{i} C_{\text{component } i}, \qquad F_{\text{WT}} = 1.1$$
 (1)

with

$$C_{\text{component }i} = C_{\text{unit }i} \times f(\text{parameter}_1, \text{parameter}_2, \dots, \text{parameter}_{ni})$$
 (2)

where $C_{\text{unit}i}$ is the unit cost per kg (\in).

The evaluation of the total cost of a project must take into account some additional costs due to:

- land purchase and development of the site;
- transport of the wind turbine;
- installation of the wind turbine;
- foundations building;
- grid connection;
- remote monitoring;
- financing and insurance;
- miscellaneous costs (engineering, unexpected costs, etc.).

These additional costs have been estimated using data from the Nordex® society and have been interpolated as a function that depends on the diameter of the rotor and on the nominal power of the wind turbine:

$$C_{\rm DV} = \sum_{i} C_{\rm additional \, cost \, i} \tag{3}$$

with

$$C_{\text{component }i} = f(P_{\text{n}}, D_{\text{R}}) \tag{4}$$

The total investment cost $C_{\rm IT}$ is the sum of the wind turbine cost and additional costs. Using the relation given in Reference 5, we can estimate the total annual cost $C_{\rm TA}$ of the project. The actualization uses a factor a that depends on the economic life of the project, n, and on the required rate of return, r. The annual operation and maintenance costs are fixed to 2.5% of the investment cost.

$$C_{\rm TA} = (a + 0.025)C_{\rm IT} \tag{5}$$

with

$$a = \frac{r}{1 - (1+r)^{-n}} \tag{6}$$

Annual Energy Output Model

Optimal design of wind turbine systems goes through an estimation of their annual energy production. Indeed, this design always takes into account the cost of electrical energy, which appears to be the main design criterion of wind turbines. An energy production model has been built that involves both wind and turbine characteristics. Wind turbine technologies considered by the model are limited to horizontal axis wind turbines with two types of regulation:

- pitch regulation with constant rotation speed or variable rotation speed;
- stall regulation with constant rotation speed.

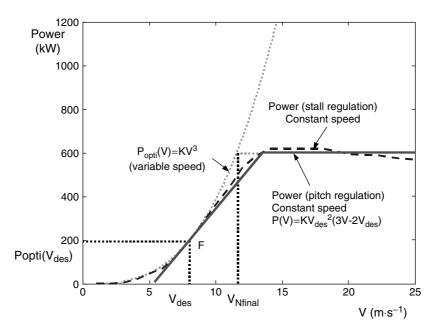


Figure 3. Power curve models

The estimation of the energy output is based on knowledge of the rotor performance or the rotor power profile. Figure 3 shows the power profiles of the different technologies used in our model. These performances are determined by a power coefficient C_P , defined by the ratio of the power P of the rotor to the kinetic power of the wind crossing the rotation surface of the rotor $(\frac{1}{2}\rho_{air}S_RV^3)$, where S_R is the swept area of the rotor). The calculation of C_P by aerodynamic methods requires a lot of data, including the complete geometry of the blade, and should be carried out for each individual wind turbine. Here we use an analytical relation given in Reference 6 to calculate the performance envelopes of C_P (the set of $C_{P \max}$):

$$C_{\text{Pmax}} = 0.593 \left(\frac{\lambda_{0 \max} \times p^{0.67}}{1.48 + (p^{0.67} - 0.04)\lambda_{0 \max} + 0.0025\lambda_{0 \max}^2} - \frac{1.92\lambda_{0 \max}^2 \times p}{1 + 2\lambda_{0 \max} \times p} \frac{C_{\text{d}}}{C_{\text{l}}} \right)$$
(7)

with

$$\lambda_{0\,\text{max}} = \frac{\omega}{V_{\text{des}}} \frac{D_{\text{R}}}{2} \tag{8}$$

The values of D_R , N and V_{des} completely determine $C_{P \text{ max}}$ and thus the optimal power profile $P_{\text{opti}} = f(V)$ (see Figure 3) that simulates the power of a variable speed wind turbine. Optimal power curves may be determined using the following relations:

$$P_{\text{opti}}(V) = KV^3 \tag{9}$$

with

$$K = \frac{1}{2}\rho\pi C_{\text{Pmax}}(D_{\text{R}}/2)^2$$
 (10)

The power curve of horizontal axis pitch-regulated wind turbines with constant rotation speed (see Figure 3) between the cut-in wind speed V_i and the nominal wind speed V_n is well approximated by means of a straight line tangent to the optimal power curve (see Figure 3); thus

$$P(V) = KV_{\text{des}}^{2}(3V - 2V_{\text{des}})$$
 (11)

Above V_n and until the cut-out wind speed V_f is attained, the wind turbine power remains fixed at the value P_n .

For stall-controlled wind turbines we use the power curve of some Nordex® wind turbines. It bounds the value of some parameters, but is necessary since this type of wind turbine performance is difficult to generalize.

With the previous relations the annual electrical energy output is determined using the integration of the wind speed distribution and the corresponding energy output during 1 year:

$$E_{\rm AP} = \frac{8760}{1000} \frac{\rho_{\rm air}}{2} \times S_{\rm R} \times \int_{V_{\rm i}}^{V_{\rm f}} V^3 f(V) C_{\rm P}(V) \eta_{\rm GB}(V) \eta_{\rm G}(V) dV$$
 (12)

Where E_{AP} is the annual electrical energy output (kWh), S_R is the swept area of the rotor (m²), f(V) is the Weibull probability density function of wind speed, η_{GB} is the gearbox efficiency and η_G is the generator efficiency.

The Weibull probability density function of wind speed depends on the parameters k and c that determine the shape and intensity of the wind during 1 year on a site:

$$f(V) = \left(\frac{k}{V}\right) \left(\frac{V}{c}\right)^k \exp\left[\left(\frac{V}{c}\right)^k\right] \tag{13}$$

where k is the Weibull shape parameter (dimensionless) and c is the Weibull scale parameter (m s⁻¹).

The parameters k and c are functions of the height of the wind turbine, since the site environment influences the wind distribution. The Weibull scale parameter is determined using a power law that involves wind speed increasing with hub height at vertical positions of the site:

$$c = c_0 (Z/Z_0)^{\alpha}$$
 at hub height Z (14)

where α is the coefficient of the power law and c_0 is the value of c at reference height Z_0 (generally 10 m). The Weibull shape parameter follows a law from Reference 7:

$$k(Z) = k_0 + \Delta k(Z) \tag{15}$$

where

$$\Delta k(Z) = \begin{cases} 0.008Z - 0.08 & \text{for } Z < 20 \text{ m} \\ 0.003Z + 0.02 & \text{for } Z \ge 20 \text{ m} \end{cases}$$

and k_0 is the value of k at reference height Z_0 .

The gearbox and generator efficiencies are calculated using equations presented in Reference 1. They depend on the wind turbine power law, on the nominal power P_n and on the type of regulation of the generator.

Optimization Variables and Constraints

The main factor that influences the design of wind turbines is the annual cost of the electrical energy produced by the wind turbine. These parameters are integrated in the mechanical laws and cost models presented above. The annual electrical energy cost is defined as the ratio of the total annual cost C_{TA} and the annual electrical energy output E_{AP} of the wind turbine:

$$C_{\rm EP} = C_{\rm TA}/E_{\rm AP} \tag{16}$$

Other factors, such as security parameters or environmental factors, also have great importance, but they are not used as optimization parameters in this article. These considerations introduce inequality constraints that bound the rotor diameter to the height of the wind turbine and limit the rotor speed. Indeed, in order to prevent security problems, the ground clearance between the blade tip and the ground is set to 15 m. Thus

$$D_{\rm R}/2 + 15 \le H_{\rm hub}$$
 (17)

Also, in order to limit noise levels and sound pollution, the maximum rotor tip speed is fixed at 80 m s $^{-1}$. Thus

$$V_{\rm tip} = 2\pi N D_{\rm R} / 120 \le 80 \text{ m s}^{-1} \tag{18}$$

Principal Parameters

The choice and number of principal parameters used for the optimization of wind turbine systems depend on the design constraints (physical constraints, technological constraints, design domain) and thus on the nature and complexity of the design model. Subsidiary parameters are determined by the model constraints or are fixed by the designer. The design model developed in this work uses eight principal parameters:

- the number of blades p;
- the rotor diameter D_R ;
- the hub height H_{hub} ;
- the rotation speed of the rotor, N;
- the nominal power P_n ;
- the design wind speed V_{des} ;
- the type of regulation (stall or pitch);
- the type of generator (asynchronous with variable or constant speed).

Most of these parameters are classically used as principal parameters for the optimization of wind turbines. However, the power characteristic curve of the wind turbine may be determined using several different parameters. The parameter used in our work is the design wind speed $V_{\rm des}$. This parameter is the wind speed for which the wind turbine power is optimal. At constant rotation speed this parameter defines a unique point F (see Figure 3) that depends on the rotor diameter and rotation speed. Point F corresponds to the maximum tip speed ratio $\lambda_{0\,\text{max}}$ (equation (8)). The parameter V_{des} greatly affects the power characteristic curve of the wind turbine by determining the slope of the power curve. Schematically, a variation in V_{des} will result in a shift of the C_p curve (Figure 4) and may be used to optimize the total energy retrieved from the kinetic energy of the wind.

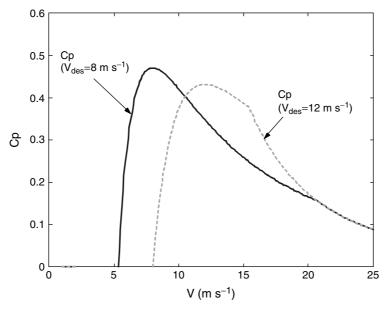


Figure 4. Influence of the parameter V_{des} on the power curve

A wind turbine configuration is defined with a set of discrete parameters (number of blades, type of regulation, type of generator) or positive real parameters (rotor diameter, hub height, rotor rotation speed, design wind speed, nominal power) evolving in a value space limited by physical or technological constraints (wind turbine cost model, annual energy output model) and design constraints (security constraints, environmental constraints). The aim of the wind turbine optimization is to find the best configuration of these parameters for a specific site by giving the minimum of an objective function. The optimization results presented in this article use the annual cost of the electrical energy produced by the wind turbine as the objective function. Owing to the nature of the design parameters and to the specificities of the design constraints, genetic algorithms have been used to solve this optimization problem.

Genetic Algorithms

Genetic algorithms, developed by Holland,⁸ are powerful tools for the global optimization of problems defined through non-continuous variables and functions. They are well adapted to the solution of non-linear problems that lead to objective functions containing many local minima or maxima.⁹ Objective functions may be non-derivable, non-continuous or ill-defined (as the result of a complex algorithm for instance), and no gradient information is required to solve them. More to the point, genetic algorithms appear to be robust and easy to implement. Classical optimization methods, e.g. gradient, Newton or quasi-Newton methods, are not suitable for the optimization of such hard-to-solve problems.

Genetic algorithm implementation is based on the evolution of a population including several individuals (10 in our application) towards a best individual by using selection and natural reproduction processes. These processes are based on the Darwinian concept of natural selection¹⁰ (see Figure 5). Individuals are defined through values of the principal parameter vector. Each principal parameter is called a gene, and individuals are completely defined by values of their own genes. Individuals evolve following an iterative

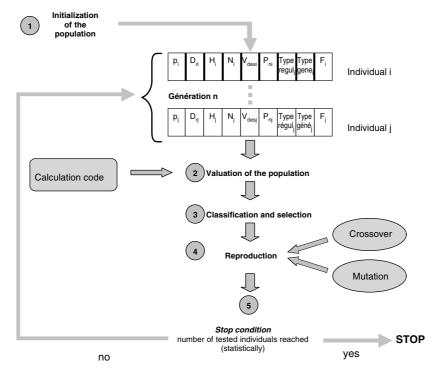


Figure 5. Global schema of the optimization process that uses genetic algorithms

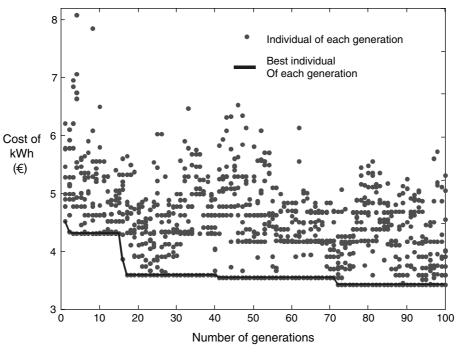


Figure 6. Example of evolution of a population

process. At each generation and for all individuals a selection probability (probability of participating in the next iteration) is calculated according to their classification in the population. This classification is based on the valuation of the objective function F(x), with $x = [p, D_R, H_{hub}, N, V_{des}, P_n]$. The selection probability is defined as¹⁰

$$p_i = \frac{1}{N_{\text{ind}}} \left(\frac{\phi - (r_i - 1)(2\phi - 2)}{N_{\text{ind}} - 1} \right)$$
 (19)

where N_{ind} is the number of individuals in the population, r_i is the rank of individual i and ϕ is the selection fitness equal to the average number of children, $\phi \in [1; 2]$.

Individuals selected according to p_i participate in the reproduction operations in order to create a new generation. These operations are the crossover operation (crossing of a few genes between two individuals) and the mutation operation (modification of genes). Crossover permits a good exploitation of the local value space, whereas mutation allows a renewal of the population and then an effective search of all the value space. The newly generated individuals are included in the generation only if the parameters check the inequality constraints. Otherwise, new crossovers or mutations are made. The number of tested individuals necessary to find the best turbine is determined statistically. The algorithm stops when this number is reached.

Figure 6 shows the evolution of a population of 10 individuals according to the objective function, which is the minimization of the electrical energy cost produced by a wind turbine. Convergence of the best individual and disparity of the individuals at each generation may be observed in the figure. A continuous line points out the best individual of each population.

Results and Discussion

Other constraints limiting the optimization domain of our application are listed in Table I. Corresponding constraints define several equipment characteristics and technological choices. These choices have been made in order to remain close to the most commonly encountered technologies in wind turbine design. Indeed, the optimization process presented in this article is concerned with optimizing interactions between

Table I. Main technological choices

Number of wind turbines on site	1			
Availability	100%			
Rotor	Horizontal axis			
Blades	2 or 3			
	Rigid blades			
	Aerofoil: NACA 63-4			
	Thickness/chord ratio = 0.18			
	$C_1/C_d = 120$			
	Stall or pitch regulation (hydraulic actuator)			
	Tip brake mechanism for stall regulation			
Blade spar and aerofoil	Glass-reinforced polyester			
Hub	Rigid for 3 blades			
	Teeter for 2 blades			
Drivetrain	Integral			
Low-speed shaft	Constant diameter			
	Internal diameter/external diameter = 0.5			
	$Length = 0.05D_{R}$			
Mechanical brake	Located on high-speed shaft for pitch regulation,			
~ .	on low-speed shaft for stall regulation			
Gearbox	2 stages epicyclic: ratio ≤ 40			
	3 stages (2 epicyclic, 1 parallel): $40 < \text{ratio} \le 80$			
	3 stages epicyclic: 80 < ratio			
Generator	Asynchronous generator			
	Constant speed: $N_{\rm G} = 1500$ rpm			
T1	Variable speed: $N_{\text{Gmax}} = 1500 \text{ rpm}$			
Electrical cables	4 copper sections, 3 cores			
Tower	Tapered steel sections			
Cut-in and cut-out speeds	3 and 25 m s^{-1}			

Table II. Variation range of principal parameters

Parameter	Domain of variation
P $D_{\rm R}$ $H_{ m hub}$ N $V_{ m des}$ $P_{ m n}$ Type of regulation	2 or 3 20-80 m (step of 5 m) 25-95 m (step of 10 m) 15-65 rpm (step of 5 rpm) 6-15 m s ⁻¹ (step of 1 m s ⁻¹) 300-2000 kW (step of 100 kW) Stall or pitch
Type of generator	Asynchronous constant or variable speed

wind turbine components. We do not perform optimization of existing blade geometry, mechanical or electrical equipment. Our investigations are limited to the optimization of the global performance of systems made with elements having known and predictable behaviours.

Optimization is performed by varying the principal parameters of a wind turbine configuration in a discretized domain. Table II presents the variation domain of these parameters and their corresponding variation step. Each principal parameter belongs to a finite set of possible values, so the possible configuration set of wind turbines is also a finite set. The design problem of wind turbines appears to be a combinatorial problem. This problem is non-polynomial owing to the non-linear structure of the model. ¹⁴ The number of possible individuals of the design problems treated in this article is 647,360.

Principal Parameters of Wind Turbine Design

A sensitivity study was performed to determine the relevance of the principal parameters and to compare the performances of optimized wind turbines with those of reference wind turbines. The reference systems have the same characteristics as existing wind turbines that are summarized in Table III. It must be noted that the reference wind turbines are installed in the north of Europe and have been chosen because we know the characteristics of their components and the costs of the project. Each wind turbine uses a different control technology (stall with constant rotation speed, pitch with constant and variable rotation speed). Energy and costs of optimized configurations put forward here are calculated using our model and using wind and site characteristics typical of Mediterranean sites.

Tables IV-X present optimization results of simulations performed by fixing one parameter and varying the principal parameters. These results may be compared with optimized wind turbines obtained by varying all the parameters. Size characteristics of the optimized wind turbines (rotor diameter, hub height) are lower

Table III. Principal parameters and optimization results for reference wind turbines

Parameter	Reference WT 1	Reference WT 2	Reference WT 3
p	3	3	3
$D_{\rm R}$ (m)	43	47	48
H_{hub} (m)	50	60	70
N (rpm)	27	26	28
$V_{\rm des}~({\rm m~s^{-1}})$	8	8	9
$P_{\rm n}$ (kW)	600	660	600
Type of regulation	Stall	Pitch	Pitch
Type of generator	Const. speed	Const. speed	Var. speed
Energy (kWh year ⁻¹)	1.6×10^{6}	1.8×10^{6}	1.9×10^{6}
Cost of WT (M€)	0.50	0.61	0.72
Cost of project (M€)	0.61	0.81	0.84
Cost of kWh (€cent)	5.5	6.4	6.3

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

Table IV. Optimization results: influence of the number of blades

Parameter	Reference WT 3	Optimized $p = 2$	Optimized $p = 3$
p	3	2	3
$D_{\rm R}$ (m)	48	35	35
H_{hub} (m)	70	35	35
N (rpm)	28	40	40
$V_{\rm des}~({\rm m~s^{-1}})$	9	14	14
$P_{\rm n}$ (kW)	600	900	1000
Type of regulation	Pitch	Pitch	Pitch
Type of generator	Var. speed	Var. speed	Var. speed
Energy (kWh year ⁻¹)	1.9×10^{6}	1.5×10^{6}	1.6×10^{6}
Cost of WT (M€)	0.72	0.34	0.37
Cost of project (M€)	0.84	0.43	0.47
Cost of kWh (€cent)	6.3	4.0	4.11
Cost reduction (%)		37	35

Wind characteristics: k=1.2, c=8 m s⁻¹. Site characteristics: $\alpha=0.12$ at 30 m.

Table V. Optimization results: influence of the rotor diameter

Parameter	Reference WT 3	Optimized $D_{\rm R} = 20 \text{m}$	Optimized $D_{\rm R} = 40 \text{m}$	Optimized $D_{\rm R} = 60 \text{m}$
p	3	2	2	2
$D_{\rm R}$ (m)	48	20	35	60
H_{hub} (m)	70	25	35	45
N (rpm)	28	65	40	15
$V_{\rm des}$ (m s ⁻¹)	9	15	14	12
$P_{\rm n}$ (kW)	600	400	900	2000
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	l Var. speed	d Var. speed	Var. speed
Energy (kWh year ⁻¹)	1.9×10^{6}	0.5×10^{6}	1.5×10^{6}	4.2×10^{6}
Cost of WT (M€)	0.72	0.11	0.34	1.05
Cost of project (M€)	0.84	0.17	0.43	1.25
Cost of kWh (€cent)	6.3	4.8	4.0	4.3
Cost reduction (%)		23	37	32

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

Table VI. Optimization results: influence of the hub height

Parameter	Reference WT 3	Optimized WT	Optimized $H_{\text{hub}} = 50 \text{ m}$	Optimized $H_{\text{hub}} = 80 \text{ m}$
p	3	2	2	2
$D_{\rm R}$ (m)	48	35	35	35
H_{hub} (m)	70	35	50	80
N (rpm)	28	40	40	40
$V_{\rm des} \ ({\rm m \ s^{-1}})$	9	14	14	15
$P_{\rm n}$ (kW)	600	1000	1000	1100
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	Var. speed	Var. speed	Var. speed
Energy (kWh year ⁻¹)	1.9×10^{6}	1.5×10^{6}	1.7×10^{6}	1.9×10^{6}
Cost of WT (M€)	0.72	0.34	0.38	0.52
Cost of project (M€)	0.84	0.43	0.49	0.63
Cost of kWh (€cent)	6.3	4.0	4.1	4.6
Cost reduction (%)		37	35	27

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

than those of the reference wind turbines, whereas parameters related to power (rotor rotation speed, design wind speed, nominal power) appear to be higher. As we shall see in the next subsection, this result is relevant to sites where the coefficient k is low and the coefficient c is high, which is typical of Mediterranean sites. Optimized and reference configurations are presented in Tables IV-X in order to compare the characteristics of wind turbines installed in the north of Europe with those of wind turbines well-adapted to Mediterranean sites. Reductions in energy cost between reference and optimized configurations generally range between 10% and 30%, thus showing the necessity of specific wind turbines for Mediterranean sites. For high turbine rotation speed the optimization process reduces the size of the gearbox, which appears to be one of the most expensive devices of wind turbine systems.

Table IV compares optimization results obtained for wind turbines with two and three blades with the performance of reference wind turbine 3. Cost reductions for the optimized wind turbines are close

Table VII. Optimization results: influence of the rotation speed

Parameter	Reference WT 3	Optimized $N = 20$ rpm	Optimized WT	Optimized $N = 50$ rpm
p	3	2	2	2
$D_{\rm R}$ (m)	48	50	35	30
H_{hub} (m)	70	45	35	35
N (rpm)	28	20	40	50
$V_{\rm des}~({\rm m~s^{-1}})$	9	13	14	14
$P_{\rm n}$ (kW)	600	1500	1000	700
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	l Var. speed	Var. speed	d Var. speed
Energy (kWh year ⁻¹)	1.9×10^{6}		1.5×10^{6}	
Cost of WT (M€)	0.72	0.81	0.34	0.24
Cost of project (M€)	0.84	0.96	0.43	0.34
Cost of kWh (€cent)	6.3	4.6	4.0	4.1
Cost reduction (%)		28	37	35

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

Table VIII. Optimization results: influence of the design speed

Parameter	Reference WT 3	Optimized $V_{\text{des}} = 6 \text{ m s}^{-1}$	Optimized $V_{\text{des}} = 10 \text{ m s}^{-1}$	Optimized WT
p	3	2	2	2
$D_{\rm R}$ (m)	48	55	35	35
H_{hub} (m)	70	45	35	35
N (rpm)	28	15	40	40
$V_{\rm des}$ (m s ⁻¹)	9	6	10	14
$P_{\rm n}$ (kW)	600	800	700	1000
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	Var. speed	Var. speed	Var. speed
Energy (kWh year ⁻¹)	1.9×10^{6}	2.0×10^{6}	1.3×10^{6}	1.5×10^{6}
Cost of WT (M€)	0.72	0.69	0.29	0.34
Cost of project (M€)	0.84	0.84	0.40	0.43
Cost of kWh (€cent)	6.3	6.1	4.2	4.0
Cost reduction (%)		3	33	37

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

(35% and 37%), irrespective of the number of blades. The number of rotor blades has a low influence on wind turbine performance. The components optimized characteristics (dimensions, technological choices) of both wind turbines are also close. Three-blade wind turbines produce more energy but appear to be more expensive than two-blade ones. Thus two-blade wind turbines are more efficient on this site.

Table V shows optimization results obtained for different values of the rotor diameter. The rotation speed and design speed of optimized wind turbines are reduced by increasing the rotor diameter. The air stream section, and thus energy production, increases with the squared rotor diameter and, as a consequence, energy may be produced with low characteristic speeds. However, the wind turbine nominal power increases with the rotor diameter. High-diameter turbines are expensive but produce a lot of energy, thus compensating for their high cost.

Table VI illustrates the influence of the hub height on wind turbine performance. Because of ground topography and roughness, wind speed and energy production increase with altitude. However, as the wind

Table IX. Optimization results: influence of the nominal power

Parameter	Reference WT 1/3	Optimized $P_{\rm n} = 300 \text{ kW}$	Optimized WT $P_n =$	Optimized 1500 kW
p	3	3	2	2
$D_{\rm R}$ (m)	43/48	25	35	35
H_{hub} (m)	50/70	35	35	35
N (rpm)	27/28	55	40	40
$V_{\rm des}~({\rm m~s^{-1}})$	8/9	10	14	15
$P_{\rm n}$ (kW)	600	300	900	1500
Type of regulation	Stall/pitch	Stall	Pitch	Pitch
Type of generator	Const./var. speed	Const. speed	Var. speed	Var. speed
Energy (kWh year ⁻¹) Cost of WT (M€) Cost of project (M€) Cost of kWh (€cent) Cost reduction (%)	$ \begin{array}{r} 1.6 \times 10^6 / 1.9 \times 10 \\ 0.5 / 0.72 \\ 0.61 / 0.84 \\ 5.5 / 6.3 \end{array} $	$ \begin{array}{ccc} 0^6 & 6.5 \times 10^5 \\ & 0.15 \\ & 0.21 \\ & 4.7 \\ & 13 \end{array} $	$ \begin{array}{c} 1.5 \times 10^6 \\ 0.34 \\ 0.43 \\ 4.0 \\ 37 \end{array} $	$ \begin{array}{c} 1.8 \times 10^{6} \\ 0.43 \\ 0.55 \\ 4.2 \\ 33 \end{array} $

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

Table X. Optimization results: influence of the regulation

Parameter	Reference WT 1/2	Optimized Pitch (N var.)	Optimized Stall (<i>N</i> const.)	Optimized Pitch (N const.)
p	3	2	3	2
$D_{\rm R}$ (m)	43/47	35	35	35
H_{hub} (m)	50/60	35	35	35
N (rpm)	27/26	40	40	40
$V_{\rm des}~({\rm m~s^{-1}})$	8	14	10	11
$P_{\rm n}$ (kW)	600/660	900	700	900
Type of regulation	Stall/pitch	Pitch	Stall	Pitch
Type of generator	Constspeed	Var∙ speed	Const· speed	Const· speed
Energy (kWh year ⁻¹) Cost of WT (M€)	$1.6 \times 10^6 / 1.6 \times 10^6$ 0.5/0	1.5×10^6 0.34	1.3×10^6 0.32	1.4×10^6 0.34
Cost of wr (M€) Cost of project (M€)	0.61/0	0.43	0.41	0.44
Cost of project (WC) Cost of kWh (€cent)	5.5/6	4.0	4.4	4.6
Cost reduction (%)	3.310	27/38	20/32	16/28

Wind characteristics: k = 1.2, c = 8 m s⁻¹. Site characteristics: $\alpha = 0.12$ at 30 m.

turbine tower becomes taller and more expensive, the increase in energy production is not sufficient to compensate the tower cost.

Table VII presents optimized wind turbines determined for different blade rotation speeds. For low rotation speed of the blades, the rotor diameter and nominal power of the wind turbine must be high in order to maintain the annual energy production at a sufficient level.

Table VIII shows optimization results obtained by fixing design wind speeds. These speeds have a very strong influence on the wind turbine performances. Design wind speed is characteristic of the blade shape and is linked to the aerodynamic phenomena that occur at the blade profile level. The aerodynamic performance of the blades determines to a large extent the technological choices and the dimensions of the entire system. On this site a high design wind speed gives the best results, but this principal parameter is very much linked to the site properties.

Table IX illustrates the influence of the nominal power of wind turbines on the produced energy cost. For low nominal powers, stall regulation appears to be more efficient than pitch regulation. Indeed, even

for low-power turbines, pitch regulation remains expensive. It must be noted that, despite the low nominal power of the 300 kW wind turbine and owing to the high wind speeds of the site investigated, optimized wind turbines appear to be better adapted to this site than reference turbines. Optimized turbines are indeed well adapted to high wind speeds.

Table X presents optimization results for stall- or pitch-regulated wind turbines with variable or constant rotation speeds. These results show that, on the site considered, the regulation has a low influence on the characteristics of the optimized systems. Produced energy costs are close and, for reasons discussed below, stall wind turbines have a slightly lower nominal power.

The sensitivity of the produced energy cost to variations in the principal parameters is summarized in Figures 7 and 8. The cost reductions range between 3% and 38%, while the principal parameters vary between

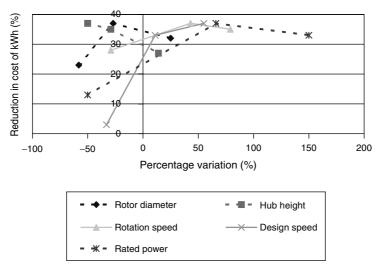


Figure 7. Influence of the principal parameters on the cost of a kWh

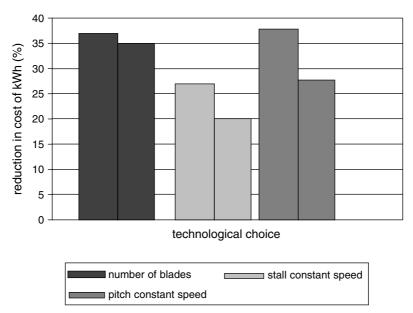


Figure 8. Influence of the principal parameters on the cost of a kWh (technological choices)

-60% and 150% of their values. These figures point out the influence of the principal parameters, which appears to be complex and non-linear, arising from interactions between components which have complex behaviours. Owing to this complexity and to the sensitivity of wind turbine performance to the principal parameters used in our model, global optimization of wind turbines appears to be highly profitable. This optimization process is linked to the installation site. The wind and site topographic characteristics also have a major influence on wind turbine performance.

Influence of Installation Site on Wind Turbine Design

About 80% of the installed power of wind turbines in Europe was located in northern Europe in 1999. However, Mediterranean sites have a great potential that remains underused owing to local politics of the Mediterranean countries during the 1980s and the 1990s. Environmental politics have been changing and these countries are interested in increasing their wind turbine parks. Our applications are mainly focused on the analysis of the design of wind turbines for Mediterranean sites.

In our model, installation site characteristics are taken into account through the annual distribution function of the wind and through the ground roughness coefficient. Mediterranean sites are characterized by Weibull parameters k which are smaller and parameters k which are larger than the corresponding parameters for northern Europe. Parameter k usually ranges between 1 and 2 and velocity k is around 9 m s⁻¹ at 10 m height in Mediterranean sites, while k is usually around 2 and velocity k ranges between 5 and 7 m s⁻¹ at 10 m height in northern European sites. Wind speed distributions may be far from a Weibull function and, owing to uneven reliefs in these regions, most Mediterranean sites have singular wind profiles. More to the point, in the presence of obstacles the wind shear coefficient k is stronger for Mediterranean sites, where it ranges between 0·16 and 0·18, than for northern European sites, where it is about 0·10. This coefficient also varies with height and wind sector. Simulation results presented as an application in this article are based on Weibull distributions.

Tables XI-XIII illustrate the influence of the Weibull parameters for wind turbines optimized for k values between $1\cdot 2$ and 2 and for c values between 6 and 12 m s⁻¹. The main trends that can be observed are linked to the size of wind turbines and to the energetic potential of sites. Indeed, wind turbines well adapted to the Mediterranean regions appear to be smaller (D_R and H_{hub}) than northern European wind turbines. The hub height and rotor diameter of Mediterranean turbines are small, but their power speed and parameters (N, V_{des} , P_n) appear to be greater. The kWh cost is much smaller for Mediterranean sites, varying between $2\cdot 9$ €cent for $k = 1\cdot 2$ and c = 12 m s⁻¹ and $4\cdot 4$ €cent for k = 2 and c = 8 m s⁻¹. Energy potential is greater for Mediterranean sites than for north European sites. Optimal design of wind turbines leads to small devices adapted to optimal energy capture and mechanical resistance in strong winds. This result is illustrated by Figure 9, showing that the smaller k is and the greater c is, the more important is the ratio P_n/S_R .

The influence of the wind shear coefficient α is presented in Figure 10. This coefficient has a great influence on the produced energy cost through the hub height and wind potential. Figure 10 shows the evolution of the

Table XI. Wind turbines optimized for k = 2 and c = 6-12 m s⁻¹ (k and c are defined at 30 m height)

Parameter	k = 2, $c = 6 \text{ m s}^{-1}$	k = 2, $c = 8 \text{ m s}^{-1}$	k = 2, $c = 10 \text{ m s}^{-1}$	k = 2, $c = 12 \text{ m s}^{-1}$
p	2	2	2	2
$D_{\rm R}$ (m)	55	35	35	30
H_{hub} (m)	45	35	35	35
N (rpm)	15	40	40	50
$V_{\rm des}~({\rm m~s^{-1}})$	9	11	13	15
$P_{\rm n}$ (kW)	600	500	800	800
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	Var. speed	Var. speed	Var. speed
Cost of kWh (€cent)	7.77	4.42	2.90	2.29

Table XII. Wind turbines optimized for k = 1.6 and c = 6-12 m s⁻¹ (k and c are defined at 30 m height)

Parameter	k = 1.6, $c = 6 \text{ m s}^{-1}$	k = 1.6, $c = 8 \text{ m s}^{-1}$	k = 1.6, $c = 10 \text{ m s}^{-1}$	k = 1.6, $c = 12 \text{ m s}^{-1}$
p	2	2	2	2
$D_{\rm R}$ (m)	60	35	30	35
H_{hub} (m)	45	35	35	35
N (rpm)	15	40	50	40
$V_{\rm des}$ (m s ⁻¹)	10	13	15	15
$P_{\rm n}$ (kW)	1100	700	800	1200
Type of regulation	Pitch	Pitch	Pitch	Pitch
Type of generator	Var. speed	Var. speed	Var. speed	Var. speed
Cost of kWh (€cent)	7.01	4.12	3.05	2.44

Table XIII. Wind turbines optimized for k = 1.2 and c = 6-12 m s⁻¹(k and c are defined at 30 m height)

Parameter	k = 1.2, $c = 6 \text{ m s}^{-1}$	$k = 1.2,$ $c = 8 \text{ m s}^{-1}$	k = 1.2, $c = 10 \text{ m s}^{-1}$	k = 1.2, $c = 12 \text{ m s}^{-1}$	
p	2	2	2	2	
$D_{\rm R}$ (m)	60	35	35	35	
H_{hub} (m)	45	35	35	35	
N (rpm)	15	40	40	40	
$V_{\rm des}$ (m s ⁻¹)	11	14	15	15	
$P_{\rm n}$ (kW)	1500	900	1200	1300	
Type of regulation	Pitch	Pitch	Pitch	Pitch	
Type of generator	Var. speed	Var. speed	Var. speed	Var. speed	
Cost of kWh (€cent)	6.10	3.96	3.20	2.90	

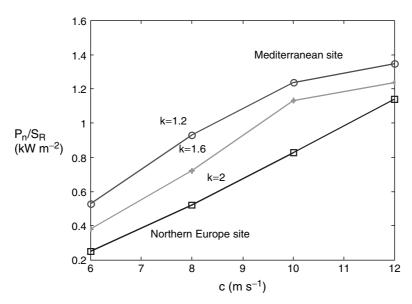


Figure 9. Ratio P_n/S_R versus Weibull law parameters

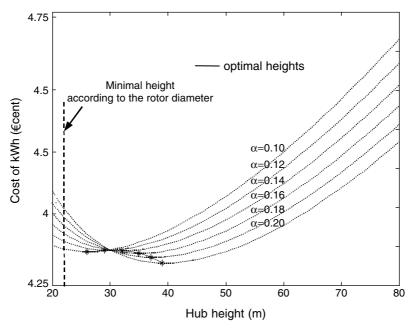


Figure 10. Evolution of the cost of a kWh versus the hub height and the wind shear coefficient

kWh cost with the hub height for values of α varying between 0·1 and 0·2. The characteristics of the wind turbine are fixed to a number of blades p=2, a rotor diameter $D_R=35$ m, a rotation speed N=40 rpm, a design speed $V_{\rm des}=14$ m s⁻¹, a nominal power $P_{\rm n}=1000$ kW, with pitch regulation and variable rotation speed. The Weibull parameters are $k=1\cdot2$ and c=8 m s⁻¹ at 30 m height. As can be seen in Reference 12, optimal hub heights corresponding to the minima of the curves presented in Figure 10 increase with the wind shear coefficient. However, the produced energy cost goes through a maximum value, since the tower cost increases with the hub height and wind speed.

Comparison of Optimized Wind Turbines and Commercial Wind Turbines

The optimized wind turbines presented in Tables XI–XIII are compared with commercial wind turbines in Figure 11. This figure points out the cost of optimized wind turbines and compares these costs with those of installed systems. Calculated costs of the reference wind turbines used in the sensitivity study are realistic compared with the costs of existing wind turbines, and our wind turbine cost model leads to realistic results. The application of this model to the optimization of wind turbine design lowers the cost of existing wind turbines. The objective function of the optimization, which is the minimization of the produced energy cost, may be attained by decreasing wind turbine costs or by increasing the amount of energy produced on the installation site. The optimization process leads to enhanced wind turbine performance by improving both factors.

The commercial wind turbines used in this article as references are well designed for northern European sites (for k values around 2), but Tables XI–XIII show that performance enhancement is more important for small values of k (k values around 1·2). Optimization of wind turbine design remains profitable for Mediterranean devices. Table XIV illustrates the performance of a wind turbine optimized for a Mediterranean configuration on a site that has typical northern European characteristics. In this configuration the energy produced by the wind turbine is much more expensive than the energy produced by reference wind turbine 3. The reference wind turbine performance can also be improved, as the kWh cost may be lowered to 7.8€cent.

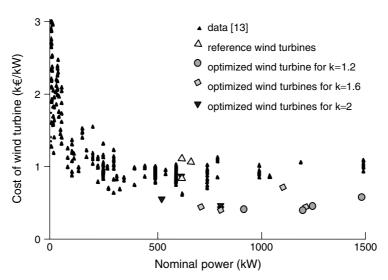


Figure 11. Wind turbine costs—comparison between optimized and commercial wind turbines

Table XIV. Comparison of the cost of a kWh for k = 2, c = 6 m s⁻¹ and $\alpha = 0.12$ (k, c and α are defined at 30 m height)

	p	$D_{\rm R}$ (m)	H_{hub} (m)	N (rpm)	$V_{\rm des}~({ m m~s^{-1}})$	$P_{\rm n}$ (kW)	Cost of kWh (€cent)
Optimized WT of Table 6	2	35	35	40	14	900	11.13
Reference WT 3	3	48	70	28	9	600	8.69
Optimized WT	2	55	45	15	9	600	7.77

Conclusion and Further Work

A method for optimizing a wind turbine system for a specific site has been presented in this article. This optimization involves a model for an estimation of the cost of electrical energy output. Using this model, a sensitivity study has shown the principal parameter influence on the produced energy cost. The optimization method is based on the use of genetic algorithms, as they appear to be suitable for the optimization of problems involving non-continuous, non-derivable and ill-defined variables and functions. A study with different Weibull parameters has been performed. It shows that wind turbine characteristics for Mediterranean sites are very different from those for northern European sites: according to our model, optimal wind turbines are smaller for Mediterranean sites and have larger power parameters. Produced energy with wind turbines optimized for Mediterranean sites is cheap.

Improvements in the model are being developed that allow for better consideration of existing technologies that are not taken into account in our model. They are focused on:

- introduction of other types of components, e.g. generators with other rotation speeds (750, 1500 rpm), dual-speed generator, direct drive generator (variable speed) or active stall regulation;
- elaboration of a control strategy model for wind turbines (starting, cut-in and cut-out wind speed, stopping) and optimization of this strategy according to the wind characteristics;
- improvement in site characterization by introducing factors related to accessibility, development (difficult for Mediterranean sites), foundation and grid connection.

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Appendix: Notation

a annualization factor

c Weibull scale parameter (m s $^{-1}$)

 c_0 reference c (m s⁻¹) $C_{\text{component }i}$ cost of component i (\in)

 $C_{\rm d}$ aerodynamic drag coefficient of blade aerofoil

 C_{EP} electrical energy cost $(\mathbf{\xi})$ total investment cost $(\mathbf{\xi})$

 C_1 aerodynamic lift coefficient of blade aerofoil

 $C_{\rm P}$ power coefficient of rotor

 $C_{\rm P\,max}$ maximum power coefficient of rotor

 C_{TA} total annual cost (\in)

 $C_{\text{unit }i}$ unit cost per kg of component $i \in \mathbb{C}$ total cost of wind turbine $\in \mathbb{C}$

D_R rotor diameter (m)

E_{AP} annual electrical energy output of wind turbine (kWh) f Weibull probability density function of wind speed

 F_{WT} calibration factor H_{hub} hub height (m)

k Weibull shape parameter

 k_0 reference k

n economic life of project (years) N rotation speed of rotor (rpm) N_G rotation speed of generator (rpm)

 $N_{\rm G\,max}$ maximum rotation speed of generator (rpm)

 $N_{\rm ind}$ number of individuals in population

p number of blades

 p_i selection probability of individual i

P power of rotor (kW)

 $P_{\rm n}$ nominal power of rotor (kW) $P_{\rm opti}$ optimum power of a rotor (kW) r required rate of return (%) $r_{\rm i}$ rank of individual i $S_{\rm R}$ swept area of rotor (m²) V wind speed (m s⁻¹)

 $V_{
m des}$ design wind speed (m s⁻¹) $V_{
m f}$ cut-out wind speed (m s⁻¹) $V_{
m i}$ cut-in wind speed (m s⁻¹) $V_{
m max}$ extreme wind speed (m s⁻¹) $V_{
m n}$ nominal wind speed (m s⁻¹) $V_{
m tip}$ rotor tip speed (m s⁻¹)

Zhub height(m) Z_0 reference Z(m) α wind shear exponent ϕ selection fitness, $\phi \in [1; 2]$

 η_{GB} gearbox efficiency generator efficiency

 $\lambda_{0\,\text{max}}$ tip speed ratio at maximum power coefficient

 $\rho_{\rm air}$ air density (kg m⁻³)

 ρ_x density material of x (kg m⁻³)

 σ_x admissible stress of material x (Pa) ω angular rotation speed of rotor (rad s⁻¹)

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