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# “Problèmes pour chercher” : Experience, Possible and Necessity in Pupils Reasonings

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## Abstract

«Problèmes pour chercher» are, literally, «problems to research» in mathematics at the primary school level. Their aim is for students to solve non classical problems, and to discuss and argue about them. The goal is not to build curricular knowledge. Hence, these problems are, by some aspects, close to the activity of mathematicians. But what is really the activity of the pupils during these classroom works? Are they working on proofs? What kind of difficulties do they have? This contribution deals with the attitude of pupils towards *Experience*, *Possible*, *Necessity* and proofs. To achieve this, we propose to analyze the productions of 8–9 year old pupils on the “No-Three-In-Line” problem.

## 1 Introduction

In France, official texts for the primary school level require teachers to propose “problème pour chercher”, literally “problems to research”, in mathematics. Their aim is for pupils to develop mathematical reasoning skills. Is it easy to develop these abilities at the end of primary school? What kind of reasoning can produce pupils? What kind of difficulties do they have? Can these “problème pour chercher” bring pupils to a mathematical activity?

In this communication, we are interested in tackling these questions. And to this means, we study the activity of 8–9 year old pupils while solving the “No-Three-In-Line” problem. In particular, we analyse their reasonings and propositions of proof, and their links with the *Possible*, the *Necessity* and proofs in mathematics. This analysis leads us to clarify the origins of the pupils’ difficulties.

## 2 Presentation of the problem and of the sequence

In this communication, we focus on the activity of pupils about the “No-Three-In-Line” problem, introduced by Henry Dudeney in 1917. It asks for the maximum number of points that can be placed on the  $n \times n$  grid so that no three points are collinear. This problem is still open, and for large  $n$  we only have conjectures (for more details, see <http://www.whomes.uni-bielefeld.de/achim/no3in/readme.html>). For  $2 \leq n \leq 32$ , we know that it is possible to put  $2n$  points on the grid.

### 2.1 Scenario of the sequence

We propose the No-Three-In-Line problem for  $n$  equal to 4 or 5 to third grade pupils (*Cours élémentaire 2*, in France). The scenario of the sequence is the following. First, the pupils have to put the maximum number of points on a  $4 \times 4$  grid. At this stage, our aim is for pupils to understand the problem and its constraints : they have to take into account not only horizontal or vertical lines, but any slanted “straight line” as well, whatever its slope. The teacher collect the pupils propositions, and pupils exclude the ones that do not obey the constraints. Then, the teacher proposes to solve a similar problem for a  $5 \times 5$  grid. This time, after a long time researching and examining the propositions, and after most of pupils have found a solution with 10 points, we ask pupils : is it worth trying to keep on researching? At this point, we expect pupils to propose some explanation. The sequence ends with a discussion about these explanations.

### 2.2 Characteristics of the problem

The No-Three-In-Line problem is not a classical problem for the pupils. Even though it may look like a training exercise to check alignments, the resolution needs more. The aim of this problem is not to build curricular knowledge but to take part in the construction of knowledge about reasoning and proof.

This problem corresponds to a “problème pour chercher”, as their characteristics are described in the official texts of the French Ministry of Education (see Appendix). It also has the main characteristics of a “problème ouvert” (Arsac, Germain, Mante, 1991).

### 2.3 How to explain it at the end of the primary school?

At the primary school level, this problem, which is a discrete geometry problem, corresponds to an optimization problem because the 8–9 year old pupils do not have any knowledge about plane transformations and collinearity. Thus, what kind of explanation<sup>1</sup> can they produce?

For 8–9 year olds, and for  $n$  equal to 4 or 5, the proof consists in three points<sup>2</sup> :

1. No row or column can have more than two points on it, so one can at most put 8 (resp. 10) points on a  $4 \times 4$  (resp.  $5 \times 5$ ) grid ;
2. On the other hand, one can indeed success in placing 8 (resp. 10) points ;
3. Consequently, the maximum number of points that one can place on a  $4 \times 4$  (resp.  $5 \times 5$ ) grid is attained.

The nature of these three elements is different. The first one is the product of a reasoning, considering the constraints of the problem and some knowledge about collinearity. The third one is the conclusion and also corresponds to a thought process. But, the second element strongly depends on the experience. Thus, this one is mostly related to the pragmatic register while the others are mostly related to the intellectual register. So, this proof rests both on pragmatic and intellectual registers.

## 3 Analysis of the productions of the pupils

The pupils are first asked to find correct and optimal configurations. Their activity at this stage is mostly related to the empirical register. When we ask them “is it worth trying to keep on researching?”, we expect it will concern the axiomatical register. Does it correspond to the reality?

In this paragraph, we study the explanations produced in a 26 pupils classroom. These pupils worked on this problem during four one-hour-classroom sessions.

### 3.1 Are pupils’ propositions related to some concept of proof? If yes, to which kind?

After some long time searching for a solution on a  $5 \times 5$  grid, the teacher asked pupils : “who thinks that it is possible to put more than 10 points on this grid? Should we believe that 10 is the maximum?” Eight pupils strongly thought that one could do better than 10. The teacher proposed them to keep on searching. She asked the other pupils to explain why it was not worth trying to keep on searching?

We have translated below the main propositions given by the pupils to explain that it is impossible to put more than 10 points on a  $5 \times 5$  grid:

1. “I think that there are no more than 10. Because I have checked for the rows, columns and vertically (*sic*) and could not find more.”
2. “Because on every column and row there are two points.”
3. “Because Jean said there are two points a row and a column.”<sup>3</sup>
4. “Because there are 5 nodes (*sic*) and we cannot place more than the double.”<sup>4</sup>
5. “Because we need a  $10 \times 10$  grid to place more than 10 points.”
6. “Because it is already very difficult to place 10 points. I think we cannot do better.”
7. “Because we tried many times and we did not do succeed.”

<sup>1</sup>In reference to the distinction of Balacheff (1982), we speak about explanation rather than of proof.

<sup>2</sup>We number these points to facilitate the recall but 1 and 2 can completely be reversed.

<sup>3</sup>Jean said this before, about the  $4 \times 4$  grid.

<sup>4</sup>In fact, these pupils confuse «nodes» for «lines».

These explanations have been produced inside small groups and accepted inside them. So they can be considered as proof at this level (Balacheff, 1982). However, the third one has a special status : the original argument is the same as the second one, but the pupils grant credit to it, because they grant credit to Jean. Can we accept this proposition as a proof? Whatever it is, these pupils do not work on mathematical reasoning. Proposition 5 is also different: the pupils did not take into account the constraints of the problem (especially the fact that they have to work with a  $5 \times 5$  grid) and proposed to change the problem.

According to Balacheff's classification (Balacheff, 1987), there are two main categories of proofs: the pragmatic ones, linked to the action, and the intellectual ones that suppose some remoteness from the action. This classification excludes the third proposition from the domain of proof. The explanations number 1, 6 and 7 are clearly pragmatic proofs, while number 2 and 4 may correspond to intellectual proofs, even if the second one can also be understood in the following way as: there are already 2 points on every line, if we add a new one there will be three points on the line. More precisely, according to Balacheff's classification of pragmatic and intellectual proofs once again, Proposition 1 matches a generic example. Indeed, the pupils consider a grid with 10 points, respecting the no-three in line constraint, and try to place another point on it. As they do not succeed, they conclude to the impossibility. So the grid which is the support of their reasoning corresponds to some generic example. In our opinion, Propositions 6 and 7 match naïve empirism, especially because of the lack of questioning about validation (it does not work because I do not succeed even with many tries). However, these two propositions seem different to us. Let us now try to analyze the pupils propositions with the help of the "problematisation" framework to make this difference precise.

### 3.2 Possible and Necessity in the pupils reasonings

This framework was developed by Orange (Orange, 2005) in the context of didactic of sciences. It is a way to think the relationships between problem and solution, as Orange said (Orange, 2005, p. 70):

« c'est une construction explicite du champ des possibles, en lien avec deux caractéristiques essentielles du travail scientifique : la relation dynamique entre savoirs et problèmes, et l'apodicticité (le caractère de nécessité) des savoirs produits »

This concept is more and more used as a theoretical framework in France, in the domain of didactic of sciences. It is used to produce teaching sequences or to analyze students and teacher activities. It does take into account neither the «suffisance» question, which is strongly linked with necessity in mathematics, nor the axiomatic step. However, it appears to us as a possible framework to study pupils' mathematical reasoning. Indeed, this framework takes into account epistemological aspects of knowledge building in mathematics: problems found mathematical research and teaching, their resolution generates mathematical knowledge. Moreover "*Possible*" and "*Necessity*" are two essential notions in mathematics.

Proposition 6 does not show any exploration of the possible (it only deals with the difficulty to place 10 points for many grids), whereas Proposition 7 is based on the exploration of many cases (we tried but ...). For this reason we think that these pupils did not build the problem the same way.

The first explanation shows that the pupils explore the possibles on a given grid, which has a generic status, with already 10 points put in place. It is a local exploration that has a generical value. The exploration leads pupils to construct a necessity: we cannot place another point, so we can only place 10 points on the grid.

In Propositions 2 and 4, a mathematical necessity appears. But, we do not have information about the way the pupils have built it.

### 3.3 The major difficulties of the problem

These propositions have been posted on the table and the whole class have to discuss about them. The question is especially to know if we accept them as a proof inside the classroom.

The discussion among the pupils shows the following elements. The pupils quickly set aside the third proposition: for them, it is neither an explanation, nor a proof. A girl said: "Jean can be wrong". They also quickly set aside the fifth one: this proposition changes the problem and does not give any explanation. But for the other propositions, the discussion was more difficult. If all the pupils agreed with the fact that it is already difficult to place 10 points, some kept on thinking that it was possible to do better. They were saying: "if we keep on searching, we will certainly find a better solution." So, for them, this explanation is not proof. However, none of them points to the fact that this pragmatic argument has no mathematical value; they only reject its conclusion. And many pupils who do not seem convinced do not manage to say why. Finally, the teacher said : "this is not a mathematical argument."

Many pupils also reject or do not understand Propositions 2 and 4. For them, these propositions use the constraint of the problem in the reasoning: saying that one cannot put more than 2 points on a line

corresponds to the main constraint of the problem. They do not see the feature of necessity restrained in these propositions whereas, for them, this feature seems to be more clear in Propositions 1, 6 and 7. We believe that there lies the major difficulty for these pupils: it prevents them to have access to an intellectual proof.

However, some pupils accepted these intellectual proofs, but they did not consider the necessity to precise that, moreover, one succeeds in placing 10 points. For them, this return to a pragmatic aspect of the problem corresponds to some strong difficulty inherent to this problem. Indeed, they could not produce a proof of this problem.

## 4 Conclusion

Then, using in a complementary way the classification of proofs of Balacheff and the framework of “problematization” (Orange, 2005), we studied the explanations produced by 8–9 year old pupils on the “No-Three-In-Line” problem and pointed out the major difficulties that forbid pupils to reach an intellectual proof. Now, according to this analysis, let us give some elements of comparison between the pupils activity and the mathematician one (Perrin, 2007). The experimental part of the pupils research is real; in particular, some of them use reasonings on many or generical examples. But, where the mathematician will easily distinguish between the constraints of the problem and the necessity, and more precisely, will change the constraints of the problem into necessities, the pupils get stuck. Moreover, the mathematician knows that 10 non-collinear points put on a  $5 \times 5$  grid can contribute to a proof only because they prove the existence of a maximal solution, whereas this game on the status of the argument is difficult for the pupils.

## 5 References

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## 6 Appendix : about «problème pour chercher»

In this paragraph, we quote extracts of the official document that defines the nature and goals of «problèmes pour chercher».

### 6.1 Their goals

Cinq objectifs différents peuvent être dégagés :

1) La pratique du « problème pour chercher » développe la capacité de l’élève à faire face à des situations inédites.

2) Dans la résolution de ces problèmes, l’élève prend conscience de la puissance de ses connaissances, même si celles-ci sont modestes. Il existe en effet toujours plusieurs moyens d’élaborer une réponse, faisant appel à des registres de connaissances différents [...].

3) L’activité de l’élève dans la résolution d’un « problème pour chercher » valorise des comportements et des méthodes essentiels pour la construction de leurs savoirs : prendre des initiatives (tenter, faire des essais... ), être critique vis-à-vis de son travail (contrôler, analyser ses erreurs... ), s’organiser, être méthodique (réduire la part du hasard, le nombre de cas à envisager), communiquer (par oral, dans le groupe et face à la classe, par écrit pour rendre compte de sa recherche).

4) Les phases d’échanges et de débats développent les capacités argumentatives de l’élève. Les débats qui s’instaurent soit dans les groupes, soit dans la classe conduisent les élèves à valider ou réfuter une proposition. Un élève qui est persuadé du bien-fondé de son idée, de l’intérêt de la piste qu’il veut explorer ou de la solution qu’il a trouvée, devra convaincre ses camarades. La raison doit l’emporter sur la passion. Pour cela, le maître gère les débats afin

que ce soit la valeur de l'argument qui l'emporte. Ni la force de conviction de celui qui le défend, ni le fait que cet argument soit accepté par la majorité des élèves ne doivent être décisifs quant à la validité d'un argument : en mathématiques, l'accord du plus grand nombre sur une proposition ne constitue pas un critère de sa validité.

5) Ce type d'activité contribue à l'éducation civique des élèves. Les moments de recherche sont plus efficaces si l'on s'entraide : les idées proposées par les uns, même erronées, alimentent celles des autres. Les moments de débat offrent également l'occasion de travailler l'écoute, la prise en compte et le respect d'autrui.

## 6.2 Their characteristics

Les élèves doivent pouvoir s'approprier facilement la situation et se représenter la tâche pour s'y engager avec leurs connaissances antérieures. La difficulté doit se situer non dans la compréhension de la situation, mais dans les moyens de répondre à la question posée.

Le problème doit être « consistant », c'est-à-dire présenter une certaine « résistance ». Il ne doit pas donner lieu à une réponse qui résulte d'un traitement immédiatement reconnu. [...]

Donner un problème de recherche, c'est lancer un défi. Il est important que les élèves s'approprient le problème et qu'ils aient envie de relever le défi. De ce point de vue, l'attitude du maître est aussi décisive que le choix du problème. La « mise en scène » qu'il a imaginée conditionne l'engagement des élèves à relever le défi. Cet engagement dans la tâche est souvent plus aisé si les élèves sont persuadés qu'il existe une solution, parce qu'ils ont vu le problème se créer [...] : ils sont ainsi mieux à même de se représenter la situation. [...]

La validation de la solution doit être le plus possible à la charge des élèves. Ils doivent pouvoir se rendre compte par eux-mêmes du bien-fondé ou non de leur réponse, par l'échange d'arguments destinés à défendre ou contredire une proposition, par des contrôles tout au long de leur recherche et, si possible, par une vérification, à la fin, sur la situation elle-même.