



HAL
open science

Asynchronous Switched Takagi-Sugeno H_∞ Filters Design For Switched Nonlinear Systems

Issam Chekakta, Djamel E.C. Belkhiat, Kevin Guelton, Koffi Motchon, Dalel
Jabri

► **To cite this version:**

Issam Chekakta, Djamel E.C. Belkhiat, Kevin Guelton, Koffi Motchon, Dalel Jabri. Asynchronous Switched Takagi-Sugeno H_∞ Filters Design For Switched Nonlinear Systems. International Conference on Advances in Control and Optimization of Dynamical Systems (ACODS), 2022, Silchar, India. pp.351-356, 10.1016/j.ifacol.2022.04.058 . hal-03663496

HAL Id: hal-03663496

<https://hal.science/hal-03663496v1>

Submitted on 10 May 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0
International License

Asynchronous Switched Takagi-Sugeno H_∞ Filters Design For Switched Nonlinear Systems

Issam Chekakta* Djamel E.C. Belkhiat* Kevin Guelton**
Koffi M. D. Motchon** Dalel Jabri*

* *Ferhat Abbas University Setif, Setif, Algeria (e-mail: {issam.chekakta,djamel.belkhiat,dalel.jabri}@univ-setif.dz).*

** *CReSTIC EA3804 - EISINe, Reims Champagne-Ardenne University, 51687 Reims, France (e-mail: {kevin.guelton,koffi.motchon}@univ-reims.fr)*

Abstract: In this paper, the H_∞ filter design is investigated for switched T-S systems. Based on a descriptor redundancy formulation of the error dynamics, new relaxed LMI-based conditions are proposed, which are suitable when the switched filter's modes can be asynchronous with regards to the system's ones. Compared with previous related literature, these conditions have the advantage of being free of any dwell-time constraints. Considering the simulation of an academic example, comparisons are done regarding to previous results, placing the proposed switched T-S filter design conditions among the less conservative, and illustrating their effectiveness, even when the filter's modes are not properly matching the system's ones.

Copyright © 2022 The Authors. This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0/>)

Keywords: H_∞ Filtering, Switched Takagi-Sugeno systems, Asynchronous switched modes, Switched Lyapunov functions, LMIs.

1. INTRODUCTION

Switched systems have attracted the attention of many researchers over the last few years, because of their generality and practical use in a wide range of applications, such like power systems, robotics, navigation, and so on (Zhao et al., 2017). The main interest of switched systems is to describe the whole dynamics of systems which involve discrete event-based interactions between continuous subsystems. In this context, many research efforts have been done to address issues such like modeling, stability analysis, state estimation and control, for both linear and nonlinear switched systems (see e.g., (Liberzon, 2003; Belkhiat et al., 2011; Iqbal et al., 2014; Li et al., 2018; Gómez-Gutiérrez et al., 2015; Zhao et al., 2017; Etienne et al., 2020; Han et al., 2020)). Furthermore, Takagi-Sugeno (T-S) model provides an attractive framework to deal with systems that exhibit nonlinear behavior (Tanaka and Wang, 2001). Extending this framework to switched nonlinear systems, several works have been carried out to deal with their stability analysis, stabilization or observer design (see e.g., (Belkhiat et al., 2014; Bouibed et al., 2014; Zhai et al., 2018; Jabri et al., 2018; Belkhiat et al., 2019; Cherifi et al., 2019; Du et al., 2020; Chekakta et al., 2021)).

Among relevant issues related to switched T-S systems, we focus in this paper on H_∞ filtering techniques. Indeed, such filters are particularly useful to estimate some of the system's outputs, which are not measured or affected by noise disturbances (Chang, 2012). Compared with other filtering approach such as Kalman filters, the main interest of H_∞ filters are their robustness against unmodeled dynamics as well as they don't require any knowledge

on the noise disturbance signals, excepted their bounded-energy condition (Tuan et al., 2001a). H_∞ filter design for switched T-S systems has been the subject of recent investigations. Synchronous H_∞ filter design for continuous-time switched T-S systems based on persistent dwell-time with a prescribed non-weighted noise attenuation performance has been investigated in (Shi et al., 2019). In (Hong et al., 2018) and (Liu et al., 2020), the asynchronous case has been considered using multiple Lyapunov functions with dwell-times conditions.

In a nutshell, the aforementioned works were only valid for the H_∞ filtering of switched T-S systems under dwell-time considerations, which may be useless when the switched sequence is uncontrolled, arbitrary or unknown. Hence, the goal of this paper is to propose dwell-time free LMI-based conditions for the design of H_∞ filters for switched T-S systems. To do so and to reduce the conservatism of the design conditions, LMI-based relaxation techniques (Peaucelle et al., 2000; Tuan et al., 2001b) are considered together with a descriptor redundancy approach (Schulte and Guelton, 2009; Guelton et al., 2009; Bouarar et al., 2013; Jabri et al., 2020) to express the filtered error dynamics, such that the considered Lyapunov function candidate only depends state and output filtering errors. The effectiveness of the proposed H_∞ filter design methodology is illustrated through the simulation of an academic example, with comparison to related results from the literature.

2. OBJECTIVES AND PRELIMINARIES

Let us consider the class of switched T-S systems given by:

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(\xi_j(t)) (A_{i_j} x(t) + B_{i_j} w(t)) \\ z(t) = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(\xi_j(t)) F_{i_j} x(t) \\ y(t) = Cx(t) + Dw(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^{v_2}$ is the measured output vector and $z(t) \in \mathbb{R}^{v_1}$ is the unmeasured output to be estimated, $w(t) \in \mathbb{R}^{v_3}$ is a time-varying L_2 norm bounded exogenous disturbance input vector. The number of switched modes is denoted by m and the number of fuzzy rules in the j^{th} mode by r_j ($j = 1, \dots, m$), $\xi_j(t)$ are the vectors of premise variables in each modes, which are assumed to be measurable in this paper, and, $\forall i_j = 1, \dots, r_j$, $h_{i_j}(\xi_j(t))$ are normalized membership functions satisfying the convex sum properties: $h_{i_j}(\xi_j(t)) > 0$ and $\sum_{i=1}^{r_j} h_{i_j}(\xi_j(t)) = 1$. The matrices $A_{i_j} \in \mathbb{R}^{n \times n}$, $B_{i_j} \in \mathbb{R}^{n \times v_3}$, $C \in \mathbb{R}^{v_2 \times n}$, $D \in \mathbb{R}^{v_2 \times v_3}$, $F_{i_j} \in \mathbb{R}^{v_1 \times n}$ define the vertices of T-S subsystems in each switched modes, $\sigma_j(t)$ is the switching law such that $\sigma_j(t) = 1$ when the active system j is in the l^{th} mode and $\sigma_j(t) = 0$ when $j \neq l$.

In the sequel, to cope with the problem of H_∞ filtering for the switched T-S fuzzy system (1), i.e. with the estimation of the unmeasured output vector $z(t)$, let us consider the following asynchronous mode-dependent T-S fuzzy filter:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \hat{\sigma}_j(t) h_{i_j}(\hat{y}(t)) (\hat{A}_{i_j} \hat{x}(t) + \hat{B}_{i_j} \hat{y}(t)) \\ \hat{z}(t) = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \hat{\sigma}_j(t) h_{i_j}(\hat{y}(t)) \hat{F}_{i_j} \hat{x}(t) \end{cases} \quad (2)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the filter's state vector, $\hat{z}(t) \in \mathbb{R}^{v_1}$ is the estimate of $z(t)$, $\hat{A}_{i_j} \in \mathbb{R}^{n \times n}$, $\hat{B}_{i_j} \in \mathbb{R}^{n \times v_2}$, $\hat{F}_{i_j} \in \mathbb{R}^{v_3 \times n}$ are gain matrices to be synthesized and, for $\hat{j} = 1, \dots, m$, $\hat{\sigma}_j(t)$ is the switching law of the filter, which may be asynchronous regarding to the system's one ($\sigma_j(t)$).

In the sequel, to lighten mathematical expressions, the time t as functions argument will be omitted when convenient and convex combinations of matrices with appropriate dimensions will be denoted by:

$$X_{h_\sigma} = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(\xi_j(t)) X_{i_j},$$

$$X_{h_{\hat{\sigma}}} = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \hat{\sigma}_j(t) h_{i_j}(\xi_j(t)) X_{i_j}.$$

Let us define the filtering errors $e_z(t) = z(t) - \hat{z}(t)$ and $e_x(t) = x(t) - \hat{x}(t)$. Then, consider the augmented vectors $e(t) = [e_x(t)^T \ e_z(t)^T]^T$ and $\tilde{w}(t) = [x(t)^T \ \hat{x}(t)^T \ w(t)^T]^T$, the dynamics of the filtering error can be written as the following descriptor:

$$E\dot{e}(t) = \tilde{A}_{h_\sigma} e(t) + \tilde{B}_{h_\sigma h_{\hat{\sigma}}} \tilde{w}(t) \quad (3)$$

with:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A}_{h_\sigma} = \begin{bmatrix} A_{h_\sigma} & 0 \\ 0 & -I \end{bmatrix},$$

$$\tilde{B}_{h_\sigma h_{\hat{\sigma}}} = \begin{bmatrix} -\hat{B}_{h_{\hat{\sigma}}} C & A_{h_\sigma} - \hat{A}_{h_{\hat{\sigma}}} & B_{h_\sigma} - \hat{B}_{h_{\hat{\sigma}}} D \\ F_{h_\sigma} & -\hat{F}_{h_{\hat{\sigma}}} & 0 \end{bmatrix}.$$

Problem statement: Provide new LMI-based conditions for the design of asynchronous switched T-S filters (2) such that:

- i. When $\tilde{w}(t) = 0$, the filtering error dynamics (3) is asymptotically converging to the origin.
- ii. $\forall \tilde{w}(t) \neq 0$, the transfer between $\tilde{w}(t)$ and the output estimation error $e_z(t)$ is minimized with an attenuation level γ defined such that:

$$\frac{\|e_z\|_2^2}{\|\tilde{w}\|_2^2} \leq \gamma^2 \quad (4)$$

that is to say, regarding to the following H_∞ criterion:

$$\int_0^\infty e_z^T(t) e_z(t) dt \leq \gamma^2 \int_0^\infty \tilde{w}^T(t) \tilde{w}(t) dt \quad (5)$$

Recall that previous related results, e.g. (Hong et al., 2018; Shi et al., 2019; Liu et al., 2020), were only available for switched T-S (3) assuming dwell-time conditions. In the sequel, thank to the descriptor form of the filtering error dynamics (3), we propose LMI-based conditions, which are independent of any dwell-time consideration, which constitutes the main contribution of this work. Furthermore, the following lemmas will be considered to provide the main result proposed in the next section.

Lemma 1. (Peaucelle et al., 2000) For any matrices N , P and Q with appropriate dimensions, the inequality $N^T P + P^T N + Q < 0$ holds if:

$$\exists R, \exists L \left[\begin{array}{cc} N^T L^T + LN + Q & (*) \\ P - L^T + R^T N & -R^T - R \end{array} \right] < 0 \quad (6)$$

Lemma 2. (Tuan et al., 2001b) $\forall (i, j) \in \{1, \dots, r\}^2$, let $\delta_i(\cdot) \geq 0$ being scalar functions satisfying the convex sum property $\sum_{i=1}^r \delta_i(\cdot) = 1$. For same sized symmetric matrices Γ_{ij} , the inequality $\sum_{i=1}^r \sum_{k=1}^r \delta_i(\cdot) \delta_j(\cdot) \Gamma_{ij} < 0$ is verified if:

$$\frac{1}{r-1} \Gamma_{ii} + \frac{1}{2} (\Gamma_{ji} + \Gamma_{ij}) < 0 \quad (7)$$

3. LMI-BASED FILTER DESIGN CONDITIONS

The following theorem summarize the proposed LMI conditions for the H_∞ design of asynchronous switched T-S filters (2) for switched T-S systems (1) (where dwell-time conditions are not required).

Theorem 3. $\forall (j, \hat{j}) \in \{1, \dots, m\}^2$, $\forall (i_j, k_j) = \{1, \dots, r_j\}^2$ and $\forall q_j \in \{1, \dots, r_j\}$, the conditions i. and ii. of the above defined problem statement of H_∞ filter (2) design are satisfied if there exist a real scalar γ and real matrices \hat{A}_{k_j} , \hat{B}_{k_j} , \hat{F}_{k_j} , G_j , $L_{k_j q_j}$, $R_{k_j q_j}$, $P_j^1 = P_j^{1T}$, P_j^2 and P_j^3 , which verify the following optimization problem:

$$\begin{cases} \min \gamma^2, \\ \text{subject to (9), (10) and } P_j^1 > 0 \end{cases} \quad (8)$$

$$P_{j^+}^1 = P_j^1 + G_j^T C + C^T G_j \quad (9)$$

$$\frac{1}{r_j-1} \Gamma_{i_j i_j q_j} + \frac{1}{2} (\Gamma_{i_j k_j q_j} + \Gamma_{k_j i_j q_j}) < 0 \quad (10)$$

with :

$$\Gamma_{i_j k_j q_j} = \begin{bmatrix} \Phi_{i_j k_j q_j}^1 & (*) & (*) & (*) \\ \Phi_{i_j k_j q_j}^2 & -R_{i_j q_j}^T - R_{i_j q_j} & (*) & (*) \\ \tilde{B}_{i_j q_j}^T & 0 & -\gamma^2 I & (*) \\ P_{i_j}^3 \phi & 0 & 0 & -I \end{bmatrix},$$

$$\begin{aligned} \Phi_{i_j k_j q_j}^1 &= \tilde{A}_{i_j} L_{k_j q_j}^T + L_{k_j q_j} \tilde{A}_{i_j}^T, \quad \Phi_{i_j k_j q_j}^2 = P_j - L_{k_j q_j}^T + \\ R_{k_j q_j}^T \tilde{A}_{i_j}^T, \quad P_j &= \begin{bmatrix} P_j^1 & 0 \\ P_j^2 & P_j^3 \end{bmatrix}, \quad \phi = [0 \ I], \quad \tilde{A}_{i_j} = \begin{bmatrix} A_{i_j} & 0 \\ 0 & -I \end{bmatrix} \text{ and} \\ \tilde{B}_{i_j q_j} &= \begin{bmatrix} -\hat{B}_{q_j} C & A_{i_j} - \hat{A}_{q_j} & B_{i_j} - \hat{B}_{q_j} D \\ F_{i_j} & -\hat{F}_{q_j} & 0 \end{bmatrix}. \end{aligned}$$

In that case, the filter's state vector is updated at the switching instants according to:

$$\hat{x}^+ = (I - Q_j^{-1} (C Q_j^{-1})^\dagger C) \hat{x} + Q_j^{-1} (C Q_j^{-1})^\dagger y \quad (11)$$

with $Q_j = \mathbf{V}_j \sqrt{\Lambda_j} \mathbf{V}_j^T$, where $\mathbf{V}_j \in \mathbb{R}^{n \times n}$ are composed of the orthonormal eigenvectors of P_j^1 , and where $\Lambda_j \in \mathbb{R}^{n \times n}$ are diagonal matrices, which entries are the eigenvalues of P_j^1 .

Proof. Let us consider a multiple Lyapunov candidate function given by:

$$V(t, e(t)) = e(t)^T E^T P_{\hat{\sigma}}^{-1} e(t) \quad (12)$$

where $EP_{\hat{\sigma}} = P_{\hat{\sigma}}^T E > 0$, $P_{\hat{\sigma}} = \sum_{j=1}^m \hat{\sigma}_j(t) P_j$ with $P_j = \begin{bmatrix} P_j^1 & 0 \\ P_j^2 & P_j^3 \end{bmatrix}$ and $P_j^1 = P_j^{1T} > 0$.

Taking the time derivative of (12), yields:

$$\begin{aligned} \dot{V}(t, e(t)) &= \dot{e}^T(t) E^T P_{\hat{\sigma}}^{-1} e(t) + e^T(t) P_{\hat{\sigma}}^{-T} E \dot{e}(t) \\ &= 2e^T(t) P_{\hat{\sigma}}^{-T} (\tilde{A}_{h_\sigma} e(t) + \tilde{B}_{h_\sigma h_{\hat{\sigma}}} \tilde{w}(t)) \end{aligned} \quad (13)$$

Hence, the items i. and ii. of the problem statement defined in section II are satisfied if there exists γ such that the following inequality is satisfied:

$$\begin{aligned} \dot{V}(t, e(t)) + e_z(t)^T e_z(t) - \gamma^2 \tilde{w}(t)^T \tilde{w}(t) \\ = \begin{bmatrix} e \\ \tilde{w} \end{bmatrix}^T \begin{bmatrix} \tilde{A}_{h_\sigma}^T P_{\hat{\sigma}}^{-1} + P_{\hat{\sigma}}^{-T} \tilde{A}_{h_\sigma} + \psi & (*) \\ \tilde{B}_{h_\sigma h_{\hat{\sigma}}}^T P_{\hat{\sigma}}^{-1} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} e \\ \tilde{w} \end{bmatrix} < 0 \end{aligned} \quad (14)$$

with $\psi = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$.

$\forall [e^T \ \tilde{w}^T] \neq 0$, taking the congruence by $\begin{bmatrix} P_{\hat{\sigma}} & 0 \\ 0 & I \end{bmatrix}$, the inequality (14) is satisfied if:

$$\begin{bmatrix} P_{\hat{\sigma}}^T \tilde{A}_{h_\sigma}^T + \tilde{A}_{h_\sigma} P_{\hat{\sigma}} + P_{\hat{\sigma}}^T \psi P_{\hat{\sigma}} & (*) \\ \tilde{B}_{h_\sigma h_{\hat{\sigma}}}^T & -\gamma^2 I \end{bmatrix} < 0 \quad (15)$$

Applying lemma 1, (15) holds if $\exists L_{h_\sigma h_{\hat{\sigma}}}$ and $\exists R_{h_\sigma h_{\hat{\sigma}}}$ such that:

$$\begin{bmatrix} \Phi_{h_\sigma h_{\hat{\sigma}}}^1 + P_{\hat{\sigma}}^T \psi P_{\hat{\sigma}} & (*) & (*) \\ \Phi_{h_\sigma h_{\hat{\sigma}}}^2 & \Phi_{h_\sigma h_{\hat{\sigma}}}^3 & (*) \\ \tilde{B}_{h_\sigma h_{\hat{\sigma}}}^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (16)$$

with $\Phi_{h_\sigma h_{\hat{\sigma}}}^1 = \tilde{A}_{h_\sigma} L_{h_\sigma h_{\hat{\sigma}}}^T + L_{h_\sigma h_{\hat{\sigma}}} \tilde{A}_{h_\sigma}^T$, $\Phi_{h_\sigma h_{\hat{\sigma}}}^2 = P_{\hat{\sigma}} - L_{h_\sigma h_{\hat{\sigma}}}^T + R_{h_\sigma h_{\hat{\sigma}}}^T \tilde{A}_{h_\sigma}^T$, $\Phi_{h_\sigma h_{\hat{\sigma}}}^3 = -R_{h_\sigma h_{\hat{\sigma}}}^T - R_{h_\sigma h_{\hat{\sigma}}}$. Then, applying the Schur complement, (16) becomes:

$$\begin{bmatrix} \Phi_{h_\sigma h_{\hat{\sigma}}}^1 & (*) & (*) & (*) \\ \Phi_{h_\sigma h_{\hat{\sigma}}}^2 & \Phi_{h_\sigma h_{\hat{\sigma}}}^3 & (*) & (*) \\ \tilde{B}_{h_\sigma h_{\hat{\sigma}}}^T & 0 & -\gamma^2 I & (*) \\ P_{\hat{\sigma}}^3 \phi & 0 & 0 & -I \end{bmatrix} < 0 \quad (17)$$

with $\phi = [0 \ I]$.

Applying Lemma 2, (17) holds if the LMIs (10) are satisfied.

Inspired by the methodology proposed in (Pettersson, 2005), to avoid any dwell time conditions, the goal is now to guarantee the decreasing of the Lyapunov function candidate (when $w(t) = 0$) at the filter's switching instants. This occurs if:

$$(x - \hat{x}^+)^T P_{j^+}^1 (x - \hat{x}^+) \leq (x - \hat{x})^T P_j^1 (x - \hat{x}) \quad (18)$$

where $\hat{x}^+(t)$ is the updated value of the state vector after the switching from j to j^+ , which denote respectively the previous and next switching mode of the filter.

Assuming that \hat{x}^+ verifies $y = C\hat{x}^+$ and, without disturbance, since $y = Cx$, we get $C(x - \hat{x}^+) = 0$. Therefore, any matrices G_j verify the following equality:

$$(x - \hat{x}^+)^T (G_j^T C + C^T G_j) (x - \hat{x}^+) = 0 \quad (19)$$

Hence, assuming (9), the inequality (18) becomes:

$$(x - \hat{x}^+)^T P_j^1 (x - \hat{x}^+) \leq (x - \hat{x})^T P_j^1 (x - \hat{x}) \quad (20)$$

This ensures the decreasing of the Lyapunov function if the filter's state is properly updated when it switches. To do so, consider the spectral decomposition $P_j^1 = Q_j^T Q_j$ with $Q_j = \mathbf{V}_j \sqrt{\Lambda_j} \mathbf{V}_j^T \in \mathbb{R}^{n \times n}$ (Pettersson, 2005). The inequality (20) is satisfied if:

$$\|Q_j(x - \hat{x}^+)\| \leq \|Q_j(x - \hat{x})\| \quad (21)$$

To compute the update value of the filter's state \hat{x}^+ belonging to the hyperplane $y = C\hat{x}^+$ and minimizing the distance $\|Q_j(\hat{x}^+ - \hat{x})\|$, the following optimization problem is considered:

$$\begin{cases} \min_{\hat{x}^+} \|Q_j(\hat{x}^+ - \hat{x})\| \\ \text{s.c. } C\hat{x}^+ = y \end{cases} \quad (22)$$

Introducing $\nu_j = Q_j(\hat{x}^+ - \hat{x})$, we get $Q_j \hat{x}^+ = \nu_j + Q_j \hat{x}$. The above defined optimization problem can be rewritten as:

$$\begin{cases} \min_{\hat{x}^+} \|\nu_j\| \\ \text{s.c. } CQ_j^{-1} \nu_j = y - C\hat{x} \end{cases} \quad (23)$$

which solution is given by:

$$\nu_j = (CQ_j^{-1})^\dagger (y - C\hat{x}) \quad (24)$$

hence $Q_j \hat{x}^+ = Q_j \hat{x} + (CQ_j^{-1})^\dagger (y - C\hat{x})$, which leads to (11) after left-multiplication by Q_j^{-1} . This ends the proof.

Remark 4. Note that the LMI-based conditions proposed in Theorem 1 are generic and holds for arbitrary switching sequences. Nevertheless, when the switching sequence is priory known, a straightforward simplification arises by reducing the number of LMI constraints with the proper switching set S_{j,j^+} , containing all admissible switches for the switched T-S system, and \hat{S}_{j,j^+} for the switched T-S filter. Moreover, the synchronous switched filter design constitutes a special case of Theorem 1 where $\hat{j} = j \in [1, m]$.

4. ILLUSTRATIVE EXAMPLE

To illustrate the effectiveness of the proposed switched T-S filter design methodology, let us consider an example of a switched T-S model (1), inspired by (Wang et al., 2016; Liu et al., 2020; Shi et al., 2019). This model, defined by the following matrices and membership functions, switches between $m = 3$ modes ($j \in \{1, 2, 3\}$), and have two fuzzy rules in each mode ($r_j = 2$).

$$A_{11} = \begin{bmatrix} 5a & 1 \\ -4.9 & 3(b+a) \end{bmatrix}, A_{21} = \begin{bmatrix} a & -b \\ -4.2 & -3 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 1 \\ -4.3 & -3.4 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 \\ 2b - 0.5 & -3.4 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0 & 1 \\ -5.5 & -3.2 \end{bmatrix}, A_{23} = \begin{bmatrix} 0 & 1 \\ -2.9 & b - 2.2 \end{bmatrix},$$

$$\forall i_j, B_{i_j} = [0 \ 0.1]^T, C = [1 \ 0], D = 0.2, F_{i_j} = [0 \ 1],$$

$$h_{1_j}(x_1(t)) = \sin^2(x_1(t)) \text{ and } h_{2_j}(x_1(t)) = \cos^2(x_1(t)).$$

Note that a and b are two extra parameters dedicated to investigate the feasibility fields of the LMI conditions ($a = 0$ and $b = -1$ recover the vertices considered in (Wang et al., 2016; Liu et al., 2020; Shi et al., 2019)). In this context, using Matlab and the YALMIP toolbox (Lofberg, 2004) with SeDuMi solvers, Fig. 1 compares the feasibility fields obtained from Theorem 3, Theorem 1 in (Hong et al., 2018), Theorem 3 in (Shi et al., 2019) and Theorem 1 in (Liu et al., 2020). We notice that over the 403 points tested to draw the feasibility fields of each theorem, the solutions obtained from Theorem 1 (56.6%, 227 solutions) in this paper are close to the one obtained from Theorem 1 in (Liu et al., 2020) (50.4%, 203 solutions). Nevertheless, it is worth to point-out that the conditions of Theorem 1 in (Liu et al., 2020) requires dwell-time prescriptions, which is not the case of the conditions proposed as Theorem 3 in this paper. This constitutes the main advantage of our proposal regarding to (Liu et al., 2020). Moreover, the feasibility fields obtained from Theorem 1 includes the ones obtained from Theorem 1 in (Hong et al., 2018) and Theorem 3 in (Shi et al., 2019), which confirms the conservatism improvement raised by the present proposal. Finally, even if the computational complexity of the LMI-based conditions proposed in Theorem 3 is higher than the ones proposed in the considered previous related studies (Hong et al., 2018; Shi et al., 2019; Liu et al., 2020), it doesn't constitute a big drawback since these conditions are usually solved offline.

Now, consider the particular case where $a = 0$, $b = -1$, using YALMIP toolbox and SeDuMi solver in Matlab to solve the conditions of Theorem 1 provides a minimized H_∞ performance index of $\gamma = 1.41$ and the filter gains given by:

$$\hat{A}_{11} = \begin{bmatrix} 0.111 & 1.0 \\ -4.243 & -3.220 \end{bmatrix}, \hat{A}_{12} = \begin{bmatrix} 0.112 & 0.996 \\ -4.244 & -3.220 \end{bmatrix},$$

$$\hat{A}_{21} = \begin{bmatrix} 0.112 & 0.994 \\ -4.245 & -3.225 \end{bmatrix}, \hat{A}_{22} = \begin{bmatrix} 0.112 & 0.995 \\ -4.242 & -3.225 \end{bmatrix},$$

$$\hat{A}_{31} = \begin{bmatrix} 0.113 & 0.995 \\ -4.244 & -3.225 \end{bmatrix}, \hat{A}_{32} = \begin{bmatrix} 0.112 & 0.994 \\ -4.243 & -3.225 \end{bmatrix},$$

$$\hat{F}_{i_j} = [0 \ 1], \hat{B}_{i_j}^T = [0.0 \ 0.5], (\forall i_j).$$

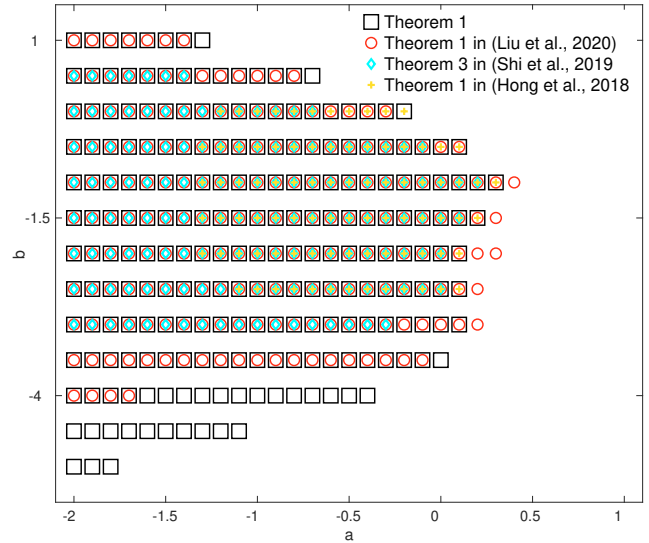


Fig. 1. Feasibility areas of Theorem 1 with respect to previous works.

In the sequel, for simulation purpose, the initial conditions of the system and the filter are assumed to be respectively $x(0) = [0.03 \ 0.1]^T$ and $\hat{x}(0) = [0.01 \ -0.2]^T$. Moreover, we apply an exogenous disturbance input signal $w(t) = 20 \cos(6t)e^{-0.4t} + v(t)$, where $v(t)$ is a noise set as normally distributed random numbers updated with a sampling period of 0.0005s.

Figures 2 to 4 show the simulation results, where a non-periodic switching signal is considered for the system. For the filter, the switching signal is a delayed replicate of the system's one (the delay is fixed at 125ms). That is to say, an asynchronous switching occurs, e.g. due to the time spent to detect or identify the active mode of the system. Fig. 2 shows the time history of the estimated output $\hat{z}(t)$ w.r.t. the system's unmeasured output state $z(t)$. The measured output of the system and the exogenous disturbance input signal are plotted in Fig. 3. Fig. 4 exhibits the estimation error $e_z(t)$ and the switching signals of both the system and the filter. Note that, in this simulation, the RMS error between the unmeasured output and the filtered one is relatively small ($RMS(z - \hat{z}) = 0.0329$). These results show that the filter provides a good estimate of the unmeasured output, despite the lag caused by the active mode detection delay and the noise introduced in the exogenous disturbance input.

Now, let us consider that the switching signal of the system is assumed to be unmeasured (nor estimated), but evolves according to several switching sets described by hyper-planes such that:

$$S_{j,j^+} = \{x \in \mathbb{R}^n | s_{j,j^+} x = 0\} \quad (25)$$

where j and j^+ denote respectively the active modes before and after switching instants, $s_{1,2} = [-0.5 \ 1]$, $s_{2,3} = [1 \ -40]$, $s_{3,1} = [1 \ 6]$, $s_{1,3} = [-1 \ 1]$, $s_{3,2} = [1 \ 10]$ and $s_{2,1} = [4 \ 1]$.

Moreover, let us assume that the filter switched modes evolve according to its own hyper-planes, which can be different, in the worst case, of the systems. The filter's hyper-planes are given by:

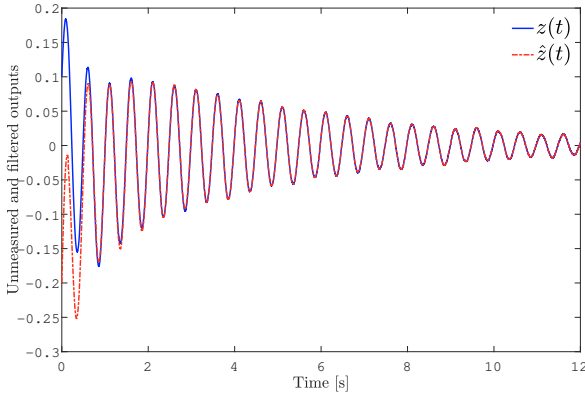


Fig. 2. Evolution of the unmeasured $z(t)$ and filtered $\hat{z}(t)$ states

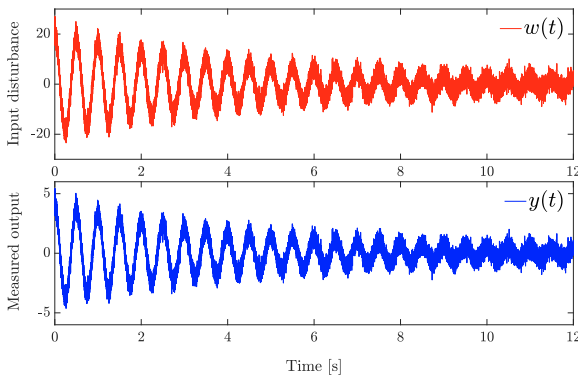


Fig. 3. Evolution of the measured output $y(t)$ and input disturbance $w(t)$

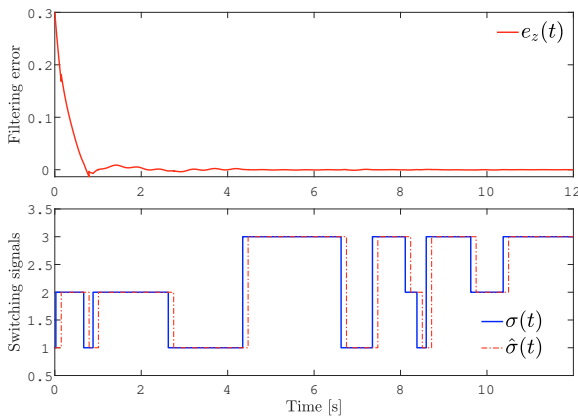


Fig. 4. Evolution of the estimation error (Top), Evolution of system's and filter's modes (Bottom).

$$\hat{S}_{j,j+} = \{\hat{x} \in \mathbb{R}^n | \hat{s}_{j,j+} \hat{x} = 0\} \quad (26)$$

with $\hat{s}_{1,2} = [-0.8 \ 1]$, $\hat{s}_{2,3} = [1 \ -50]$, $\hat{s}_{3,1} = [3 \ 8]$, $\hat{s}_{1,3} = [-1.5 \ 1]$, $\hat{s}_{3,2} = [2 \ 15]$ and $\hat{s}_{2,1} = [4 \ 2]$.

For this worst case, the simulation results are shown in figures 5 and 6. Fig. 5 exhibits the unmeasured output $z(t)$ and its estimate $\hat{z}(t)$. The estimation error $e_z(t)$ and the switching signals for both the system and the filter are depicted in Fig. 6. We also note that, in this case, the RMS error between the unmeasured output and the filtered one

remains relatively small ($RMS(z - \hat{z}) = 0.0348$). This clearly shows that the designed filter provides a good estimation of the unmeasured output, despite the fact that asynchronous switching mechanism are arbitrarily chosen. This confirm the effectiveness of the proposed asynchronous filter design methodology.

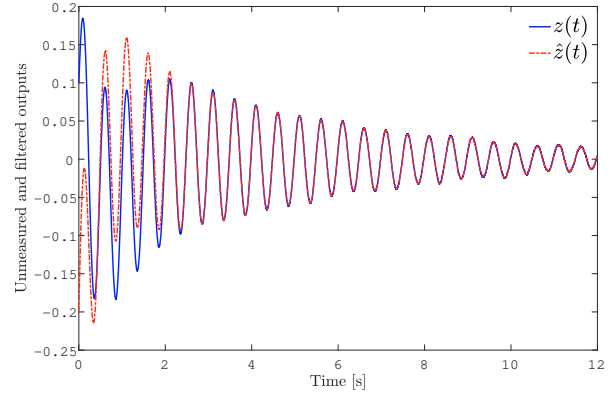


Fig. 5. Evolution of the unmeasured $z(t)$ and filtered $\hat{z}(t)$ states

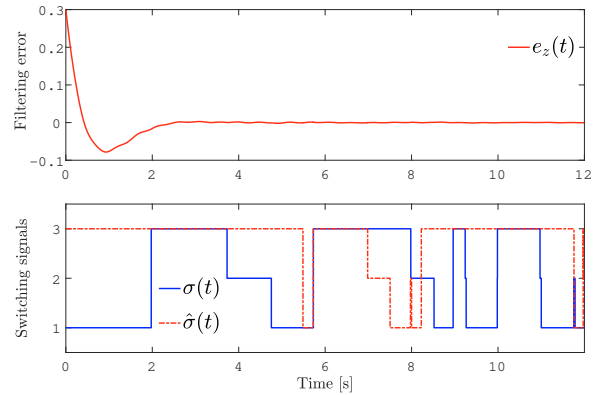


Fig. 6. Evolution of the estimation error (Top), Evolution of system's and filter's modes (Bottom).

5. CONCLUSION

New LMI-based conditions for the design of asynchronous switched T-S H_∞ filters, for switched T-S systems without dwell-time constraints, have been proposed with the help of the descriptor redundancy approach. Multiple-switched Lyapunov functions, with the satisfaction of H_∞ disturbance attenuation performances, have been considered to provide relaxed design conditions. Considering an academic example, two simulations have been presented to evaluate the conservatism of the proposed approach and the filter's effectiveness with regards to previous related results. Our further prospects will be to generalize the proposed results to the case where the premises variable are not measured. Moreover, it is worth to be noticed that, with the proposed H_∞ filtering technique and similarly to previous related results, the considered original system is required to be stable. Hence, another perspective will aim at extending this work to the class of unstable or marginally stable systems.

REFERENCES

- Belkhiat, D.E.C., Jabri, D., and Fourati, H. (2014). Robust H_∞ tracking control design for a class of switched linear systems using descriptor redundancy approach. *2014 European Control Conference, ECC 2014*, 2248–2253.
- Belkhiat, D.E.C., Jabri, D., Guelton, K., Manamanni, N., and Chekakta, I. (2019). Asynchronous Switched Observers Design for Switched Takagi-Sugeno Systems Subject to Output Disturbances. *IFAC-PapersOnLine*, 52(11), 49–54.
- Belkhiat, D.E.C., Messai, N., and Manamanni, N. (2011). Design of a robust fault detection based observer for linear switched systems with external disturbances. *Nonlinear Analysis: Hybrid Systems*, 5(2), 206–219.
- Bouarar, T., Guelton, K., and Manamanni, N. (2013). Robust non-quadratic static output feedback controller design for takagi-sugeno systems using descriptor redundancy. *Engineering Applications of Artificial Intelligence*, 26(2), 739–756.
- Bouibed, K., Seddiki, L., Guelton, K., and Akdag, H. (2014). Actuator and sensor fault detection and isolation of an actuated seat via nonlinear multi-observers. *Systems Science & Control Engineering*, 2(1), 150–160.
- Chang, X.H. (2012). *Takagi-sugeno fuzzy systems non-fragile H-infinity filtering*, volume 282. Springer.
- Chekakta, I., Belkhiat, D.E.C., Guelton, K., Jabri, D., and Manamanni, N. (2021). Asynchronous observer design for switched T-S systems with unmeasurable premises and switching mismatches. *Engineering Applications of Artificial Intelligence*, 104, 104371.
- Cherifi, A., Guelton, K., Arcese, L., and Leite, V.J. (2019). Global non-quadratic d-stabilization of takagi-sugeno systems with piecewise continuous membership functions. *Applied Mathematics and Computation*, 351, 23–36.
- Du, S., Li, X., Sun, S., and Li, X. (2020). Stability analysis and stabilization of discrete-time switched takagi-sugeno fuzzy systems. *ISA Transactions*.
- Etienne, L., Motchon, K.M.D., Duviella, E., and Guelton, K. (2020). Impulsive observer design for switched linear systems with time varying sampling and (a)synchronous switching rules. In *21st IFAC World Congress*, 1–6.
- Gómez-Gutiérrez, D., Čelikovský, S., Ramírez-Treviño, A., and Castillo-Toledo, B. (2015). On the observer design problem for continuous-time switched linear systems with unknown switchings. *Journal of the Franklin Institute*, 352(4), 1595–1612.
- Guelton, K., Bouarar, T., and Manamanni, N. (2009). Robust dynamic output feedback fuzzy lyapunov stabilization of takagi-sugeno systems—a descriptor redundancy approach. *Fuzzy sets and systems*, 160(19), 2796–2811.
- Han, M., Zhang, R., Zhang, L., Zhao, Y., and Pan, W. (2020). Asynchronous observer design for switched linear systems: A tube-based approach. *IEEE/CAA Journal of Automatica Sinica*, 7(1), 70–81.
- Hong, Y., Zhang, H., and Zheng, Q. (2018). Asynchronous H_∞ Filtering for Switched T-S Fuzzy Systems and Its Application to the Continuous Stirred Tank Reactor. *International Journal of Fuzzy Systems*, 20(5), 1470–1482.
- Iqbal, M.N., Xiao, J., and Xiang, W. (2014). Finite-time H_∞ state estimation for discrete-time switched control systems under asynchronous switching. *Asian Journal of Control*, 16(4), 1112–1121.
- Jabri, D., Guelton, K., Belkhiat, D.E., and Manamanni, N. (2020). Decentralized static output tracking control of interconnected and disturbed takagi-sugeno systems. *International Journal of Applied Mathematics and Computer Science*, 30(2), 225–238.
- Jabri, D., Guelton, K., Chouaib Belkhiat, D.E., and Manamanni, N. (2018). Output-Tracking Controller Design for Switched T-S Systems Subject to External Disturbances. In *2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 1–6. IEEE.
- Li, S., Ahn, C.K., and Xiang, Z. (2018). Sampled-Data Adaptive Output Feedback Fuzzy Stabilization for Switched Nonlinear Systems with Asynchronous Switching. *IEEE Transactions on Fuzzy Systems*, PP(c), 1.
- Liberzon, D. (2003). *Switching in systems and control*. Birkhäuser Basel.
- Liu, C., Li, Y., Zheng, Q., and Zhang, H. (2020). Non-weighted Asynchronous H_∞ Filtering for Continuous-Time Switched Fuzzy Systems. *International Journal of Fuzzy Systems*, 22(6), 1892–1904.
- Lofberg, J. (2004). YALMIP : a toolbox for modeling and optimization in MATLAB. In *2004 IEEE International Conference on Robotics and Automation (IEEE Cat. No.04CH37508)*, 284–289.
- Peaucelle, D., Arzelier, D., Bachelier, O., and Bernussou, J. (2000). A new robust script D sign-stability condition for real convex polytopic uncertainty. *Systems and Control Letters*, 40(1), 21–30.
- Pettersson, S. (2005). Switched state jump observer for switched systems. *IFAC Proceedings Volumes*, 38(1), 127–132. 16th IFAC World Congress.
- Schulte, H. and Guelton, K. (2009). Descriptor modelling towards control of a two link pneumatic robot manipulator: A T-S multimodel approach. *Nonlinear Analysis: Hybrid Systems*, 3(2), 124 – 132.
- Shi, S., Fei, Z., Wang, T., and Xu, Y. (2019). Filtering for switched T-S fuzzy systems with persistent dwell time. *IEEE Transactions on Cybernetics*, 49(5), 1923–1931.
- Tanaka, K. and Wang, H.O. (2001). *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, Inc.
- Tuan, H.D., Apkarian, P., and Nguyen, T.Q. (2001a). Robust and reduced-order filtering: New LMI-based characterizations and methods. *IEEE Transactions on Signal Processing*, 49(12), 2975–2984.
- Tuan, H.D., Apkarian, P., Narikiyo, T., and Yamamoto, Y. (2001b). Parameterized linear matrix inequality techniques in fuzzy control system design. *IEEE Transactions on fuzzy systems*, 9(2), 324–332.
- Wang, G., Xie, R., Zhang, H., Yu, G., and Dang, C. (2016). Robust Exponential H_∞ Filtering for Discrete-Time Switched Fuzzy Systems with Time-Varying Delay. *Circuits, Systems, and Signal Processing*, 35(1), 117–138.
- Zhai, D., Lu, A.Y., Dong, J., and Zhang, Q. (2018). Adaptive Tracking Control for a Class of Switched Nonlinear Systems Under Asynchronous Switching. *IEEE Transactions on Fuzzy Systems*, 26(3), 1245–1256.
- Zhao, X., Kao, Y., Niu, B., and Wu, T. (2017). *Control synthesis of switched systems*. Springer International Publishing.