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▶ To cite this version:

HAL Id: hal-03667175

https://hal.science/hal-03667175

Submitted on 13 May 2022

Topology Optimization of Heat Exchangers: A Review

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Abstract

The critical importance of heat exchangers (HXs) on energy systems has been widely recognized, which can largely determine the overall efficiency. With the rise of the topology optimization (TO) and additive manufacturing techniques, it is anticipated that the TO may became a leading optimization technique in designing HX structures for the heat transfer intensification. Various algorithms for TO of HXs are dispersed in the literature, while a comprehensive and comparative review on their features, advantages, disadvantages, and limitations, is still lacking. Therefore, this paper aims at filling the literature gap by providing a comprehensive state-of-the art review on the TO for HXs over the past decades, so as to indicate the most promising technology roadmap. Each stage of the TO, i.e. the design parametrization, the heat transfer modeling, the optimization, and the final realization, is analyzed carefully in the corresponding section, highlighting the major pros, cons and challenges. Our statistics demonstrate that the current TO, though well-developed and fast improved, still have numerous limitations in handling the industrial HXs that hold the complicate structures and flow patterns. Eventually, three emerging schemes, i.e. machine learning, model order reduction, and moving morphable components, aimed to improve the efficiency of TO are also discussed.

Key Words: Topology optimization, Heat exchangers, Conjugate heat transfer, Additive manufacturing, Machine learning.

Nomenclature

γ	Densities (Design variables)
α	Inverse permeability
λ	Physical properties
$\emptyset$	Level Set function
q	Penalization factor
v	Velocity (m/s)
T	Temperature (K)
ρ_f	Fluid density (kg/m^3)
ρ_s	Solid density (kg/m^3)
C_{pf}	Specific heat capacity for fluid (J.kg^{-1}.K^{-1})
C_{ps}	Specific heat capacity for solid (J.kg^{-1}.K^{-1})
μ_f	Fluid dynamic viscosity (Pa.s)
P	Pressure (Pa)
f	Fictious force
Q	Heat transfer rate (W)
U	Overall heat transfer coefficient (W.m^{-2}.K^{-1})
ΔT_m	Mean temperature difference (K)
t	Time (s)
S	Source Term (W)
k_s	Solid thermal conductivity (W.m^{-1}.K^{-1})
k_f	Fluid thermal conductivity (W.m^{-1}.K^{-1})
A	Heat transfer surface area (m^2)
X	Position vector

Abbreviations

TO	Topology Optimization
HX	Heat Exchanger
AM	Additive Manufacturing
RAMP	Rational Approximation of Material Properties
SIMP	Solid Isotropic Material with Penalization
NS	Navier-Stokes
CM	Conventional Manufacturing

LBM	Lattice Boltzmann Method
XFEM	Extended Finite Element Method
VOF	Volume of Fluid method
BCM	Building Cube Method
GA	Genetic Algorithm
RANS	Reynolds Averaged Navier Stokes
MMA	Method of Moving Asymptotes
GCMMA	Globally Convergent MMA
SQL	Sequential Quadratic Programming
MOSQL	Multi-objective SQL
NSA	Null Space Algorithm
SLP	Sequential Linear Programming
SLA	Stereolithography
DLP	Digital Light Processing
EDM	Electrical Discharge Machining
NSGA	Nondominated Sorting Genetic Algorithm
CFD	Computational Fluid Dynamics
CNC	Computer Numerical Control
CV	Control Volume
LES	Large Eddy Simulation
DES	Detached Eddy Simulation
BO	Bayesian Optimization
ML	Machine Learning
MMC	Moving Morphable Component
MOR	Model Order Reduction
S-D	Steepest Descent Method
H-J	Hamilton-Jacobi equation
R-D	Reaction-Diffusion equation
FVM	Finite Volume Method
FEM	Finite Element Method
LSF	Level Set Function
LSM	Level Set Method
EA	Evolutionary Algorithm
GP	Gaussian Process
AI	Artificial intelligence

1. Introduction

Energy, environment and sustainable development are closely related topics, while energy is at the center of the sustainable development paradigm. All energy conversion systems involve the heat transfer via fluid flows. More than two thirds of energy is lost in the energy conversion chain, from capture, conversion, transport, production, distribution, storage to end use. Increasing energy efficiency has been identified as one of the main challenges for energy systems and has attracted increasing attention from the academic and industrial communities [1]–[3].

Heat exchanger (HX) is a classical component [4]–[6] and the basic element not only for all systems and processes of energy conversion, production and use but also for many industries (food, cosmetics, medical, textile, chemical, metallurgical, materials, building, embedded systems, aeronautics, aerospace...). HXs are everywhere, indispensable, in different forms, to meet various needs, and are often subject to strong functional and operational constraints. It is a highly applied research topic that requires fundamental sciences such as thermodynamics, transport phenomena, fluid mechanics, materials, combined with high-performance numerical methods and optimization tools. The objective is to increase their overall performance. The key points are the intensification of heat transfer on the one hand, and the optimized management of fluid flows on the other hand, at each structural and temporary scale [6]–[9]. Therefore, how to improve the thermal performance of HXs has long been a hot topic in the research community of energy engineering.

Many theorems and methodologies have been developed for enhancing the heat

transfer rate of HXs at the given pressure loss [10]–[14]. Starting with the basic heat transfer equation for HXs [10]:

$$Q = UA\Delta T_m \tag{1}$$

where Q is the heat transfer rate (W), U is the overall heat transfer coefficient (W.m^{-2}.K^{-1}), A is the heat transfer surface area (m^2) and ΔT_m is the mean temperature difference or the heat flux driving force (K). U is composed of conduction and convection coefficients which are associated to the transport properties. Both heat transfer mechanisms (conduction and convection) could be magnified by enhancing the thermal properties of the HX material and by affecting the fluid flow pattern near to the heat transfer surfaces, respectively. Moreover, it is evident that increasing A and better allocating the heat transfer driving force (ΔT_m) will also intensify the heat transfer. For all these three aspects, a determinant factor is the shape/form/arrangement of the solid-fluid interface within the HX, on which the size/shape/topology optimization methods could be employed to play a critical role.

In general, the optimization of HXs can be classified into three types: the size optimization, the shape optimization and the topology optimization (TO). The size/shape optimization has been well developed for years [15]–[17], which refers to the design process that searches for the optimal size or shape in the given configuration or arrangement for a specific HX [18]. In practice, good performance improvement can still be achieved with the careful selection of the initial structures and optimization criteria [19]. Nevertheless, the size/shape optimization could not significantly change the prescribed configuration or arrangement of HXs set by designers, which may limit

the optimization performance. Different from the size/shape optimization, the TO act directly on the topology of the (interface) geometry by spatially optimizing the distribution of fluid or solid phase and their connectivity, within a defined domain, which may attain any topology that minimize/maximize the optimization objective under some constraints. In theory, it holds the possibly maximum degrees of freedom in optimization, though in practice, the optimization objectives and constraints can also have a significant influence on the final results. In recent years, the TO has been regarded as a groundbreaking technique to obtain the innovative designs of HXs with greatly improved effectiveness, and has drawn more and more attention of researchers.
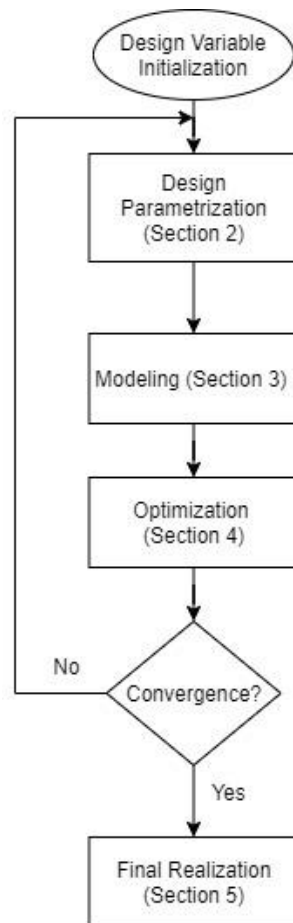

Figure 1: The basic stages (the corresponding section) in the TO process

Figure 1 shows a representative workflow of TO that includes four basic stages: (1) Design parametrization, (2) Heat transfer modeling, (3) Optimization process, and (4) Final realization. Compared to structural TO for mechanics, the issues that limit the TO's utilization for the HXs can emerge in each stage of the TO process. The HXs involve the conjugate flow and heat transfer [20]. Thus, the fluid problems should be solved during the iteration process of TO, leading to large computational expenses. This is actually the major obstacle for the practical utilization of TO for the real HXs of which the intermediate or final interface structures/topologies can be really complicated. Meanwhile, the mixing among different flows should be avoided by carefully designing the parametrization scheme, when updating the geometry of the solid phase that separates different fluids [21]. Additionally, maximizing the heat transfer rate is not always the only goal when designing HXs; the pressure loss should also be considered. To address this issue, the weighted-sum objective function [22] or a multiple-objective optimizer, such as NSGA-II [23], should be employed. Moreover, even if the rapid development of additive manufacturing (AM) techniques has greatly improved the ability to realize the optimized designs obtained by the TO, there are still some manufacturing constraints when applying a specific AM technique [24], and the proper post-treatments on the TO-derived structures are highly needed [25]. Actually, there are few researches that consider the fabricating constraints directly into the TO of HXs. Researchers have proposed some specific solutions for the issues mentioned above, which are dispersed among the literature. In the past years, several review articles were

published to cover the literature of TO for microfluid devices [26], heat transfer systems [27], and fluid-based problems [28]. However, a comprehensive and comparative review on different TO stage's features, advantages, disadvantages, and limitations, is still lacking particularly for HXs.

Here, we will analyze and compare the researches on the TO for HXs in the most recent years with the main objectives of defining a research guideline for the development and improvement of TO for HXs. A brief understanding of different methods employed in all TO stages is intended to be provided which will help and clarify the implementation procedure for interested readers. However, there have been few TO researches handling such practical multi-flow HXs at the component level. In fact, most of the TO papers just deal with a specific element (such as a duct) within the whole HX structure. In order to extend the coverage of our review, the papers for single-flow heat sinks and fins that involve the physics of conjugate heat transfer are also included, while the pure heat conduction, the radiation, the phase change (evaporation, condensation), transient operations (thermal energy storage for example), and exothermic/endothermic reaction problems are excluded for clarity. According to this inclusion criterion, 91 studies published in the past fifteen years are covered in this review, which can well reflect the mostly-recent progress in the TO for HXs.

The present paper is organized following the procedure of TO, that is, each stage of TO will be discussed in the corresponding section. Those common approaches are presented, with emphasis in the issues that limit their utilization for the practical HXs. Afterwards, some new trends in this area aimed to improve the efficiency, like the

integration of machine learning techniques, will also be covered in Section 6. A series

of statistics, comparative tables and figures will be given in each section to demonstrate

the features, advantages, disadvantages, and limitations of the developed schemes in

the TO of HXs.

2. Design Parametrization

Design parametrization refers to the representation of optimization variables

determining the design configurations that establish the relationship between the design

variables (e.g., the density distribution describing the flow paths in the density-based

TO problems) and the physical properties by the interpolation functions. Its sensitive

representation strongly affects the TO's output results [29]. Furthermore, the design

parametrization can vary from TO types, problem descriptions, and physical

phenomena. It should be carefully chosen according to the problem's features,

considering both efficiency and accuracy. As given in Tab. 1, there are three main types

of parametrization methods: Density-based, Level set and Direct explicit.

Table 1: Summary and comparison of design parametrization methods

Parametrization	Advantages	Disadvantages
Density-based	• Fixed mesh; • Well developed in TO for years.	• No interface described; • Numerical instabilities; • Modified governing equations.
Level-set	• Crisp description of interface profile; • No re-meshing in general.	• Slow convergence; • Results dependent on initial configurations; • Numerical artifacts.

Direct explicit	- Interface described explicitly; - Straight-forward & relatively simple.	Applicable only for simple geometries.

2.1 Density-based method

The density-based method is the most popular means, which was first proposed by Bendsøe [30] in 1989. It is based on representing the design domain by densities or porosities to parametrize the fluid and solid phases. Researches started with the single-flow problems from Borrvall and Petersson [31]. Their representation of design variables (the density γ) consists of assigning $\gamma = 0$ for the solid phase or non-existing fluid phase, and $\gamma = 1$ for the fluid phase. The governing equation need be modified by introducing a fictitious force (the detailed equations will be given in Sec.3), which is determined by an inverse permeability (α) for each element, with α_{min} corresponding to $\gamma = 1$ (fluid region) and α_{max} corresponding to $\gamma = 0$ (solid region). In the elements of solid with $\gamma = 0$, the fictitious force is maximum to block the flowing of fluid. During the iteration process, the inverse permeability (α) is changing continuously in every element, which is determined by an interpolation function. Taking the widely-used SIMP (solid isotropic material with penalization) [30] as an example, it is given by,

$$\lambda(\rho) = \lambda_{max} + (\lambda_{min} - \lambda_{max})\gamma^q \qquad (2)$$

where $q > 1$ is a penalization coefficient to minimize the presence of the gray elements, i.e. the ones of partial density from 0 to 1, $\lambda_{min}, \lambda_{max}$ are the minimum and maximum physical properties values (e.g., thermal conductivity), respectively. Note that the gray elements are usually regarded to hold no physical meaning and thus should be avoided by adjusting q. Thereafter, the above density-based representation with the different

penalization functions, including SIMP and RAMP (rational approximation of material properties), etc., have been utilized in a wide range of single-flow HX problems [32]–[85]. Additionally, some other researchers [86]–[101] used an opposite representation of design parametrization by assigning $\gamma = 1$ for the solid phase or non-existing fluid phase and $\gamma = 0$ for the fluid phase. Note that no evidence demonstrates that such different representation of solid and liquid phases will significantly affect the solutions or efficiency of TO in the single-flow heat transfer problems. Currently, the first kind of representation accounts for the largest portion in the published articles, as shown in Fig.2b.
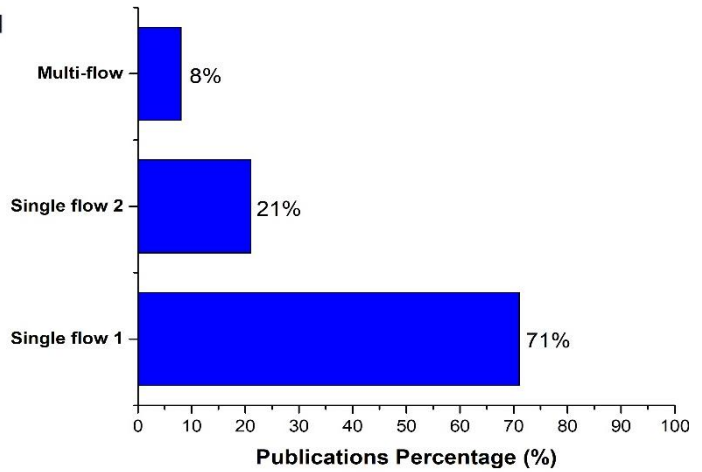
Furthermore, the density-based method was extended to the multi-flow HX problems, which involve two or more fluids separated by one or more solid phases [21], [101]–[105]. For instance, Kobayashi et al. [105] used one density (γ) to describe the multi-fluid problem by assigning $\gamma = 0$ for fluid 1, $\gamma = 1$ for fluid 2 and intermediate values ($0 < \gamma < 1$) for the solid phase. Tang et al. [101] divided a dual flow heat transfer problem into two independent one-flow and one-solid sub-problems, and thus one design variable (density) is used for both sub-problems.

The density-based method has shown its high efficiency by avoiding the re-meshing process at each iteration. As for the interpolation functions, the RAMP has proved the ability to penalize the larger range of design variables compared to the SIMP function [106]. In fact, the majority of TO publications are based on the density-based method, as shown in Fig. 2a. However, some numerical issues are usually encountered, like the mesh-dependent results, the bad formation of solid/fluid cells in the optimized structure

in which they are ordered similarly to the checkerboard configurations, and the intermediate densities values, etc. [107]. To remedy these instabilities, densities filters and projections should be implemented [107],[108]. More importantly, it is not able to exactly describe the interface between different phases due to the element by element updating procedure, and thus not suitable for the problems where the interfacial profiles or the properties near the interfaces are important [109].
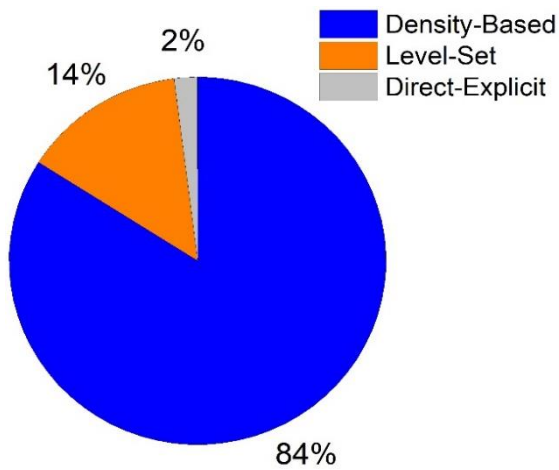
| (2.a) Publication percentage of different design parametrization methods | (2.b) Publication percentage of Density-based different design parametrization. Single flow 1 (γ=1 fluids, γ=0 solids). Single flow 2 (γ=0 fluids, γ=1 solids) |

Figure 2:Publications statistics for design parametrization methods in TO of heat exchangers (until 1-March-2022)

2.2 Level set method

The level-set method (LSM) was first developed by Osher and Sethian [110], for the purpose of well defining the interface between phases. Most of the time, it implicitly describes the interface between multiple phases by a level-set function (LSF) [111]–[113], which allows a clear description of the interfaces and improves the accuracy of

the responses captured at the boundaries. The design parametrization of LSM is given

by,

$$
\begin{cases}
\emptyset(\boldsymbol{X}) > 0 & \Leftrightarrow \text{(Material Phase)} \\
\emptyset(\boldsymbol{X}) = 0 & \Leftrightarrow \text{(Interface)} \\
\emptyset(\boldsymbol{X}) < 0 & \Leftrightarrow \text{(Void)}
\end{cases}
\tag{3}
$$

where $\emptyset$ is the level-set function and $\boldsymbol{X}$ is the location vector of the design domain.

The LSF is set to be zero at the interface, and the nodal values of LSF can be solved

based on a governing equation or interpolated on the computational domain by a space

function called "basis functions". The LSM has also been applied in the TO of HXs

[96], [104], [114]–[123]. Feppon et al. [117] even used the LSM to deal with the 2D

and 3D HXs involving two fluids. Furthermore, the LSF can be described in an explicit

way [120],[124]. Li et al. [124] suggested a component-based level-set parametrization

to describe explicitly the solid/fluid interface for the TO of a micro-channel heat sink.

The clear and crisp description of interface in the LSM makes it a good option for

the problems where the interfacial profiles or the properties near the interfaces really

matter. Generally in the LSM, re-meshing is not needed, except in the case of

conforming discretization (referring to the conforming discretization section in Ref.

[125]). In this sense, the LSM can be well suitable for the HXs where heat transfer rate

is largely determined by the flow velocity and temperature fields near the solid-liquid

or liquid-solid-liquid interfaces. However, the dependence of output results on the

initial configurations can significantly affect the accuracy and efficiency of LSM [125].

Another disadvantage is the slow convergence compared to the density-based method

[125]. Moreover, similar to the density-based method, the regularization techniques are

always necessary to avoid numerical artifacts and enhance the convergence rate in the

LSM [125].

2.3 Direct explicit method

Direct explicit parametrization permits to describe interfaces in a direct way. One or several functions or arrays are used to describe the interfacial profiles explicitly. Among the literature, the direct explicit parametrization is very well established for shape optimization [126]–[128], but only applicable for some simple problems. As for TO of HXs, it is infrequently applied (see Figure 2.a), though it can eliminate the numerical artifacts encountered by the implicit representations [129]. For example, Mekki et al. [130] proposed an explicit voxel parametrization for optimizing the 2D fins of a HX: each voxel can represent either solid or liquid, and can be iteratively switched during the optimization process. Moreover, Shimoyama and Komiya [131] suggested a new explicit parametrization by representing the 3D lattice-structured heat sink using a point/edge system. However, up to date, there has been no published research that used the direct explicit method in the TO for complex HX problems.

3. Heat transfer modeling

The conjugate heat transfer in HXs is characterized by four equations, i.e., continuity eq. (4a), (b) momentum eq. (4b), and energy balance eq. (4c) for fluids, (4d) for solids,

$$\frac{\partial \rho_f}{\partial t} + \nabla . (\rho_f v) = 0 \qquad (a)$$

$$\rho_f \left(\frac{\partial v}{\partial t} + v . \nabla v \right) = -\nabla P + \mu_f . \nabla^2 v + f \qquad (b)$$

$$\rho_f C_{pf} \frac{\partial T}{\partial t} + \rho_f C_{pf} v . \nabla T = \nabla . (k_f \nabla T) + S \qquad (c)$$

$$\rho_s C_{ps} \frac{\partial T}{\partial t} = \nabla . (k_s \nabla T) + S \qquad (d)$$

(4)

where C_{pf} and C_{ps} are the specific heat at constant pressure for fluid and solid phases

respectively (J.kg^{-1}.K^{-1}), k_s and k_f are the thermal conductivities for solid and fluid

phase respectively (W.m^{-1}.K^{-1}), v the velocity (m.s^{-1}), T the temperature (K), P the

pressure (Pa), μ_f the fluid dynamic viscosity (Pa.s), ρ_f and ρ_s are the fluid and

solid phases densities (kg.m^{-3}), t is the time (s), S is the heat source term (W) and

f is the fictious force equal to -αv, where α is the inverse permeability. This fictious

force is an indispensable term specifically in density-based TO which represents the

solid phase force on the fluid phase. Nevertheless, this fictious term is infrequently used

in level set TO; in the direct explicit case, it is not needed. In the TO, the governing

equations should be solved at each iteration to compute the objective function values.

Apparently, the solver efficiency and accuracy will greatly affect the performance of

TO. Furthermore, these numerical solvers encountered some difficulties to correctly

and efficiently simulate turbulent flows which are usually described by the velocity,

pressure chaotic changes and unsteady eddies. In decades, several solvers have been

developed to solve Eq. (4), as given in Tab. 2. In the majority of HXs applications, some

acceptable simplifications and assumptions are made to simplify the numerical

modeling e.g., steady-state, temperature independent thermophysical properties for

fluid and solid phases, incompressible flows, etc. Additionally, some rare exceptional

studies dealt with some more complicated conditions, e.g. temperature dependent

thermophysical properties [60].

Table 2: Summary and comparison of the solvers in TO of HXs

Methods	Advantages	Disadvantages
FEM	• High availability in TO; • Flexible with a wide range of physics.	• Not ensuring the conservation law locally; • Numerical instabilities for convection.
FVM	• Ensuring the conservation law locally; • Suitable for CFD problems.	• Relatively low availability in TO; • Tough to design higher order schemes with high accuracy; • High requirement of mesh quality especially for complex geometries.
XFEM	• Well capturing interfaces.	• Very low availability in TO; • Not ensuring the conservation law locally.
LBM	• Ensuring the conservation law locally; • Able to consider the size effects at microscale; • Easy-meshing.	• Very low availability in TO; • Difficulties in handling the multiphase flow, compressibility and 3D extension; • Memory intensive.

3.1 Finite element method (FEM)

The FEM is one of the well-developed techniques for solving partial differential

equations. It was first proposed by Hrennikoff [132] and McHenry [133] on structural

problems. FEM consists of discretizing the domains into small domains called "finite

elements" to transform a continuous problem into a discrete one. Thereafter, the

governing equations are integrated over each element by the weighted residual methods

[134], e.g., Galerkin method. The elemental matrices are then formulated and assembled into the global discretized system of equations that enable to calculate the unknown variables at each node. FEM has proved its high flexibility of being applied to a wide range of physics with highly accurate results [135].

In the TO of HXs, the solvers based on the FEM have been extensively used for the steady-state laminar flow in both 2D and 3D cases [21],[32]–[37], [44],[46],[48]–[55], [59]–[62], [64]–[78], [80], [82]–[84], [86], [89], [91], [94]–[100], [103]–[105], [116]–[119], [121], [122], [136]–[138]. As for the single-flow HXs, for instance, Dede et al. [46],[89] used the FEM solver in the TO of a liquid cooled heat sink, and Matsumori et al. [32] optimized the channels of a HX using the FEM-integrated TO. In the multi-flow HX cases, the FEM was adopted in the TO by Papazoglou [21]. Sun et al. [36] executed the TO on a fin and tube HX using the FEM-based COMSOL Multiphysics software. Different from the preceding references, under laminar transient conditions, Zeng et al. [56] performed a TO on a 3D heat sink using a finite element solver. On the other hand, a few researches on the TO of HXs involving the turbulent flow have also been conducted mainly using the FEM to solve the RANS (Reynolds averaged Navier-Stokes) equations [40],[61],[85],[87],[124]. For example, Zhao et al. [87] adopted the Darcy-flow and RANS models for the TO of cooling channels problems under steady state conditions: the FEM-based commercial software (COMSOL Multiphysics) was employed to simulate the turbulent flow in the channels.

The combination of density-based method and FEM is the most convenient transfer from the TO of structural mechanics to that of conjugated heat transfer. In decades, a

series of algorithms and codes have been developed, and recently the TO module has even be integrated in the FEM-based commercial software. Owing to such high availability, the FEM is currently the mostly-used solver in the TO of HXs, as illustrated in Fig. 3. However, in the FEM, the conservation law is not well guaranteed locally for each finite element [139]. This may lead to the numerical instabilities of the conjugate heat transfer problems [140], which can largely affect the performance of the TO of HXs.

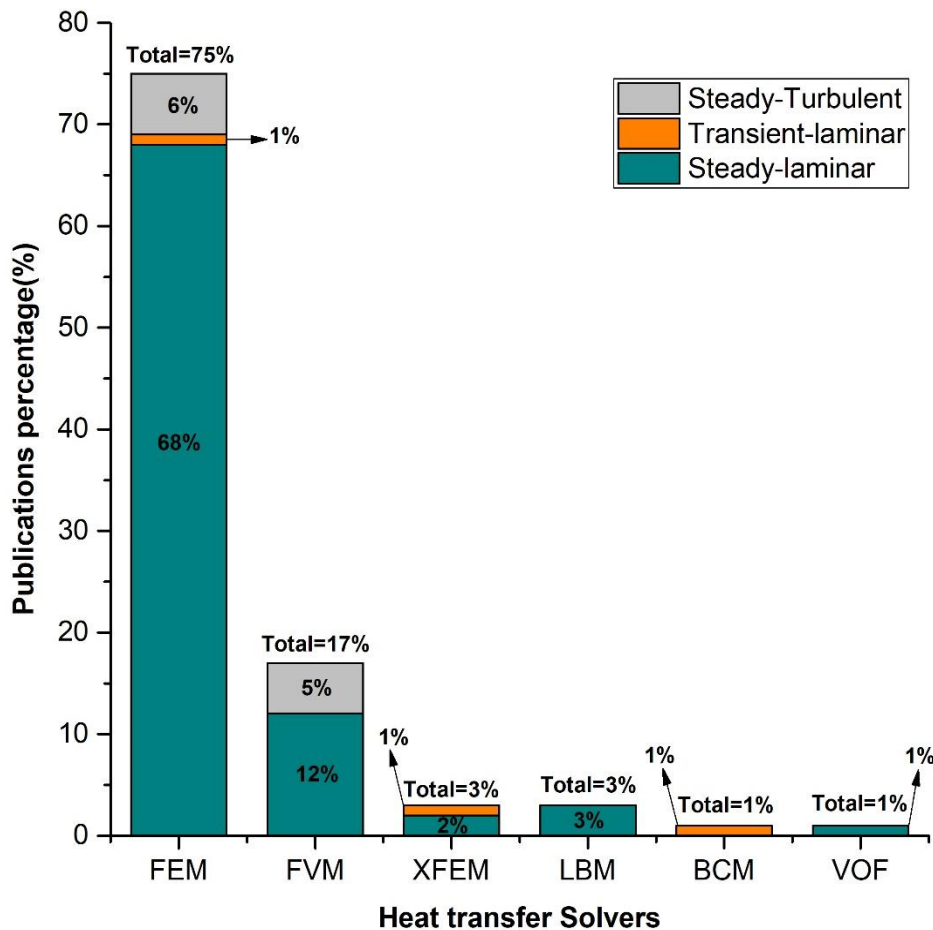

Figure 3:Publications percentages for different solvers used in the TO of HXs under different conditions (until 1-March-2022)

3.2 Finite volume method (FVM)

The FVM is discretizing the design domain into a group of control-volumes (CVs)

by directly integrating the governing equations over each CV, and the divergence theorem is applied to transform the CV integration into boundaries summation over each CV [141]–[143]. It has shown its robustness and stability in CFD (computational fluid dynamics) problems [144].

FVM was first used in the TO of heat conduction by Gersborg-Hansen et al. [145]. Then, it was implemented in a TO algorithm for the steady-state laminar flow HXs [38],[43],[45],[58],[63],[81],[90],[101],[102],[130],[131]. Tawk et al. [102] optimized both parallel and counter-flow HXs using the FVM-based TO for thermo-hydraulic enhancement purposes. Recently, the open source library OpenFoam based on FVM becomes popular to solve the flow problems. It has also been applied for TO of HXs [45],[130] on a few cases.

As for the HXs involving turbulent flow, Kontoleontos et al. [88] used the FVM to solve the Spalart-Allmaras turbulence model in the TO of a thermal-fluid problem. In the same idea, Dilgen et al. [42] studied the turbulence effect on a heat sink using the k-w model at steady state conditions. With the intention of studying the turbulence effects inside a square tube HX, the FVM was applied by Pietropaoli et al. [93] to solve the RANS equations in the TO; then, they carried out a detached eddy simulation (DES) to evaluate the thermal performance of optimized structure. Ghosh et al. [79] used a FVM-based software (OpenFoam) to model the turbulent flow inside a cooling duct.

The FVM ensures the conservativeness over every CV [143], which makes it a good option for CFD problems. In fact, the majority of authors that used the FEM-based TO mentioned in Subsection 3.1 performed the CFD analysis using FVM solvers

to evaluate the performance of the TO-derived structures, which underscores the advantages of the FVM over the FEM in CFD applications. However, the portion of the FVM-based TO of HXs to date happens to be rather small, as shown in Fig. 3. The low availability of the FVM-based TO programs may be the major reason for that. Moreover, the optimized results in the FVM-based TO can be mesh dependent, without integrating a proper filter [27]; it becomes difficult to design high order schemes that obtain a good accuracy using the FVM [146].

3.3 Extended finite element method (XFEM)

The XFEM extends the approach of the FEM by adding enrichment degrees of freedom on the nodes near the discontinuities to improve the description of discontinuities [147],[148]. The XFEM was first used on the 2D cracks by Belytschko and Black [149] to study the crack propagation and interfaces.

As a very valuable attempt, Coffin and Maute [120] combined the XFEM and the LSM in the TO for the 2D and 3D, steady-state and transient single-flow heat transfer problems dominated by natural convection. Recently, Lin et al. [123] performed a topology optimization using the LSM-XFEM coupling to optimize the channel topology for a 2D heat sink under steady-state conditions. Thanks to the features of XFEM and LSM, the interface is well captured during the iterative optimization process, while the computational burden increases at the same time. In the XFEM-based TO, the description of the interface can be improved, however adding new degrees of freedom at the nodes near the interfaces induces a high algorithmic complexity that strongly

increases the computational time. As demonstrated by Fig. 3, XFEM was scarcely used as a numerical solver for the TO of HXs. The same as the FEM, some instability problems are encountered due to the deficiency of conservative fluxes at each element [139]. Additionally, particularly for transient problems due to the rapid change of the physical properties (like temperature jumps near the interface), small time steps are required to capture it when using the XFEM [150].

3.4 Lattice Boltzmann method (LBM)

The LBM is a mesoscale method used for solving transport governing equations described at the macroscopic scale [151],[152]. A set of Boltzmann transport equations are designed to correspond to the macroscopic governing equations, and then are solved in the representation of lattice gas.

The LBM is a relatively young technique compared to the FEM and the FVM. There are few researchers attempted to integrate the LBM in the TO for HX problems [39],[57],[115]. Yaji et al. [57] implemented a TO based on LBM to optimize the flow channels topology of a 2D thermo-fluid problem. In 2018, the LBM was adopted by Dugast et al. [115] for the TO of a 2D thermal fluid problem.

The LBM has showed its robustness and accuracy in the heat and mass transfer problems, particularly in the micro-scale cases where the size effects become significant [153]. It ensures the local conservation law, and has the advantages when dealing with the problems of complicated interfaces and size effects at the microscale. In this sense, the LBM-based TO may be promising for the multiscale HXs. However,

the LBM has difficulties in handling with the multiphase flow, compressibility and 3D extension. Moreover, due to the iterative propagation step, the LBM is a memory-intensive method [154]. Importantly, the integration of LBM with TO is still at a starting stage, which is not enough mature for practical applications.

4. Optimization

After computing the objective function(s) using the heat transfer solvers, an optimization process is conducted to renew the design variables (defined in the parametrization stage), in order to minimize or maximize the objective function(s) under specific constraints. The objective functions serve as the optimization criteria and may influence the final topologies. Regarding the thermal performance, there have been at least 10 different objective functions among the literature of TO, including minimizing average temperature rise, minimizing thermal compliance, minimizing thermal resistance, maximizing exchanged heat, and maximizing recoverable thermal power. Optimization criteria of HXs where the subject of long discussions in the community of heat transfer [11],[13],[155]. For instance, in the view of thermodynamics, minimizing the exergy destruction can also be an objective function of heat transfer optimization. However, to our best knowledge, there has been no research that carefully investigates the influence of objective functions on the TO of HXs up to date. Additionally, the hydraulic performance of HXs can serve as either the constraint or one of the objective functions. Regarding the hydraulic performance, the choices of optimization criteria (or constraints) are not that diverse: minimizing the

pressure drop and the energy dissipation (loss) of flow are the common ones. In addition, as illustrated by [Fig. 4b], the majority of the researchers dealt with single objective functions. Some of these studies take an advantage to enhance the thermo-hydraulic performance simultaneously by dealing with a single objective function and setting the other objective as an optimization constraint. On the other hand, other researchers employed the weighted sum or the true multi-objective optimization for the same purpose of intensifying the heat transfer and improving the hydraulic performance concurrently of HXs. The optimizer, which is the core part of TO algorithm, determines the evolution of design domains and thus the final output result by the TO. [Table 3] lists some commonly-used optimizers.

Table 3: Summary and comparison of the optimizers in TO of HXs

Optimizers	Advantages	Disadvantages
Gradient-based	• Mostly efficient for the large-design-variable-number problems; • High availability in TO.	• Deficiency in multi-objective problems; • Local optima.
GA (Genetic Algorithm)	• Gradient-free; • Global optima; • Efficient in multi-objective problems.	• Slow convergence; • Randomness.
Bayesian	• Efficient in big data problems.	• Expensive and complex computation; • Scalability weakness with the number of objective function evaluations.
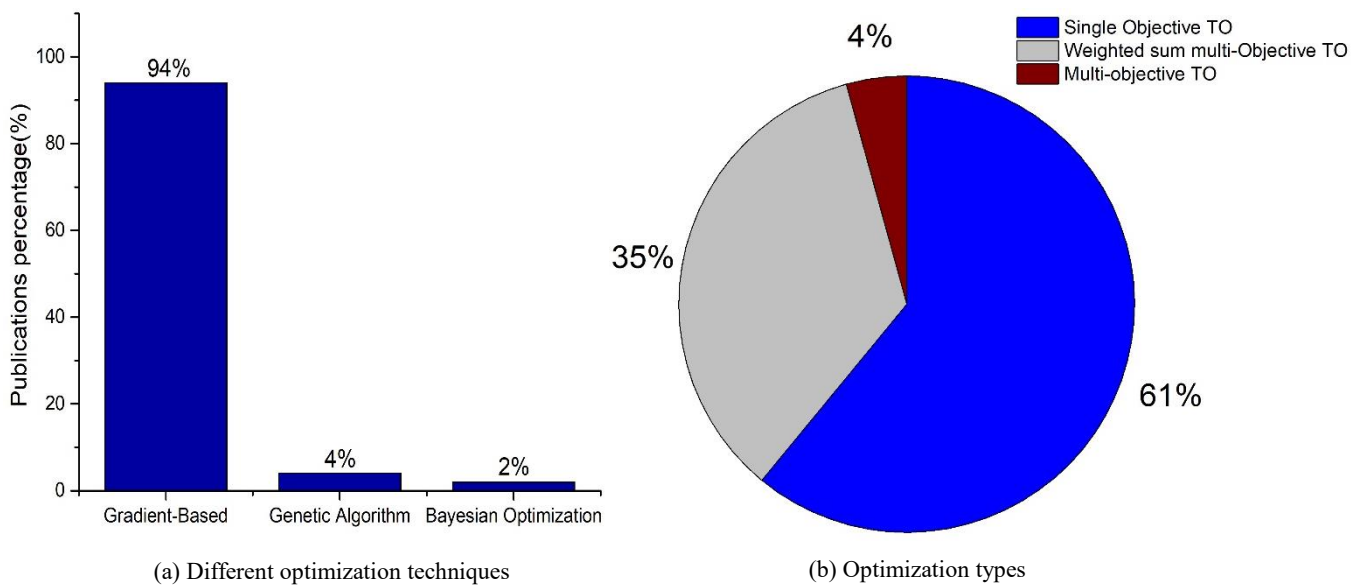
Figure 4: Publications percentages for different optimization techniques in the TO of heat exchangers (until 1-March-2022): (a) Different optimization techniques ; (b) Optimization types.

4.1 Gradient-based optimization

The gradient-based method also called "sensitivity analysis" is based on computing the gradients of the objective functions with respect to the design variables. These gradients represent the variation of the objective function with respect to the design variables at each iteration and are often solved using the adjoint method [156]. The adjoint method has shown its high efficiency in computing the objective function gradients [157]. The optimizer renews the design variables based on these gradient values.
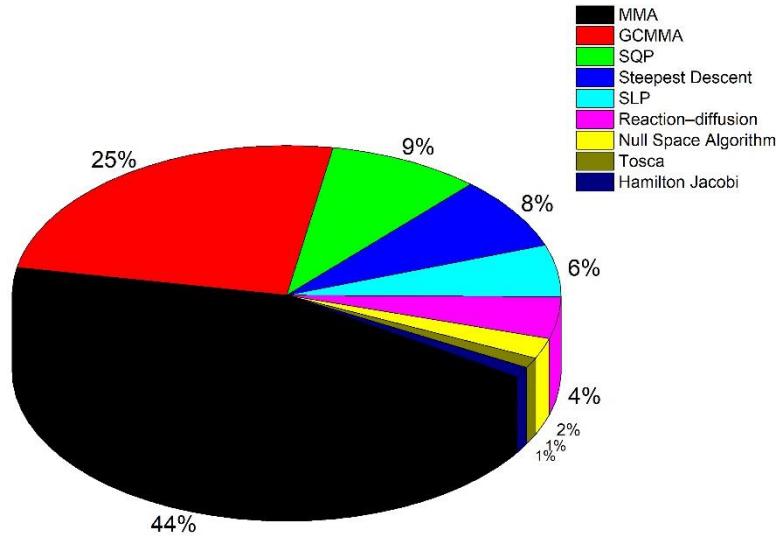
Figure 5: Publications percentages of the optimizers used in gradient-based TO of heat exchangers (until 1-March-2022)

As illustrated in Fig. 5, several gradient-based optimizers are utilized in the TO of HXs, including MMA (Method of Moving Asymptotes) [158](44%), GCMMA (Globally convergent MMA) [159](25%), SLP (Sequential linear programming) [160](6%), SQP (Sequential quadratic programming) [161](9%), Steepest descent [162](8%), Tosca [163](1%), Reaction-diffusion [164](4%), Hamilton-Jacobi [96](1%) optimizers, and Null Space algorithm [165](2%). These gradient- based optimizers hold the different mathematical natures and thus the distinct applications. The detailed explanation on their mathematical characteristics are beyond the scope of our review, and can be found in the relevant references.

The utilization of gradient-based optimizers is the mainstream in the TO not limited to the problems of HXs [21],[32]–[105],[115]–[124],[136],[137]. According to Fig. 4a, more than 90% of papers on the TO of HXs utilize the gradient-based optimizers. This is mainly because of its good efficiency in handling problems involving such large number of design variables (usually equal to the number of nodes in the solver) [166].

Recently, some gradient-based multi-objective algorithms have been developed, like MOSQP (Multi-objective SQP) [167], which has been employed in the structural TO [99]; however, up to date there has been no published work on the TO of HXs using such algorithm. Additionally, the gradient-based optimization can converge to a local optimum when the objective function has several local optimums [134], and thus re-optimization is needed by setting different initial configurations.

4.2 Genetic algorithm (GA)

The GA is a stochastic evolutionary algorithm (EA) based on the biology of chromosomes and genes [168]. This evolutionary algorithm obtains the optimized solution(s) after several generations. Each generation starts by generating the initial population randomly to increase its diversity. Then, the fitness values are evaluated for each chromosome in the population using fitness function(s), i.e. objective function(s). The parent chromosomes are selected from the initial population using natural selection processes, e.g. roulette wheel [169]. The children are then obtained by the combination of two parents using crossover [170]. Thereafter, the mutation process based on randomness is applied on the children to mutate one or more of their genes before moving to the next generation. Finally, the elitism stage [171] moves one chromosome to the next generation without being edited by the crossover and mutation.

The GA has been developed by many researchers in different fields including heat transfer [172],[173]. As for the TO of HXs, few researchers implemented the GA for generating optimized topologies [61],[114],[130],[131]. Yaji et al. [61] proposed a

multi-fidelity TO for a heat sink using EA main stages (selection, crossover, mutation). They first performed a low fidelity optimization problem based on Darcy flow model using ε-constrained method [174]. According to low fidelity results, a high-fidelity evaluation was executed using Navier–Stokes equations. Then, a non-dominated sorting strategy (NSGA II) was employed to select the optimal pareto front.

The GA method avoids the gradient computation of the objective function (s) at each iteration. In theory, it will obtain the global optima, and screen the influence from the initial guess [172]. Moreover, the GA is a good option for the multi-objective problems, since it handles a group of candidates simultaneously, which is of advantage to derive the Pareto front [23]. Having similar stochastic behaviors, other evolutionary algorithms (e.g., Particle Swarm Optimization) could also be tested with the TO of HXs which may possess more efficiency than the GA in some cases. Despite the merits of GA, it has been rarely coupled with the TO of HXs, as shown in Fig. 4a. This is mainly attributed to the slow convergence of GA [172], which significantly increases the computational time of the TO.

4.3 Bayesian optimization (BO)

The BO is an optimization technique based on machine learning concept. It initially rose thanks to the work by Kushner [175], Zhilinskas [176] and Mockus [177]. Then it was popularized after the paper by Jones et al. [178]. The BO is composed of two main parts: statistical modeling and acquisition function. In the Bayesian statistical modeling section, a random set is initially generated. After that, the mean vector and the

covariance matrix are calculated based on Gaussian process (GP) regression for the whole set. The acquisition function is then calculated and its optimum value is used to optimize the objective function for the next step (more details referring to section 4 in Ref. [179]).

The BO is a sequential optimization method that solves tasks in a sequence way. Due to its high data efficiency structure, the BO has shown its robustness in the big data applications [180]. Some structural TO problems have been studied by integrating the BO; for example, Lynch et al. [181] investigated a simple structural TO problem (i.e., minimizing the compliance of a 2D beam) to show the possibility of integrating BO in the TO for HXs. In fact, the concept of BO was also employed by Yoshimura et al. [114] and Shimoyama and Komiya [131] to handle HX problems. Both references built a Kriging surrogate model [182] to efficiently evaluate an approximated objective function which will emphatically diminishes the computational time. Apparently, the utilization of BO in the TO of HXs is very limited, and even less than that of GA up to date. The expensive and complex computation of the acquisition function optimization procedure at each iteration [183] may be a reason. Indeed, another disadvantage of BO is the scalability weakness which is represented by the asymptotically increase of the computational time when evaluating the objective function for a new sampling point or when computing the objective function derivatives [184].

5. Final Realization

The optimized complex structures obtained by the TO are generally difficult to be

fabricated using the conventional manufacturing (CM) techniques. Therefore, additive manufacturing (AM) techniques also called as 3D printing have been applied to manufacture those very complex structures [25],[185]. Table 4 compares CM and AM for fabricating the TO-derived structures of HXs. AM is an additive technique that build the structure by adding layers, while the CM techniques are subtractive, which remove material from the structure. Generally, the AM can remove the fabrication shackles of the CMs, but the equipment and materials of metallic AM are still very expensive currently [186]. Importantly, due to the restrictions of AM accuracy, some constraints, including length scale, connectivity, and overhang constraints, etc., should be subjected to the optimized structures obtained by the TO [24]. Those constraints are critically essential to eliminate the un-manufacturable features of the optimized structures. More details of those constraints can be found in Ref.[25].

Table 4: Comparison between AM and CM for TO of HXs

Techniques	Advantages	Disadvantages
AM	• Manufacturing ability for complex geometries; • High manufacturability efficiency for complex geometries.	• Relative limited understanding on the manufacturing constraints on the TO optimized structures; • Expensive equipment and materials; • Limited to prototype fabrication; • Limited choices of materials.
CM	• High availability; • High productivity; • Cheap equipment compared to AM.	• Limited manufacturability for TO-derived structures; • Slow fabrication and repairing process for complex TO-derived structures

As for the area of HXs optimization, referring to Fig.6, only few researchers (about 18%) have manufactured the optimized structures obtained by the TO and tested them

in practice. As a tradeoff, some researchers realized and tested the engineering simplified version of TO-resulted geometry due to the fabrication difficulty, the advantages of TO being partially or totally lost [58].
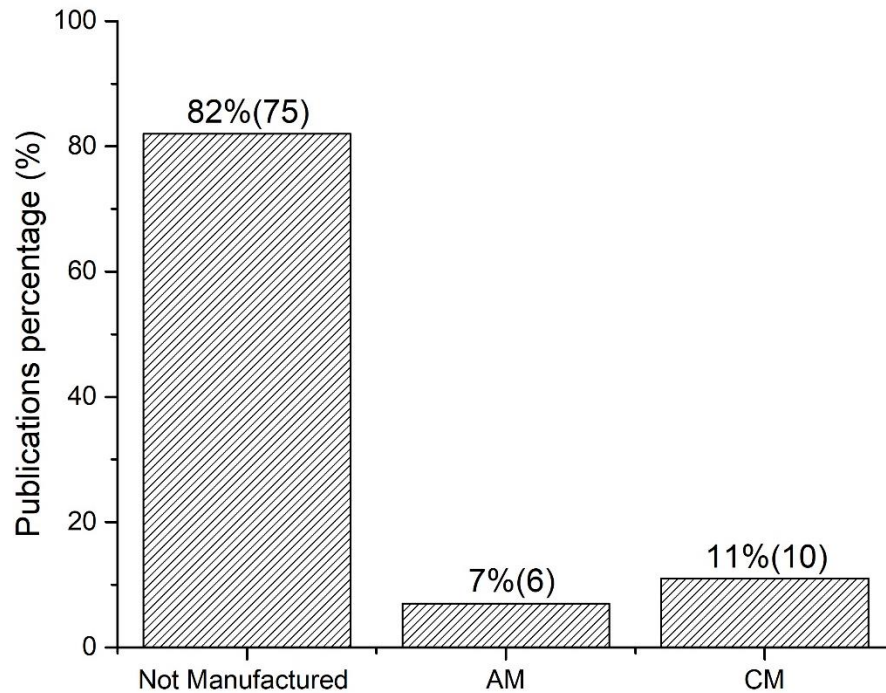
Figure 6: Publication statistics for final realization statuses of optimized HXs (until 1-March-2022). Percentage (Numbers) of publications

Using AM techniques, some researchers fabricated the TO-optimized HXs to validate their numerical results [41],[54],[68],[83],[99],[137]. For example, Lei et al. [68] manufactured the optimized structure of a passive HX by the TO using 3D stereolithography (SLA) printing technique assisted with investment casting process. On the other hand, the CM methods [33],[44],[49],[51],[58]–[60],[74],[89],[124], have also been utilized to fabricate some HXs obtained by the TO (mainly the 2D topologies, such as the 2D heat sinks). For instance, as referred by Koga et al. [33], the electrical discharge machining (EDM) was used to manufacture the optimized structure of a heat

sink with the help of CNC (computer numerical control) milling. Figures 7(a) and (b) illustrate the heat sinks fabricated by the AM [68] and the CM [51] techniques, respectively. Apparently, the AM method can attain more complex structures especially in the 3D case. Note that since the majority of current researches on the TO of HXs does not conduct the final realization of designed structures, there have been rare discussions on the fabricating constraints on the TO-optimized HXs, which should be improved in the further work. One recent paper mentions precisely the integration between AM and TO and the implementation of the overhang constraint in the TO for a fluidic problem [187].
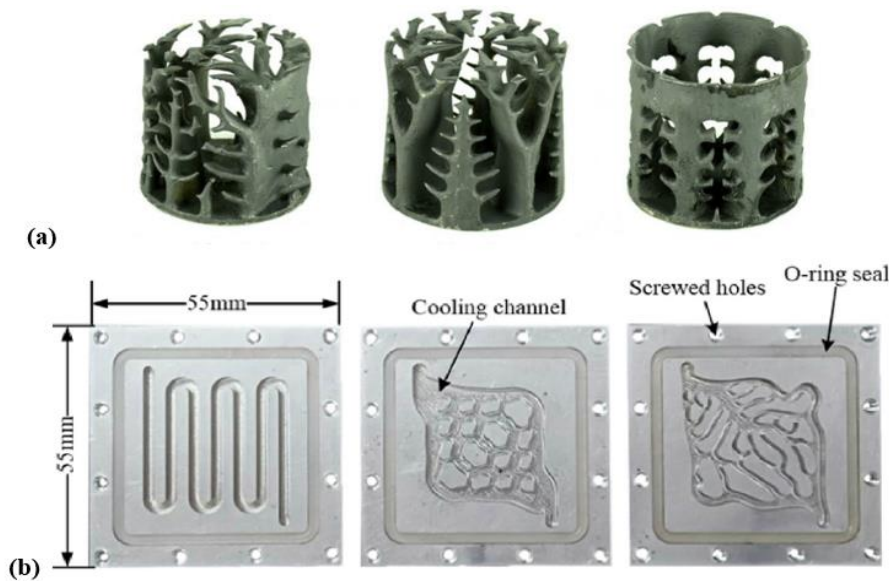
Figure 7: (a) Heat sinks manufactured using the AM technique (the SLA printing assisted with investment casting) [68]; (b) Heat sink channels manufactured using the CM technique (the CNC) [51].

Furthermore, in order to give a complete comparison among the existing literature, the papers on the TO of HXs analyzed in the sections above, i.e. Design parametrization, Heat transfer modeling, Optimization, and Final realization, are summarized in Tab. 5.

Table 5: Summary of researches on TO of HXs analyzed in the sections above.

No.	Reference	Year	Parametrization	Solver	Objective Function	Optimizer	Final Realization
1	Dede [46]	2009	Density (SF1)[1]	FEM	Min (Mean Temperature & Energy Dissipation)	Gradient (MMA)	Not [4]
2	Yoon [95]	2010	Density (SF2)[2]	FEM	Min (Thermal Compliance)	Gradient (MMA)	Not
3	Dede [89]	2012	Density (SF2)	FEM	Min (Mean Temperature & Energy Dissipation)	Gradient (MMA)	CM (N/A)
4	Kontoleontos et al. [88]	2012	Density (SF2)	FVM	Min (Pressure Drop) & Max (Temperature Difference)	Gradient(S-D)	Not
5	Matsumori et al. [32]	2013	Density (SF1)	FEM	Max (Heat Generation)	Gradient (SQP)	Not
6	Marck et al. [63]	2013	Density (SF1)	FVM	Min (Pressure Drop) & Max (The Recoverable Thermal Power)	Gradient (MMA)	Not
7	Koga et al. [33]	2013	Density (SF1)	FEM	Min (Pressure Drop) & Max (Dissipated Heat)	Gradient (SLP)	CM (EDM, CNC)
8	Oevelen et al. [43]	2014	Density (SF1)	FVM	Min (Thermal Resistance)	Gradient (MMA)	Not
9	Alexandersen et al. [64]	2014	Density (SF1)	FEM	Min (Thermal Compliance)	Gradient (MMA)	Not
10	Yaji et al. [119]	2015	LSM	FEM	Max (Heat Generation)	Gradient (R-D)	Not
11	Papazoglou [21]	2015	Density (Multi-flow)	FEM	Max (Exchanged Heat)	Gradient (MMA)	Not
12	Yaji et al. [57]	2015	Density (SF1)	LBM	Min (Pressure Drop) & Max (Exchanged Heat)	Gradient (MMA)	Not
13	Coffin and Maute [120]	2015	LSM	XFEM	Min (Average Temperature)	Gradient (GCMMA)	Not
14	Łaniewski-Wołłk et al. [39]	2016	Density (SF1)	LBM	Max (Exchanged Heat)	Gradient (MMA)	Not
15	Qian and Dede [65]	2016	Density (SF1)	FEM	Min (Average Temperature & Dissipation Energy)	Gradient (MMA)	Not
16	Zhou et al. [136]	2016	Parametrization of [188]	FEM	Max (Reaction Flux)	Gradient (TOSCA)	Not
17	Alexendersen et al. [66]	2016	Density (SF1)	FEM	Min (Thermal Compliance)	Gradient (MMA)	Not
18	Li et al. [138]	2016	Density (N/A[3])	FEM	Min (Heat Potential Capacity)	N/A	Not
19	Yoshimura et al. [114]	2017	LSM	BCM	Min (Pressure Drop) & Max (Bulk Mean Temperature)	GA & BO	Not
20	Haertel and Nellis [34]	2017	Density (SF1)	FEM	Max (Thermal Conductance)	Gradient (GCMMA)	Not
21	Zhao et al. [87]	2017	Density (SF2)	FEM	Min (Mean Temperature)	Gradient (MMA)	Not
22	Qian et al. [67]	2017	Density (SF1)	FEM	Min (RMS Temperature & Energy Dissipation)	Gradient (MMA)	Not

No.	Reference	Year	Parametrization	Solver	Objective Function	Optimizer	Final Realization
23	Sato et al. [121]	2018	LSM	FEM	Max (Heat Generation) & Min (Energy Dissipation)	Gradient (R-D)	Not
24	Haertel et al. [35]	2018	Density (SF1)	FEM	Min (Thermal Resistance)	Gradient (GCMMA)	Not
25	Zeng et al. [44]	2018	Density (SF1)	FEM	Min (Pressure Drop)	Gradient (GCMMA)	CM (CNC)
26	Dilgen et al. [42]	2018	Density (SF1)	FVM	Min (Average Temperature)	Gradient (MMA)	Not
27	Dugast et al. [115]	2018	LSM	LBM	Min (Mean Temperature) & Max (Exchanged Heat)	Gradient(S-D)	Not
28	Ramalingom et al. [90]	2018	Density (SF2)	FVM	Min (Pressure Drop) & Max (Recoverable Thermal Power)	Gradient(S-D)	Not
29	Santhanakrishnan et al. [96]	2018	Density (SF2), LSM	FEM	Min (Thermal Compliance)	Gradient (MMA, H-J)	Not
30	Lei et al. [68]	2018	Density (SF1)	FEM	Min (Thermal Compliance)	Gradient (MMA)	AM (SLA)
31	Sun et al. [36]	2018	Density (SF1)	FEM	Min (Pressure Drop)	Gradient (GCMMA)	Not
32	Lurie et al. [97]	2018	Density (SF2)	FEM	Min (Pressure Drop & Energy Dissipation)	Gradient (MMA)	Not
33	Saglietti et al. [37]	2018	Density (SF1)	FEM	Max (Exchanged Heat)	Gradient (MMA)	Not
34	Pietropaoli et al. [92]	2018	Density (SF2)	VOF	Min (Stagnation Pressure Drop) & Max (Temperature Gain)	Gradient(S-D)	Not
35	Makhija and Beran [86]	2018	Density (SF2)	FEM	Min (Average Temperature)	Gradient (GCMMA)	Not
36	Lv and Liu [47]	2018	Density (SF1)	N/A	Max (Heat Dissipation) & Min (Energy Dissipation)	Gradient (MMA)	Not
37	Subramaniam et al. [38]	2019	Density (SF1)	FVM	Min (Pressure Drop) Max (Recoverable Thermal Power)	Gradient (MMA)	Not
38	Saviers et al. [137]	2019	N/A	FEM	Max (Exchanged Heat)	Gradient (GCMMA)	AM (SLA)
39	Yu et al. [69]	2019	Density (SF1)	FEM	Min (Thermal Compliance & Energy Dissipation)	Gradient (MMA)	Not
40	Asmussen et al. [91]	2019	Density (SF2)	FEM	Min (Thermal Compliance)	Gradient (MMA)	Not

No.	Reference	Year	Parametrization	Solver	Objective Function	Optimizer	Final Realization
41	Zhang and Gao [52]	2019	Density (SF1)	FEM	Max (Heat Generation)	Gradient (MMA)	Not
42	Jahan et al. [98]	2019	Density (SF2)	FEM	Min (Thermal Compliance)	Gradient (MMA)	Not
43	Kobayashi et al. [48]	2019	Density (SF1)	FEM	Max (Heat Extraction)	Gradient (SLP)	Not
44	Tawk et al. [102]	2019	Density (Multi-flow)	FVM	Min (Pressure Drop) & Max (Exchanged Heat)	Gradient (MMA)	Not
45	Zeng and Lee [49]	2019	Density (SF1)	FEM	Min (Pressure Drop)	Gradient (GCMMA)	CM (CNC)
46	Yan et al. [50]	2019	Density (SF1)	FEM	Min (Maximum Temperature)	Gradient (MMA)	Not
47	Li et al. [59]	2019	Density (SF1)	FEM	Min (Pressure Drop) & Max (Exchanged Heat)	Gradient (SQP)	CM (CNC)
48	Li et al. [51]	2019	Density (SF1)	FEM	Min (Dissipation Energy) & Max (Exchanged Heat)	Gradient (SQP)	CM (CNC, Milling)
49	Dong and Liu [70]	2019	Density (SF1)	FEM	Min (Thermal Resistance & Pressure Drop & Energy Dissipation)	Gradient (SQP)	Not
50	Ghosh and Kapat [45]	2019	Density (SF1)	FVM	Min (Pressure Drop) & Max (Temperature Rise)	Gradient(S-D)	Not
51	Hu et al. [71]	2019	Density (SF1)	FEM	Min (Mean Temperature & Energy Dissipation)	Gradient (GCMMA)	Not
52	Kambampati et al. [116]	2020	LSM	FEM	Min (Thermal Compliance)	Gradient (SLP)	Not
53	Zhang et al. [53]	2020	Density (SF1)	FEM	Max (Exchanged Heat)	Gradient (GCMMA)	Not
54	Zeng et al. [56]	2020	Density (SF1)	FEM	Min (Average Temperature)	Gradient (GCMMA)	Not
55	Sun et al. [72]	2020	Density (SF1)	FEM	Min (Average Temperature)	Gradient (MMA)	Not
56	Zhang et al. [73]	2020	Density (SF1)	FEM	Min (Average Temperature)	Gradient (GCMMA)	Not
57	Francisco et al. [99]	2020	Density (SF2)	FEM	Max (Thermal Conductivity)	Gradient (MMA, MOSQP)	AM (N/A)
58	Høghøj et al. [103]	2020	Density (Multi-flow)	FEM	Min (Enthalpy Difference)	Gradient (MMA)	Not
59	Troya et al. [104]	2020	Multi-flow, LSM	FEM	Max (Exchanged Heat)	Gradient (MMA, NSA)	Not

No.	Reference	Year	Parametrization	Solver	Objective Function	Optimizer	Final Realization
60	Lee et al. [100]	2020	Density (SF2)	FEM	Min (Thermal Resistance)	Gradient (MMA)	Not
61	Feppon et al. [117]	2021	LSM	FEM	Max (Exchanged Heat)	Gradient (NSA)	Not
62	Pietropaoli et al. [93]	2021	Density (SF2)	FVM	Min (Stagnation Pressure Drop) & Max (Temperature Rise)	Gradient (S-D)	Not
63	Dong and Liu [62]	2021	Density (SF1)	FEM	Min (Energy Dissipation) & Max (Recoverable Thermal Power)	Gradient (GCMMA)	Not
64	Kobayashi et al. [105]	2021	Density (Multi-flow)	FEM	Max (Exchanged Heat)	Gradient (SLP)	Not
65	Mekki et al. [130]	2021	Explicit	FVM	Max (Exchanged Heat) & Min (Pressure Drop)	GA	Not
66	Lee et al. [58]	2021	Density (SF1)	FVM	Min (Average Temperature & Dissipation Energy)	Gradient (GCMMA)	CM (Laser cutting)
67	Mario et al. [118]	2021	LSM	FEM	Max (Exchanged Heat) & Min (Energy Dissipation)	Gradient (R-D)	Not
68	Zhao et al. [76]	2021	Density (SF1)	FEM	Min (Average Temperature)	Gradient (GCMMA)	Not
69	Zhou et al. [74]	2021	Density (SF1)	FEM	Min (Temperature Difference, Average Temperature, Energy Dissipation)	Gradient (MMA)	CM (Machining)
70	Han et al. [54]	2021	Density (SF1)	FEM	Min (Temperature Difference & Energy Dissipation & Average Temperature)	Gradient (GCMMA)	AM (N/A)
71	Mo et al. [41]	2021	Density (SF1)	N/A	Min (Average Temperature & Energy Dissipation)	Gradient (MMA)	AM (N/A)
72	Ghasemi and Elham [94]	2021	Density (SF2)	FEM	Min (Thermal Resistance & Pressure Drop)	Gradient (GCMMA)	Not
73	Liu et al. [75]	2021	Density (SF1)	FEM	Min (Pumping Power, Mean and Standard Deviation of The Temperature)	Gradient (GCMMA)	Not
74	Liu et al. [55]	2021	Density (SF1)	FEM	Max (Exchanged Heat) & Min (Energy Dissipation)	Gradient (MMA)	Not
75	Zhao et al. [40]	2021	Density (SF1)	FEM	Min (Average Temperature Rise)	Gradient (MMA)	Not
76	Tang et al. [101]	2021	Density (SF2, Multi-flow)	FVM	Min (Mean Temperature & Pressure Drop)	Gradient (MMA)	Not
77	Chen et al. [77]	2021	Density (SF1)	FEM	Max (Heat Generation)	Gradient (SQP)	Not

No.	Reference	Year	Parametrization	Solver	Objective Function	Optimizer	Final Realization
78	Yaji et al. [61]	2021	Density (SF1)	FEM	Max (Exchanged Heat) & Min (Energy Dissipation)	Gradient (SLP), GA	Not
79	Li et al. [124]	2021	LSM	FEM	Min (Average Temperature)	Gradient (MMA)	CM (Milling)
80	Ghosh et al. [79]	2021	Density (SF1)	FVM	Max (Gained energy) & Min (Power lost)	Gradient (S-D)	Not
81	Qian et al. [60]	2021	Density (SF1)	FEM	Min (RMS[5] Temperature & Energy Dissipation)	Gradient (MMA)	CM (CNC)
82	Shimoyama and Komiya [131]	2022	Explicit	FVM	Max (Heat transfer rate) & Min (Material cost)	GA & BO	Not
83	Li et al. [122]	2022	LSM	FEM	Min (Thermal Compliance)	Gradient (R-D)	Not
84	Yu et al. [81]	2022	Density (SF1)	FVM	Min (Maximum Temperature)	Gradient (MMA)	Not
85	Zhou et al. [83]	2022	Density (SF1)	FEM	Min (Energy Dissipation, Average Temperature & Temperature Difference)	Gradient (GCMMA)	AM (N/A)
86	Zou et al. [82]	2022	Density (SF1)	FEM	Min (Average temperature & Pumping power)	Gradient (SQP)	Not
87	Marshall and Lee [84]	2022	Density (SF1)	FEM	Min (Pressure in fin area of the fluid)	Gradient (N/A)	Not
88	Yeranee et al. [85]	2022	Density (SF1)	FEM	Min (Pressure Drop)	Gradient (GCMMA)	Not
89	Lin et al. [123]	2022	LSM	XFEM	Min (Average Temperature)	Gradient (GCMMA)	Not
90	Xie et al. [78]	2022	Density (SF1)	FEM	Min (Pressure Drop)	Gradient (GCMMA)	Not
91	Huang et al. [80]	2022	Density (SF1)	FEM	Min (Average Temperature)	Gradient (SQP)	Not

[1] SF1: single flow with γ=1 fluids, γ=0 solids; [2] SF2: single flow with γ=0 fluids, γ=1 solids; [3] N/A: Not Announced; [4] Not: Not Manufactured; [5] RMS: Root mean square

6. Some New Trends

Here, we will move to discuss some emerging schemes aimed to improve the efficiency (i.e. reducing computational time and memory storage) of TO not limited to the area of HXs. Currently many of these novel schemes are designed for the structural TO problems; nevertheless, it is possible to transfer some of them to deal with the HXs problems.

6.1 Machine learning (ML)

With the rapid development of artificial intelligence (AI) in the recent years, the ML technique (a subset of AI) has become a powerful tool to handle various engineering problems. As for transport phenomena, the ML has exhibited the ability of predicting their solutions [189]–[192], due to its high potential of learning from existing data-sets. Using different strategies, the ML algorithms can be coupled with the density-based TO mainly for the structural problems at the present stage [193]–[196]. As for the TO of HXs, in a recent study, a data driven TO based on EA was suggested by Yaji et al. [61] for a heat sink under forced convection. In this research, a variational autoencoder [197] was implemented to perform the crossover operation by generating a new dataset. The ML proved the ability of increasing the TO efficiency by predicting the optimized structures for heat transfer problems with negligible time [198]. However, currently the research combining the ML and the TO for HXs is still rare, which may be attributed to the complexity of conjugating heat transfer (especially the fluid flow part) and the complicated structures of HXs. It requires more studies to clarify how to integrate the

ML in the TO involving fluids and whether the ML can improve the efficiency.

6.2 Model order reduction (MOR)

The MOR is an approach aiming to decrease the complexity of models. It reduces the full original model into a reduced one by capturing the fundamental characteristics and neglecting the unimportant ones under certain accuracy [199]. The MOR methods were first proposed in 1980s [200]–[202]. Recently, Zhao et al. [87] proposed a poor man's approach to reduce the computational time for the TO of cooling channels: a simplified model was derived by imposing the Darcy flow model on NS equation and neglecting the effect of body force term. In the TO of a heat sink, Asmussen et al. [91] suggested a reduced order model by making some assumptions on the governing equations, which significantly reduce the number of degrees of freedom. Even with a high simplification, the reduced model obtains the subjectively analogous results with the full-based model [203]. Nevertheless, the simplifications are largely dependent on the knowledge of designers, and a poor simplification may degrade the accuracy and even cause severe computational issues represented mainly by unrealistic results and modeling problems. For instance, when the MOR technique is applied to NS equations under unsteady conditions or for turbulent flows, some computational problems frequently emerge [204].

6.3 Moving morphable components (MMC)

The MMC-based TO was originally proposed in 2014 by Guo et al. [205] then enhanced by Zhang et al. [206] for the 2D structural problems. Thereafter, the MMC

method is extended to the 3D TO [207],[208]. It represents the design domain by using several structural components. In the optimization process, the mathematical features described by center coordinates, width, length and inclination angles of these components are updated for achieving an optimized structure. In 2019, Yu et al. [69] proposed a density-based TO for a 2D heat transfer problem using the MMC. To take an advantage over the traditional MMC, Li et al. [124] utilized the quadratic Bézier curve permitting for more movement flexibility of the component. In a recent attempt, Yu et al. [81] suggested a component-based representation of the heat source distribution with various-intensities for the TO of a liquid cooled heat sink. The MMC-based TO can obtain optimized shapes in an explicit way, which can avoid the post-processing problems, like intermediate design variables, and checkboards, etc. [205]. Furthermore, the number of design variables is reduced compared to other implicit techniques, which may decrease the computational cost [206]. However, the design (including its geometry and initial distribution) of the domain components, that significant influences both the efficiency and the accuracy of the MMC-based TO, largely depends on the experience of designers. Moreover, as yet, it has been limited to some 2D HX problems, which may be attributed to the difficulties of explicit methods in describing the solid/fluid interface for 3D complex structures [129].

7. Conclusions and Perspectives

The present paper provides a comprehensive review on the literature of the TO for HXs in the most recent years. Each stage of the TO is analyzed carefully with the

statistical figures and comparison tables. Our review shows that the current TO methods are not powerful enough yet to handle the industrial HXs in energy systems: (a) the majority of the researches only deal with the 2D single-flow problems with single objective, limited to the relatively simple flow patterns (like the laminar flow and the turbulence described by the Reynolds-averaged models); (b) merely a small portion of work has manufactured and tested the TO-obtained structures in practice, and (c) few discussions have been conducted on the fabricating constraints of the TO-obtained HXs. Currently, a combination of the density-based method, the FEM, and the gradient-based optimization is the most popular TO method for HXs, since it is a straight-forward transfer from the structural mechanics to the conjugate heat transfer. However, the conjugate heat transfer holds some very different features compared to the structural mechanics, which may not be well addressed in the framework initially developed for the structural TO. Furthermore, three emerging schemes, i.e. ML, MOR, and MMC, aimed to improve the efficiency of TO are discussed. They are initially designed for the structural TO problems and show good performance in that area. Some of them have been extended to handle some simple heat transfer problems, and there has been limited evidence to prove that they are effective in improving the efficiency of the TO for conjugate heat transfer.

Apparently, future effort is still required for the TO of HXs, particularly to: (a) implement the suitable parametrization schemes that can well address the influence of interfaces in the conjugate heat transfer, (b) utilize solvers able to predict more complex flow problems, (c) develop optimizers able to handle multiple objectives, integrating

the fabrication constraints, and (d) increase the realization of component level TO-optimized HXs, their experimental testing and validation.

Acknowledgements

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] O. P. Arsenyeva, L. L. Tovazhnyansky, P. O. Kapustenko, and G. L. Khavin, "Optimal design of plate-and-frame heat exchangers for efficient heat recovery in process industries," *Energy*, vol. 36, no. 8, pp. 4588–4598, Aug. 2011, doi: 10.1016/j.energy.2011.03.022.

[2] H. Jouhara, S. Almahmoud, A. Chauhan, B. Delpech, G. Bianchi, S. Tassou, R. Llera, F. Lago and J. Arribas, "Experimental and theoretical investigation of a flat heat pipe heat exchanger for waste heat recovery in the steel industry," *Energy*, vol. 141, pp. 1928–1939, Dec. 2017, doi: 10.1016/j.energy.2017.10.142.

[3] H. Jouhara, D. Bertrand, B. Axcell, L. Montorsi, M. Venturelli, S. Almahmoud, M. Milani, L. Ahmad and A. Chauhan, "Investigation on a full-scale heat pipe heat exchanger in the ceramics industry for waste heat recovery," *Energy*, vol. 223, p. 120037, May 2021, doi: 10.1016/j.energy.2021.120037.

[4] Q. Li, G. Flamant, X. Yuan, P. Neveu, and L. Luo, "Compact heat exchangers: A review and future applications for a new generation of high temperature solar receivers," *Renewable and Sustainable Energy Reviews*, vol. 15, no. 9, pp. 4855–4875, Dec. 2011, doi: 10.1016/j.rser.2011.07.066.

[5] L. Luo, B. Hoareau, U. D'Ortona, D. Tondeur, H. L. Gall, and S. Corbel, "Design, Fabrication And Experimental Study Of New Compact Mini Heat-Exchangers," in *Microreaction Technology*, M. Matlosz, W. Ehrfeld, and J. P. Baselt, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2001, pp. 68–69. doi: 10.1007/978-3-642-56763-6_7.

[6] L. Luo, Y. Fan, and D. Tondeur, "Heat exchanger: from micro- to multi-

scale design optimization," *Int. J. Energy Res.*, vol. 31, no. 13, pp. 1266–1274, Oct. 2007, doi: 10.1002/er.1298.

[7] Y. Fan and L. Luo, "Recent Applications of Advances in Microchannel Heat Exchangers and Multi-Scale Design Optimization," *Heat Transfer Engineering*, vol. 29, no. 5, pp. 461–474, May 2008, doi: 10.1080/01457630701850968.

[8] L. Luo, Y. Fan, W. Zhang, X. Yuan, and N. Midoux, "Integration of constructal distributors to a mini crossflow heat exchanger and their assembly configuration optimization," *Chemical Engineering Science*, vol. 62, no. 13, pp. 3605–3619, Jul. 2007, doi: 10.1016/j.ces.2007.02.058.

[9] Y. Li, S. Roux, C. Castelain, L. Luo, and Y. Fan, "Tailoring the fluid flow distribution in a parallel mini-channel heat sink under multiple-peak heat flux," *Thermal Science and Engineering Progress*, vol. 29, p. 101182, Mar. 2022, doi: 10.1016/j.tsep.2021.101182.

[10] L. Luo, Ed., *Heat and Mass Transfer Intensification and Shape Optimization*. London: Springer London, 2013. doi: 10.1007/978-1-4471-4742-8.

[11] D. Tondeur and E. Kvaalen, "Equipartition of entropy production. An optimality criterion for transfer and separation processes," *Ind. Eng. Chem. Res.*, vol. 26, no. 1, pp. 50–56, Jan. 1987, doi: 10.1021/ie00061a010.

[12] A. E. Bergles, "ExHFT for fourth generation heat transfer technology," *Experimental Thermal and Fluid Science*, vol. 26, no. 2–4, pp. 335–344, Jun. 2002, doi: 10.1016/S0894-1777(02)00145-0.

[13] Z. Y. Guo, W. Q. Tao, and R. K. Shah, "The field synergy (coordination) principle and its applications in enhancing single phase convective heat transfer," *International Journal of Heat and Mass Transfer*, vol. 48, no. 9, pp. 1797–1807, Apr. 2005, doi: 10.1016/j.ijheatmasstransfer.2004.11.007.

[14] H. S. Dizaji, S. Pourhedayat, F. Aldawi, H. Moria, A. E. Anqi, and F. Jarad, "Proposing an innovative and explicit economic criterion for all passive heat transfer enhancement techniques of heat exchangers," *Energy*, vol. 239, p. 122271, Jan. 2022, doi: 10.1016/j.energy.2021.122271.

[15] R. Boichot and L. Luo, "A simple Cellular Automaton algorithm to optimise heat transfer in complex configurations," *International Journal of Exergy*, vol. 7, pp. 51–64, 2010.

[16] L. Luo, M. Wei, Y. Fan, and G. Flamant, "Heuristic shape optimization of baffled fluid distributor for uniform flow distribution," *Chemical Engineering Science*, vol. 123, pp. 542–556, Feb. 2015, doi: 10.1016/j.ces.2014.11.051.

[17] Y. Du, C. Hu, C. Yang, H. Wang, and W. Dong, "Size optimization of heat exchanger and thermoeconomic assessment for supercritical CO2 recompression Brayton cycle applied in marine," *Energy*, vol. 239, p. 122306, Jan. 2022, doi: 10.1016/j.energy.2021.122306.

[18] R. Hilbert, G. Janiga, R. Baron, and D. Thévenin, "Multi-objective shape optimization of a heat exchanger using parallel genetic algorithms," *International Journal of Heat and Mass Transfer*, vol. 49, no. 15–16, pp. 2567–2577, Jul. 2006, doi: 10.1016/j.ijheatmasstransfer.2005.12.015.

[19] L. Wang, Y. Fan, and L. Luo, "Lattice Boltzmann method for shape optimization of fluid distributor," *Computers & Fluids*, vol. 94, pp. 49–57, May 2014, doi: 10.1016/j.compfluid.2014.01.034.

[20] L.-Z. Zhang, "Conjugate Heat Transfer in Plate-Fin and Tube Heat Exchangers," in *Conjugate Heat and Mass Transfer in Heat Mass Exchanger Ducts*, Elsevier, 2013, pp. 255–274. doi: 10.1016/B978-0-12-407782-9.00009-5.

[21] P. Papazoglou, "Topology Optimization of Heat Exchangers," Master's thesis, Delft University of Technology, 2015.

[22] R. T. Marler and J. S. Arora, "The weighted sum method for multi-objective optimization: new insights," *Struct Multidisc Optim*, vol. 41, no. 6, pp. 853–862, Jun. 2010, doi: 10.1007/s00158-009-0460-7.

[23] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Computat.*, vol. 6, no. 2, pp. 182–197, Apr. 2002, doi: 10.1109/4235.996017.

[24] M. Mani, P. Witherell, and H. Jee, "Design Rules for Additive Manufacturing: A Categorization," in *Volume 1: 37th Computers and Information in Engineering Conference*, Cleveland, Ohio, USA, Aug. 2017, p. V001T02A035. doi: 10.1115/DETC2017-68446.

[25] J. Zhu, H. Zhou, C. Wang, L. Zhou, S. Yuan, and W. Zhang, "A review of topology optimization for additive manufacturing: Status and challenges," *Chinese Journal of Aeronautics*, vol. 34, no. 1, pp. 91–110, Jan. 2021, doi: 10.1016/j.cja.2020.09.020.

[26] X. Chen, "Topology optimization of microfluidics — A review," *Microchemical Journal*, vol. 127, pp. 52–61, Jul. 2016, doi: 10.1016/j.microc.2016.02.005.

[27] T. Dbouk, "A review about the engineering design of optimal heat transfer systems using topology optimization," *Applied Thermal Engineering*, vol. 112, pp. 841–854, Feb. 2017, doi: 10.1016/j.applthermaleng.2016.10.134.

[28] J. Alexandersen and C. S. Andreasen, "A Review of Topology Optimisation for Fluid-Based Problems," *Fluids*, vol. 5, no. 1, p. 29, Mar. 2020, doi: 10.3390/fluids5010029.

[29] N. van Dijk, M. Langelaar, and F. Keulen, "Critical study of design parameterization in topology optimization; The influence of design parameterization on local minima," in *TU Delft Research Portal*, Lisbon, Portugal, 2010, p. 11. [Online]. Available: https://research.tudelft.nl/en/publications/critical-study-of-design-parameterization-in-topology-optimizatio

[30] M. P. Bendsøe, "Optimal shape design as a material distribution problem," *Structural Optimization*, vol. 1, no. 4, Art. no. 4, Dec. 1989, doi: 10.1007/BF01650949.

[31] T. Borrvall and J. Petersson, "Topology optimization of fluids in Stokes flow," *Int. J. Numer. Meth. Fluids*, vol. 41, no. 1, pp. 77–107, Jan. 2003, doi: 10.1002/fld.426.

[32] T. Matsumori, T. Kondoh, A. Kawamoto, and T. Nomura, "Topology optimization for fluid‑thermal interaction problems under constant input power," *Struct Multidisc Optim*, vol. 47, no. 4, pp. 571–581, Apr. 2013, doi: 10.1007/s00158-013-0887-8.

[33] A. A. Koga, E. C. C. Lopes, H. F. Villa Nova, C. R. de Lima, and E. C. N. Silva, "Development of heat sink device by using topology optimization," *International Journal of Heat and Mass Transfer*, vol. 64, pp. 759–772, Sep. 2013, doi: 10.1016/j.ijheatmasstransfer.2013.05.007.

[34] J. H. K. Haertel and G. F. Nellis, "A fully developed flow thermofluid model for topology optimization of 3D-printed air-cooled heat exchangers," *Applied Thermal Engineering*, vol. 119, pp. 10–24, Jun. 2017, doi: 10.1016/j.applthermaleng.2017.03.030.

[35] J. H. K. Haertel, K. Engelbrecht, B. S. Lazarov, and O. Sigmund, "Topology optimization of a pseudo 3D thermofluid heat sink model," *International Journal of Heat and Mass Transfer*, vol. 121, pp. 1073–1088, Jun. 2018, doi: 10.1016/j.ijheatmasstransfer.2018.01.078.

[36] C. Sun, N. Lewpiriyawong, K. L. Khoo, S. Zeng, and P. S. Lee, "Thermal enhancement of fin and tube heat exchanger with guiding channels and topology optimisation," *International Journal of Heat and Mass Transfer*, vol. 127, pp. 1001–1013, Dec. 2018, doi: 10.1016/j.ijheatmasstransfer.2018.08.093.

[37] C. Saglietti, P. Schlatter, E. Wadbro, M. Berggren, and D. S. Henningson, "Topology optimization of heat sinks in a square differentially heated cavity," *International Journal of Heat and Fluid Flow*, vol. 74, pp. 36–52, Dec. 2018, doi: 10.1016/j.ijheatfluidflow.2018.08.004.

[38] V. Subramaniam, T. Dbouk, and J.-L. Harion, "Topology optimization of conjugate heat transfer systems: A competition between heat transfer enhancement and pressure drop reduction," *International Journal of Heat and Fluid Flow*, vol. 75, pp. 165–184, Feb. 2019, doi: 10.1016/j.ijheatfluidflow.2019.01.002.

[39] Ł. Łaniewski-Wołłk and J. Rokicki, "Adjoint Lattice Boltzmann for topology optimization on multi-GPU architecture," *Computers & Mathematics with Applications*, vol. 71, no. 3, pp. 833–848, Feb. 2016, doi: 10.1016/j.camwa.2015.12.043.

[40] J. Zhao, M. Zhang, Y. Zhu, R. Cheng, and L. Wang, "Topology optimization of turbulent forced convective heat sinks using a multi-layer thermofluid model," *Struct Multidisc Optim*, Sep. 2021, doi: 10.1007/s00158-021-03064-1.

[41] X. Mo, H. Zhi, Y. Xiao, H. Hua, and L. He, "Topology optimization of cooling plates for battery thermal management," *International Journal of Heat and Mass Transfer*, vol. 178, p. 121612, Oct. 2021, doi: 10.1016/j.ijheatmasstransfer.2021.121612.

[42] S. B. Dilgen, C. B. Dilgen, D. R. Fuhrman, O. Sigmund, and B. S. Lazarov, "Density based topology optimization of turbulent flow heat transfer systems," *Struct Multidisc Optim*, vol. 57, no. 5, pp. 1905–1918, May 2018,

doi: 10.1007/s00158-018-1967-6.

[43] T. V. Oevelen and M. Baelmans, "APPLICATION OF TOPOLOGY OPTIMIZATION IN A CONJUGATE HEAT TRANSFER PROBLEM," presented at the International Conference on Engineering and Applied Sciences Optimization, Kos Island, Greece, Jun. 2014.

[44] S. Zeng, B. Kanargi, and P. S. Lee, "Experimental and numerical investigation of a mini channel forced air heat sink designed by topology optimization," *International Journal of Heat and Mass Transfer*, vol. 121, pp. 663–679, Jun. 2018, doi: 10.1016/j.ijheatmasstransfer.2018.01.039.

[45] S. Ghosh and J. S. Kapat, "Topology Optimization of Serpentine Channels for Minimization of Pressure Loss and Maximization of Heat Transfer Performance As Applied for Additive Manufacturing," in *Volume 5B: Heat Transfer*, Phoenix, Arizona, USA, Jun. 2019, p. V05BT21A006. doi: 10.1115/GT2019-91057.

[46] E. M. Dede, "Multiphysics Topology Optimization of Heat Transfer and Fluid Flow Systems," 2009.

[47] Y. Lv and S. Liu, "Topology optimization and heat dissipation performance analysis of a micro-channel heat sink," *Meccanica*, vol. 53, no. 15, pp. 3693–3708, Dec. 2018, doi: 10.1007/s11012-018-0918-z.

[48] H. Kobayashi, K. Yaji, S. Yamasaki, and K. Fujita, "Freeform winglet design of fin-and-tube heat exchangers guided by topology optimization," *Applied Thermal Engineering*, vol. 161, p. 114020, Oct. 2019, doi: 10.1016/j.applthermaleng.2019.114020.

[49] S. Zeng and P. S. Lee, "Topology optimization of liquid-cooled microchannel heat sinks: An experimental and numerical study," *International Journal of Heat and Mass Transfer*, vol. 142, p. 118401, Oct. 2019, doi: 10.1016/j.ijheatmasstransfer.2019.07.051.

[50] S. Yan, F. Wang, J. Hong, and O. Sigmund, "Topology optimization of microchannel heat sinks using a two-layer model," *International Journal of Heat and Mass Transfer*, vol. 143, p. 118462, Nov. 2019, doi: 10.1016/j.ijheatmasstransfer.2019.118462.

[51] H. Li, X. Ding, D. Jing, M. Xiong, and F. Meng, "Experimental and numerical investigation of liquid-cooled heat sinks designed by topology optimization," *International Journal of Thermal Sciences*, vol. 146, p. 106065, Dec. 2019, doi: 10.1016/j.ijthermalsci.2019.106065.

[52] B. Zhang and L. Gao, "Topology optimization of convective heat transfer problems for non-Newtonian fluids," *Struct Multidisc Optim*, vol. 60, no. 5, pp. 1821–1840, Nov. 2019, doi: 10.1007/s00158-019-02296-6.

[53] B. Zhang, J. Zhu, and L. Gao, "Topology optimization design of nanofluid-cooled microchannel heat sink with temperature-dependent fluid properties," *Applied Thermal Engineering*, vol. 176, p. 115354, Jul. 2020, doi: 10.1016/j.applthermaleng.2020.115354.

[54] X. Han, H. Liu, G. Xie, L. Sang, and J. Zhou, "Topology optimization for spider web heat sinks for electronic cooling," *Applied Thermal Engineering*,

vol. 195, p. 117154, Aug. 2021, doi: 10.1016/j.applthermaleng.2021.117154.

[55] A. Liu, G. Wang, D. Wang, X. Peng, and H. Yuan, "Study on the thermal and hydraulic performance of fin-and-tube heat exchanger based on topology optimization," *Applied Thermal Engineering*, vol. 197, p. 117380, Oct. 2021, doi: 10.1016/j.applthermaleng.2021.117380.

[56] T. Zeng, H. Wang, M. Yang, and J. Alexandersen, "Topology optimization of heat sinks for instantaneous chip cooling using a transient pseudo-3D thermofluid model," *International Journal of Heat and Mass Transfer*, vol. 154, p. 119681, Jun. 2020, doi: 10.1016/j.ijheatmasstransfer.2020.119681.

[57] K. Yaji, T. Yamada, M. Yoshino, T. Matsumoto, K. Izui, and S. Nishiwaki, "Topology optimization in thermal-fluid flow using the lattice Boltzmann method," *Journal of Computational Physics*, vol. 307, pp. 355–377, Feb. 2016, doi: 10.1016/j.jcp.2015.12.008.

[58] J. S. Lee, S. Y. Yoon, B. Kim, H. Lee, M. Y. Ha, and J. K. Min, "A topology optimization based design of a liquid-cooled heat sink with cylindrical pin fins having varying pitch," *International Journal of Heat and Mass Transfer*, vol. 172, p. 121172, Jun. 2021, doi: 10.1016/j.ijheatmasstransfer.2021.121172.

[59] H. Li, X. Ding, F. Meng, D. Jing, and M. Xiong, "Optimal design and thermal modelling for liquid-cooled heat sink based on multi-objective topology optimization: An experimental and numerical study," *International Journal of Heat and Mass Transfer*, vol. 144, p. 118638, Dec. 2019, doi: 10.1016/j.ijheatmasstransfer.2019.118638.

[60] S. Qian, S. Lou, C. Ge, W. Wang, X. Tian, and Y. Cai, "The influence of temperature dependent fluid properties on topology optimization of conjugate heat transfer," *International Journal of Thermal Sciences*, vol. 173, p. 107424, Mar. 2022, doi: 10.1016/j.ijthermalsci.2021.107424.

[61] K. Yaji, S. Yamasaki, and K. Fujita, "Data-driven multifidelity topology design using a deep generative model: Application to forced convection heat transfer problems," *Computer Methods in Applied Mechanics and Engineering*, vol. 388, p. 114284, Jan. 2022, doi: 10.1016/j.cma.2021.114284.

[62] X. Dong and X. Liu, "Multi-objective optimization of heat transfer in microchannel for non-Newtonian fluid," *Chemical Engineering Journal*, vol. 412, p. 128594, May 2021, doi: 10.1016/j.cej.2021.128594.

[63] G. Marck, M. Nemer, and J.-L. Harion, "Topology Optimization of Heat and Mass Transfer Problems: Laminar Flow," *Numerical Heat Transfer, Part B: Fundamentals*, vol. 63, no. 6, pp. 508–539, Jun. 2013, doi: 10.1080/10407790.2013.772001.

[64] J. Alexandersen, N. Aage, C. S. Andreasen, and O. Sigmund, "Topology optimisation for natural convection problems: TOPOLOGY OPTIMISATION FOR NATURAL CONVECTION PROBLEMS," *Int. J. Numer. Meth. Fluids*, vol. 76, no. 10, pp. 699–721, Dec. 2014, doi: 10.1002/fld.3954.

[65] X. Qian and E. M. Dede, "Topology optimization of a coupled thermal-fluid system under a tangential thermal gradient constraint," *Struct Multidisc*

Optim, vol. 54, no. 3, pp. 531–551, Sep. 2016, doi: 10.1007/s00158-016-1421-6.

[66] J. Alexandersen, O. Sigmund, and N. Aage, "Large scale three-dimensional topology optimisation of heat sinks cooled by natural convection," *International Journal of Heat and Mass Transfer*, vol. 100, pp. 876–891, Sep. 2016, doi: 10.1016/j.ijheatmasstransfer.2016.05.013.

[67] S. Qian, W. Wang, C. Ge, S. Lou, E. Miao, and B. Tang, "Topology optimization of fluid flow channel in cold plate for active phased array antenna," *Struct Multidisc Optim*, vol. 57, no. 6, pp. 2223–2232, Jun. 2018, doi: 10.1007/s00158-017-1852-8.

[68] T. Lei *et al.*, "Investment casting and experimental testing of heat sinks designed by topology optimization," *International Journal of Heat and Mass Transfer*, vol. 127, pp. 396–412, Dec. 2018, doi: 10.1016/j.ijheatmasstransfer.2018.07.060.

[69] M. Yu, S. Ruan, X. Wang, Z. Li, and C. Shen, "Topology optimization of thermal–fluid problem using the MMC-based approach," *Struct Multidisc Optim*, vol. 60, no. 1, pp. 151–165, Jul. 2019, doi: 10.1007/s00158-019-02206-w.

[70] X. Dong and X. Liu, "Multi-objective optimal design of microchannel cooling heat sink using topology optimization method," *Numerical Heat Transfer, Part A: Applications*, vol. 77, no. 1, pp. 90–104, Jan. 2020, doi: 10.1080/10407782.2019.1682872.

[71] D. Hu, Z. Zhang, and Q. Li, "Numerical study on flow and heat transfer characteristics of microchannel designed using topological optimizations method," *Sci. China Technol. Sci.*, vol. 63, no. 1, pp. 105–115, Jan. 2020, doi: 10.1007/s11431-019-1438-8.

[72] S. Sun, P. Liebersbach, and X. Qian, "3D topology optimization of heat sinks for liquid cooling," *Applied Thermal Engineering*, vol. 178, p. 115540, Sep. 2020, doi: 10.1016/j.applthermaleng.2020.115540.

[73] B. Zhang, J. Zhu, G. Xiang, and L. Gao, "Design of nanofluid-cooled heat sink using topology optimization," *Chinese Journal of Aeronautics*, vol. 34, no. 2, pp. 301–317, Feb. 2021, doi: 10.1016/j.cja.2020.05.023.

[74] T. Zhou, B. Chen, and H. Liu, "Study of the Performance of a Novel Radiator with Three Inlets and One Outlet Based on Topology Optimization," *Micromachines*, vol. 12, no. 6, p. 594, May 2021, doi: 10.3390/mi12060594.

[75] X. Liu, L. Chen, M. Peng, W.-T. Ji, and W.-Q. Tao, "Topology optimization of the manifold microchannels with triple-objective functions," *Numerical Heat Transfer, Part B: Fundamentals*, vol. 80, no. 5–6, pp. 89–114, Dec. 2021, doi: 10.1080/10407790.2021.1946309.

[76] J. Zhao, M. Zhang, Y. Zhu, R. Cheng, and L. Wang, "Topology optimization of planar cooling channels using a three-layer thermofluid model in fully developed laminar flow problems," *Struct Multidisc Optim*, vol. 63, no. 6, pp. 2789–2809, Jun. 2021, doi: 10.1007/s00158-021-02842-1.

[77] F. Chen, J. Wang, and X. Yang, "Topology optimization design and numerical

analysis on cold plates for lithium-ion battery thermal management," *International Journal of Heat and Mass Transfer*, vol. 183, p. 122087, Feb. 2022, doi: 10.1016/j.ijheatmasstransfer.2021.122087.

[78] L. Xie, Y. Zhang, M. Ge, and Y. Zhao, "Topology optimization of heat sink based on variable density method," *Energy Reports*, vol. 8, pp. 718–726, Apr. 2022, doi: 10.1016/j.egyr.2021.11.214.

[79] S. Ghosh, E. Fernandez, and J. Kapat, "Fluid-Thermal Topology Optimization of Gas Turbine Blade Internal Cooling Ducts," *Journal of Mechanical Design*, vol. 144, no. 5, p. 051703, May 2022, doi: 10.1115/1.4053042.

[80] P. Huang, S. Yang, and M. Pan, "Pseudo 3D topology optimization of microchannel heat sink with an auxiliary objective," *International Journal of Heat and Mass Transfer*, vol. 187, p. 122526, May 2022, doi: 10.1016/j.ijheatmasstransfer.2022.122526.

[81] M. Yu *et al.*, "A synergic topology optimization approach on distribution of cooling channels and diverse-intensity heat sources for liquid-cooled heat sink," *Struct Multidisc Optim*, vol. 65, no. 2, p. 48, Feb. 2022, doi: 10.1007/s00158-021-03113-9.

[82] A. Zou, R. Chuan, F. Qian, W. Zhang, Q. Wang, and C. Zhao, "Topology optimization for a water-cooled heat sink in micro-electronics based on Pareto frontier," *Applied Thermal Engineering*, vol. 207, p. 118128, May 2022, doi: 10.1016/j.applthermaleng.2022.118128.

[83] T. Zhou, C. Guo, and X. Shao, "A Novel Mini-Channel Heat Sink Design with Arc-Type Design Domain by Topology Optimization," *Micromachines*, vol. 13, no. 2, p. 180, Jan. 2022, doi: 10.3390/mi13020180.

[84] S. D. Marshall and P. S. Lee, "3d Topology Optimisation of Liquid-Cooled Microchannel Heat Sinks," *SSRN Journal*, 2022, doi: 10.2139/ssrn.4031308.

[85] K. Yeranee, Y. Rao, Y. Li, and H. Li, "Design of a Serpentine Cooling Channel Under Turbulent Flow Using Density-Based Topology Optimization," *SSRN Journal*, 2022, doi: 10.2139/ssrn.4001458.

[86] D. S. Makhija and P. S. Beran, "Concurrent shape and topology optimization for steady conjugate heat transfer," *Struct Multidisc Optim*, vol. 59, no. 3, pp. 919–940, Mar. 2019, doi: 10.1007/s00158-018-2110-4.

[87] X. Zhao, M. Zhou, O. Sigmund, and C. S. Andreasen, "A 'poor man's approach' to topology optimization of cooling channels based on a Darcy flow model," *International Journal of Heat and Mass Transfer*, vol. 116, pp. 1108–1123, Jan. 2018, doi: 10.1016/j.ijheatmasstransfer.2017.09.090.

[88] E. A. Kontoleontos, E. M. Papoutsis-Kiachagias, A. S. Zymaris, D. I. Papadimitriou, and K. C. Giannakoglou, "Adjoint-based constrained topology optimization for viscous flows, including heat transfer," *Engineering Optimization*, vol. 45, no. 8, pp. 941–961, Aug. 2013, doi: 10.1080/0305215X.2012.717074.

[89] E. M. Dede, "Optimization and Design of a Multipass Branching Microchannel Heat Sink for Electronics Cooling," *Journal of Electronic Packaging*, vol. 134, no. 4, p. 041001, Dec. 2012, doi: 10.1115/1.4007159.

[90] D. Ramalingom, P.-H. Cocquet, and A. Bastide, "A new interpolation technique to deal with fluid-porous media interfaces for topology optimization of heat transfer," *Computers & Fluids*, vol. 168, pp. 144–158, May 2018, doi: 10.1016/j.compfluid.2018.04.005.

[91] J. Asmussen, J. Alexandersen, O. Sigmund, and C. S. Andreasen, "A 'poor man's' approach to topology optimization of natural convection problems," *Struct Multidisc Optim*, vol. 59, no. 4, pp. 1105–1124, Apr. 2019, doi: 10.1007/s00158-019-02215-9.

[92] M. Pietropaoli, F. Montomoli, and A. Gaymann, "Three-dimensional fluid topology optimization for heat transfer," *Struct Multidisc Optim*, vol. 59, no. 3, pp. 801–812, Mar. 2019, doi: 10.1007/s00158-018-2102-4.

[93] M. Pietropaoli, A. Gaymann, and F. Montomoli, "Three-Dimensional Fluid Topology Optimization and Validation of a Heat Exchanger With Turbulent Flow," in *Volume 7A: Heat Transfer*, Virtual, Online, Sep. 2020, p. V07AT15A006. doi: 10.1115/GT2020-14479.

[94] A. Ghasemi and A. Elham, "Multi-objective topology optimization of pin-fin heat exchangers using spectral and finite-element methods," *Struct Multidisc Optim*, vol. 64, no. 4, pp. 2075–2095, Oct. 2021, doi: 10.1007/s00158-021-02964-6.

[95] G. H. Yoon, "Topological design of heat dissipating structure with forced convective heat transfer," *J Mech Sci Technol*, vol. 24, no. 6, Art. no. 6, Jun. 2010, doi: 10.1007/s12206-010-0328-1.

[96] M. S. Santhanakrishnan, T. Tilford, and C. Bailey, "Performance assessment of density and level-set topology optimisation methods for three dimensional heat sink design," *Journal of Algorithms & Computational Technology*, vol. 12, no. 3, pp. 273–287, Sep. 2018, doi: 10.1177/1748301818779019.

[97] S. A. Lurie, L. N. Rabinskiy, and Y. O. Solyaev, "Topology optimization of the wick geometry in a flat plate heat pipe," *International Journal of Heat and Mass Transfer*, vol. 128, pp. 239–247, Jan. 2019, doi: 10.1016/j.ijheatmasstransfer.2018.08.125.

[98] S. Jahan, T. Wu, Y. Shin, A. Tovar, and H. El-Mounayri, "Thermo-fluid Topology Optimization and Experimental Study of Conformal Cooling Channels for 3D Printed Plastic Injection Molds," *Procedia Manufacturing*, vol. 34, pp. 631–639, 2019, doi: 10.1016/j.promfg.2019.06.120.

[99] P. Francisco, L. Faria, and R. Simões, "Multi-objective and multi-load topology optimization and experimental validation of homogenized coupled fluid flow and heat transfer and structural stiffness," *Struct Multidisc Optim*, vol. 62, no. 5, pp. 2571–2598, Nov. 2020, doi: 10.1007/s00158-020-02625-0.

[100] G. Lee, I. Lee, and S. J. Kim, "Topology optimization of a heat sink with an axially uniform cross-section cooled by forced convection," *International Journal of Heat and Mass Transfer*, vol. 168, p. 120732, Apr. 2021, doi: 10.1016/j.ijheatmasstransfer.2020.120732.

[101] T. L. E. Tang, S. Xia, P. Rop, S. de Wispelaere, R. Subramanian, and B.

Koos, "Multi-Physics Topology Optimization for Thermal-Flow Problems Applied to Additively Manufactured Heat Exchangers," in *Volume 2D: Turbomachinery — Multidisciplinary Design Approaches, Optimization, and Uncertainty Quantification; Radial Turbomachinery Aerodynamics; Unsteady Flows in Turbomachinery*, Virtual, Online, Jun. 2021, p. V02DT36A019. doi: 10.1115/GT2021-60336.

[102] R. Tawk, B. Ghannam, and M. Nemer, "Topology optimization of heat and mass transfer problems in two fluids—one solid domains," *Numerical Heat Transfer, Part B: Fundamentals*, vol. 76, no. 3, pp. 130–151, Sep. 2019, doi: 10.1080/10407790.2019.1644919.

[103] L. C. Høghøj, D. R. Nørhave, J. Alexandersen, O. Sigmund, and C. S. Andreasen, "Topology optimization of two fluid heat exchangers," *International Journal of Heat and Mass Transfer*, vol. 163, p. 120543, Dec. 2020, doi: 10.1016/j.ijheatmasstransfer.2020.120543.

[104] M. Troya, D. Tortorelli, and V. Beck, "Two Dimensional Topology Optimization of Heat Exchangers with the Density and Level-Set Methods," presented at the 14th WCCM-ECCOMAS Congress, 2021. doi: 10.23967/wccm-eccomas.2020.345.

[105] H. Kobayashi, K. Yaji, S. Yamasaki, and K. Fujita, "Topology design of two-fluid heat exchange," *Struct Multidisc Optim*, vol. 63, no. 2, pp. 821–834, Feb. 2021, doi: 10.1007/s00158-020-02736-8.

[106] C. F. Hvejsel and E. Lund, "Material interpolation schemes for unified topology and multi-material optimization," *Struct Multidisc Optim*, vol. 43, no. 6, pp. 811–825, Jun. 2011, doi: 10.1007/s00158-011-0625-z.

[107] O. Sigmund and J. Petersson, "Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima," *Structural Optimization*, vol. 16, no. 1, pp. 68–75, Aug. 1998, doi: 10.1007/BF01214002.

[108] F. Wang, B. S. Lazarov, and O. Sigmund, "On projection methods, convergence and robust formulations in topology optimization," *Struct Multidisc Optim*, vol. 43, no. 6, pp. 767–784, Jun. 2011, doi: 10.1007/s00158-010-0602-y.

[109] N. Vermaak *et al.*, "Topological Optimization with Interfaces," in *Architectured Materials in Nature and Engineering*, vol. 282, Y. Estrin, Y. Bréchet, J. Dunlop, and P. Fratzl, Eds. Cham: Springer International Publishing, 2019, pp. 173–193. doi: 10.1007/978-3-030-11942-3_6.

[110] S. Osher and J. A. Sethian, "Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations," *Journal of Computational Physics*, vol. 79, no. 1, pp. 12–49, Nov. 1988, doi: 10.1016/0021-9991(88)90002-2.

[111] J. A. Sethian and A. Wiegmann, "Structural Boundary Design via Level Set and Immersed Interface Methods," *Journal of Computational Physics*, vol. 163, no. 2, pp. 489–528, Sep. 2000, doi: 10.1006/jcph.2000.6581.

[112] M. Y. Wang, X. Wang, and D. Guo, "A level set method for structural

topology optimization," *Computer Methods in Applied Mechanics and Engineering*, vol. 192, no. 1–2, pp. 227–246, Jan. 2003, doi: 10.1016/S0045-7825(02)00559-5.

[113] G. Allaire, F. Jouve, and A.-M. Toader, "Structural optimization using sensitivity analysis and a level-set method," *Journal of Computational Physics*, vol. 194, no. 1, pp. 363–393, Feb. 2004, doi: 10.1016/j.jcp.2003.09.032.

[114] M. Yoshimura, K. Shimoyama, T. Misaka, and S. Obayashi, "Topology optimization of fluid problems using genetic algorithm assisted by the Kriging model: TOPOLOGY OPTIMIZATION USING A KRIGING-BASED GENETIC ALGORITHM," *Int. J. Numer. Meth. Engng*, vol. 109, no. 4, pp. 514–532, Jan. 2017, doi: 10.1002/nme.5295.

[115] F. Dugast, Y. Favennec, C. Josset, Y. Fan, and L. Luo, "Topology optimization of thermal fluid flows with an adjoint Lattice Boltzmann Method," *Journal of Computational Physics*, vol. 365, pp. 376–404, Jul. 2018, doi: 10.1016/j.jcp.2018.03.040.

[116] S. Kambampati, J. S. Gray, and H. A. Kim, "Level Set Topology Optimization of Load Carrying Heat Exchangers," presented at the AIAA Scitech 2020 Forum, Orlando, FL, Jan. 2020. doi: 10.2514/6.2020-0891.

[117] F. Feppon, G. Allaire, C. Dapogny, and P. Jolivet, "Body-fitted topology optimization of 2D and 3D fluid-to-fluid heat exchangers," *Computer Methods in Applied Mechanics and Engineering*, vol. 376, p. 113638, Apr. 2021, doi: 10.1016/j.cma.2020.113638.

[118] P. Mario, K. Tsuguo, and F. Kenichiro, "Optimization of Fin Topology for Heat Transfer Within Lightweight Plate-Fin Heat Exchangers," *Journal of Thermophysics and Heat Transfer*, vol. 35, no. 4, pp. 677–685, Oct. 2021, doi: 10.2514/1.T6121.

[119] K. Yaji, T. Yamada, S. Kubo, K. Izui, and S. Nishiwaki, "A topology optimization method for a coupled thermal–fluid problem using level set boundary expressions," *International Journal of Heat and Mass Transfer*, vol. 81, pp. 878–888, Feb. 2015, doi: 10.1016/j.ijheatmasstransfer.2014.11.005.

[120] P. Coffin and K. Maute, "A level-set method for steady-state and transient natural convection problems," *Struct Multidisc Optim*, vol. 53, no. 5, Art. no. 5, May 2016, doi: 10.1007/s00158-015-1377-y.

[121] Y. Sato, K. Yaji, K. Izui, T. Yamada, and S. Nishiwaki, "An Optimum Design Method for a Thermal-Fluid Device Incorporating Multiobjective Topology Optimization With an Adaptive Weighting Scheme," *Journal of Mechanical Design*, vol. 140, no. 3, p. 031402, Mar. 2018, doi: 10.1115/1.4038209.

[122] H. Li *et al.*, "Optimum design and thermal modeling for 2D and 3D natural convection problems incorporating level set–based topology optimization with body–fitted mesh†," *Numerical Meth Engineering*, p. nme.6923, Jan. 2022, doi: 10.1002/nme.6923.

[123] Y. Lin, W. Zhu, J. Li, and Y. Ke, "A stabilized parametric level‑set XFEM topology optimization method for thermal‑fluid problem," *Numerical Meth Engineering*, vol. 123, no. 4, pp. 924–952, Feb. 2022, doi: 10.1002/nme.6883.

[124] X. Li, L. Zhang, and B. Li, "Heat transfer augmentation in microchannel heat sink based on isogeometric topology optimization framework," *Applied Mathematical Modelling*, vol. 104, pp. 163–187, Apr. 2022, doi: 10.1016/j.apm.2021.11.021.

[125] N. P. van Dijk, K. Maute, M. Langelaar, and F. van Keulen, "Level-set methods for structural topology optimization: a review," *Struct Multidisc Optim*, vol. 48, no. 3, pp. 437–472, Sep. 2013, doi: 10.1007/s00158-013-0912-y.

[126] G. Fabbri, "A genetic algorithm for fin profile optimization," *International Journal of Heat and Mass Transfer*, vol. 40, no. 9, pp. 2165–2172, Jun. 1997, doi: 10.1016/S0017-9310(96)00294-3.

[127] E. Duplain and B. R. Baliga, "Computational optimization of the thermal performance of internally finned ducts," *International Journal of Heat and Mass Transfer*, vol. 52, no. 17–18, pp. 3929–3942, Aug. 2009, doi: 10.1016/j.ijheatmasstransfer.2009.03.030.

[128] H. Moon, K. Boyina, N. Miljkovic, and W. P. King, "Heat Transfer Enhancement of Single-Phase Internal Flows using Shape Optimization and Additively Manufactured Flow Structures," *International Journal of Heat and Mass Transfer*, vol. 177, p. 121510, Oct. 2021, doi: 10.1016/j.ijheatmasstransfer.2021.121510.

[129] P. Rostami and J. Marzbanrad, "Cooperative coevolutionary topology optimization using moving morphable components," *Engineering Optimization*, vol. 53, no. 6, pp. 962–983, Jun. 2021, doi: 10.1080/0305215X.2020.1759579.

[130] B. S. Mekki, J. Langer, and S. Lynch, "Genetic algorithm based topology optimization of heat exchanger fins used in aerospace applications," *International Journal of Heat and Mass Transfer*, vol. 170, p. 121002, May 2021, doi: 10.1016/j.ijheatmasstransfer.2021.121002.

[131] K. Shimoyama and A. Komiya, "Multi-objective Bayesian topology optimization of a lattice-structured heat sink in natural convection," *Struct Multidisc Optim*, vol. 65, no. 1, p. 1, Jan. 2022, doi: 10.1007/s00158-021-03092-x.

[132] A. Hrennikoff, "Solution of Problems of Elasticity by the Framework Method," *Journal of Applied Mechanics*, vol. 8, no. 4, pp. A169–A175, Dec. 1941, doi: 10.1115/1.4009129.

[133] D. McHenry, "Discussion: 'Solution of Problems of Elasticity by the Framework Method' (Hrennikoff, A., 1941, ASME J. Appl. Mech., 8, pp. A169–A175)," *Journal of Applied Mechanics*, vol. 9, no. 3, pp. A144–A145, Sep. 1942, doi: 10.1115/1.4009210.

[134] S. R. Idelsohn and E. Oñate, "Finite volumes and finite elements: Two 'good friends,' " *Int. J. Numer. Meth. Engng.*, vol. 37, no. 19, pp. 3323–

3341, Oct. 1994, doi: 10.1002/nme.1620371908.

[135] G. Dhatt, E. Lefranois, and G. Touzot, *Finite Element Method.* London: Wiley, 2013. Accessed: Nov. 09, 2021. [Online]. Available: http://public.ebookcentral.proquest.com/choice/publicfullrecord.aspx?p=112 0636

[136] M. Zhou, J. Alexandersen, O. Sigmund, and C. B. W. Pedersen, "Industrial application of topology optimization for combined conductive and convective heat transfer problems," *Struct Multidisc Optim*, vol. 54, no. 4, pp. 1045–1060, Oct. 2016, doi: 10.1007/s00158-016-1433-2.

[137] K. R. Saviers, R. Ranjan, and R. Mahmoudi, "Design and validation of topology optimized heat exchangers," presented at the AIAA Scitech 2019 Forum, San Diego, California, Jan. 2019. doi: 10.2514/6.2019-1465.

[138] T. Li, T. Wu, X. Ding, H. Chen, and L. Wang, "Design of an internally cooled turning tool based on topology optimization and CFD simulation," *Int J Adv Manuf Technol*, vol. 91, no. 1–4, pp. 1327–1337, Jul. 2017, doi: 10.1007/s00170-016-9804-9.

[139] L. Bush and V. Ginting, "On the Application of the Continuous Galerkin Finite Element Method for Conservation Problems," *SIAM J. Sci. Comput.*, vol. 35, no. 6, pp. A2953–A2975, Jan. 2013, doi: 10.1137/120900393.

[140] T. E. Bruns, "Topology optimization of convection-dominated, steady-state heat transfer problems," *International Journal of Heat and Mass Transfer*, vol. 50, no. 15–16, pp. 2859–2873, Jul. 2007, doi: 10.1016/j.ijheatmasstransfer.2007.01.039.

[141] P. W. McDonald, "The Computation of Transonic Flow Through Two-Dimensional Gas Turbine Cascades," in *ASME 1971 International Gas Turbine Conference and Products Show*, Houston, Texas, USA, Mar. 1971, p. V001T01A089. doi: 10.1115/71-GT-89.

[142] A. W. Rizzi and M. Inouye, "Time-Split Finite-Volume Method for Three-Dimensional Blunt-Body Flow," *AIAA Journal*, vol. 11, no. 11, pp. 1478–1485, Nov. 1973, doi: 10.2514/3.50614.

[143] R. Eymard, T. Gallouët, and R. Herbin, "Finite volume methods," in *Handbook of Numerical Analysis*, vol. 7, Elsevier, 2000, pp. 713–1018. doi: 10.1016/S1570-8659(00)07005-8.

[144] F. Moukalled, L. Mangani, and M. Darwish, *The Finite Volume Method in Computational Fluid Dynamics: An Advanced Introduction with OpenFOAM® and Matlab*, vol. 113. Cham: Springer International Publishing, 2016. doi: 10.1007/978-3-319-16874-6.

[145] A. Gersborg-Hansen, M. P. Bendsøe, and O. Sigmund, "Topology optimization of heat conduction problems using the finite volume method," *Struct Multidisc Optim*, vol. 31, no. 4, pp. 251–259, Apr. 2006, doi: 10.1007/s00158-005-0584-3.

[146] Z. J. Wang *et al.*, "High-order CFD methods: current status and perspective: HIGH-ORDER CFD METHODS," *Int. J. Numer. Meth. Fluids*, vol. 72, no. 8, pp. 811–845, Jul. 2013, doi: 10.1002/fld.3767.

[147]   A. R. Khoei, *Extended finite element method: theory and applications*. Chichester, West Sussex: John Wiley & Sons, Inc, 2015.

[148]   A. Riccio, U. Caruso, A. Raimondo, and A. Sellitto, "Robustness of XFEM Method for the Simulation of Cracks Propagation in Fracture Mechanics Problems," *American Journal of Engineering and Applied Sciences*, vol. 9, no. 3, pp. 599–610, Mar. 2016, doi: 10.3844/ajeassp.2016.599.610.

[149]   T. Belytschko and T. BLACK, "Elastic crack growth in finite elements with minimal remeshing," *International Journal for Numerical Methods in Engineering*, vol. 45, no. 5, pp. 601–620, 1999.

[150]   R. Merle and J. Dolbow, "Solving thermal and phase change problems with the eXtended finite element method," *Computational Mechanics*, vol. 28, no. 5, pp. 339–350, May 2002, doi: 10.1007/s00466-002-0298-y.

[151]   A. A. Mohamad, *Lattice Boltzmann Method: Fundamentals and Engineering Applications with Computer Codes*. London: Springer London, 2019. doi: 10.1007/978-1-4471-7423-3.

[152]   G. R. McNamara and G. Zanetti, "Use of the Boltzmann Equation to Simulate Lattice-Gas Automata," *Phys. Rev. Lett.*, vol. 61, no. 20, pp. 2332–2335, Nov. 1988, doi: 10.1103/PhysRevLett.61.2332.

[153]   J. Zhang, "Lattice Boltzmann method for microfluidics: models and applications," *Microfluid Nanofluid*, vol. 10, no. 1, pp. 1–28, Jan. 2011, doi: 10.1007/s10404-010-0624-1.

[154]   T. Krüger, H. Kusumaatmaja, A. Kuzmin, O. Shardt, G. Silva, and E. M. Viggen, *The Lattice Boltzmann Method: Principles and Practice*. Cham: Springer International Publishing, 2017. doi: 10.1007/978-3-319-44649-3.

[155]   D. Tarlet, Y. Fan, S. Roux, and L. Luo, "Entropy generation analysis of a mini heat exchanger for heat transfer intensification," *Experimental Thermal and Fluid Science*, vol. 53, pp. 119–126, Feb. 2014, doi: 10.1016/j.expthermflusci.2013.11.016.

[156]   M. Gunzburger, "Adjoint Equation-Based Methods for Control Problems in Incompressible, Viscous Flows," *Flow, Turbulence and Combustion*, vol. 65, no. 3/4, pp. 249–272, 2000, doi: 10.1023/A:1011455900396.

[157]   M. B. Giles and N. A. Pierce, "An Introduction to the Adjoint Approach to Design," *Flow, Turbulence and Combustion*, vol. 65, no. 3/4, pp. 393–415, 2000, doi: 10.1023/A:1011430410075.

[158]   K. Svanberg, "The method of moving asymptotes—a new method for structural optimization," *Int. J. Numer. Meth. Engng.*, vol. 24, no. 2, pp. 359–373, Feb. 1987, doi: 10.1002/nme.1620240207.

[159]   K. Svanberg, "A Class of Globally Convergent Optimization Methods Based on Conservative Convex Separable Approximations," *SIAM J. Optim.*, vol. 12, no. 2, pp. 555–573, Jan. 2002, doi: 10.1137/S1052623499362822.

[160]   J. S. Arora, *Introduction to Optimum Design*. 2004.

[161]   P. T. Boggs and J. W. Tolle, "Sequential Quadratic Programming," *Acta Numerica*, vol. 4, pp. 1–51, Jan. 1995, doi: 10.1017/S0962492900002518.

[162]   J. C. Meza, "Steepest descent," *WIREs Comp Stat*, vol. 2, no. 6, pp.

719–722, Nov. 2010, doi: 10.1002/wics.117.

[163]   SIMULIA-Tosca Manual, Dassault Systemes Simulia, 2015.

[164]   T. Yamada, K. Izui, S. Nishiwaki, and A. Takezawa, "A topology optimization method based on the level set method incorporating a fictitious interface energy," *Computer Methods in Applied Mechanics and Engineering*, vol. 199, no. 45–48, pp. 2876–2891, Nov. 2010, doi: 10.1016/j.cma.2010.05.013.

[165]   F. Feppon, G. Allaire, and C. Dapogny, "Null space gradient flows for constrained optimization with applications to shape optimization," *ESAIM: COCV*, vol. 26, p. 90, 2020, doi: 10.1051/cocv/2020015.

[166]   G. Venter, "Review of Optimization Techniques," in *Encyclopedia of Aerospace Engineering*, R. Blockley and W. Shyy, Eds. Chichester, UK: John Wiley & Sons, Ltd, 2010, p. eae495. doi: 10.1002/9780470686652.eae495.

[167]   J. Fliege and A. I. F. Vaz, "A Method for Constrained Multiobjective Optimization Based on SQP Techniques," *SIAM J. Optim.*, vol. 26, no. 4, pp. 2091–2119, Jan. 2016, doi: 10.1137/15M1016424.

[168]   J. H. Holland, "Genetic Algorithms," *Sci Am*, vol. 267, no. 1, pp. 66–72, Jul. 1992, doi: 10.1038/scientificamerican0792-66.

[169]   A. Lipowski and D. Lipowska, "Roulette-wheel selection via stochastic acceptance," *Physica A: Statistical Mechanics and its Applications*, vol. 391, no. 6, pp. 2193–2196, Mar. 2012, doi: 10.1016/j.physa.2011.12.004.

[170]   S. Mirjalili, "Genetic Algorithm," in *Evolutionary Algorithms and Neural Networks*, vol. 780, Cham: Springer International Publishing, 2019, pp. 43–55. doi: 10.1007/978-3-319-93025-1_4.

[171]   Chang Wook Ahn and R. S. Ramakrishna, "Elitism-based compact genetic algorithms," *IEEE Trans. Evol. Computat.*, vol. 7, no. 4, pp. 367–385, Aug. 2003, doi: 10.1109/TEVC.2003.814633.

[172]   L. Gosselin, M. Tye-Gingras, and F. Mathieu-Potvin, "Review of utilization of genetic algorithms in heat transfer problems," *International Journal of Heat and Mass Transfer*, vol. 52, no. 9–10, pp. 2169–2188, Apr. 2009, doi: 10.1016/j.ijheatmasstransfer.2008.11.015.

[173]   R. Boichot and Y. Fan, "A genetic algorithm for topology optimization of area-to-point heat conduction problem," *International Journal of Thermal Sciences*, vol. 108, pp. 209–217, Oct. 2016, doi: 10.1016/j.ijthermalsci.2016.05.015.

[174]   Y. Haimes, L. Lasdon, and D. Da, "On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-1, no. 3, pp. 296–297, Jul. 1971, doi: 10.1109/TSMC.1971.4308298.

[175]   H. J. Kushner, "A New Method of Locating the Maximum Point of an Arbitrary Multipeak Curve in the Presence of Noise," *Journal of Basic Engineering*, vol. 86, no. 1, pp. 97–106, Mar. 1964, doi: 10.1115/1.3653121.

[176]   A. G. Zhilinskas, "Single-step Bayesian search method for an extremum of functions of a single variable," *Cybern Syst Anal*, vol. 11, no. 1, pp.

160–166, 1976, doi: 10.1007/BF01069961.

[177] J. Mockus, *Bayesian Approach to Global Optimization: Theory and Applications*, vol. 37. Dordrecht: Springer Netherlands, 1989. doi: 10.1007/978-94-009-0909-0.

[178] D. R. Jones, M. Schonlau, and W. J. Welch, "Efficient Global Optimization of Expensive Black-Box Functions," *Journal of Global Optimization*, vol. 13, no. 4, pp. 455–492, 1998, doi: 10.1023/A:1008306431147.

[179] P. I. Frazier, "Bayesian Optimization," in *Recent Advances in Optimization and Modeling of Contemporary Problems*, E. Gel, L. Ntaimo, D. Shier, and H. J. Greenberg, Eds. INFORMS, 2018, pp. 255–278. doi: 10.1287/educ.2018.0188.

[180] A. Klein, S. Bartels, F. Falkner, P. Hennig, and F. Hutter, "Towards efficient Bayesian Optimization for Big Data," 2015.

[181] M. E. Lynch, S. Sarkar, and K. Maute, "Machine Learning to Aid Tuning of Numerical Parameters in Topology Optimization," *Journal of Mechanical Design*, vol. 141, no. 11, p. 114502, Nov. 2019, doi: 10.1115/1.4044228.

[182] J. Sacks, W. J. Welch, T. J. Mitchell, and H. P. Wynn, "Design and Analysis of Computer Experiments," *Statist. Sci.*, vol. 4, no. 4, Nov. 1989, doi: 10.1214/ss/1177012413.

[183] E. Merrill, A. Fern, X. Fern, and N. Dolatnia, "An Empirical Study of Bayesian Optimization: Acquisition Versus Partition," *Journal of Machine Learning Research,22*, 2021.

[184] X. Garcia-Santiago, S. Burger, C. Rockstuhl, and P.-I. Schneider, "Bayesian Optimization With Improved Scalability and Derivative Information for Efficient Design of Nanophotonic Structures," *J. Lightwave Technol.*, vol. 39, no. 1, pp. 167–177, Jan. 2021, doi: 10.1109/JLT.2020.3023450.

[185] S. Ford and M. Despeisse, "Additive manufacturing and sustainability: an exploratory study of the advantages and challenges," *Journal of Cleaner Production*, vol. 137, pp. 1573–1587, Nov. 2016, doi: 10.1016/j.jclepro.2016.04.150.

[186] M. Attaran, "The rise of 3-D printing: The advantages of additive manufacturing over traditional manufacturing," *Business Horizons*, vol. 60, no. 5, pp. 677–688, Sep. 2017, doi: 10.1016/j.bushor.2017.05.011.

[187] R. Behrou, K. Kirsch, R. Ranjan, and J. K. Guest, "Topology optimization of additively manufactured fluidic components free of internal support structures," *Computer Methods in Applied Mechanics and Engineering*, p. 114270, Nov. 2021, doi: 10.1016/j.cma.2021.114270.

[188] J. Alexandersen, "Topology Optimisation for Axisymmetric Convection Problems," Technical University of Denmark, 2011a. [Online]. Available: https://backend.orbit.dtu.dk/ws/portalfiles/portal/155591760/Alexandersen2011a.pdf

[189] A. B. Farimani, J. Gomes, and V. S. Pande, "Deep Learning the Physics of Transport Phenomena," *arXiv:1709.02432 [physics]*, Sep. 2017, Accessed:

Sep. 09, 2021. [Online]. Available: http://arxiv.org/abs/1709.02432

[190] B. Kwon, F. Ejaz, and L. K. Hwang, "Machine learning for heat transfer correlations," *International Communications in Heat and Mass Transfer*, vol. 116, p. 104694, Jul. 2020, doi: 10.1016/j.icheatmasstransfer.2020.104694.

[191] L. Zhou, D. Garg, Y. Qiu, S.-M. Kim, I. Mudawar, and C. R. Kharangate, "Machine learning algorithms to predict flow condensation heat transfer coefficient in mini/micro-channel utilizing universal data," *International Journal of Heat and Mass Transfer*, vol. 162, p. 120351, Dec. 2020, doi: 10.1016/j.ijheatmasstransfer.2020.120351.

[192] G. Zhu, T. Wen, and D. Zhang, "Machine learning based approach for the prediction of flow boiling/condensation heat transfer performance in mini channels with serrated fins," *International Journal of Heat and Mass Transfer*, vol. 166, p. 120783, Feb. 2021, doi: 10.1016/j.ijheatmasstransfer.2020.120783.

[193] I. Sosnovik and I. Oseledets, "Neural networks for topology optimization," *Russian Journal of Numerical Analysis and Mathematical Modelling*, vol. 34, no. 4, pp. 215–223, Aug. 2019, doi: 10.1515/rnam-2019-0018.

[194] D. W. Abueidda, S. Koric, and N. A. Sobh, "Topology optimization of 2D structures with nonlinearities using deep learning," *Computers & Structures*, vol. 237, p. 106283, Sep. 2020, doi: 10.1016/j.compstruc.2020.106283.

[195] A. Chandrasekhar and K. Suresh, "TOuNN: Topology Optimization using Neural Networks," *Struct Multidisc Optim*, vol. 63, no. 3, pp. 1135–1149, Mar. 2021, doi: 10.1007/s00158-020-02748-4.

[196] H. Chi *et al.*, "Universal machine learning for topology optimization," *Computer Methods in Applied Mechanics and Engineering*, vol. 375, p. 112739, Mar. 2021, doi: 10.1016/j.cma.2019.112739.

[197] D. P. Kingma and M. Welling, "Auto-Encoding Variational Bayes," *arXiv:1312.6114 [cs, stat]*, May 2014, Accessed: Dec. 27, 2021. [Online]. Available: http://arxiv.org/abs/1312.6114

[198] Q. Lin, J. Hong, Z. Liu, B. Li, and J. Wang, "Investigation into the topology optimization for conductive heat transfer based on deep learning approach," *International Communications in Heat and Mass Transfer*, vol. 97, pp. 103–109, Oct. 2018, doi: 10.1016/j.icheatmasstransfer.2018.07.001.

[199] W. H. A. Schilders, H. A. van der Vorst, and J. Rommes, Eds., *Model Order Reduction: Theory, Research Aspects and Applications*, vol. 13. Berlin, Heidelberg: Springer Berlin Heidelberg, 2008. doi: 10.1007/978-3-540-78841-6.

[200] B. Moore, "Principal component analysis in linear systems: Controllability, observability, and model reduction," *IEEE Trans. Automat. Contr.*, vol. 26, no. 1, pp. 17–32, Feb. 1981, doi: 10.1109/TAC.1981.1102568.

[201] L. Sirovich, "Turbulence and the dynamics of coherent structures. I. Coherent structures," *Quart. Appl. Math.*, vol. 45, no. 3, pp. 561–571, Oct. 1987, doi: 10.1090/qam/910462.

[202]　K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L , $^\infty$ -error bounds†," *International Journal of Control*, vol. 39, no. 6, pp. 1115–1193, Jun. 1984, doi: 10.1080/00207178408933239.

[203]　M. Behbahani-Nejad and Y. Shekari, "The accuracy and efficiency of a reduced-order model for transient flow analysis in gas pipelines," *Journal of Petroleum Science and Engineering*, vol. 73, no. 1–2, pp. 13–19, Aug. 2010, doi: 10.1016/j.petrol.2010.05.001.

[204]　T. Lassila, A. Manzoni, A. Quarteroni, and G. Rozza, "Model Order Reduction in Fluid Dynamics: Challenges and Perspectives," in *Reduced Order Methods for Modeling and Computational Reduction*, A. Quarteroni and G. Rozza, Eds. Cham: Springer International Publishing, 2014, pp. 235–273. doi: 10.1007/978-3-319-02090-7_9.

[205]　X. Guo, W. Zhang, and W. Zhong, "Doing Topology Optimization Explicitly and Geometrically—A New Moving Morphable Components Based Framework," *Journal of Applied Mechanics*, vol. 81, no. 8, p. 081009, Aug. 2014, doi: 10.1115/1.4027609.

[206]　W. Zhang, J. Yuan, J. Zhang, and X. Guo, "A new topology optimization approach based on Moving Morphable Components (MMC) and the ersatz material model," *Struct Multidisc Optim*, vol. 53, no. 6, pp. 1243–1260, Jun. 2016, doi: 10.1007/s00158-015-1372-3.

[207]　W. Zhang, D. Li, J. Yuan, J. Song, and X. Guo, "A new three-dimensional topology optimization method based on moving morphable components (MMCs)," *Comput Mech*, vol. 59, no. 4, pp. 647–665, Apr. 2017, doi: 10.1007/s00466-016-1365-0.

[208]　J. Sun, Q. Tian, and H. Hu, "Topology Optimization of a Three-Dimensional Flexible Multibody System Via Moving Morphable Components," *Journal of Computational and Nonlinear Dynamics*, vol. 13, no. 2, p. 021010, Feb. 2018, doi: 10.1115/1.4038142.