

Flow in highly permeable porous media Michel Quintard

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4th Cargèse Summer School, 2018

Flow in High Permeability Porous Media

M. Quintard

https://www.dropbox.com/s/6v6sfi3ngzk5qri/two_phase_high_perm_car_00.pdf?dl=0

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Transport in Porous Media

Outline (emphasis on theoretical aspects)

- Introduction: background, multiple-scale, averaging
- One-Phase Flows:
 - Phenomenology:
 - Departure from Darcy's law
 - Weak inertia regimes
 - Strong Inertia regimes
 - Transitions
 - Turbulence: pore-scale or large-scale?
 - Upscaling?
- Two-Phase Flows: introduction to several models
- Conclusions



Structured and Packed bed reactors, nuclear safety, fractured media, karsts, ...



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Upscaling



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- Separation of scales: $l_{\beta}, l_{\sigma} \ll L$
- Objectives of a macroscale theory?
 - Smoothing operator → macro-scale equations and BCs
 - Link between micro- and macro-scale
 - Effective properties

Upscaling: different point of Views

Heuristic: Darcy's law (1856)!



Upscaling with closure: homogenization theory (Sanchez-Palencia, Bensoussans et al.,...**), volume averaging (**Whitaker, ... and variants**),**

$$p_{\beta} = \left\langle p_{\beta} \right\rangle^{\beta} + \tilde{p}_{\beta} \qquad \qquad \tilde{p}_{\beta} \stackrel{?}{=} f\left(\left\langle p_{\beta} \right\rangle^{\beta}, \nabla \left\langle p_{\beta} \right\rangle^{\beta}, \ldots\right)$$

- Stochastic theories (Matheron, Dagan, Gelhar, ...), ...
- Mixture theories: volume averaging + irreversible thermodynamics (Marle, Hassanizadeh and Gray, Bowen, ...)
- Other point of views...: dual-phase-lagging heat conduction (Wang et al., 2008; Vadasz, 2005...); mixed models; fractional derivatives (Néel,...); CTRW; ...





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Upscaling with *closure*: a schematic view for diffusion in a heterogeneous medium



One-Phase Flow: Phenomenolgy

• Pore-scale equation $\nabla \cdot \mathbf{v}_{\beta} = 0$

 $\rho_{\beta} \frac{\partial \mathbf{v}_{\beta}}{\partial t} + \rho_{\beta} \mathbf{v}_{\beta} \cdot \nabla \mathbf{v}_{\beta} = -\nabla p_{\beta} + \rho_{\beta} \mathbf{g} + \mu_{\beta} \nabla^2 \mathbf{v}_{\beta} \qquad \text{B.C.1 } \mathbf{v}_{\beta} = 0 \quad \text{at } A_{\beta\sigma}$ $\operatorname{Re} = \frac{\rho_{\beta} U_r \ell_{\beta}}{\mu_{\beta}}; \ p_{\beta}^c = \frac{p_{\beta} \ell_{\beta}}{\mu_{\beta} U_r}; \ \operatorname{Fr} = \frac{U_r}{\sqrt{\ell_{\beta} g}} \quad \Longrightarrow \quad \operatorname{Re} \left(\frac{\partial \mathbf{v}_l^{'}}{\partial t^{'}} + \mathbf{v}_l^{'} \cdot \nabla^{'} \mathbf{v}_l^{'} \right) = -\nabla^{'} p_l^c + \frac{\operatorname{Re}}{\operatorname{Fr}^2} \frac{\mathbf{g}}{g} + \nabla^{'2} \mathbf{v}_l^{'}$

Pore-Scale regimes

- -Creeping (Re \rightarrow 0): leads to Darcy's law

Laminar
 Turbulent
 Various regimes and transitions

Upscaling One-Phase Flow

• Averaging
$$p_{\beta} = \langle p_{\beta} \rangle^{\beta} + \tilde{p}_{\beta} ; \mathbf{v}_{\beta} = \langle \mathbf{v}_{\beta} \rangle^{\beta} + \tilde{\mathbf{v}}_{\beta}$$

 $\nabla \cdot \langle \mathbf{v}_{\beta} \rangle = 0$
B.C.1 $\tilde{\mathbf{v}}_{\beta} = -\langle \mathbf{v}_{\beta} \rangle^{\beta}$ at $A_{\beta\sigma}$
 $\rho_{\beta} \frac{\partial \langle \mathbf{v}_{\beta} \rangle}{\partial t} + \rho_{\beta} \langle \mathbf{v}_{\beta} \rangle \cdot \nabla \langle \mathbf{v}_{\beta} \rangle^{\beta} + \nabla \cdot (\rho_{\beta} \langle \tilde{\mathbf{v}}_{\beta} \tilde{\mathbf{v}}_{\beta} \rangle) =$
 $-\varepsilon_{\beta} \nabla \langle p_{\beta} \rangle^{\beta} + \varepsilon_{\beta} \rho_{\beta} \mathbf{g} + \underbrace{\mu_{\beta} \nabla^{2} \langle \mathbf{v}_{\beta} \rangle - \mu_{\beta} \nabla \varepsilon_{\beta} \cdot \nabla \langle \mathbf{v}_{\beta} \rangle^{\beta}}_{\text{Brinkman terms}} + \frac{1}{V} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\tilde{p}_{\beta} \mathbf{I} + \mu_{\beta} \nabla \tilde{\mathbf{v}}_{\beta}) dA$

Remark: $\tilde{\mathbf{v}}_{\beta} \sim \langle \mathbf{v}_{\beta} \rangle^{\beta}$

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Various Regimes: Creeping

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 $Re \sim 0$

Various Regimes: Laminar Inertial



Still
$$0 = -\varepsilon_{\beta} \nabla \langle p_{\beta} \rangle^{\beta} + \varepsilon_{\beta} \rho_{\beta} \mathbf{g} + \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot \left(-\tilde{p}_{\beta} \mathbf{I} + \mu_{\beta} \nabla \tilde{\mathbf{v}}_{\beta}\right) dA$$

But non-linear relationship between average velocity and pressure drop!



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Various Regimes: towards turbulence

(v)=steady-state



- Macro-Scale Turbulence? (see Jin et al., 2015, DNS)
 - Localized turbulence (nearly periodic over *N*-UC) need $\overline{\langle v_{\beta} \rangle}$

 Macro-scale turbulence? Ex.: entrance regions (D'Hueppe et al., ...)

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• Various types of bifurcations:



Ex.: array of cylinders – not same Re_c for flow // or \perp to axis

- Bifurcations depends on Re and number of UC (Agnaou et al., 2016)...and topology! \rightarrow REV size=f($\langle v_{\beta} \rangle^{\beta}$)
- Difficult to estimate order of magnitude when fully non-linear \rightarrow rely on DNS

RESULT: $(\nabla \langle p_{\beta} \rangle^{\beta} - \rho_{\beta} \mathbf{g})$ and $\langle \mathbf{v}_{\beta} \rangle$ nearly constant over REV!

and up to localized turbulence one has

 $\overline{(\nabla \langle p_{\beta} \rangle^{\beta} - \rho_{\beta} \mathbf{g})}$ and $\overline{\langle \mathbf{v}_{\beta} \rangle}$ nearly constant over REV!

Closure: Darcy regime (Stokes problem=creeping flow) Re~0

PDEs for deviations

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Closure in the linear case (3 elementary solutions for e_i)

$$\tilde{p}_{\beta} = \mu_{\beta} \mathbf{b}. \langle \mathbf{v}_{\beta} \rangle^{\beta} + ...; \mathbf{\tilde{v}} = \mathbf{B}. \langle \mathbf{v}_{\beta} \rangle^{\beta} + ...$$

Closure and Macro-Scale Equation (Darcy regime) $\nabla \mathbf{B} = 0$ $-\nabla \mathbf{b} + \nabla^2 \mathbf{B} = \varepsilon_{\beta}^{-1} \frac{1}{\mathcal{V}} \int \mathbf{n}_{\beta\sigma} \cdot (-\mathbf{I}\mathbf{b} + \nabla \mathbf{B}) \, dA$ $A_{\beta\sigma}$ $\mathbf{B} = -\mathbf{I} \quad \text{at } A_{\beta\sigma} \ , \langle \mathbf{b} \rangle^{\beta} = 0$ $\mathbf{b}(\mathbf{r} + \mathbf{l}_i) = \mathbf{b}(\mathbf{r})$ $\mathbf{B}(\mathbf{r} + \mathbf{l}_i) = \mathbf{B}(\mathbf{r})$ $\langle \mathbf{v}_{\beta} \rangle = -\frac{1}{\mu_{\beta}} \mathbf{K} \cdot \left(\nabla \left\langle p_{\beta} \right\rangle^{\beta} - \rho_{\beta} \mathbf{g} \right)$ $\mathbf{K}^{-1} = -\varepsilon_{\beta}^{-2} \frac{1}{\mathcal{V}} \int \mathbf{n}_{\beta\sigma} \cdot (-\mathbf{I}\mathbf{b} + \nabla \mathbf{B}) \, dA$ $A_{\beta\sigma}$

K intrinsic permeability

See Sanchez-Palencia (homogenization), Whitaker, ...

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Other version of closure: permeameter-like problems

Note: also suitable for non-linear problems (non-newtonian flows, etc...)

$$\rho_{\beta} \mathbf{v}_{\beta} \cdot \nabla \tilde{\mathbf{v}}_{\beta} = -\nabla \tilde{p}_{\beta} + \nabla \cdot \left(\mu_{\beta} \left(\nabla \tilde{\mathbf{v}}_{\beta} + \nabla \tilde{\mathbf{v}}_{\beta}^{T} \right) \right) - \left(\nabla \left\langle p_{\beta} \right\rangle^{\beta} - \rho_{\beta} \mathbf{g} \right)$$

=0, Darcy regime $\nabla \cdot (\tilde{\mathbf{v}}_{\beta}) = 0$ $\tilde{\mathbf{v}} = -\left\langle \mathbf{v}_{\beta} \right\rangle^{\beta}$ at A

=0, Darcy regime

layer, ...

BCs (see Guibert et al., 2016)?

- Periodicity conditions: good for anisotropic media, but percolation problem if "periodized" media
- Permeameter (various types: classical, Bamberger, ...)
- Bordering media: fluid or porous



 $\tilde{\mathbf{v}} = -\langle \mathbf{v}_{\beta} \rangle^{\rho}$ at $A_{\beta\sigma}$

Non Darcean regimes (Re>1)

• Heuristic: Forchheimer, Ergun, ...

 $0 = -\nabla P_{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} K^{-1} \mathbf{V}_{\beta} - \rho_{\beta} \eta^{-1} \| \mathbf{V}_{\beta} \| \mathbf{V}_{\beta} \qquad \text{with } \mathbf{V}_{\beta} = \langle \mathbf{v}_{\beta} \rangle^{\rho}$ $K = \frac{\varepsilon^3 d^2}{h_{\kappa} (1 - \varepsilon)^2} \quad ; \quad \eta = \frac{\varepsilon^3 d}{h_n (1 - \varepsilon)}$ weak strong regime regime 10^{0} (passability) quadratic 10-1 Darcy regime regime **Upscaling**? Ц transition regime 10^{-2} Macro-scale $0 = -\nabla \left\langle p_{\beta} \right\rangle^{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \mathbf{K}_{app}^{-1} \cdot \left\langle \mathbf{v}_{\beta} \right\rangle$ turbulence 10^{-3} $0 = -\nabla \left\langle p_{\beta} \right\rangle^{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \mathbf{K}^{-1} \cdot \left\langle \mathbf{v}_{\beta} \right\rangle - \mathbf{F} \left(\left\langle \mathbf{v}_{\beta} \right\rangle \right) \cdot \left\langle \mathbf{v}_{\beta} \right\rangle$ 10-4 10° 10^{-2} 10⁻¹ 10¹ 10^{2} Re_{k}

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Non Darcean regimes (Re>1)

Iaminar inertia effects: theory → generalized
 Forchheimer equation

$$0 = -\nabla \left\langle p_{\beta} \right\rangle^{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \mathbf{K}^{-1} \cdot \left\langle \mathbf{v}_{\beta} \right\rangle - \mathbf{F} \left(\left\langle \mathbf{v}_{\beta} \right\rangle \right) \cdot \left\langle \mathbf{v}_{\beta} \right\rangle$$

- Re \rightarrow 0: Darcy, F=0
- Re ~ 0: weak inertia, F. $\langle v_{\beta} \rangle \sim \langle v_{\beta} \rangle^{3}$ (Levy, Mei & Auriault, Firdaouss, ...)
- Re > 0: strong inertia, $F.\langle v_{\beta} \rangle \sim \langle v_{\beta} \rangle^2$ (Whitaker, 1996; Lasseux et al., 2011;...)

Weak Inertia

Solution following asymptotic expansions in Re $\delta = \frac{\ell}{\nu_{\beta}}v = \text{Re}$

$$\mathbf{v}^* = \frac{\tilde{\mathbf{v}}_{\beta}}{v} + \lambda, \ p^* = \ell_{\beta} \frac{\tilde{p}_{\beta}}{\mu_{\beta} v}$$

$$\mathbf{v}^{*} = \mathbf{v}_{0}^{*} + \delta \mathbf{v}_{1}^{*} + \delta^{2} \mathbf{v}_{2}^{*} + O\left(\delta^{3}\right)$$
$$p^{*} = p_{0}^{*} + \delta p_{1}^{*} + \delta^{2} p_{2}^{*} + O\left(\delta^{3}\right)$$

1st correction?

– Periodic media: $F_{\parallel} \sim v^2$

-Periodic media+reversibility: $F_{\parallel}^{(1)} = F_{\perp}^{(1)} = 0$

Wodié et Lévy, 1991; Mei et Auriault, 1991; Firdaouss et al., 1997 revisited by Pauthenet et al., 2017



Strong Inertia

• Solution for
$$\nabla \langle p_{\beta} \rangle^{\beta} - \rho_{\beta} \mathbf{g} \Rightarrow (\tilde{p}_{\beta}, \mathbf{v}_{\beta}) \Rightarrow \langle \mathbf{v}_{\beta} \rangle^{\beta} = v\lambda$$

 $0 = -\nabla \langle p_{\beta} \rangle^{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \mathbf{K}_{app}^{-1} \cdot \langle \mathbf{v}_{\beta} \rangle$
 $0 = -\nabla \langle p_{\beta} \rangle^{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} (\mathbf{I} + F\mathbf{P}) \cdot \mathbf{K}^{-1} \cdot \langle \mathbf{v}_{\beta} \rangle$
or $\nabla \langle p_{\beta} \rangle^{\beta} - \rho_{\beta} \mathbf{g} = -\varepsilon_{\beta} \mu_{\beta} (\mathbf{K}_{D}^{-1} \cdot \lambda + \mathbf{f}) v$
P: rotation
F: non-linear scaling

 $\mathbf{f} = K_{\lambda}^{-1} \left(F_{\parallel} \boldsymbol{\lambda} + F_{\perp} \left(\cos \left(\boldsymbol{\omega} \right) \mathbf{n}_{1} + \sin \left(\boldsymbol{\omega} \right) \mathbf{n}_{2} \right) \right)$

$$\mathbf{n}_{1} \times \mathbf{n}_{2} = \lambda \qquad F_{\parallel} = \frac{\lambda \cdot \mathbf{f}}{K_{\lambda}^{-1}} \qquad K_{\lambda}^{-1} = ||\mathbf{K}_{D}^{-1} \cdot \lambda|$$
$$F_{\perp} = ||(\mathbf{I} - \lambda\lambda) \cdot \frac{\mathbf{f}}{K_{\lambda}^{-1}}|| \qquad \omega = \left((\mathbf{I} - \lambda\lambda) \cdot \frac{\nabla \langle p_{\beta} \rangle^{\beta} - \rho_{\beta} \mathbf{g}}{\varepsilon_{\beta} \mu_{\beta} K_{\lambda}^{-1} v}, \mathbf{n}_{1}\right)$$

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Transition to inertia regimes: which Re number? $\operatorname{Re} = \frac{\left\langle \mathbf{v}_{\beta} \right\rangle^{\beta} \ell}{v_{\beta}}$

- Literature discussion (see polemic in Lage et Antohe, 2000; etc...)
- **ℓ**=√K ?
 - $K = \epsilon d^2/32$ for a bundle of tubes (transition would change with porosity! \rightarrow better \sqrt{K}

$$\ell = \sqrt{\frac{K}{\varepsilon}} \to \operatorname{Re}_k$$

■ Topological problem: array of cylinders → transition depends on the orientation → need a topological information!

Transition to inertia effect: effect of topology (Pauthenet et al., 2017)

If Re_k





(various media, rocks, ...: Muljadi et al., 2016; etc...)



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Topology effect (Ex.: array of Cylinder)

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Tentative (open question)

from Darcy closure $C_{\lambda} = \frac{\zeta(\mathbf{v}_{0}^{*} \cdot \nabla^{*} \mathbf{v}_{0}^{*})}{\sqrt{\langle \nabla^{*} \mathbf{v}_{0}^{*} : T \nabla^{*} \mathbf{v}_{0}^{*} \rangle^{\beta}}} \sqrt{\frac{\lambda \cdot \mathbf{K}_{D}^{-1} \cdot \lambda}{K_{\lambda}^{-1}}}$ $\zeta(\mathbf{v}_{0}^{*} \cdot \nabla^{*} \mathbf{v}_{0}^{*}) = \langle ||\mathbf{v}_{0}^{*} \cdot \nabla^{*} \mathbf{v}_{0}^{*}||^{n} \rangle^{\beta^{1/n}}$ 0.4 $\alpha=0, \perp$ to cyl 5 0.2 0 40 60 0 20 80 $\alpha(^{\circ})$

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Example: canopy flow

Honami generation (Pauthenet, 2017) ...need \mathbf{K}_{app} for all fiber directions and \mathbf{V}_{β}





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Experimental Evidence: inertia regime



Turbulent flows in porous media (Re>>1)

• Turbulence: time and spatial averaging (see book De Lemos, 2006; …)

- Time and spatial averaging commute!
- However: not necessarily the same result if sequential closure!?

I.
$$\mathbf{v}_{\beta} \to \langle \mathbf{v}_{\beta} \rangle \to \overline{\langle \mathbf{v}_{\beta} \rangle}$$

II. $\mathbf{v}_{\beta} \to \overline{\mathbf{v}_{\beta}} (RANS, ...) \to \langle \overline{\mathbf{v}_{\beta}} \rangle$

- For one-phase flow: scheme "II" seems preferable: contrary to Antohe & Lage (1997), Getachew et al. (2000), see discussion in Nakayama & Kuwahara (1999), Pedras and de Lemos (2001), etc...
- Open: Simultaneous closure over $[R_3 \times R_t]$? More complex sequential closures (t \rightarrow x \rightarrow t \rightarrow ...) depending on the hierarchy of scales?
- Open question for multiphase flow?

Turbulent flows in porous media (continued)

- Localized Turbulence: i.e., nearly periodic (see Jin et al., 2015 for DNS results)
 - Slow unsteady flows \rightarrow generalized Forchheimer for time averaged pressure and velocity $\langle v_{\beta} \rangle$ or $\langle \overline{v_{\beta}} \rangle$
 - Spatial averaging of RANS models \rightarrow generalized Forchheimer equation, *F* not necessarily $\sim \langle v_{\beta} \rangle$ or $\langle v_{\beta} \rangle^2$
- Example: structured packings

$$K_{yy}^* = \frac{K_0}{1 + \gamma_l \sqrt{Re}} \text{ (laminar)} ; \quad K_{yy}^* = \frac{K_1}{1 + \beta_t Re} \text{ (turbulent)} \text{ Soulaine and used and used of the second second$$

- Porous media turbulence models (beyond the scope of the lecture)?
 - Pedras & De Lemos, Masuoka & Takatsu (1996), Nakayama & Kuwahara (1999),
 ...
 - note: useful for fluid/porous medium interface (D'Hueppe et al., 2012), deep inside the p.m. one may recover Darcy-Forchheimer regimes

Example for Structured Packings



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Two-Phase Flow

Pore-scale

$$\nabla \cdot \mathbf{v}_{\alpha} = 0$$

$$\frac{\partial \rho_{\alpha} \mathbf{v}_{\alpha}}{\partial t} + \nabla \cdot (\rho_{\alpha} \mathbf{v}_{\alpha} \mathbf{v}_{\alpha}) =$$

$$-\nabla p_{\alpha} + \rho_{\alpha} \mathbf{g} + \mu_{\alpha} \nabla^{2} \mathbf{v}_{\alpha} \text{ in } V_{\alpha} \qquad \alpha = \beta, \gamma$$
B.C. 1 $\mathbf{v}_{\beta} = 0$ at $A_{\beta r}(t)$
B.C. 2 $\mathbf{v}_{\gamma} = 0$ at $A_{\gamma \sigma}(t)$
B.C. 3 $\mathbf{v}_{\beta} = \mathbf{v}_{\gamma}$ at $A_{\beta \gamma}(t)$
B.C. 4
$$-\mathbf{n}_{\beta \gamma} p_{\beta} + \mu_{\beta} \mathbf{n}_{\beta \gamma} \cdot (\nabla \mathbf{v}_{\beta} + (\nabla \mathbf{v}_{\beta})^{T}) =$$

$$-\mathbf{n}_{\beta \gamma} p_{\gamma} + \mu_{\gamma} \mathbf{n}_{\beta \gamma} \cdot (\nabla \mathbf{v}_{\gamma} + (\nabla \mathbf{v}_{\gamma})^{T}) + 2\sigma H_{\beta \gamma} \mathbf{n}_{\beta \gamma} \text{ at } A_{\beta \gamma}(t)$$

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Note: potentially unsteady flows

- Front propagation is in general time dependent!
- Specific features: Haines jumps, snap-off, ...
- Boiling, ...

• Common practice: quasi-static approximation... $\langle \phi \rangle$ and not $\overline{\langle \phi \rangle}$ Justification: Ergodicity (not validated)?

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Horgue et al.



See paper Gourbil et al.

Sapin et al., 2016



DNS

- VOF, LBM, Cahn-Hilliard, SPH,: spurious velocities & convergence for high density and viscosity ratios, problems with wettability and Capillary number.
- Penalization (work with a phase indicator for the solid phase also; Raeni et al., 2012; Horgue et al., 2012): work in progress
- Achieving numerical convergence is very difficult, though averages (e.g. effective properties) may be good for statistical reasons (one phase flow: Romeu & Noetinger, 1995; may contribute to some success stories for kr,Pc estimations)
- Need large computational resources
- Alternative to DNS?
 - Macro-scale models (various types)

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– PNM (pore network models), dynamic PNM, Hybrid PNM



Fig. 6. Simulation of drainage with spurious currents along the meniscus curvature (HRIC, Co = 1.00, Ca =0.03), (a) volume fraction, (b) velocity profile, (c) pressure distribution.

Horgue et al., 2012



Upscaling: quasi-static theory

Whitaker, 1986; Auriault, 1987; Lasseux et al., 1996; ...



Macro-Scale Models : Quasi-Static $V_{\alpha} = \langle v_{\alpha} \rangle ; P_{\alpha} = \langle p_{\alpha} \rangle^{\alpha} ; \varepsilon_{\alpha} = \varepsilon S_{\alpha} \qquad \alpha = \beta, \gamma$

Heuristic (Muskat): generalized Darcy's laws

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Macro-Scale Models : Quasi-Static, viscous coupling

 Model with viscous coupling (upscaling, also heuristic models,...)

(Ex. : Whitaker, 1986)

$$\begin{bmatrix} \tilde{\mathbf{v}}_{\alpha} = \mathbf{A}_{\alpha\beta} \cdot \left\langle \mathbf{v}_{\beta} \right\rangle^{\beta} + \mathbf{A}_{\alpha\gamma} \cdot \left\langle \mathbf{v}_{\gamma} \right\rangle^{\gamma} & \alpha = \beta, \gamma \\ \tilde{p}_{\alpha} = \left(\mathbf{a}_{\alpha} \cdot \left\langle \mathbf{v}_{\beta} \right\rangle^{\beta} + \mathbf{a}_{\gamma} \cdot \left\langle \mathbf{v}_{\gamma} \right\rangle^{\gamma} \right) & \alpha = \beta, \gamma \\ \end{bmatrix}$$

$$\mathbf{V}_{\alpha} = -\frac{1}{\mu_{\alpha}} \mathbf{K}_{\alpha} . (\nabla P_{\alpha} - \rho_{\alpha} \mathbf{g}) + \mathbf{K}_{\alpha \kappa} . \mathbf{V}_{\kappa} \qquad \alpha, \kappa = \beta, \gamma \qquad \alpha \neq \kappa$$
Phase interaction

 $\begin{array}{ll} \underline{\mathrm{Ex.:Two-Phase Poiseuille Flow}} & \frac{\mu_g}{\mu_l} \to 0 \\ K_{ll} \approx K\alpha^2 & K_{lg} \approx \frac{\mu_g}{\mu_l} \frac{(1-\alpha)^2}{\alpha} & \alpha = S_{\gamma} \\ K_{gg} \approx K\alpha^2 & K_{gl} \approx \frac{\alpha}{1-\alpha} \end{array}$



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Inertia Effects

Ergun (Heuristic): no-phase interaction terms

$$0 = -\nabla P_{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \frac{1}{K(k_{r\beta})} \mathbf{V}_{\beta} - \frac{\rho_{\beta}}{\eta_{\beta}} \| \mathbf{V}_{\beta} \| \mathbf{V}_{\beta} \|$$

Schulenberg and Muler (1987) (Heuristic and <a>?)

Similar to theory for fluidized beds

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$$0 = -\nabla P_{\beta} + \rho_{\beta} \mathbf{g} - \mu_{\beta} \frac{1}{K k_{r\beta}} \mathbf{V}_{\beta} - \frac{\rho_{\beta}}{\eta \eta_{\beta}} \left\| \mathbf{V}_{\beta} \right\| \mathbf{V}_{\beta} + \frac{\mathbf{F}_{\beta\gamma}^{S}}{S_{\beta}}$$

Upscaling (Lasseux et al., 2008; ...)

$$\mathbf{V}_{\alpha} = -\frac{1}{\mu_{\alpha}} \mathbf{K}_{\alpha} \cdot (\nabla P_{\alpha} - \rho_{\alpha} \mathbf{g}) - \mathbf{F}_{\alpha\alpha} \cdot \mathbf{V}_{\alpha} + \mathbf{K}_{\alpha\kappa} \cdot \mathbf{V}_{\kappa} - \mathbf{F}_{\alpha\kappa} \cdot \mathbf{V}_{\kappa} \qquad \alpha, \kappa = \beta, \gamma \qquad \alpha \neq \kappa$$

Transfert de chaleur

T-S

Importance of Cross-Terms, and Non-Linear Effects





from Clavier et al. (2015) (IRSN: context of nuclear reactor severe accident)

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See also Taherzadeh & Saidi (2015)

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Importance of Cross-Terms, and Non-Linear Effects $t \rightarrow \infty$

Models without cross-terms



$$l \to \infty$$

$$\frac{\partial P_{\beta}}{\partial z} - \rho_{\beta} \mathbf{g} \cdot \mathbf{e}_{z} = -\mu_{\beta} K_{\beta}^{-1} \left(1 + F_{\beta\beta}\right) V_{\beta}$$

$$+\mu_{\beta} K_{\beta}^{-1} K_{\beta\gamma} \left(1 - K_{\beta\gamma}^{-1} F_{\beta\gamma}\right) V_{\gamma}$$

$$\frac{\partial P_{\gamma}}{\partial z} - \rho_{\gamma} \mathbf{g} \cdot \mathbf{e}_{z} = -\mu_{\gamma} K_{\gamma}^{-1} \left(1 + F_{\gamma\gamma}\right) V_{\gamma}$$

$$+\mu_{\gamma} K_{\gamma}^{-1} K_{\gamma\beta} \left(1 - K_{\gamma\beta}^{-1} F_{\gamma\beta}\right) V_{\beta}$$

$$Case \ V_{\beta} = 0 :$$

$$\left(\frac{\partial P_{\beta}}{\partial z} - \rho_{\beta} \mathbf{g} \cdot \mathbf{e}_{z}\right) = +\mu_{\beta} K_{\beta}^{-1} K_{\beta\gamma} \left(1 - K_{\beta\gamma}^{-1} F_{\beta\gamma}\right)$$

from Clavier et al. (2015), Chikhi et al. (2016)

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see also Taherzadeh & Saidi (2015) using Tutu et al. experiments

Transfert de chaleur

 V_{γ}

Impact of K₀ on cross-term effect

 $K_{ij} = \beta K_{ij}$ (annular 2-phase flow)

(Pasquier et al., 2017)



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Effect of cross terms on the flow dynamic



(a) Configuration 1: Imbibition

adapted Buckley-Leverett theory (*Pasquier et al., 2017*)

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(h) $\frac{1}{N_g} = 1.1$

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More: Dynamic Models

- impact of $\partial S / \partial t$, V_{α} , a_{v} ...:
 - Quintard & Whitaker (1990, from large-scale heterogeneity effects and multi-zone)
 - -Hilfer (1998, multi-zone)

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- Panfilov & Panfilova (2005, meniscus)
- Hassanizadeh and Gray (Irr. Therm., a_v as state variable, 1993), also Kalaydjian (1987)...
- Phase field, Cahn-Hilliard (Cueto-Felgueroso & Juanes, 2009)...

Examples of dynamic equations

 Quintard & Whitaker, 1990

$$\mathbf{V}_{\beta} = -\frac{1}{\mu_{\beta}} \mathbf{K}_{\beta}^{*} \cdot \left(\nabla P_{\beta} - \rho_{\beta} \mathbf{g}\right) - \mathbf{u}_{\beta} \frac{\partial \varepsilon S_{\beta}}{\partial t} - \mathbf{U}_{\beta} \cdot \nabla \frac{\partial \varepsilon S_{\beta}}{\partial t} - \frac{1}{\mu_{\beta}} \mathbf{M}_{\beta} : \nabla \nabla P_{\beta} - \frac{1}{\mu_{\beta}} \mathbf{\Phi}_{\beta} - \frac{1}{\mu_{\beta}} \mathbf{R}_{\beta} : \nabla \mathbf{\Phi}_{\beta}$$

$$p_{c} = \mathcal{F}\left(S_{\beta}, \left(\rho_{\gamma} - \rho_{\beta}\right) \mathbf{g}, \nabla P_{\beta}, \frac{\partial \varepsilon S_{\beta}}{\partial t}, \ldots\right)$$

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 Kalaydjian, Hassanizadeh & Gray, ...

$$P_{\gamma} - P_{\beta} = p_c - L_1 \frac{\partial \varepsilon S_{\beta}}{\partial t}$$

- ...see also Petroleum engng literature on pseudo-functions!
- Usefulness for highly prmeable media?

PNM (Pore Network Model)

(reviews: Dullien, 1992;...; Jockar-Niasar & Hassanizadeh, 2012)



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Phase repartition according to capillary equilibrium

- Quasi-static rules → **Percolation theory**: vast literature and important results about flow patterns, etc.
- Used also to estimate k_r , P_c
- Drawback: structural properties are lost for other phases (e.g. solid phase)

PNM – Dynamic (See review: Jockar-Niasar & Hassanizadeh, 2012)





Pore-scale rules (analytical: e.g. Poiseuille)

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- Time dependent solution, solve for pressure field with approximate solutions for describing flow in connections
- Used to estimate k_r , P_c and other relations (hysteresis)

Example: application to the interpretation of tomographic images in the inertia regime



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Transport in Porous Media

Mixed or Hybrid Models

- Network models → a mesoscale representation!
- If low Re, Ca, Bo → percolation theory

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 Otherwise: Coupling network model and Dynamic rules (which may come from local VOF simulations)

Melli & Scriven, 1991; Horgue et al. (PhD CIFRE/IFP/IMFT), 2012

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Trickle Bed (X-ray, IFP)



Micromodel experiments



Mixed or Hybrid Models



 Mass and momentum
 balance for the network
 Dynamic rules
 Dynamic rules
 Coming from local
 VOF simulations
 (or from
 experiments)



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Macro-Scale Models with Phase "Splitting"

 Example: Flow through Structured Media



Mahr and Mewes (2007)

MellaPak (Sulzer Chemtech)





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Macro-Scale Models with Phase "Splitting"

$$\varepsilon \frac{\partial S_{\gamma}}{\partial t} + \nabla \mathbf{U}_{\gamma} = 0,$$

$$\varepsilon \frac{\partial S_{\beta_1}}{\partial t} + \nabla \mathbf{U}_{\beta_1} = \dot{m},$$

$$\varepsilon \frac{\partial S_{\beta_2}}{\partial t} + \nabla \mathbf{U}_{\beta_2} = -\dot{m}.$$

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+ 3 momentum equations

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Model with liquid phase splitting, noexchange



 Comparison with Fourati et al. (2012) experiments (Soulaine et al. 2014) → calibration of exchange term on the 1st stack



Conclusions

- One-Phase Flow:
 - Generalized Forchheimer equations → a practical model for laminar inertia regimes or localized turbulence
 - Porous Media Turbulence models
- Two-Phase Flow:
 - Importance of cross-terms, Experimental determination?
 - Extensions to more dynamic flows:
 - Extended generalized Darcy's laws
 - Models with phase splitting
 - Hybrid models
 - Highly transient effects, complex time and space averaging?
- Coupling with other transport mechanisms (dispersion, heat transfer, ...)?