Differentiation and Integration of Annamalai's Binomial Expansion

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Abstract: Nowadays, the growing complexity of computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. This paper presents the differential and integral equations based on the Annamalai's binomial expansion.

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Annamalai's Binomial Expansion:

The following binomial expansion uses Annamalai's binomial coefficient and identity [2-10].

$$\sum_{r=k}^{n} V_{r-k}^{n} x^{i} = \prod_{i=1}^{n} \frac{(r-k+i)}{n!} x^{i} \quad (k, n, r \in N = \{1, 2, 3, \cdots\}),$$

where $V_{r-k}^{n} = \dfrac{(r-k+1)(r-k+2)(r-k+3)\cdots(r-k+n)}{n!}$.

$$If\ k = 0, \sum_{r=0}^{n} V_{r}^{n} x^{i} = \prod_{i=1}^{n} \frac{(r+i)}{n!} x^{i},$$

$$\text{where } V_{r}^{n} = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!}.$$

Differentiation:

$\mathrm{a} + \mathrm{ax} + \mathrm{a}x^2 + ax^3 + \cdots + ax^n$ is a geometric series, which is used in the effective medicine dosage [1].

Let $\mathrm{y} = 1 + \mathrm{x} + x^2 + x^3 + \cdots + x^n = \sum_{i=0}^{n} V_i^0 x^i$, where $V_n^0 = V_0^n - 1$ & $V_0^0 = 1$.

Differentiation of y is as follows:

$$\frac{1}{1!}\frac{dy}{dx} = \sum_{i=0}^{n-1} V_i^1 x^i; \quad \frac{1}{2!}\frac{d^2y}{dx^2} = \sum_{i=0}^{n-2} V_i^2 x^i; \quad \dots; \quad \frac{1}{p!}\frac{d^py}{dx^p} = \sum_{i=0}^{n-p} V_i^p x^i,$$

where $p \in N = \{1, 2, 3, \cdots\}$.

Integration:

The following equations are Annamalai's binomial expansions [2-10].

$$\sum_{i=0}^{n} V_i^p x^i = 1 + \frac{(p+1)}{1!}x + \frac{(p+1)(p+2)}{2!}x^2 + \cdots + \frac{(n+1)(n+2)\ldots(n+p)}{p!}x^n .$$

$$\sum_{i=0}^{n-1} V_i^{p+1} x^i = 1 + \frac{(p+2)}{1!}x + \frac{(p+2)(p+3)}{2!}x^2 + \cdots + \frac{n(n+1)(n+2)\ldots(n+p)}{(p+1)!}x^{n-1}.$$

Prove that following integral equation and its results using the above-mentioned Annamalai's binomial expansions [2-10]:

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx + C = (p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx + 1 = \sum_{i=0}^{n} V_i^p x^i \;,$$

where C=1 and C is a constant of Integration.

Proof: The integration of Annamalai's binomial expansion [2-10] is given below:

$$\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx = x + \frac{(p+2)}{1!}\frac{x^2}{2} + \frac{(p+2)(p+3)}{2!}\frac{x^3}{3} + \cdots + \frac{n(n+1)\ldots(n+p)}{(p+1)!}\frac{x^n}{n} + C.$$

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx = 1 + \frac{(p+1)}{1!}x + \frac{(p+1)(p+2)}{2!}x^2 + \frac{(p+1)(p+2)(p+3)}{3!}x^3 + \cdots + \frac{(n+1)(n+2)\ldots(n+p)}{p!}x^n, \qquad \text{where } C = 1.$$

$$(p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx + C = (p+1)\int \sum_{i=0}^{n-1} V_i^{p+1} x^i \; dx + 1 = \sum_{i=0}^{n} V_i^p x^i \;.$$

Hence, it is proved.

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