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# Modeling and Observer Design for Aluminum Manufacturing <sup>\*</sup>

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## Abstract

This paper addresses modeling, identification, and observer design of the Hall-Héroult process. This process is the main core of aluminum manufacturing. More specifically, by building upon a blend of physical-chemical constitutive relations and system identification tools, this paper first provides a model describing the interplay of Anode-Cathode Distance (ACD), alumina concentration, and pot pseudo-resistance. The proposed model is then used to tune a linear Kalman filter generating online estimates of the plant state variables. The proposed approach is validated via data taken from a real industrial plant. *Keywords:* physical modeling, parameter identification, observer design, Kalman Filter, industrial application

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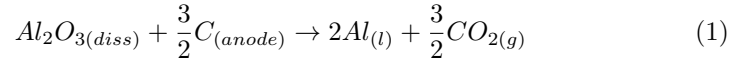
## 1. Introduction

Aluminum is produced in large scale via the Hall-Heroult electrolysis process since the 19th century [1]. This process is carried out by dipping carbon anodes into a cryolite bath solution that contains dissolved alumina ( $Al_2O_3$ ). A high intensity electric current is applied to the system and the chemical process produces liquid aluminum cumulating at the bottom of the cell, and releases

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carbon dioxide. The process is summarized by the following chemical reaction:



A schematic representation of a simple pot cell is shown in Figure 1. To enhance production, industrial plants have a large number of anodes connected in parallel by a common bar for each electrolysis cell. The height of this bar can be adjusted to change the Anode-Cathode Distance (*ACD*). This distance is not constant during the operation due to the chemical reaction. Indeed, the carbon anodes are consumed and they are replaced after some time, while the liquid aluminum layer increases because of the production. Furthermore, perturbations of the current and bath composition can affect the *ACD*. However, the hazardous conditions inside the pot make it impossible to develop a sensor for continuous measurement of the system state. The *ACD* value is critical since a large distance decreases the pot cell efficiency and a small value can cause a short-circuit between the produced aluminum and the anode [2, 3]. A recent study shows that an effective *ACD* regulation can improve cell energy consumption [4] and consequently increase the efficiency as shown in [5]. Unfortunately, the mechanisms behind this process are complex and only a few papers have modeled this dynamical behavior in detail [6, 7].

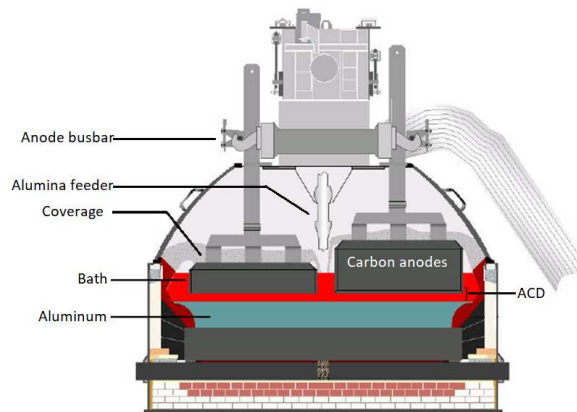


Figure 1: Pot Schematic View

The dissolved alumina concentration ( $wAl_2O_3$ ) is also an important quantity that is not continuously measured. Usually, alumina is injected in powder state by individual feeders distributed along the cell according to a predefined sequence. Low values of alumina concentration can cause the so-called *anode effect*, a deleterious phenomenon leading to the production of greenhouse gases [8]. However, a large alumina powder injection does not induce an instantaneous change in the concentration and it can possibly produce sludge phenomena. This is an undesired condition since the accumulation of undissolved alumina in the bath can lead to cell damages [9]. Commonly, just a few measurements of the alumina concentration per week are manually taken, which makes it difficult to obtain an experimental model.

In actual plants, only the line current applied to the system ( $I$ ), the pot cell voltage ( $V$ ), the busbar motion ( $BM$ ), and the frequency of the alumina feeding ( $F$ ) are continuously collected by sensors. For the system regulation, an indirect measurement called “pseudo-resistance” ( $R$ ) is commonly used to adjust  $ACD$  and  $wAl_2O_3$ . Based on the pseudo-resistance value, the alumina feeding frequency is modified, alternating between two pre-determined periods to have faster or slower feeding [7]. Moreover, the pseudo-resistance is adjusted via the  $ACD$  regulation to ensure system stability and obtain a good current efficiency [10]. From this complex relation, it is not easy to obtain information about the instantaneous values of  $ACD$  and  $wAl_2O_3$ . Therefore, it is important to model this process to be able to generate online estimations for those quantities.

Some researchers have been developing nonlinear estimators to overcome this information limitation and obtain the desired process states [11, 6, 12]. However, those works are mostly based on the *Extended Kalman Filter* [13], i.e. on approximate model linearization on the one hand (facing all related drawbacks), and do not rely much on the underlying physics of the process on the other hand.

In this context, and on the basis of a plant in actual industrial operation, the present paper proposes a modeling methodology for Hall-Hérault process, which enables real time estimation of  $ACD$  and  $wAl_2O_3$ , as an extension of our pre-

liminary works of [14, 15]. Despite its simplicity, our model captures the main  
50 features of regular operations conditions by combining physical-chemical aspects  
with experimental models. Furthermore, in spite of its nonlinearity, we show  
how tools from *linear* systems theory can be used by separating the estimation  
problem in two steps: identification in a first one, with least square optimiza-  
tion, and state observer design in a second one, via an appropriate rewriting and  
55 an exact linear Kalman Filter approach (referring to formal results of [16] for  
instance). Finally, the proposed method is validated on industrial data, taken in  
actual operational conditions. This paper extends our previous work in multiple  
directions [14, 15]:

First, the *ACD* physical model is here explained in details by a chemical balance  
60 analysis.

Additionally, a simplification in the alumina concentration is done by avoiding  
any time delay. This is reasonable since the available alumina measurements  
have been taken at intervals of several hours.

Moreover, the alumina dissolving dynamics has a time response of several min-  
65 utes. Thus, a delay of one minute, or even slightly more, is not expected to  
relevantly affect the value of the alumina at the measurement instants and then  
a relatively accurate alumina model can be identified even by neglecting the  
delay.

Furthermore, the pot pseudo-resistance is represented by a specific polynomial  
70 function with new identification procedure.

Finally, validation tests are carried out with new sets of data to ensure ro-  
bustness, collected from industrial APXe pot cell of Rio Tinto Laboratoire des  
Recherches de Fabrications (LRF) located in Saint Jean de Maurienne, France,  
in regular operation. The data used in this paper was collected during daily  
75 industrial operations. Therefore, all the parameters identification and state es-  
timation procedures are performed with real signals that are generated aiming  
at operation safety and production goals. Notice that for confidentiality rea-  
sons, the data details in the y-axis of all plots and the values of the estimated  
coefficients are not shown.

80 The paper is organized as follows: Section 2 presents the proposed physical-based modeling equations, and Section 3 discusses the related identification approach. Section 4 shows the use of this model for state estimation. The conclusions are presented in Section 5.

## 2. Modeling

85 The complex electrochemical process under consideration is modeled in discrete-time since the goal is a real-time application of Linear Kalman Filter tools. It is aimed to capture the average dynamics of the pot cell. Hence, the respective states represent global indicators of the plant. This means that local conditions such as the current distribution and anode change effect are not taken into account explicitly. In this context, the models of the unmeasured quantities  $ACD$  90 and  $wAl_2O_3$  are related to the available inputs, i.e. beam motion and frequency of feeders, as well as the measured disturbance of the electric line current. Then, the system output, i.e. the pseudo-resistance, is related to the modeled states. The model is obtained using a hybrid approach that combines physical-chemical 95 aspects and information extracted from data. Moreover, the sampling time of 1 minute is selected because the time constant for the dissolution of alumina is in this order of magnitude and to avoid the noise influence [17].

### 2.1. Anode-Cathode Distance

From the problem description, it is possible to define the derivative of the  $ACD$  ( $\frac{d}{dt}ACD$ ) as a result of the height variations of the aluminum ( $\frac{d}{dt}Al_{height}$ ), the carbon ( $\frac{d}{dt}C_{height}$ ) layer, and the anodes busbar beam position ( $\frac{d}{dt}BM$ ):

$$\frac{d}{dt}ACD = \frac{d}{dt}Al_{height} - \frac{d}{dt}C_{height} + \frac{d}{dt}BM \quad (2)$$

Each layer height can be written as a function of the mass production using the corresponding densities. Hence, the aluminum and carbon height variations become:

$$\frac{d}{dt}Al_{height} = \frac{1}{\rho_{Al}S} \left[ \frac{d}{dt}m_{Al} \right] \quad (3)$$

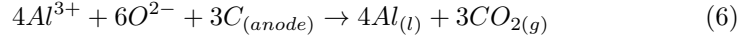
$$\frac{d}{dt}C_{height} = \frac{1}{\rho_C S} \left[ \frac{d}{dt}m_C \right] \quad (4)$$

where  $\rho_{Al}$  is the liquid aluminum density,  $m_{Al}$  is the aluminum produced mass,  $\rho_C$  is the carbon density,  $m_C$  is the carbon produced mass, and  $S$  is the average reaction surface area.

The produced aluminum mass rate ( $\frac{d}{dt}m_{Al}$ ) is given by Faraday's law of electrolysis [18]:

$$\frac{d}{dt}m_{Al} = \frac{C_e Al_m}{3\mathfrak{F}} I \quad (5)$$

where  $C_e$  is the current efficiency,  $Al_m$  is the aluminum molar mass,  $I$  is the line current,  $\mathfrak{F}$  is the Faraday's constant, and 3 corresponds to the valency number of aluminum ions obtained via the following reaction:



From the chemical balance in (1), it is possible to relate the aluminum mass production with the carbon mass consumption. This leads to the following expression for the carbon mass consumption rate  $\frac{d}{dt}m_C$ :

$$\frac{d}{dt}m_C = \frac{C_m}{4\mathfrak{F}} I \quad (7)$$

where  $C_m$  is the carbon molar mass and 4 is the stoichiometric coefficient.

Hence, equation (2) can be rewritten using the above expressions as:

$$\frac{d}{dt}ACD = \frac{1}{S\mathfrak{F}} \left( \frac{C_e Al_m}{3\rho_{Al}} - \frac{C_m}{4\rho_C} \right) I + \frac{d}{dt}BM \quad (8)$$

The beam position variation is considered as one of the system inputs ( $U_1$ ):

$$U_1 := \frac{d}{dt}BM \quad (9)$$

Hence, by defining  $\beta$  as:

$$\beta := \frac{1}{S\mathfrak{F}} \left( \frac{C_e Al_m}{3\rho_{Al}} - \frac{C_m}{4\rho_C} \right) \quad (10)$$

and using equation (10) in (8), the  $ACD$  dynamics result as:

$$\frac{d}{dt}ACD = \beta I + U_1 \quad (11)$$

In discrete-time, equation (11) becomes:

$$ACD[n+1] = ACD[n] + \beta i[n] + u_1[n] \quad (12)$$

$ACD[n]$  stands for  $ACD(nT_s)$  for a sampling time  $T_s$ ,  $i$  and  $u_1$  are discretized integrals of  $I$  and  $U_1$  respectively over  $T_s$  with  $N_s$  samples, as:

$$u_1[n] = \sum_{k=1}^{N_s} U_1 \left( \left( \frac{k}{N_s} + n \right) T_s \right) \quad (13)$$

$$i[n] = \sum_{k=1}^{N_s} I \left( \left( \frac{k}{N_s} + n \right) T_s \right) \quad (14)$$

**Remark 1.** As it is not possible to measure the  $ACD$  during the operation of the plant,  $\beta$  cannot be obtained experimentally. Hence, this parameter is calculated using theoretical values for a regular pot operation. 105

## 2.2. Alumina Concentration

The variation of the alumina concentration  $wAl_2O_3$  can be modeled as the difference between the quantity injected by the feeders ( $wAl_2O_{3in}$ ) and the one consumed by the chemical reaction ( $wAl_2O_{3cons}$ ):

$$\frac{d}{dt} wAl_2O_3 = wAl_2O_{3in} - wAl_2O_{3cons} \quad (15)$$

The quantity injected by the feeders at time  $t$  can be represented by:

$$wAl_2O_{3in} = \frac{N_g m_{in}}{M} F \quad (16)$$

where  $N_g$  is the number of feeders in the pot,  $m_{in}$  is the amount of mass injected by feeders,  $M$  is the total bath mass, and  $F$  is the frequency of feeders. As the sampling time for this process is typically large, we assume that the alumina dissolving dynamics can be neglected. 110

The frequency of the feeders is considered as a second system input:

$$U_2 := F \quad (17)$$



Then, equation (16) becomes:

$$wAl_2O_{3in} = \frac{N_g m_{in}}{M} U_2 \quad (18)$$

The  $Al_2O_3$  consumption is given by the Faraday's law divided by the bath mass:

$$wAl_2O_{3cons} = \frac{Al_2O_{3m} C_e}{6\mathfrak{F}M} I \quad (19)$$

where  $Al_2O_{3m}$  is the alumina molar mass and 6 is the number of electrons required for the electrolysis to perform. Replacing equations (18) and (19) into (15):

$$\frac{d}{dt} wAl_2O_3 = \frac{N_g m_{in}}{M} U_2 - \frac{Al_2O_{3m} C_e}{6\mathfrak{F}M} I \quad (20)$$

and defining:

$$\alpha_1 = \frac{N_g m_{in}}{M}, \quad \alpha_2 = \frac{Al_2O_{3m} C_e}{6\mathfrak{F}M} \quad (21)$$

equation (20) can be written as:

$$\frac{d}{dt} wAl_2O_3 = \alpha_1 U_2 - \alpha_2 I \quad (22)$$

A discrete-time model for  $wAl_2O_3$  can then be obtained using a similar procedure for  $ACD$ :

$$wAl_2O_3[n+1] = wAl_2O_3[n] + \alpha_1 u_2[n] - \alpha_2 i[n] \quad (23)$$

with same notations as before, and:

$$u_2[n] = \sum_{k=1}^{N_s} U_2 \left( \left( \frac{k}{N_s} + n \right) T_s \right) \quad (24)$$

### 2.3. Pseudo-Resistance

Now that  $ACD$  and  $wAl_2O_3$  models are available, it is necessary to relate them with the output signal, pseudo-resistance  $R$ . From the physical modeling described in [19], it is possible to plot a curve that relates  $ACD$ ,  $wAl_2O_3$  and  $R$  as shown in Figure 2.

However, this relation is complex and takes into account other quantities that cannot be continuously measured. One possible solution to directly relate  $ACD$

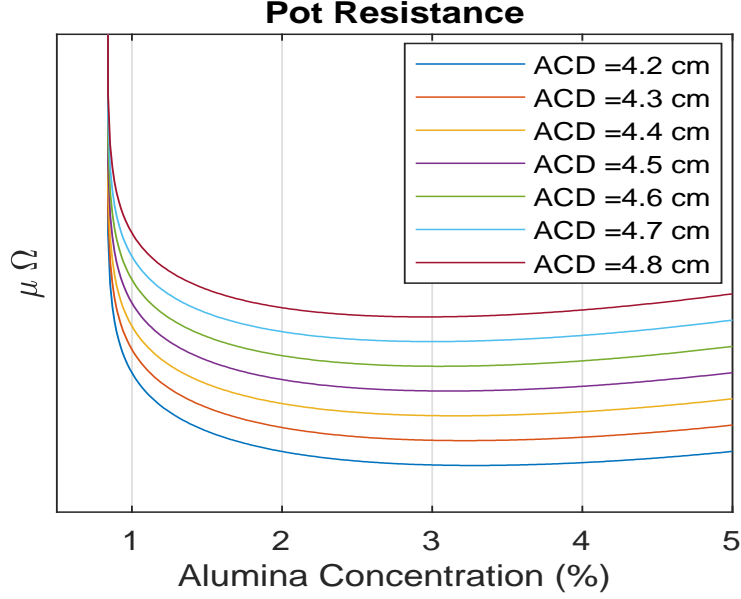


Figure 2: Typical pot resistance curve as a function of alumina concentration and  $ACD$

and  $wAl_2O_3$  is to approximate the curve with a polynomial function around the desired operational range. In particular, by denoting the  $ACD$  as  $z_1$ ,  $wAl_2O_3$  as  $z_2$  and considering an alumina concentration range between 2% and 4%, the curve can be approximated as follows:

$$R = c(z_2)^2 + (d + ez_1)z_2 + (f + gz_1) \quad (25)$$

where  $c$ ,  $d$ ,  $e$ ,  $f$ , and  $g$  are constant parameters to be determined. By differentiating equation (25), parameter  $f$  disappears and it is possible to obtain a dynamical model for the variation of pseudo-resistance as:

$$\frac{d}{dt}R = ez_1 \frac{d}{dt}z_2 + \left( 2c \frac{d}{dt}z_2 + e \frac{d}{dt}z_1 \right) z_2 + d \frac{d}{dt}z_2 + g \frac{d}{dt}z_1 \quad (26)$$

From equations (11) and (22), the pseudo-resistance as a function of the available inputs is given by:

$$\begin{aligned} \frac{d}{dt}R = ez_1 (\alpha_1 u_2 - \alpha_2 i) + [2c(\alpha_1 u_2 - \alpha_2 i) + e(\beta i + u_1)] z_2 \\ + d(\alpha_1 u_2 - \alpha_2 i) + g(\beta i + u_1) \end{aligned} \quad (27)$$

Using the discretization method described in previous subsections, it is possible to obtain the following model for the resistance variations:

$$\begin{aligned} R[n+1] = R[n] + e\Delta z_2[n]z_1[n] + (2c\Delta z_2[n] + e\Delta z_1[n])z_2[n] \\ + d\Delta z_2[n] + g\Delta z_1[n] \end{aligned} \quad (28)$$

where:

$$\Delta z_1[n] := u_1[n] + \beta i[n] \quad (29)$$

$$\Delta z_2[n] := \alpha_1 u_2[n] - \alpha_2 i[n] \quad (30)$$

### 3. Identification

Using the data collected from daily operational conditions, it is possible to identify parameters  $\alpha_1$  and  $\alpha_2$  in  $wAl_2O_3$  model equation (23). Then, the estimation can be performed for parameters  $c$ ,  $d$ ,  $e$  and  $g$  for  $R$  model equation (28). Next, it is presented an identification procedure, along with the corresponding validation.

#### 3.1. Alumina Concentration

During an operational day, just few alumina concentration measurements are recorded, and without constant sampling time. Hence, to identify parameters  $\alpha_1$  and  $\alpha_2$ , using the data collected at  $N_a$  times  $[n_1 T_s \quad n_2 T_s \quad \dots \quad n_{N_a} T_s]$  in seconds, equation (23) can be arranged in matrix form using the accumulation

of  $u_2$  and  $i$  signals in these intervals, as follows:

$$\begin{bmatrix} wAl_2O_3[n_2] - wAl_2O_3[n_1] \\ wAl_2O_3[n_3] - wAl_2O_3[n_2] \\ \vdots \\ wAl_2O_3[n_N] - wAl_2O_3[n_{N-1}] \end{bmatrix} = \begin{bmatrix} \sum_{k=n_1}^{n_2-1} u_2[k] & - \sum_{k=n_1}^{n_2-1} i[k] \\ \sum_{k=n_2}^{n_3-1} u_2[k] & - \sum_{k=n_2}^{n_3-1} i[k] \\ \vdots & \vdots \\ \sum_{k=n_{N-1}}^{n_N-1} u_2[k] & - \sum_{k=n_{N-1}}^{n_N-1} i[k] \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad (31)$$

125 Based on the structure of equation (31), it is possible to perform a least square estimation for parameters  $\alpha_1$  and  $\alpha_2$ . The experimental model is validated by a comparison between the simulated values of alumina concentration with estimated parameters and the alumina concentration values collected on the plant. Notice that the data used for validation are different from those used  
130 for the identification. The model is initialized with a real measurement. For every new alumina concentration sample collected, the model is reinitialized to improve the accuracy.

Figure 3 shows first the comparison between simulated and alumina concentration values, then the absolute relative error at measurement times, and the  
135 frequency of feeders and the line current. The model is initialized with measurement and the simulation is started using signals  $u_2$  and  $i$ . Every time a new alumina concentration measurement is available, the model is reinitialized to improve the accuracy of the prediction provided by the model.

By analyzing the absolute relative error plot and the inputs, it turns out  
140 that the model presents a very good fit for regular operation, when current is not affected by a disturbance or absence of feeding. In particular, the mean absolute relative error is smaller than 1 % for this condition. For the general situation, over the full period of time which is displayed, the model presents an average error of 9.2376%. Therefore, this model should be used for a regular  
145 pot operation to estimate the alumina concentration in the collected position.

### 3.2. Pseudo-Resistance

In equation (28), the parameter identification requires initial knowledge. On  $wAl_2O_3$ , which can be available at some specific times but also on  $ACD$ ,

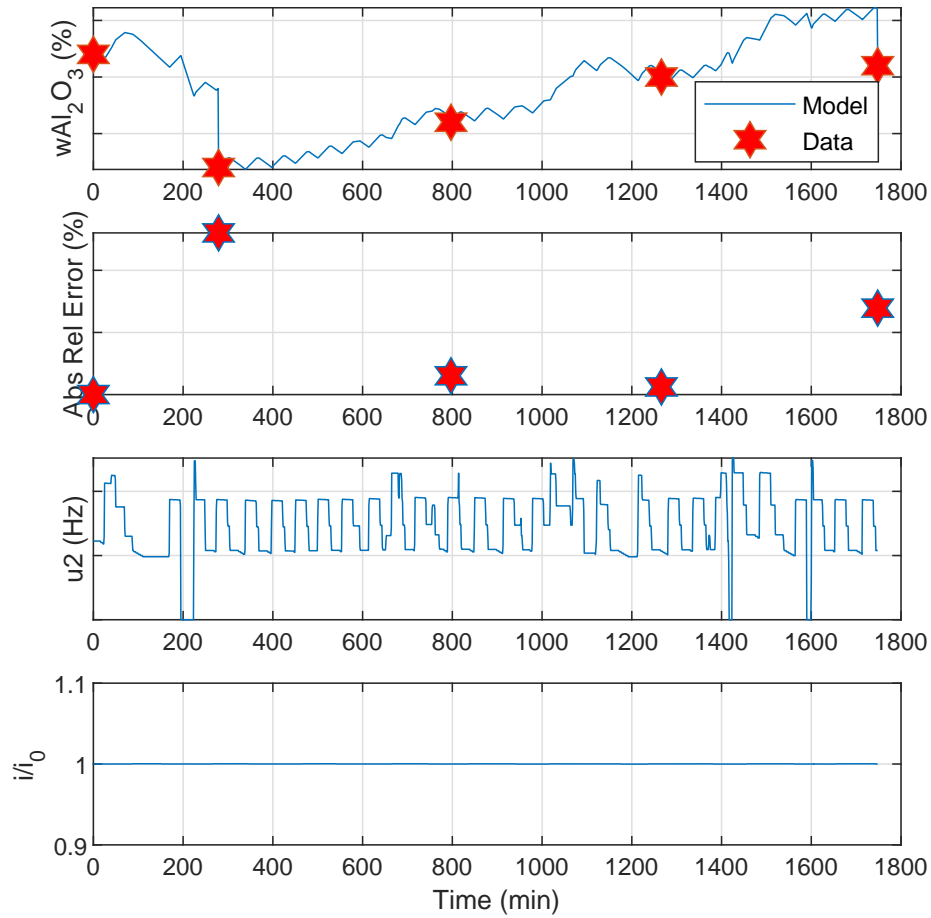


Figure 3: Alumina Concentration Model Validation

150 which is not measurable during usual operation. One solution is to initialize  
 the procedure at a time when  $wAl_2O_3$  is reliable, and  $ACD$  can be inferred  
 from specific physical knowledge, in a similar way as in as [19] for instance.  
 Then, using  $N$  samples at times as  $\left[0, T_s, \dots, (N-1)T_s\right]$ , it is possible to

organize equation (28) as follows:

$$\begin{bmatrix} \Delta R[1] \\ \vdots \\ \Delta R[N] \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ \vdots & \vdots & \vdots & \vdots \\ a_{N1} & a_{N2} & a_{N3} & a_{N4} \end{bmatrix} \begin{bmatrix} c \\ d \\ e \\ g \end{bmatrix} \quad (32)$$

where:

$$a_{11} = 2\Delta z_2[0]z_2[0]$$

$$a_{12} = \Delta z_2[0]$$

$$a_{13} = \Delta z_2[0]z_1[0] + \Delta z_1[0]z_2[0]$$

$$a_{14} = \Delta z_1[0]$$

$\vdots$

$$a_{N1} = 2\Delta z_2[N-1] \left( z_2[0] + \sum_{k=0}^{N-1} (\alpha_1 u_2[k] - \alpha_2 i[k]) \right)$$

$$a_{N2} = \Delta z_2[N-1]$$

$$a_{N3} = \Delta z_2[N-1] \left( z_1[0] + \sum_{k=0}^{N-1} (u_1[k] + \beta i[k]) \right) + \Delta z_1[N-1] \left( z_2[0] + \sum_{k=0}^{N-1} (\alpha_1 u_2[k] - \alpha_2 i[k]) \right)$$

$$a_{N4} = \Delta z_1[N-1]$$

with notations  $\Delta z_1$ ,  $\Delta z_2$  of equations (29)-(30), and:

$$\Delta R[k] = R[k] - R[k-1] \quad (33)$$

for  $k \geq 1$ .

155 At this stage, the parameters can be identified using again a least square estimation algorithm. In Figure 4, a comparison between the model output and actual measurements is shown.

The mean absolute error computed for this dataset is 2.21%. However, Figure 4 shows a small drift. This happens because the model (28) is an integrator.

160 Therefore, the difference between the model and the measurements accumulates

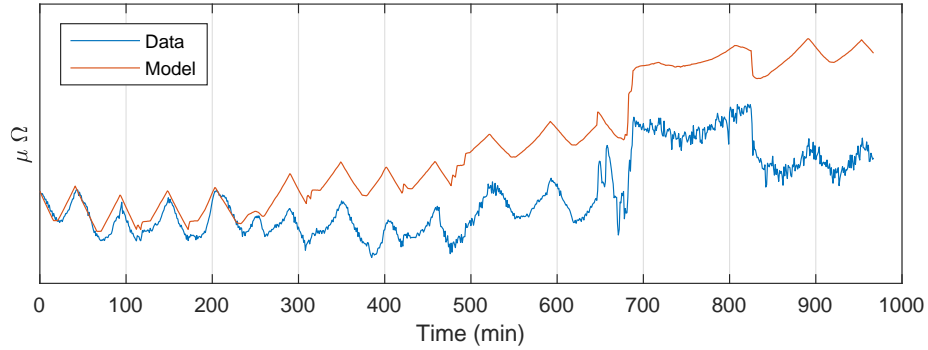


Figure 4: Resistance Model Validation

because of the integrator dynamics. Apart from this, it can be concluded that the proposed estimations are accurate.

#### 4. State Estimation

Based on the models established in Section 2 with the parameter identified  
 165 in Section 3, the problem of state estimation can now be addressed. Considering the current intensity as a measured disturbance in the system, it is possible to define a discrete-time time varying state-space model of the plant as follows:

$$\left\{ \begin{array}{l} \mathbf{x}[n+1] = \underbrace{\begin{bmatrix} 1 & a_{12}[n] & a_{13}[n] \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A[n]} \mathbf{x}[n] + \underbrace{\begin{bmatrix} b_1[n] \\ b_2[n] \\ b_3[n] \end{bmatrix}}_{B[n]} \\ y[n] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_C \mathbf{x}[n] \end{array} \right. \quad (34)$$

where:

$$a_{12}[n] = e(\alpha_1 u_2[n] - \alpha_2 i[n]) \quad (35)$$

$$a_{13}[n] = (2c(\alpha_1 u_2[n] - \alpha_2 i[n]) + e(u_1[n] + \beta i[n])) \quad (36)$$

$$b_1[n] = (d(\alpha_1 u_2[n] - \alpha_2 i[n]) + g(u_1[n] + \beta i[n])) \quad (37)$$

$$b_2[n] = (u_1[n] + \beta i[n]) \quad (38)$$

$$b_3[n] = (\alpha_1 u_2[n] - \alpha_2 i[n]) \quad (39)$$

with all those varying parameters being known quantities and the state vector given by:

$$\mathbf{x}[n] = \begin{bmatrix} R[n] \\ ACD[n] \\ wAl_2O_3[n] \end{bmatrix} \in \mathbb{R}^3 \quad (40)$$

Notice that model (34) is affine in the state. This makes it possible to design an exact linear observer for the nonlinear model by feeding it with measured quantities of equations (35)-(39). To perform this state estimation, a Time-Varying Linear Kalman Filter [20] is used as a state observer. In particular, Kalman Filter equations are given as follows:

*Prediction:*

$$\hat{\mathbf{x}}[n+1]^- = A[n]\hat{\mathbf{x}}[n] + B[n]$$

$$P[n+1|n] = A[n]P[n|n]A[n]^T + Q_{noise}$$

*Update:*

$$K[n] = P[n|n-1]C^T(CP[n|n-1]C^T + R_{noise})^{-1}$$

$$\hat{\mathbf{x}}[n] = \hat{\mathbf{x}}[n]^- + K[n](y[k] - C\hat{\mathbf{x}}[n]^-)$$

$$P[n|n] = (I - K[n]C)P[n|n-1]$$

where  $\hat{\mathbf{x}}$  is the state estimate vector,  $P$  is the covariance matrix,  $Q_{noise}$  is the process noise intensity matrix and  $R_{noise}$  is the measurement noise matrix. To implement the observer, it is necessary to tune the covariance and noise matrices.



For the considered data, the initial covariance matrix  $P[0|0]$  is set to:

$$P[0|0] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10^1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (41)$$

The largest entry in the  $P$  matrix is related to the  $ACD$  [14]. As it is not possible to measure this state, it is given a larger priority. The process and measurement noise matrices,  $Q_{noise}$  and  $R_{noise}$  respectively are chosen from various trials (as in [14])

$$Q_{noise} = \begin{bmatrix} 10^{-5} & 0 & 0 \\ 0 & 10^{-5} & 0 \\ 0 & 0 & 10^{-5} \end{bmatrix}, \quad R_{noise} = 10^{-5} \quad (42)$$

The proposed observer is tested on data sets with different initial conditions. For each test, a different combination of  $ACD$  and  $wAl_2O_3$  initial values is used in a certain operational range, while the values of  $R$  are initialized using real measurements. The resulting estimates are shown in Figure 5 with a zoom on initial times in Figure 6. The solid line is the pot resistance measurement, the dashed lines are the states estimates and the “x” markers are the alumina concentration measurements.

From both figures, it is possible to notice a fast convergence, for all estimates regardless of the initial conditions. In addition, Figure 5 also shows the estimates provided by the observer over a long period and compares those with the measurements of  $wAl_2O_3$ . The values of the mean absolute error reported in Table 1 confirm the effectiveness of the proposed estimation strategy: good results are indeed achieved jointly for resistance and alumina concentration estimation, while relying on a single model combining their evolution. This validates the overall model and makes the estimation results for  $ACD$  (which are consistent with usual industrial knowledge) quite trustable.

## 5. Conclusions

185 In this paper, a modeling approach to capture the main dynamics of the Hall-  
Héroult process has been proposed. In particular, a state affine representation  
has been obtained, that combines physical-chemical aspects and experimental  
data. On this basis, a Linear Kalman filter is provided to estimate the states  
of the system. The model and observer have been validated with experimental  
190 data. The proposed observer strategy proved to be reliable in providing accurate  
estimates of the plant states for unknown initial conditions. The use of these  
results for monitoring and closed-loop control are part of our ongoing work.

## References

- [1] K. Grjotheim, B. J. Welch, Aluminium Smelter Technology: A Pure and  
195 Applied Approach, Aluminium-Verlag; 2nd edition, 1980.
- [2] J. T. Keniry, G. C. Barber, M. P. Taylor, B. J. Welch, Digital processing  
of anode current signals: an opportunity for improved cell diagnosis and  
control, Light Metals (2001) 1225–1232.
- [3] P. Côté, O. Martin, B. Allano, V. Dassylva-Raymond, Predicting instability  
200 and current efficiency of industrial cells, in: Light Metals 2017, Springer,  
2017, pp. 623–629.
- [4] M. Dupuis, V. Bojarevics, Demo retrofit study of a chinese inspired cell  
technology, in: Light Metals 2020, Springer, 2020, pp. 495–509.
- [5] D. S. Wong, G. Matthews, A. T. Tabereaux, T. Buckley, M. M. Dorreen,

Table 1: Estimations Final Values Comparison

	Mean Absolute Relative Error
$R$	0.07%
$wAl_2O_3$	4.19 %

- 205 The Australian energy crisis, its impact on domestic aluminium smelting and potential solutions, in: *Light Metals 2020*, Springer, 2020, pp. 791–802.
- [6] S. R. Jakobsen, K. Hestetun, M. Hovd, I. Solberg, Estimating alumina concentration distribution in aluminium electrolysis cells, *IFAC Proceedings Volumes* 34 (18) (2001) 303–308.
- 210 [7] Y. Yao, C.-Y. Cheung, J. Bao, M. Skyllas-Kazacos, B. J. Welch, S. Akhmetov, Estimation of spatial alumina concentration in an aluminum reduction cell using a multilevel state observer, *AIChE Journal* 63 (7) (2017) 2806–2818.
- [8] G. P. Bearne, The development of aluminum reduction cell process control, 215 *JOM* 51 (5) (1999) 16–22.
- [9] P. Biedler, Modeling of an aluminum reduction cell for the development of a state estimator, Ph.D. thesis, West Virginia University (2003).
- [10] P. Homsy, J.-M. Peyneau, M. Reverdy, Overview of process control in reduction cells and potlines, in: *Essential Readings in Light Metals*, Springer, 220 2016, pp. 739–746.
- [11] S. Kolås, B. Foss, T. S. Schei, State estimation is the real challenge in NMPC, in: *International workshop on assessment and future directions of nonlinear model predictive control*, Pavia, Italy, 2008.
- [12] K. Hestetun, M. Hovd, Detecting abnormal feed rate in aluminium electrolysis using extended Kalman filter, *IFAC Proceedings Volumes* 38 (1) 225 (2005) 85–90.
- [13] J. B. Moore, B. Anderson, *Optimal filtering*, Prentice-Hall, 1979.
- [14] L. J. da Silva Moreira, G. Besançon, F. Ferrante, M. Fiacchini, H. Roustan, Model based approach for online monitoring of aluminum production 230 process, in: *Light Metals 2020*, Springer, 2020, pp. 566–571.

- [15] L. J. da Silva Moreira, M. Fiacchini, G. Besançon, F. Ferrante, H. Roustan, State affine modeling and observer design for Hall-Hérault process, in: IFAC World Congress 2020, 2020.
- [16] A. Țiclea, G. Besançon, Exponential forgetting factor observer in discrete  
235 time, *Systems & Control Letters* 62 (9) (2013) 756–763.
- [17] S.-q. Zhan, L. Mao, J.-m. Zhou, J.-h. Yang, Y.-w. Zhou, Analysis and modeling of alumina dissolution based on heat and mass transfer, *Transactions of nonferrous metals society of China* 25 (5) (2015) 1648–1656.
- [18] C. G. Zoski, *Handbook of electrochemistry*, Elsevier, 2006.
- [19] W. Haupin, Interpreting the components of cell voltage, in: *Essential Readings in Light Metals*, Springer, 2016, pp. 153–159.  
240
- [20] R. E. Kalman, A new approach to linear filtering and prediction problems, *Journal of basic Engineering* 82 (1) (1960) 35–45.

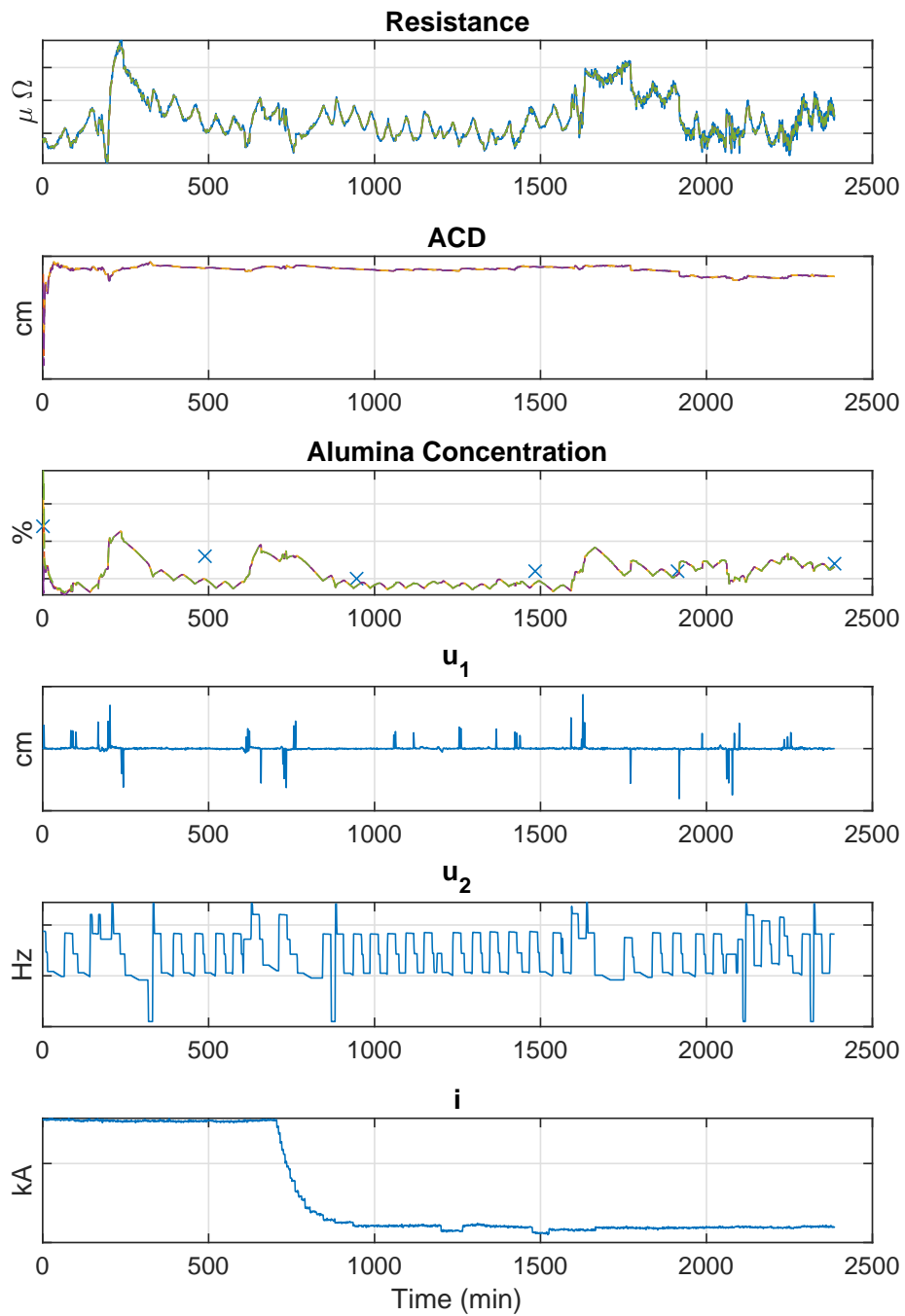


Figure 5: State Estimation for a long period - Each dashed line is a state estimation with different initial condition and the solid line is the measurement.

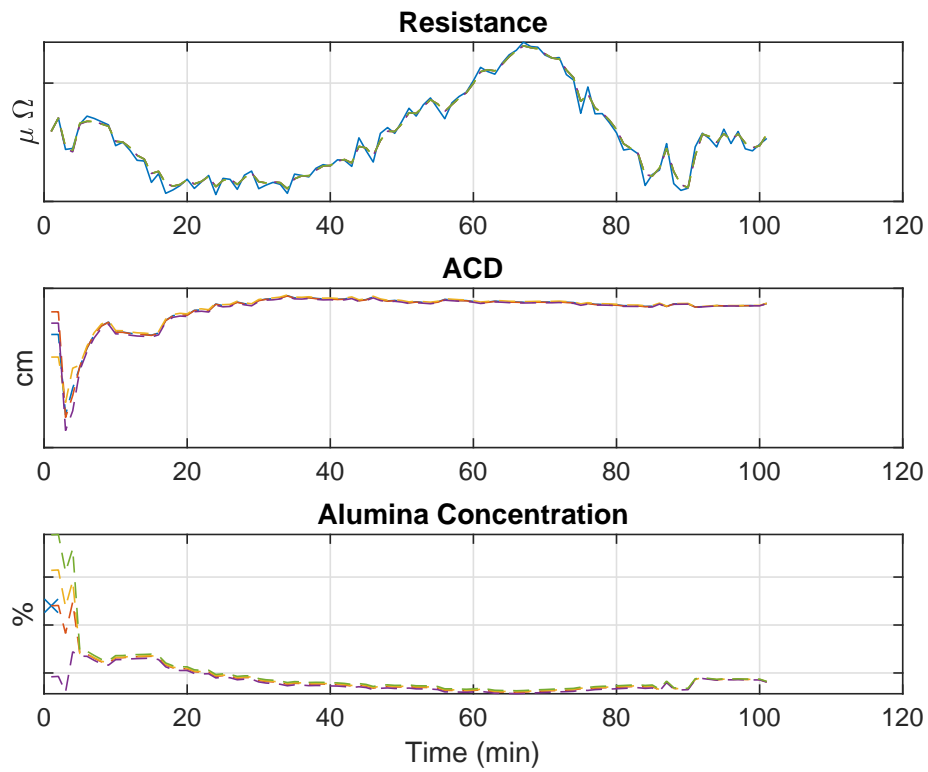


Figure 6: State Estimation for a short period - Each dashed line is a state estimation with different initial condition and the solid line is the measurement.