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Optical Flow Estimation in Ultrasound Images Using a Sparse Representation

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Abstract—This paper introduces a 2D optical flow estimation method for cardiac ultrasound imaging based on a sparse representation. The optical flow problem is regularized using a classical gradient-based smoothness term combined with a sparsity inducing regularization that uses a learned cardiac flow dictionary. A particular emphasis is put on the influence of the spatial and sparse regularizations on the optical flow estimation problem. A comparison with state-of-the-art methods using realistic simulations shows the competitiveness of the proposed method for cardiac motion estimation in ultrasound images.

Keywords—Optical flow, sparse representations, cardiac ultrasound, motion estimation, dictionary learning.

I. INTRODUCTION

Optical flow (OF) aims at estimating the pixel motion or flow between a pair of consecutive images. The differential OF methods, also known as gradient-based OF, assume that the intensity of a particular pixel is constant across consecutive frames [1], [2] and estimate the flow using the spatial and temporal image intensity variations. OF methods have been used successfully in a large variety of applications ranging from computer vision [3] to more specific ones such as atmospheric motion estimation in meteorology [4]. These methods have also been investigated for various medical imaging modalities (including ultrasound imaging [2], [5], [6], magnetic resonance imaging (MRI) [7] and computed tomography (CT) [8]) and for different clinical applications requiring cardiac motion estimation [2], [5].

Because OF problems are ill-posed, the constant intensity assumption also known as *brightness constancy* requires additional constraints to provide a unique solution. These constraints are generally expressed as global [1] or local [2] regularizations. Global OF regularization schemes aim at estimating a dense flow, while local methods use windowing to constraint the flow in smaller regions. However, both approaches typically assume spatial smoothness of the flow field. Spatial regularization can be introduced by means of parametric models of the flow, e.g., based on affine motions [2], [9] or B-splines [2].

Sparse representations have been shown to be powerful tools for regularization in various problems. They have gained a lot of attention, especially for image denoising [10], but also for other image processing problems such as inpainting

or demosaicing [11]. Sparse representations assume that an image (or an image patch) can be sparsely represented in a suitable predefined or learned dictionary. It has been shown that the dictionaries learned from the data using algorithms such as K-SVD [12] and online dictionary learning (ODL) [13] can outperform the predefined dictionaries based on wavelets, curvelets, ... [13].

In the context of cardiac ultrasound imaging, sparse representations have been successfully combined with spatial smoothness constraints to regularize cardiac motion estimation [14], [15]. However, these works have never been formulated within a general OF framework. Instead, they have been investigated using specific assumptions about the ultrasound image distribution in order to construct the motion estimation problem. Motivated by the success of the sparsity-based prior for cardiac ultrasound (US), the objective of this work is to propose a cardiac flow estimation method within a general OF framework combining regularizations based on the sparsity of the flow field in an appropriate dictionary and its spatial smoothness.

The paper is organized as follows. Section II formulates the general OF estimation problem with spatial regularization. The proposed OF method with sparse and spatial regularizations is introduced in Section III. Experimental results are presented and discussed in Section IV whereas concluding remarks are reported in Section V.

II. BACKGROUND ON OPTICAL FLOW

Differential OF methods rely on the brightness constancy and temporal consistence assumptions. Based on these assumptions, the spatial and temporal image intensity variations are linked to the flow field, leading to the so-called optical flow constraint (OFC) equations

$$\partial_t \mathbf{I} + \nabla \mathbf{I}^T \mathbf{U} = 0 \quad (1)$$

where N is the number of image pixels, $\mathbf{U} = (\mathbf{u}^T, \mathbf{v}^T)^T \in \mathbb{R}^{2N}$ is the 2D flow field with $\mathbf{u} \in \mathbb{R}^N$ and $\mathbf{v} \in \mathbb{R}^N$ the vectorized horizontal and vertical velocities. $\mathbf{I} \in \mathbb{R}^N$ contains the vectorized image intensities, $\nabla \mathbf{I} = (\mathbf{I}_x, \mathbf{I}_y)^T \in \mathbb{R}^{2N}$ is the spatial intensity gradient in both directions and $\partial_t \mathbf{I}$ is the temporal derivative at time t . Note that according to (1), the flow field \mathbf{U} only depends on $\partial_t \mathbf{I}$ and $\nabla \mathbf{I}$.

A standard approach used to overcome the ill-posed nature of the OF estimation problem (1) consists in introducing assumptions about the spatial behavior of the flow [1]. One

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way of explicitly incorporating these spatial constraints in the flow estimation problem is through the minimization of an appropriate energy function defined as

$$E(\mathbf{U}, \mathbf{I}) = E_D(\mathbf{U}, \mathbf{I}) + \lambda_S E_S(\mathbf{U}) \quad (2)$$

where E_D results from (1) and is called the data fidelity term and E_S stands for the spatial constraint promoting the smoothness of the flow (also known as *spatial coherence*) and controlled by the regularization parameter $\lambda_S \in \mathbb{R}^+$. In this work, we start from the well-known energy minimization formulation proposed by Horn and Schunck [1], which minimizes the quadratic error of the OF-based data fidelity and the gradient-based spatial regularization, leading to the following problem

$$\min_{\mathbf{U}} \left\{ \|\partial_t \mathbf{I} + \nabla \mathbf{I}^T \mathbf{U}\|_2^2 + \lambda_S \|\nabla \mathbf{U}\|_2^2 \right\} \quad (3)$$

where $\nabla \mathbf{U} = (\mathbf{U}_x, \mathbf{U}_y)^T \in \mathbb{R}^{4N}$, with \mathbf{U}_x and \mathbf{U}_y the horizontal and vertical spatial gradients of the horizontal and vertical flows. The Horn-Schunck (HS) method related to the problem (3) has been proved to be a simple and efficient flow estimation method [16]. However, the reliance on a global (piece-wise) and purely geometrical prior, which does not hold in many cases, such as motion boundaries, makes it inadequate for the estimation of complex or multiple motions. In the next section, a sparsity-based regularization strategy is introduced in order to bypass these shortcomings.

III. PROPOSED OPTICAL FLOW WITH SPARSE AND SPATIAL REGULARIZATIONS

A. Sparse regularization

Recent advances have made possible to simulate realistic cardiac US image sequences with ground-truth [17], enabling the use of learning-based methods for cardiac flow estimation. In this context, we propose to use a learned flow dictionary that captures typical patterns of cardiac motion. Typically, the dictionary \mathbf{D} is overcomplete, leading to a sparse representation of the flow when decomposed on \mathbf{D} . A way of exploiting the sparse properties of the flow is to extract overlapping flow patches from the global flow field. Each flow patch is then expressed as a weighted linear combination of a few elements of the dictionary \mathbf{D} . This patch-wise approach is motivated by the fact that it allows meaningful local cardiac motion patterns to be captured. The resulting problem is called the *sparse coding* problem and can be performed jointly or separately for the horizontal and vertical flow components. In this work, the sparse coding is performed separately for \mathbf{u} and \mathbf{v} . We also use separate dictionaries $\mathbf{D}_u \in \mathbb{R}^{n \times q}$ and $\mathbf{D}_v \in \mathbb{R}^{n \times q}$ associated with the horizontal and vertical flow components \mathbf{u} and \mathbf{v} [14], [18], where n is the patch size and q is the number of elements in the dictionaries \mathbf{D}_u and \mathbf{D}_v . For example, the sparse coding problem for the horizontal flow component \mathbf{u} is

$$\min_{\alpha_{u,i}} \|\alpha_{u,i}\|_0 \text{ subject to } \|\mathbf{P}_i \mathbf{u} - \mathbf{D}_u \alpha_{u,i}\|_2^2 < \epsilon \quad (4)$$

where $i = 1, \dots, N_p$ with N_p the total number of patches, $\|\cdot\|_0$ is the l_0 pseudo-norm (which counts the number of non-zero elements of a vector), $\mathbf{P}_i \in \mathbb{R}^{n \times N}$ is an operator that extracts the i th patch from \mathbf{u} , $\alpha_{u,i} \in \mathbb{R}^q$ is the corresponding sparse vector with $q > n$ the number of atoms in the dictionary \mathbf{D}_u and ϵ is an *a priori* fixed constant. Note that the sparse coding of the vertical flow field \mathbf{v} is performed similarly, using the

dictionary \mathbf{D}_v . More details about how to solve the problem (4) will be provided in Section III-B.

In addition to the classical spatial smoothness constraint employed for the OF problem (3), we propose to introduce a patch-wise sparse regularization term E_P defined as

$$E_P(\mathbf{U}, \alpha) = \sum_{i=1}^{N_p} \|\mathbf{Q}_i \mathbf{U} - \mathbf{D} \alpha_i\|_2^2 \quad (5)$$

where $\mathbf{Q}_i \in \mathbb{R}^{2n \times 2N}$ is an operator that extracts the i th patches in the horizontal and vertical directions from \mathbf{U}^1 , $\mathbf{D} \in \mathbb{R}^{2n \times 2q}$ is a block diagonal matrix whose blocks are \mathbf{D}_u and \mathbf{D}_v , i.e.,

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_u & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_v \end{bmatrix}$$

and $\alpha \in \mathbb{R}^{2q \times 2N_p}$ is a matrix whose columns are $\alpha_i = (\alpha_{u,i}^T, \alpha_{v,i}^T)^T$. The sparse regularization term (5) constrains each flow patch $\mathbf{Q}_i \mathbf{U}$ to be sparsely represented in the learned dictionary \mathbf{D} of typical flow patterns. In contrast with the purely geometrical assumptions used for the regularization term E_S , the learned flow patterns encode more complex and general behaviors of the cardiac flow (including motion discontinuities inside the myocardium), leading to a spatially more flexible and cardiac motion-specific prior. Note that the use of overlapping patches in (5) introduces an implicit inter-patch regularization in accordance with the *expected patch log-likelihood* framework [19]. This regularization is due to the fact that a single pixel is counted multiple times.

Finally, after combining the data fidelity term and the spatial constraint in (3) with the prior (5), the global energy function is

$$E(\mathbf{U}, \mathbf{I}, \alpha) = E_D(\mathbf{U}, \mathbf{I}) + \lambda_S E_S(\mathbf{U}) + \lambda_P E_P(\mathbf{U}, \alpha) \quad (6)$$

where $\lambda_P \in \mathbb{R}^+$ controls the importance of the sparse regularization term and E_S corresponds to the spatial regularization in (3). Note that the interest of introducing a sparsity-based prior for the flow estimation has been previously shown in [18], [20] on the Middlebury dataset [21]. Note also that it has been successfully employed for cardiac US motion estimation in [14], [15]. However, in these studies, the sparse regularization was used with a specific data fidelity term (based on a Rayleigh noise model) different from the one investigated in this paper. In this work, we seek to highlight the benefits of using a combination of spatial and sparse regularizations for cardiac flow estimation within a more general OF-based framework.

B. Estimation

1) *Dictionary learning*: The proposed method requires a training step during which the dictionaries \mathbf{D}_u and \mathbf{D}_v are learnt using a set of ground-truth cardiac motion files. The dictionary learning was performed offline using the ODL algorithm [13], which iterates between a sparse coding step (\mathbf{D}_u and \mathbf{D}_v fixed) and a dictionary update step ($\alpha_{u,i}$ and $\alpha_{v,i}$ fixed). Details about the parameter choices for the dictionary learning are provided in Section IV-C. Note that the horizontal and vertical dictionaries were learnt separately (see Section III-A). Note also that the dictionaries \mathbf{D}_u and \mathbf{D}_v could be updated in an adaptive way [14] during the flow estimation step (see Section III-B2). However, we have not observed

¹ \mathbf{Q}_i is a block diagonal matrix whose blocks are \mathbf{P}_i , i.e., $\mathbf{Q}_i = \mathbf{P}_i \otimes \mathbf{I}_2$, with \mathbf{I}_2 the 2×2 identity matrix and \otimes the Kronecker product.

significant improvements with this adaptive scheme for cardiac flow estimation in ultrasound imaging.

2) *Flow estimation*: Once the dictionaries D_u and D_v have been learned using a set of training flow fields, the OF estimation problem reduces to the following optimization problem with respect to U and α_i

$$\min_{\alpha_i, U} \left\{ E_D(U, I) + \lambda_S \|\nabla U\|_2^2 + \lambda_P \sum_{i=1}^{N_p} \|Q_i U - D \alpha_i\|_2^2 \right\}$$

subject to $\forall i = 1, \dots, N_p, \|\alpha_{u,i}\|_0 \leq K$ and $\|\alpha_{v,i}\|_0 \leq K$. (7)

Since (7) is hard to solve directly, we use an optimization strategy that alternates optimizations with respect to α_i and U . This process is repeated during a few iterations for fixed values of λ_S and λ_P (typically 4 iterations, as in [22]), after which the sparsity parameter λ_P is increased. The optimization with respect to α_i is a sparse coding problem, which is known to be NP-hard. However, good approximate solutions can be achieved using greedy algorithms (such as the orthogonal matching pursuit (OMP) [23]) or convex relaxation methods (such as LASSO). The optimization with respect to U can be solved by setting the gradient to zero. These two steps are detailed below.

- **Sparse coding**: the sparse coding problem is addressed using OMP and is performed separately for the horizontal and vertical directions. The minimization problem with respect to $\alpha_{u,i}$ for the horizontal flow field is

$$\min_{\alpha_{u,i}} \sum_{i=1}^{N_p} \|P_i u - D_u \alpha_{u,i}\|_2^2 \text{ subject to } \forall i, \|\alpha_{u,i}\|_0 \leq K$$

(8)

where K is the maximum number of non-zero coefficients. The same approach is used for the vertical flow component.

- **Flow estimation**: after determining the sparse coefficients α_i , the optimization is conducted with respect to the flow field U . The corresponding minimization problem is

$$\min_U \left\{ E_D(U, I) + \lambda_S \|\nabla U\|_2^2 + \lambda_P \sum_{i=1}^{N_p} \|Q_i U - D \alpha_i\|_2^2 \right\}$$

(9)

Since the cost function in (9) is differentiable, its solution can be found by equating the gradient to zero and using the optimization approach studied in [24].

IV. EXPERIMENTAL RESULTS

This section evaluates the performance of the proposed method using realistic simulated sequences with available ground-truth. In the following section we analyze the effect of the spatial and sparse regularizations when used separately and combined as in (6). The performance of the proposed method is then compared to two state-of-the-art OF algorithms. The first method uses the standard HS formulation (3), whereas the second approach considers a phase-based data term (based on the monogenic signal used in [9]) and a patch-wise affine motion parameterization.

A. Realistic Simulations

The synthetic evaluation dataset provided in [17] contains simulated US sequences, which are to our knowledge among the most realistic in the recent US literature. The ground-truth flow fields provide the possibility of training the flow dictionaries as well as evaluating the OF estimation accuracy. In this work, we choose to use the LADdist sequence for learning the dictionaries D_u and D_v and the LADprox sequence for evaluating the estimation performance. In both sequences, the image size is 224×208 , with a pixel size of $0.7 \times 0.6 \text{ mm}^2$, and a frame rate of 22 Hz. The sequences represent a full cardiac cycle of 34 consecutive frames (see Fig. 1 for an example). Finally, note that all the sequences presented in this section simulate cardiac pathologies².

In order to evaluate the performance using these realistic datasets, we use the endpoint error described in [9]. For each pixel k , this error is computed as $e(k) = \sqrt{[u(k) - \hat{u}(k)]^2 + [v(k) - \hat{v}(k)]^2}$, where $u(k), v(k)$ and $\hat{u}(k), \hat{v}(k)$ are the true and estimated horizontal and vertical flow fields at pixel k .

B. Sparse and spatial regularizations

In this section, we analyze the effects of the spatial and sparse regularization terms when used separately and combined as in (6). In order to have a visual interpretation of the

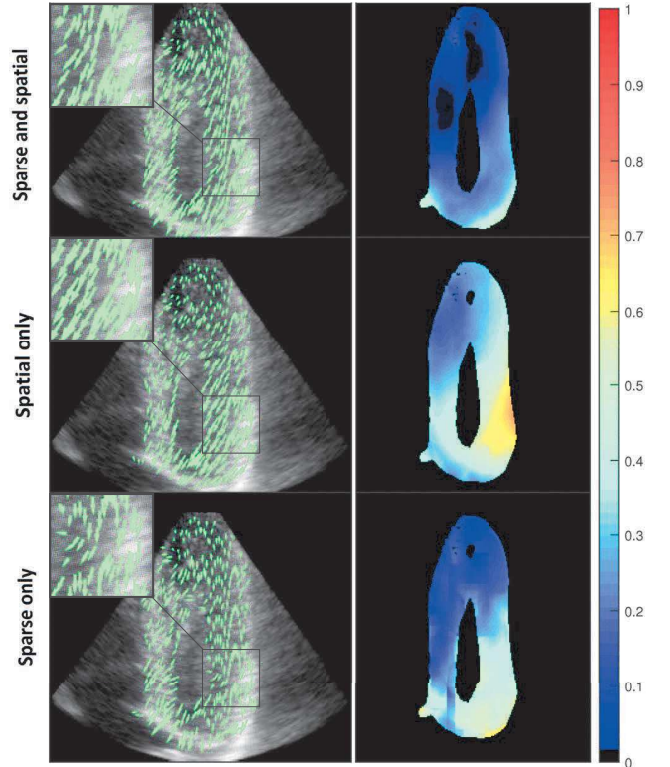


Fig. 1: The estimated flow fields (left) and the corresponding error maps (right) for the 20th frame of the LADprox sequence with separated and combined sparse and spatial regularizations.

²More details about these pathologies and the data generation process can be found at <https://team.inria.fr/asclepios/data/straus/>.

influence of each regularization term on the estimation process, the estimated flow fields between a pair of consecutive diastolic frames (20th and 21st frames) of the LADprox sequence as well as the corresponding error maps are illustrated in Fig. 1. The error maps correspond to the endpoint error for the displacements of each pixel in the 20th frame. It is clear from the error maps (right) that the combined use of the spatial and sparse regularizations provides the smallest error for this frame. The estimated motion fields (left) show that the flow estimated with the spatial term alone is over-smoothed and lacks structure, while the flow resulting from the sparse regularization alone lacks smoothness. This is for example the case for some patch borders that create nonexistent motion boundaries.

These conclusions are confirmed in terms of flow estimation accuracy for the entire sequence, detailed in Fig. 2 and Table I. Fig. 2 shows that the combination of the sparse and spatial regularizations provides the smallest mean endpoint errors for almost all the frames of the cardiac cycle. Table I confirms that the combination of the two regularization terms provides the best performance in terms of average mean and standard deviation (std) for the 34 frames of the sequence. Details about the parameter choices for the proposed OF estimation method are given in Section IV-C.

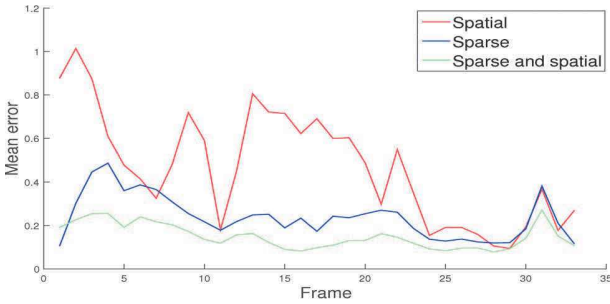


Fig. 2: Mean endpoint error for the LADprox sequence with separated and combined sparse and spatial regularizations.

Error	Both	Spatial	Sparse
Mean	0.15	0.46	0.24
\pm Std	0.10	0.40	0.14

TABLE I: Average error means and stds for the LADprox sequence using spatial, sparse and both regularizations.

C. Comparison with state-of-the-art methods

The parameters used for the OF estimation of the LADprox sequence were selected by cross-validation. The size of the dictionaries D_u and D_v was 256×384 (corresponding to patches of size 16×16 pixels for each direction) and the maximum number of non-zero sparse coefficients was set to $K = 5$. Generally, the parameter K is much smaller than the number of atoms q in the dictionary.

As seen in Section III-B, the sparse regularization parameter λ_p of the proposed method was logarithmically increased from 10^{-3} to 10^2 in 6 outer iterations. The spatial parameter was adjusted to $\lambda_s = 0.1$. In the same way, the parameters giving the best performance for the state-of-the-art algorithms were selected. Those parameters were 0.75 for the spatial

regularization of the HS method (3) and 0.25 for the initial wavelength of the monogenic signal algorithm.

Table II shows the average means and stds of the endpoint error for the 34 frames of the LADprox sequence. The proposed method provides smaller average endpoint errors and competitive estimation stds, resulting in a better performance for the considered sequence. These results are confirmed for

Error	Proposed	HS	Monogenic
Mean	0.15	0.22	0.30
\pm Std	0.10	0.08	0.18

TABLE II: Average endpoint error for the simulation sequence LADprox.

the entire cardiac cycle as shown in Fig 3. The plots in Fig 3 show that the proposed method provides smaller errors for the majority of the frames of the entire cardiac cycle. Less differences can be observed at the end of the sequence (the end of the cardiac cycle), where the displacements are relatively small. Note that contrary to the proposed method, both the HS and the monogenic signal methods use a coarse-to-fine estimation scheme to cope with large displacements. Note also that this coarse-to-fine implementation is the main difference between the HS method used in this section and the proposed approach with $\lambda_p = 0$ (*i.e.*, proposed flow estimation method with spatial regularization alone) studied in Section 1.

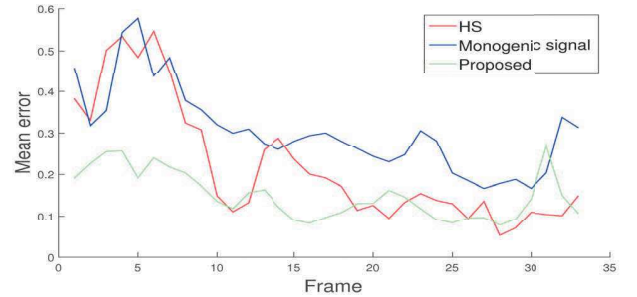


Fig. 3: Mean endpoint error for the LADprox sequence.

V. CONCLUSIONS

This paper introduced a new method for optical flow estimation based on a classical spatial smoothness prior and a sparsity-based regularization term incorporated into a standard variational optical flow problem. The results obtained in this paper showed the effectiveness of the combined spatial and sparse regularizations, with competitive results when compared to state-of-the-art algorithms in cardiac ultrasound imaging.

For future work, it would be interesting to conduct more experiments, particularly for *in vivo* data. Since the proposed approach considers a standard optical flow-based data fidelity term, it could be applied to other imaging modalities (*e.g.*, MRI) or could be used to learn dictionaries for different flow types in the context of other clinical applications. Furthermore, the temporal aspect could be incorporated in the optical flow problem by exploiting more than a pair of images. Also, it would be interesting to consider a fully robust approach in order to handle potential data outliers and violations of the smoothness assumption across myocardium borders.

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