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# Pickup and delivery problems with autonomous vehicles on rings

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## 1 Introduction

In this work we investigate on-demand transportation services carried out using autonomous vehicles on circular networks. We called these problems Pickup and Delivery Problems on Rings (PDP-R). Concerning problems defined on circles, the existing literature is limited. In [1] the multiple capacity nonpreemptive vehicle routing problem on cycles is studied. Gendreau et al. [2] developed a linear time exact algorithm for the Single-Vehicle Pickup and Delivery Problem defined on a cycle graph. Tzoreff et al. [3] studied the Vehicle Routing Problem with Pickup and Delivery on some special graphs.

## 2 Problem description, classification scheme and complexity results

PDP-R have been introduced in [4] and can be described as follows. The context is that of a circular layout (a ring) with  $m$  stations, numbered from 0 to  $m - 1$  in a clockwise direction. The distance from station  $j$  to station  $j + 1$  is denoted as  $\delta_{j,j+1}$  and  $L =$

$\sum_{j=0}^{m-2} \delta_{j,j+1} + \delta_{m-1,0}$  is the total length of the ring. The problem consists in scheduling a set  $\mathcal{R}$  of  $n$  transportation requests where each request  $i \in \mathcal{R}$  asks for the transportation of a quantity of goods (or number of people)  $q_i$  from a pickup station  $s_i$  to a delivery station  $t_i$ . Each request specifies also a release date  $r_i$  and a due date  $d_i$ , that correspond, respectively, to the earliest time for pickup and the latest time for delivery. A fleet of  $V$  vehicles of capacity  $Q$  is available at the depot (station 0) to serve the transportation requests. All the vehicles start their tours from the depot and get back to it when all requests are served. One of the contributions of this work is a classification scheme for PDP-R problems based on three fields  $\alpha|\beta|\gamma$  where  $\alpha$  contains information about the number of vehicles and vehicle capacity,  $\beta$  specifies whether vehicles travel in one or both directions, whether release and due dates are defined and whether request have unitary demands, and  $\gamma$  corresponds to the objective function. Table 1 summarizes the words that compose the fields together with their meaning.

Field	Name	Description
$\alpha$	1 or $V > 1$	number of available vehicles
	1 or $Q > 1$ or $Q_v$	vehicle capacity ( $Q_v$ if they have different capacities)
$\beta$	<i>sd</i>	single-direction (clockwise or counterclockwise)
	<i>md</i>	mixed single-direction (each vehicle follows a single direction)
	<i>bd</i>	both directions
	$r_i$	pickups are subject to release dates
	$d_i$	deliveries are subject to due dates
$\gamma$	$u$	demands are unitary
	$C_{max}$	makespan
	$\sum C_i$	total completion time
	$CLT$	closing time

Table 1: Name and description of the various words of the fields

We obtained the following results related to the computational complexity of different classes of problems.

Name	Complexity	Proof
$1, 1 sd, u CLT$	P	Exact algorithm
$V, 1 sd, u CLT$	NP-hard	Reduction from 3-SAT
$1, Q sd, u CLT$	NP-hard	Equivalent to $V, 1 sd, u CLT$
$1, 1 sd, u, r_i, d_i CLT$	NP-hard	Reduction from $(\gamma, \mu)$ -Colouring
$1, 1 sd, u \sum C_i$	NP-hard	Reduction from Sum Colouring
$1, 1 bd, u CLT$	P	Exact algorithm(s)
$1, Q bd, u CLT$	NP-hard	[1]

Table 2: Summary of the complexity results of the problems addressed

### 3 Single direction and closing time objective function

The simplest variant in this class is  $1, 1|sd, u|CLT$ . We proved it has polynomial-time complexity by describing an exact algorithm. We proved that all possible generalizations

of this problem are NP-hard. We developed an ILP model to solve the most general variant  $(V, Q|sd, r_i, d_i|CLT)$ . In this model there is only one kind of decision variable  $x_{ikv}$  equal to 1 if request  $i$  is started in tour number  $k$  and assigned to vehicle  $v$ . The cost  $c_{ik}$  of starting request  $i$  in tour  $k$  is either  $k$  (if  $i$  is completed in tour  $k$ ) or  $k + 1$  if it is completed in tour  $k + 1$ . Then, the objective is to minimize  $CLT$  where  $CLT \geq \sum_{v=1}^V \sum_{k=1}^K c_{ik} x_{ikv}$  ( $1 \leq i \leq n$ ) subject to the constraints that all requests must be satisfied and that vehicle capacities cannot be exceeded.  $K$  represents an upper bound on the number of tours needed. We then strengthened the formulation with some symmetry-breaking constraints imposing that the  $i$  first requests have to be assigned to vehicles in set  $\{1, \dots, i\}$ . We generated randomly 5960 instances. We set  $m = 10$  stations,  $n$  requests ( $n \in \{20, 40, 80, 160\}$ ) random distances between consecutive stations in  $[1, 10]$ ,  $V \in \{1, 2, 3\}$  and  $Q \in \{1, 2, 4, 8\}$ , random demands in  $[1, Q]$  and four combinations for release and due dates: t-t, t-w, w-t, w-w (with t = tight and w = wide). We compared the performances of the formulation with a simple greedy algorithm in which vehicles turn all together along the ring and if a vehicle is at a station  $m'$  and a request  $i$  with starting station  $m'$  can be served,  $i$  is assigned to  $v$ . If multiple requests can be served at the same time, the algorithm breaks ties by sorting them according to the following criteria (taken in this order): earliest due date, largest demand, longest distance to reach the delivery node. We solved all instances (to optimality or proven infeasible) with CPLEX 12.9.0. Table 3 summarizes the computational results.

$\beta$	max CPU(s)	max greedy GAP(%)
$sd, u$	40	25
$sd, u, r_i, d_i$	2.2	50
$sd$	454	26.7
$sd, r_i, d_i$	365.6	33.3

Table 3: Maximum CPU time and greedy GAP for CLT instances

## 4 Single direction and total completion time objective function

We proved that the simplest variant in this class  $(1, 1|sd, u|\sum C_i)$  is NP-hard by reduction from the Sum Colouring Problem that is NP-complete. As a consequence, all possible generalizations are NP-hard. We used an ILP model based on the model described in section 3 with a different objective function, *i.e.*  $\min \sum_{i \in \mathcal{R}} \sum_{v=1}^V \sum_{k=1}^K c_{ik} x_{ikv}$ . We ran computational tests on the same instances used for  $V, Q|sd, r_i, d_i|CLT$ , but this time we set a time limit of 30 minutes. CPLEX solved 3014 out of 3123 feasible instances (a feasible solution or infeasibility proven for all instances). We compared the performances of the model with a greedy algorithm similar to the one described in section 3 that breaks ties

among requests by sorting them according to the following criteria (taken in this order): earliest due date, shortest distance to reach the delivery node, largest demand. Table 4 summarizes the computational results.

$\beta$	max CPU(s)	max greedy GAP(%) (opt.)	max greedy GAP(%) (feas.)
$sd, u$	3.3	6.3	-
$sd, u, r_i, d_i$	1.6	46.7	-
$sd$	1800	17.5	16.5
$sd, r_i, d_i$	1800	30.2	10.9

Table 4: Maximum CPU time and greedy GAP for  $\sum C_i$  instances

## 5 Both directions and closing time objective function

The simplest variant in this class is  $1, 1|bd, u|CLT$  and it is also known as the Stacker Crane Problem on a circle. This problem is polynomial-time solvable [5]. We propose an alternative algorithm for it. The variant where vehicles can have arbitrary capacity is denoted by  $1, Q|bd, u|CLT$ . This variant is also known as the multiple capacity nonpreemptive vehicle routing problem on cycles. In [1] it is proven NP-hard. As a consequence, all variants where vehicles have capacity more than 1 are NP-hard. We proposed an ILP formulation for variant  $V, Q|bd, u, r_i, d_i|CLT$  and 2 MILP formulations for variant  $V, Q|bd, r_i, d_i|CLT$ .

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