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► **To cite this version:**

John Tichy, Marine Menut, Camella Oumahi, Sandrine Muller, Benyebka Bou-Saïd. Grease flow based on a two-component mixture model. Tribology International, 2021, 153, pp.106638. 10.1016/j.triboint.2020.106638 . hal-03660562

**HAL Id: hal-03660562**

**<https://hal.science/hal-03660562>**

Submitted on 21 Nov 2023

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## Grease Flow based on a Two-Component Mixture Model

by

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Keywords: grease, lubrication, non-Newtonian

## Abstract

Rolling element bearings are the most common type of bearings and about 90 % of them are lubricated with grease. In flow modeling grease has almost always been treated as a homogeneous single-component material described by a shear thinning rheological model. The present modeling paper attempts to incorporate key features not addressed by existing models: the two-component nature of the grease, and the fact that the thickener is a sort of delivery system of oil to the contact. In the present approach, the thickener is modeled as a medium containing the oil. Oil flows through the thickener according to the Darcy-Brinkman law. The thickener is regarded as a sort of porous medium, but the key feature is a linear interaction force between the two components. The model is conceptual and speculative but could provide inspiration for alternate ways of thinking about grease flow behavior. Three idealized cases are considered: flow in simple configurations of rheological devices, flow in a contact completely composed of grease, and flow in a contact with a layer of grease and a layer of oil. For illustrative purposes the rigid cylinder-plane contact is considered, and a beneficial effect of the grease is seen. Many extensions of the model itself are possible, and it can be applied to more realistic problems. Although validation is not provided, the conceptual framework could likely be tested by relatively straightforward experiments.

## I. Introduction

Grease is a widely used complex material. Many authors point out that grease is a two-component mixture - a base oil and a thickener. Likewise, it is commonly noted that the thickener (usually a polymer fiber or a soap) may constitute a delivery system for the oil to the contact. The oil, in turn, provides the essential lubrication - be it hydrodynamic or boundary. Despite its industrial and economic importance, and despite the fact that understanding grease lubrication presents an interesting intellectual challenge, these aspects (the two-component nature and the oil delivery system) are rarely addressed in a mathematical model with predictive ability in flow problems. Indeed, the mechanisms may be so different from application to application, that it is not evident a unified conceptual approach can be applied. Clearly, better grease models would be desirable. Please note throughout that by ‘modeling’ we mean only flow modeling, not modeling of the bleed process, evaporation, aging, oxidation, etc.

This paper owes a heavy debt to a 1999 paper by P Cann [1]. She notes the importance of grease, and the lack of satisfactory models for grease lubrication. She goes on to present a heuristic physical description of grease lubrication in rolling contacts that we loosely try to implement in the present paper. In her description, the grease layer is deposited in the contact track, repetitive passes of the roller releases oil from the grease, which, in turn improves the hydrodynamic film.

There are numerous papers in the literature applying mathematical models of grease to lubrication contacts as a flow boundary value problem. However, as to predictive flow models, we could only find ones which consider the grease as a homogeneous single component material. The thickener is generally a very viscous non-Newtonian fluid, which may be purely viscous (so-called generalized non-Newtonian) or viscoelastic. Non-Newtonian fluid models and rheology principles are discussed in the text by Bird et al. [2]. Typically, the grease at low shear is some 1000 times more viscous than the base oil, which is probably close to

Newtonian in nature.

In the following background description, we do not try to be comprehensive but rather to give the reader a flavor as to what is in the current literature. Probably being discouraged by the problem complexity, the general nature of grease flow models has changed very little over 50 years. For example in 1965, Batra [3] applied the Bingham model to a journal bearing. The Bingham model describes a fluid that behaves as a rigid solid when the shear stress magnitude is lower than a yield stress, and flows as a quasi-Newtonian material at higher stresses. Similarly, in 1973 Wada et al. [4] published a series of papers applying the Bingham model to several bearing configurations. Through the intervening years, there are a number of similar papers.

Jumping forward to more recent times, in 2008 Gertzog et al. [5] applied computational fluid dynamics (CFD) methods for the Bingham material to journal bearings, while in 2017 Westerberg et al. [6] describe grease flow in seals by the Herschel-Bulkley fluid (another yield stress model) using CFD. Zhang et al. [7] apply several generalized Newtonian models to simulate grease flow to an elastohydrodynamic lubrication (EHL) journal bearing. A concentric cylinder geometry is both experimentally and theoretically studied by Li et al., [8], using the Carreau model and the Herschel-Bulkley model. Augusto et al. in 2020 [9] apply sophisticated CFD methods and the Herschel-Bulkley model to the labyrinth seal problem.

There are a number of papers in the literature which can be retrieved searching under ‘grease’ and ‘model’ whose intention is to predict grease behavior in transient shear strain, strain relaxation and the like, rather than prediction of behavior in lubrication flow. See for example, the studies of Madieto et al. [10]. Cyriac et al. [11], have written more recent papers on grease lubrication and flow.

A paper with some direct relevance to the present work is from 2010 due to Baart et al. [12]. The effect of porosity in grease is studied, but during ‘bleeding’ conditions, i.e., in storage as the oil separates from the thickener. A 2017 paper by Saatchi et al. [13] use a two-component flow analysis similar to that used here to describe the oil release mechanism in greases in somewhat the same fashion as the present work.

It can be safely stated that grease lubrication is still not thoroughly understood. There are a number of recent excellent papers on grease behavior which bear mentioning, to give the interested reader a feeling for the current state of affairs. In no special order, Cann and Lubrecht 2007 [14] discuss grease performance and design, Cousseau et al. 2015 [15] study the grease aging process, Venner et al. 2012 [16] treat grease film behavior in an EHL contact, while Couronné et al. 2003, [17] correlate grease properties with EHL film thickness. Cousseau et al. 2012 [18] correlate grease flow rheology to frictional torque in thrust ball bearings.

Rheological behavior of many polymer greases was reported in 2015 by Gonçalves et al. 2015 [19]

## II. Analysis

### A. Grease flow modeled as a mixture

The proposed model assumes that the flow of grease consists of an oil component, denoted by the superscript ‘ $O$ ’, and a thickener component which we denote by superscript ‘ $T$ ’. We use the precepts of mixture theory, outlined in the 2017 review article of Siddique et al. [20] for porous media. The components have different properties that interact, but the mixture may be treated as single fluid with homogeneous properties. The thickener can consist of materials such as a polymer fiber matrix, or lithium soap. Thus we describe a two-component material with a linearly weighted average:

$$\mathbf{v} = \phi \mathbf{v}^O + (1 - \phi) \mathbf{v}^T, \quad \boldsymbol{\tau} = \phi \boldsymbol{\tau}^O + (1 - \phi) \boldsymbol{\tau}^T$$

$$\mathbf{v}^O = v_x^O \mathbf{i} + v_y^O \mathbf{j} + v_z^O \mathbf{k}, \quad \boldsymbol{\tau}^O = \tau_{xx}^O \mathbf{ii} + \tau_{xy}^O \mathbf{ij} + \tau_{xz}^O \mathbf{iz} + \tau_{yx}^O \mathbf{ji} + \dots, \quad \text{etc.} \quad (1)$$

The components  $\mathbf{v}^O$  and  $\mathbf{v}^T$  represent the velocity of a volume element if it were entirely oil  $O$  or thickener  $T$ . Instead, each volume element contains the two components treated as an equivalent single component homogeneous material. The symbol  $\phi$  denotes the oil component volume fraction, and thus  $1 - \phi$  is the thickener volume fraction. The volume-averaged velocity of the mixture is  $\mathbf{v}$ . The stress tensor is  $\boldsymbol{\tau}$  shown in dyadic form. The primary flow direction along the film is  $x$  and the direction across the thin film is  $z$ . The surfaces separated by the film reside primarily in the  $x - y$  plane. In the present paper, we restrict ourselves to two-dimensional flow (a one-dimensional contact) to illustrate the basic concepts, although the model can be extended to the three-dimensional case.

At the boundaries of the mixture, the individual mixture components match the velocity of the opposing material. For example, at a boundary  $z = z_b$ , where  $v_x = V_b$  and  $\tau_{zx} = \tau_b$

$$v_x^O(z_b) = V_b, \quad v_x^T(z_b) = V_b, \quad \phi \tau_{zx}^O(z_b) + (1 - \phi) \tau_{zx}^T(z_b) = \tau_b \quad (2)$$

#### 1. Thickener modeled as a generalized Newtonian fluid, oil as a Newtonian fluid

The purely viscous or generalized Newtonian case, is much simpler than the viscoelastic case, often used to describe grease. The shear stresses are obtained from a model such as Carreau-Yasuda as described in the text of Bird et al. [2]:

$$\tau_{zx}^T = \eta^T \frac{\partial v_x^T}{\partial z}, \quad \dot{\gamma}^T = \sqrt{\left(\frac{\partial v_x^T}{\partial z}\right)^2}$$

$$\eta^T = \eta_\infty^T + (\eta_0^T - \eta_\infty^T) (1 + \lambda^2 \dot{\gamma}^{T2})^{(n-1)/2} \quad (3)$$

where  $\dot{\gamma}^T$  is the thickener shear rate,  $\eta^T$  is the thickener non-Newtonian viscosity; and  $\eta_0^T$  and  $\eta_\infty^T$  are the thickener low and high shear viscosities, respectively. The symbols  $\lambda$  and  $n$  are model parameters, the former having dimensions of time, the latter being dimensionless.

The thickener has very high viscosity at low shear ( $\eta_\infty^T$  large). In the limit the behavior of an extremely viscous fluid approaches that of a rigid solid. The particular model for the thickener is not crucial to the paper, only that its viscosity is much greater than that of the oil.

## 2. Grease modeled as a porous medium mixture

We treat the oil phase as Newtonian, obeying the Darcy-Brinkman law [21]. Likewise in what follows we treat the thickener as Newtonian. We assume that the thickener shearing is slow and its viscosity can be considered to be the low-shear viscosity  $\eta^T = \eta_0^T$ . Both bulk soap and an array of fibers, although solid-like materials, can evidence flow properties, see for example Castro et al. [22] (on soft soaps) and du Roure et al. [23] (on fiber arrays).

$$\frac{\partial p^O}{\partial x} = \frac{\partial p^T}{\partial x} = \frac{\partial p}{\partial x} \quad (4)$$

$$\frac{\partial p}{\partial x} = \frac{\eta^O}{k} (v_x^O - v_x^T) + \eta^O \frac{\partial^2 v_x^O}{\partial z^2} \quad (5)$$

$$\frac{\partial p}{\partial x} = -\frac{\eta^O}{k} (v_x^O - v_x^T) + \eta^T \frac{\partial^2 v_x^T}{\partial z^2} \quad (6)$$

The pressure is identical for the oil and thickener components, Eq. (4). The thickener component is considered to be a porous network causing a drag on the oil flow passing through it, Eq. (5). The drag is proportional to the velocity difference  $v_x^O - v_x^T$ , the oil viscosity  $\eta^O$  and inversely proportional to the permeability  $k$ . Permeability has units of  $\text{m}^2$  and  $\sqrt{k}$  roughly corresponds to the pore size. There is an equal and opposite force of the oil on the thickener, Eq. (6). The second derivative terms are due to Brinkman and account for shear stress at the boundaries. These terms allow boundary conditions to be imposed in shear flow.

In the end, the important thing is that the local interaction force between the oil and thickener is linearly proportional to the oil viscosity and the component relative velocity and inversely proportional to a characteristic pore size. It is not required that the pore be of a certain size or shape.

## B. Rheological Behavior

In a *viscometric* shear flow, each fluid element undergoes an identical simple shearing motion, regardless of the constitutive behavior. Through various arguments made in rheology texts (see Bird et al., page 100, for example), Couette (simple shearing) flow is found to be viscometric. The present case is that of two parallel plates separated by a constant gap  $H$ , one of which is sliding at velocity  $V$ . The material is undergoing a viscometric flow of shear rate  $\dot{\gamma} = V/H$ . Cone and plate flow is also viscometric in that the shear rate likewise constant throughout. The gap is thin  $H \ll L$ , where  $L$  is the characteristic length in the direction

of flow. Strictly speaking, viscometric flow is required for rheological measurements because any homogeneous fluid (Newtonian or otherwise) will have a known velocity field. Other configurations, such as Poiseuille flow between parallel plates driven by a pressure gradient, are sometimes used for viscosity measurement (similar to the the capillary viscometer) but to a lesser degree of certitude because the velocity field may vary depending on non-Newtonian fluid type.

### 1. Couette or Simple Shearing Flow

As described above, there is a lower stationary surface at  $z = 0$  and an upper surface at  $z = H$  sliding at velocity  $V$ , with uniform pressure. Wall boundary conditions for the mixture are treated just as for a single-component fluid. For these conditions, the solutions to Eqs. (4)-(6) are

$$v_x^T = V \frac{z}{H}, \quad v_x^O = V \frac{z}{H} \quad (7)$$

According to the model, the shear stress is,

$$\tau_{zx}^O = \eta^O \frac{V}{H}, \quad \tau_{zx}^T = \eta^T \frac{V}{H}$$

$$\tau_{zx} = \phi \tau_{zx}^O + (1 - \phi) \tau_{zx}^T = \eta^{eff} \frac{V}{H}, \quad \eta^{eff} = \phi \eta^O + (1 - \phi) \eta^T. \quad (8)$$

The effective viscosity  $\eta^{eff}$  is a weighted sum of the viscosities of the two components. Since  $\eta^T \gg \eta^O$  if the model is to be believed, such a viscometer measures an effective viscosity proportional close to that of the thickener.

### 2. Poiseuille or Simple Channel Flow

Consider the case of an infinitely wide channel of constant gap height  $H$ . The channel is of length  $L \gg H$ , so thin film flow conditions are in force. The flow is thus steady and fully developed. A pressure  $p_0$  is imposed on the left hand entry side, while the exiting pressure on the right hand side,  $p_L = 0$ , thus  $\partial p / \partial x = -p_0 / L$ . As usual for thin film flows, the pressure does not vary across the gap,  $\partial p / \partial y = 0$ . Classical no-slip conditions are assumed to be in force on both surfaces for both  $v_x^T$  and  $v_x^O$ . This flow through a thin slit is the rectilinear analogue to flow through a capillary which is commonly used as a viscometer, especially in adverse conditions such as at high temperatures or pressure.

We further assume that the flow is slow (low Reynolds number) such that inertia can be neglected. The no-slip boundary conditions on the two phases are  $v_x^T(0) = v_x^O(0) = v_x^T(H) = v_x^O(H) = 0$ .

Equations (4)-(6) can be solved in closed form for the two velocity components:

$$v_x^T = -\frac{p_0}{2\eta_0^T L} (z^2 - zH) \quad (9)$$

$$z^* = \frac{z}{\sqrt{k}}, \quad H^* = \frac{H}{\sqrt{k}} \quad (10)$$

$$v_x^O = -\frac{p_0 \exp(-z^*)}{2L \eta^O (1 + \exp H^*)} \left[ (\exp z^* + \exp(z^* + H^*)) \frac{\eta^O}{\eta_0^T} z^* (z^* - H^*) + A \right]$$

$$A = 2 [-\exp H^* + \exp z^* - \exp 2z^* + \exp(H^* + z^*)] k \left( \frac{\eta^O}{\eta_0^T} - 1 \right) \quad (11)$$

An expression for the permeability in terms of the porosity is

$$k = \frac{\phi D_p^2}{32T}, \quad (12)$$

where  $D_p$  is an effective pore size and  $T$  is the tortuosity (arc length to chord length) [24]. This equation is derived by modeling the thickener medium as an array of thin tubes oriented in the direction of flow.

For the limiting case of very small oil viscosity, the equations simplify considerably:

$$v_x^O = -\frac{p_0 k}{L \eta^O} \frac{\exp z^*}{1 + \exp H^*} (1 - \exp z^*) (\exp z^* - \exp H^*) \quad (13)$$

For the following conditions, the velocity field is shown in Fig. 1:  $H = 1$  mm,  $L = 100$  mm,  $p_0 = 10$  kPa,  $\eta^O = 1$  Pa.s,  $\eta_0^T = 1000$  Pa.s,  $\phi = 0.7$ , and  $k = 10^{-8}$  m<sup>2</sup>. The thickener flows slowly in a parabolic Newtonian fashion, while the oil is flowing much faster. Most of the oil flow resistance comes from the porosity effect. The volume-averaged flow is much closer to that of the oil than the thickener. The densities of typical oils and polymer thickeners are not that different, thus mass-averaged and volume average quantities would be nearly the same.

The flow behavior is strongly dependent on the ratio  $\sqrt{k}/H$  which roughly corresponds to the pore size relative to the lubricant gap size. If this dimensionless parameter is small (relative to one), the oil encounters high resistance to flow through the thickener and the two would have nearly identical velocities. By contrast, if this ratio is large, meaning of order one, the oil flows easily through the thickener and the oil flow profile is nearly Newtonian. These latter conditions are portrayed in Fig. 1. We are not aware of permeability measurements for grease thickener, so for the purposes of this paper we have selected values of  $k$  to best illustrate the orders of magnitude and the trends predicted by the proposed model for the various cases.

The oil flow rate  $q^O$  and thickener flow rate  $q^T$  (per unit depth in the  $y$ -direction) are shown below. These quantities represent the flow rate of thickener or oil if the volume element contained the pure components. The mixture volume flow rate  $q$  is also given



$$q^O = \int_0^H v_x^O dz, \quad q^T = \int_0^H v_x^T dz, \quad q = \phi q^O + (1 - \phi) q^T \quad (14)$$

$$q^T = \frac{H^3}{12\eta^T} \frac{p_0}{L},$$

$$q^O = -\frac{k}{\eta^O} \frac{p_0}{L} \frac{1}{1 + \exp H^*} \left( H (1 + \exp H^*) - 2\sqrt{k} (-1 + \exp H^*) \right) \quad (15)$$

For channel flow, effective viscosity based on the Newtonian profile is as follows,

$$\eta^{eff} = -\frac{H^3}{12q} \frac{p_0}{L}. \quad (16)$$

For the above parameter values, the effective viscosity as measured in this ‘thin slit Poiseuille viscometer’ would be  $\eta^{eff} = 14.8$  Pa.s. The slit viscometer takes into account the flow of oil through the thickener as well as flow of the thickener itself. The viscosity of the same grease measured in a Couette viscometer would be  $\eta^{eff} = (1 - \phi) \eta^T = 300$  Pa.s. In the limit of large  $\sqrt{k}/H$  the effective viscosity becomes simply the viscosity of the base oil.

## C. The Flow of Grease in a Lubricated Contact

### 1. The case of a full grease contact

This situation is schematically shown in Fig. 2. The film thickness for a rigid cylinder-plane contact is described by:

$$h = h_0 + \frac{x^2}{2R} \quad (17)$$

Note, we are not attempting to model an EHL contact with elastically deforming surfaces. Due to pure rolling conditions the contact points of the two surfaces are equal, and by suitable placement of the coordinates, the velocity of both surfaces can be represented as  $V$ . This is highly simplified illustrative example just to show to how the model behaves. Further refinements such as a layer attached to each surface could be added. We admit to many unrealistic assumptions (such as flooded conditions, half-Sommerfeld exit boundary conditions, a sort of rebound of the grease layer after the minimum point, etc.). We are not trying to solve a complex EHL problem, although there is no reason, in principle, this could not be done.

The viscosity of the thickener is often some 1000 times greater than that of the oil. It can be shown formally through a regular perturbation of a small parameter  $\eta^O/\eta^T$ , that Eqs. (6) and (5), respectively, reduce to:

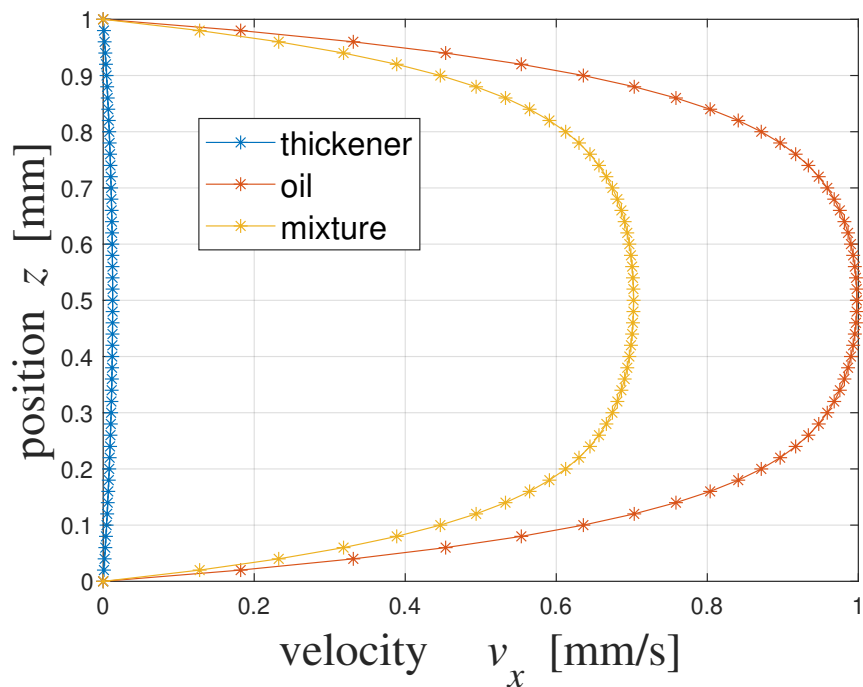


Figure 1: Velocity profiles for channel flow. Parameters:  $p_0 = 10$  kPa,  $L = 100$  mm,  $\eta^O = 1$  Pa.s,  $\eta_0^T = 1000$  Pa.s,  $\phi = 0.7$ , and  $k = 10^{-8}$  m<sup>2</sup>.

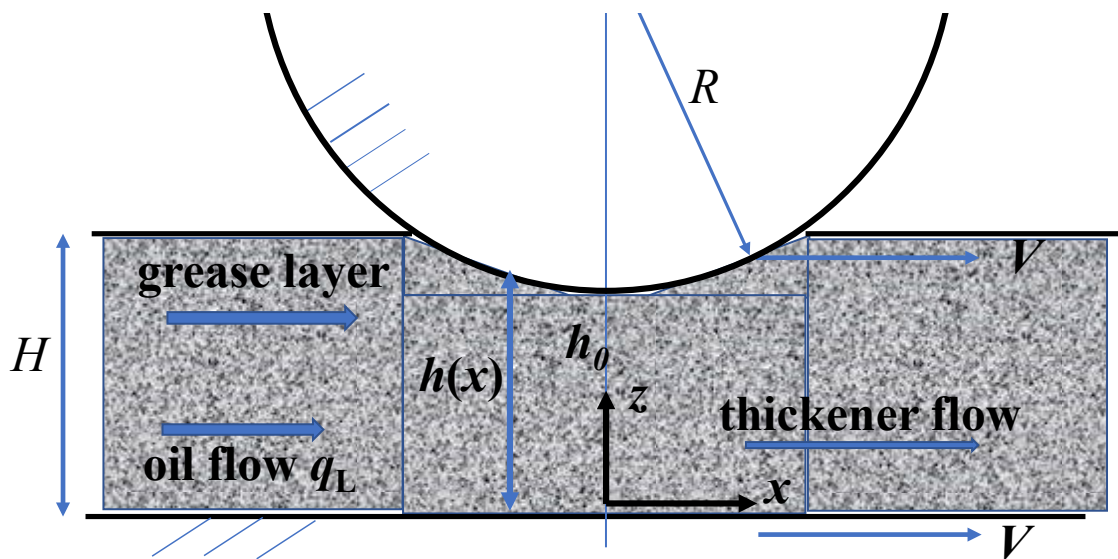


Figure 2: Schematic of rolling contact filled with grease

$$0 = \eta^T \frac{\partial^2 v_x^T}{\partial z^2} \quad (18)$$

$$\frac{\partial p}{\partial x} = \frac{\eta^O}{k} (v_x^O - v_x^T) + \eta^O \frac{\partial^2 v_x^O}{\partial z^2} \quad (19)$$

It is reasonable that the Darcy drag of the oil on the thickener (proportional to  $\eta^O$ ) is much less than the boundary shear stress (proportional to  $\eta^T$ ). The general solution to Eq. (18) is

$$v_x^T = V_1 + (V_2 - V_1) \frac{z}{h(x)} \quad (20)$$

however, for this rolling contact ( $V_2 = V_1$ ) =  $V$  and thus  $v_x^T = V$ .

Substituting  $v_x^T = V$  into Eq.(19) and solving the differential equation by standard methods we obtain,

$$v_x^O = \frac{dp}{dx} \frac{k}{\eta^O} \frac{\exp z^*}{1 + \exp h^*} (1 - \exp h^*) (\exp z^* - \exp h^*) + v_x^T \quad (21)$$

and integration across the film for the oil volume flow rate gives

$$q^O = \frac{k}{\eta^O} \frac{dp}{dx} \frac{1}{1 + \exp h^*} \left( -h (1 + \exp h^*) + 2\sqrt{k} (-1 + \exp h^*) \right) + \frac{h}{2} (V_1 + V_2) \quad (22)$$

The incompressible oil flow rate is constant. Respecting global continuity as in all developments of Reynolds equation, we differentiate  $q^O$  with respect to  $x$ , and a modified Reynolds equation is obtained:

$$q^O = -f_P[h(x), k, \eta^O] \frac{dp}{dx} + f_V[h(x)]V, \quad V = \frac{V_1 + V_2}{2}$$

$$\frac{dq}{dx} = 0, \quad \frac{d}{dx} \left( f_P \frac{dp}{dx} \right) = V f_V, \quad (23)$$

Typical pressure profiles are shown in Fig. 3. The parameter values used are representative of conditions at a position along a high speed train wheel/rail contact are as follows:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa.s}$ ,  $\eta^T = 100 \text{ Pa.s}$ . The lower and upper curves are for permeability  $k = 10^{-6} \text{ m}^2$  and  $k = 10^{-8} \text{ m}^2$ , respectively. The effect of the grease is to increase lubricant flow resistance and thus increase the hydrodynamic pressure generated.

## 2. The case of a contact with a grease layer

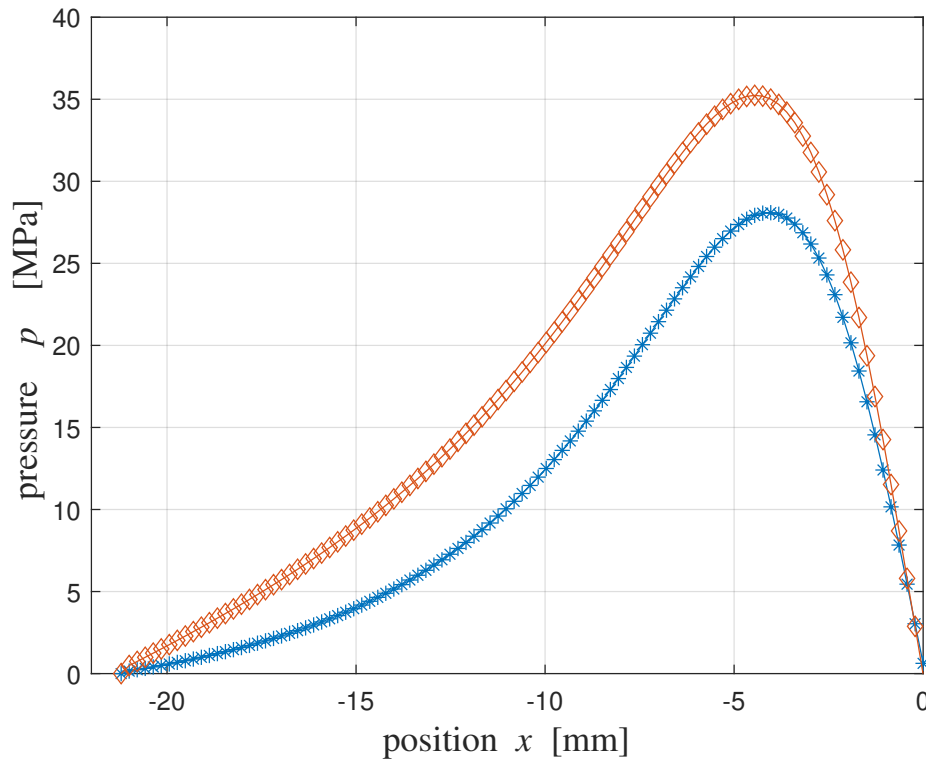


Figure 3: Pressure profiles, full grease contact. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa.s}$ ,  $\eta^T = 100 \text{ Pa.s}$ . The lower and upper curves are for permeability  $k = 10^{-6} \text{ m}^2$  and  $k = 10^{-8} \text{ m}^2$ , respectively.

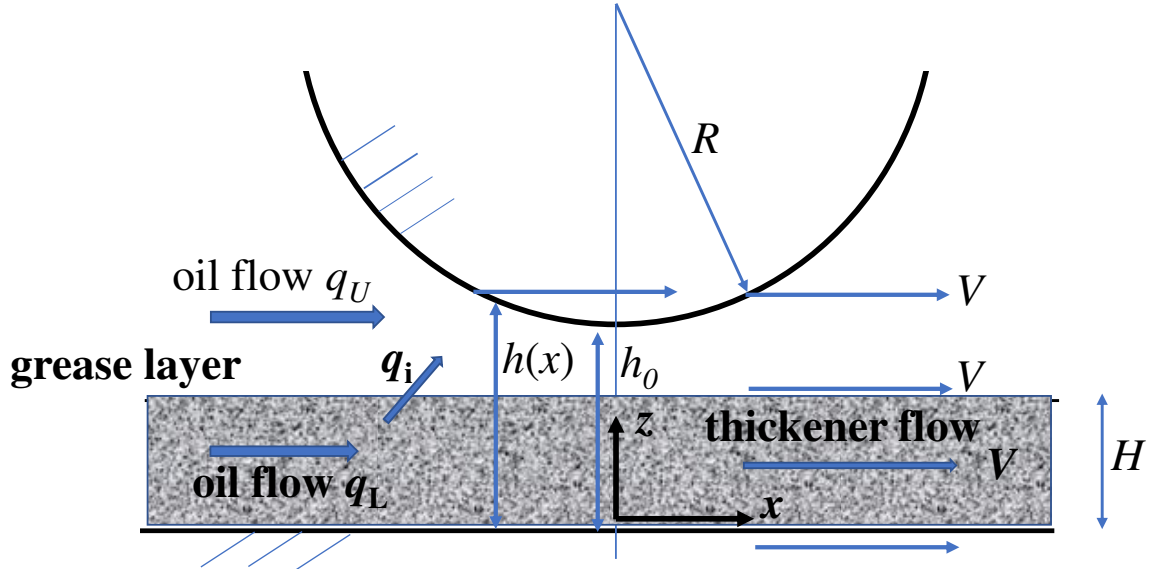


Figure 4: Schematic of rolling contact with a grease layer.

The following analysis builds on a ideas put forth in a paper in 1996 by P. Cann [1], as mentioned. She idealizes a grease lubricated EHL rolling contact as a grease layer (thickener plus base oil) deposited in the contact track and a layer of bled oil released by the grease. Repeated passes of the cylinder serve to extract the base oil from grease mixture, and the oil serves as the lubricant.

We consider a two-dimensional incompressible flow (1-D bearing) with rigid surfaces. The caveats regarding over-simplification discussed in the previous section are still in force. The grease layer (oil-thickener mixture) of thickness  $H(x)$  is attached to the lower surface. These conditions are noted schematically in Fig. 4 for the case of a cylinder-plane rolling contact.

The oil in the upper region obeys,

$$\frac{dp}{dx} = \frac{\partial \tau^U}{\partial z}, \quad \tau^U = \eta^O \frac{\partial v^U}{\partial z}. \quad (24)$$

Likewise the oil in the lower porous medium region (i.e., in the grease) obeys,

$$\frac{dp}{dx} = -\frac{\eta^O(v^L - V)}{k} + \frac{\partial \tau^L}{\partial z}, \quad \tau^L = \eta^O \frac{\partial v^L}{\partial z}. \quad (25)$$

By the usual lubrication assumptions, it can be shown that the pressure gradient is constant across the film. The solution of these equations has the form

$$v^U = \frac{z^2}{2\eta^O} \frac{dp}{dx} + z C_4 + C_3 \quad (26)$$

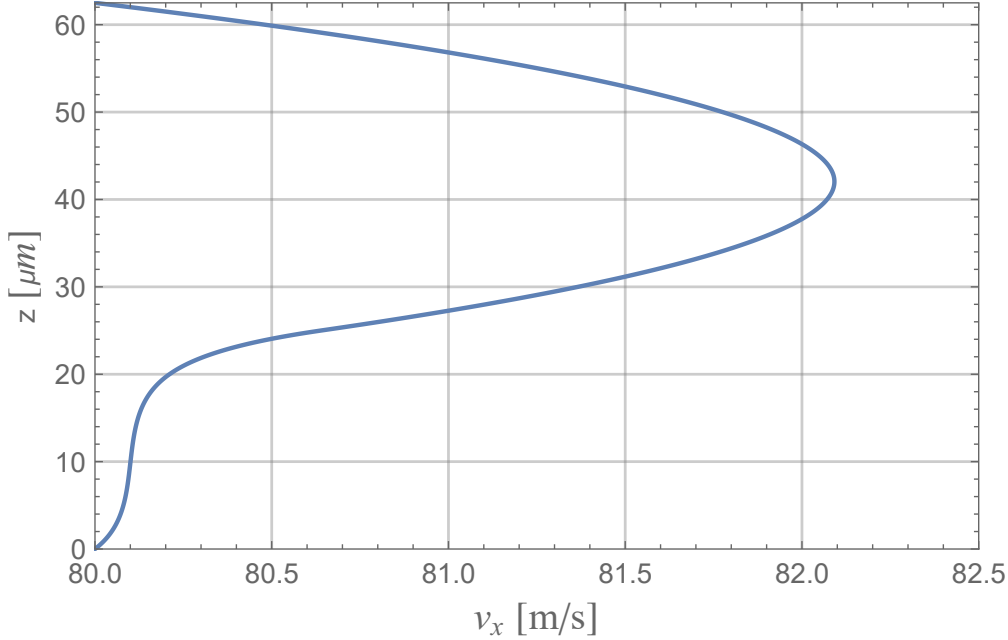


Figure 5: Oil velocity profile, film with grease layer. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $h = 62 \mu\text{m}$ ,  $H = 31 \mu\text{m}$ ,  $\eta^O = 0.1 \text{ Pa}\cdot\text{s}$ ,  $k = 2.0 \cdot 10^{-8} \text{ m}^2$ , and  $dp/dx = -1.0 \cdot 10^9 \text{ N/m}^3$ .

$$v^L = \frac{k}{\eta^O} \frac{dp}{dx} + C_1 \exp(z/\sqrt{k}) + C_2 \exp(-z/\sqrt{k}) + V \quad (27)$$

The boundary conditions on the oil flow are,

$$z = 0 : v^L = V; \quad z = h : v^U = V^U; \quad z = H : v^L = v^U; \quad \tau^L = \tau^U \quad (28)$$

For pure rolling contact the upper surface boundary condition  $v^U = V^U = V$ , while for pure sliding  $V^U = 0$ . Solving for the four constants is tedious but straightforward, and the resulting expressions are complicated, take up a lot of space, and are not shown.

Some computed results are shown in Fig. 5. The parameter values used are representative of conditions at a position along a high speed train wheel/rail contact, and noted are as follows:  $L = \sqrt{2Rh_0}$ ,  $x = -L/2$ ,  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5\text{m}$ ,  $V = 80 \text{ m/s}$ ,  $h = 62 \mu\text{m}$ ,  $H = 31 \mu\text{m}$ ,  $\eta^O = 0.1\text{Pa}\cdot\text{s}$ ,  $k = 2.0 \cdot 10^{-8} \text{ m}^2$ , and  $dp/dx = -1.0 \cdot 10^9 \text{ N/m}^3$ . The negative pressure gradient propels the oil forward relative to the sliding speed. Grease is present in the lower half of the figure, where the oil velocity is retarded due to the Darcy porous medium effect.

A steady 1-D incompressible modified Reynolds equation is now developed for this grease layer configuration. The volume flow rate is,

$$q = \int_0^H v^L dz + \int_H^h v^U dz = -f_P[H(x), h(x), k, \eta^O] \frac{dp}{dx} + f_V[H(x), h(x), k] V \quad (29)$$

Reynolds equation is determined from the constancy of the flow rate:

$$\frac{dq}{dx} = 0, \quad \frac{d}{dx} \left( f_P \frac{dp}{dx} \right) = V f_V = V \frac{dh}{dx}, \quad (30)$$

As above, determination of  $f_P = f_P(H, h, k, \eta_0)$  is straightforward but complicated, as are the resulting expressions.

An approximate version of Reynolds equation for the grease layer can be written as follows:

$$\frac{d}{dx} \left( \frac{H^3}{12\eta^T} \frac{dp}{dx} \right) + \frac{H^3}{12\eta^T} \frac{p}{W^2} = V \frac{dH}{dx} \quad (31)$$

The second term on the left hand side approximately represents loss of grease from the ends (into and out of the plane of the paper). This flow of grease is assumed to be governed by the thickener viscosity at low rates of shear.

In performing the computations that follow, Eq. (31) is rewritten as

$$\frac{dH}{dx} = \left( \frac{dp^2}{dx^2} + \frac{p}{W^2} \right) / \left( 12 \frac{\eta^T V}{H^3} - \frac{1}{3H} \frac{dp}{dx} \right) \quad (32)$$

and the right hand side terms are treated as known from the conditions upstream. The solution thus involves numerical integration of Eqs. (31) and (32), subject to initial conditions:

$$x = x_{in} : H = H_{in}, \quad p = 0, \quad dp/dx = p_{g,in} \quad (33)$$

This is a so-called ‘shooting problem’ in which  $p_{g,in}$  is guessed until  $p(x_{out}) = 0$ .

### III. Results

#### A. Cylinder-plane rolling contact, constant grease layer thickness

To portray the overall model behavior, we use the same simplified representative problem for rigid surfaces and an isoviscous oil. For the first example, we assume the grease layer height  $H$  is fixed. In the calculations, we decouple Eq. (32) from Eq. (31). The boundary conditions used are,

$$L = \sqrt{2Rh_0}, \quad x = -3L : p = 0; \quad x = 0, p = 0 \quad (34)$$

The inlet is taken sufficiently far to the left such that the local film thickness  $h_{in}$  is much greater than the minimum thickness at the midplane  $h_0$ . Because our goal is to portray the basic model behavior, we assume the pressure in the diverging region is zero (ambient), i.e., the half-Somerfeld condition. Again, we realize these conditions are oversimplified, but they

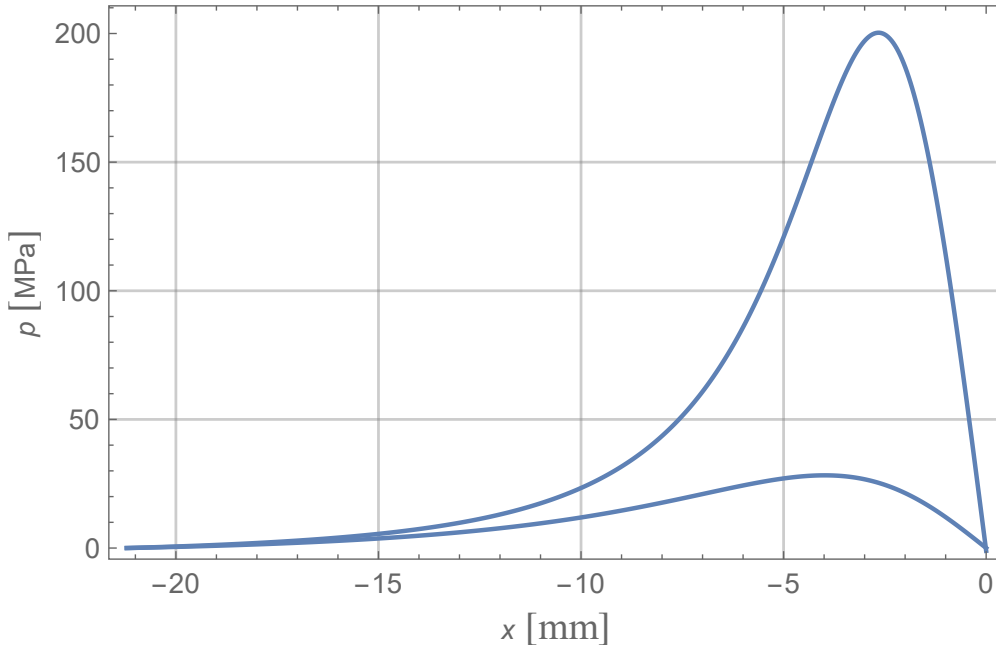


Figure 6: Rolling contact pressure distribution, with and without grease layer. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa.s}$ . The lower curve is for no grease layer, the upper curve is for  $H = 45 \mu\text{m}$ , and  $k = 2.0 \cdot 10^{-10} \text{ m}^2$ .

are useful to illustrate the grease model behavior. Computed pressure profiles are shown in Fig. 6. The following conditions are in force:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$  and  $V = 80 \text{ m/s}$ .

The lower pressure curve is the solution of Reynolds equation for a film entirely of oil, while the upper is for a grease layer with layer thickness  $H = 45 \mu\text{m}$ , and permeability coefficient  $k = 2.0 \cdot 10^{-8} \text{ m}^2$ . The pressure, and thus the load carrying capacity, is considerably enhanced by the grease layer. If the film is same order as the pore size, the model effective viscosity becomes that of the base oil. This result is consistent with the experimental observation of Cousseau et al. 2012 that for EHL (very thin films) the base oil viscosity is a reasonable predictor friction and load.

Another interesting result is shown in Fig. 7. For the same conditions, the figure shows the flow supply from the grease layer to the upper pure bled oil region ( $q_i$ ), as a fraction of total oil flow ( $q_O$ ). In the region of positive pressure gradient toward the left side, the grease is supplying oil to the upper layer, while toward the minimum film, oil is being squeezed back into the grease.

## B. Cylinder-Plane Rolling Contact, varying grease layer thickness

In this case, we use Eqs. (31)-(32) and (34), along with the left-hand-side boundary condition  $x = -3L : H = H_{in}$ . Parameter values are as follows:  $L = \sqrt{2Rh_0}$ ,  $x = -L/2$ ,  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa.s}$ . An inlet condition on the grease layer is needed:  $H_{in} = 0.9h_0 = 45 \mu\text{m}$ .

Figure 8 shows the pressure behavior for two cases: permeability  $k = 10^{-9} \text{ m}^2$  (lower curve)



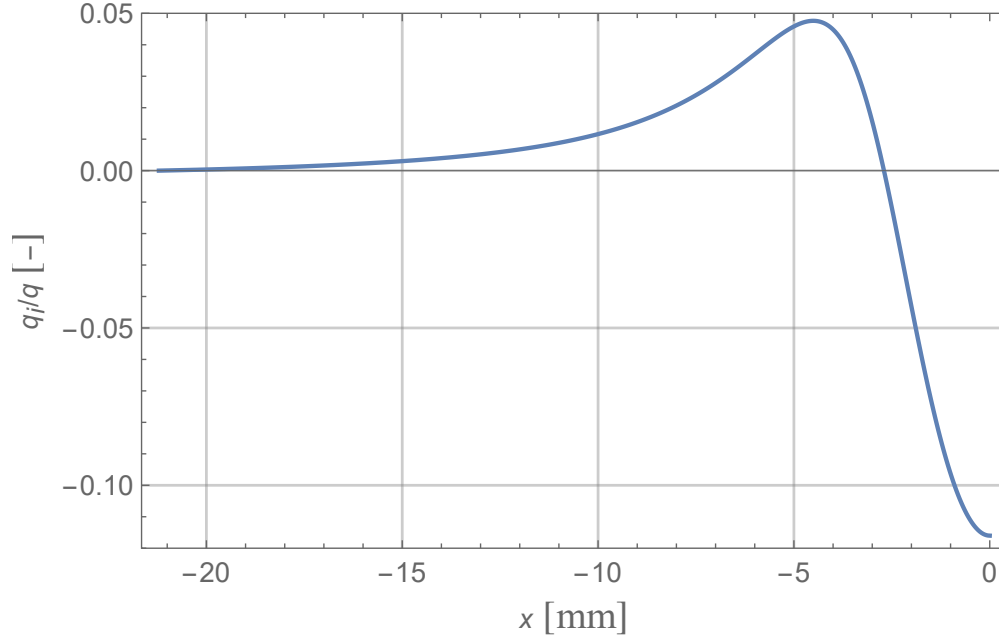


Figure 7: Rolling contact, oil flow from grease layer to upper oil layer. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5\text{m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa}\cdot\text{s}$ ,  $H = 45 \mu\text{m}$ , and  $k = 2.0 \cdot 10^{-10} \text{ m}^2$ .

and  $k = 10^{-11} \text{ m}^2$  (upper curve). As in the cases above, porous flow resistance in the grease causes increased pressure. Figure 9 shows the variation of the grease layer under the bearing conditions for the same two cases. In the case of lower permeability, the grease layer thickness is reduced into the contact.

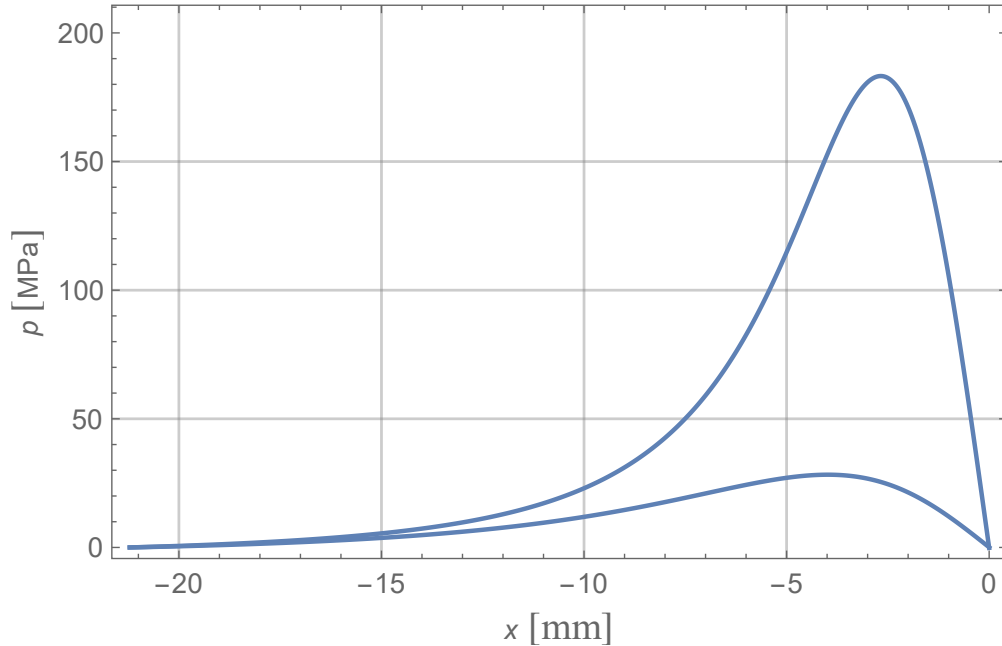


Figure 8: Rolling contact pressure distribution in which the grease layer has variable thickness, effect of permeability. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa}\cdot\text{s}$ ,  $H_{in} = 45 \mu\text{m}$ , permeability  $k = 10^{-9} \text{ m}^2$  (lower curve) and  $k = 10^{-11} \text{ m}^2$  (upper curve).

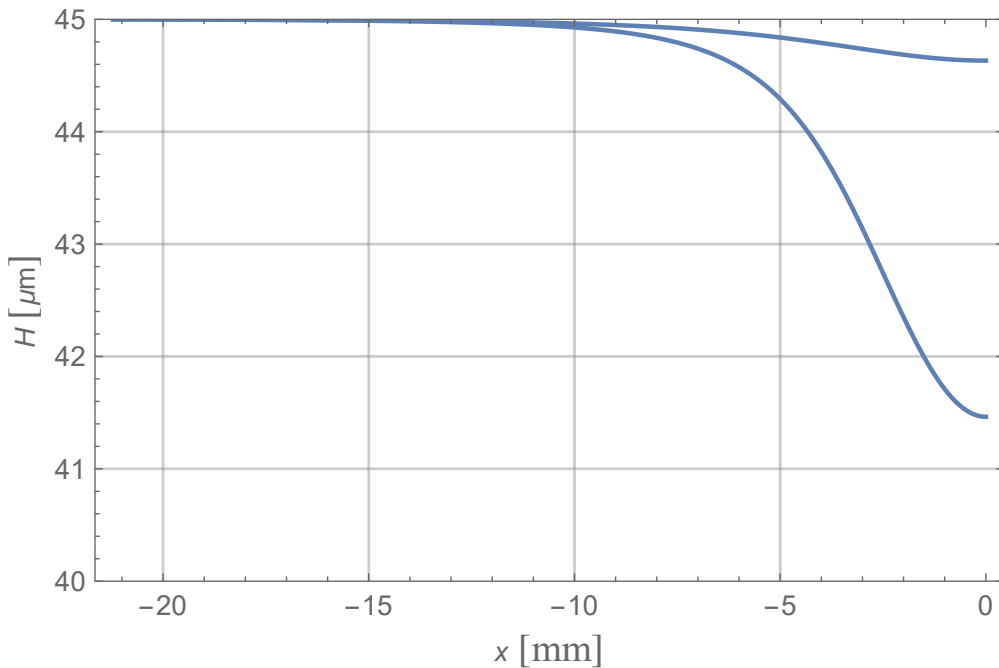


Figure 9: Rolling contact, grease layer thickness variation, effect of permeability. Parameter values:  $h_0 = 50 \mu\text{m}$ ,  $R = 0.5 \text{ m}$ ,  $V = 80 \text{ m/s}$ ,  $\eta^O = 0.1 \text{ Pa}\cdot\text{s}$ ,  $H_{in} = 45 \mu\text{m}$ . Permeability  $k = 10^{-9} \text{ m}^2$  (lower curve) and  $k = 10^{-11} \text{ m}^2$  (upper curve).

## IV. Conclusions

A new model for grease lubrication is proposed, which takes into account the fact that grease is a two-component mixture consisting of a thickener and a base oil. The thickener, rather than providing lubrication itself, is a sort of delivery system for the base oil. The thickener and the base oil interact according to the Darcy-Brinkman porous media model. The goal of the effort is to develop a model with predictive ability in engineering design, rather than to describe grease lubrication in scientific detail for numerical simulations. It is possible that grease lubrication is too complex and too varied from application to application and from lubricant to lubricant that anything other than highly empirical approaches will prove futile. The one-dimensional bearing case (two-dimensional thin film shearing flow) is addressed to focus on the basic physical aspects of the model, but it can be readily extended to the three-dimensional case.

The basic equations of the model are developed consisting of two coupled second order differential equations, one each for the thickener and the oil. If a Newtonian fluid is assumed, the equations are linear and can be solved in closed form. The linear equations are applied to Couette and Poiseuille flow, each of which is used as a device to measure flow properties. A Poiseuille flow device would be the more useful for grease as it would shed some light on the porous flow interactions between the components.

The model is then applied to highly idealized and simplified lubricated contacts. Two cases are noted, one in which the entire contact is flooded with grease, and a second in which the contact contains both a layer of grease and a layer of oil. The layered system loosely follows Cann's physical description of lubricated rolling contact. An increase in pressure (and consequently load carrying capacity) due to the grease is predicted. This increase is less notable as the film thickness approaches the pore size, and in the limit, the base oil behavior is obtained.

Only very basic and highly simplified demonstrative cases are addressed in the present work. Extensive parametric studies would best await a more highly evolved state of affairs. In particular, experiments to determine the permeability  $k$  would seem to be feasible. The basic model physics should be testable by viscosity determination of pure thickener and pure oil, followed by relatively straightforward measurements of grease behavior in Poiseuille flow.

Any number of extensions to the model can be foreseen (3D flow, non-Newtonian flow, property change with pressure and temperature, aging of the thickener properties, etc.), and many additional problems can be attacked (other types of bearings, elastic compliance of surfaces, etc.)

Whether such further studies prove worth the effort is yet to be determined. However, the present model does provide a new framework about which to consider grease modeling.

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