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## Flow in highly permeable porous media

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4th Cargèse Summer School, 2018

# Flow in High Permeability Porous Media

**M. Quintard**

[https://www.dropbox.com/s/6v6sfi3ngzk5qri/two\\_phase\\_high\\_perm\\_car\\_00.pdf?dl=0](https://www.dropbox.com/s/6v6sfi3ngzk5qri/two_phase_high_perm_car_00.pdf?dl=0)

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# Outline (emphasis on *theoretical aspects*)

- Introduction: background, multiple-scale, averaging
- One-Phase Flows:
  - Phenomenology:
    - Departure from Darcy's law
    - Weak inertia regimes
    - Strong Inertia regimes
    - Transitions
    - Turbulence: pore-scale or large-scale?
  - Upscaling?
- Two-Phase Flows: introduction to several models
- Conclusions



# Examples:

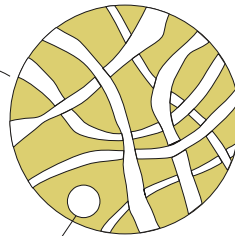
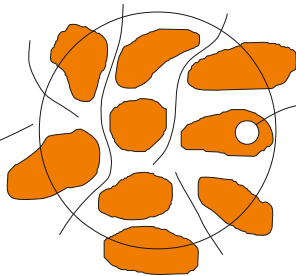
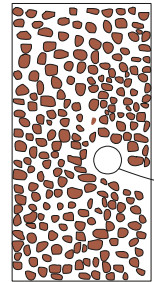
Structured and Packed bed reactors, nuclear safety, fractured media, karsts, ...



Packed Bed Reactor

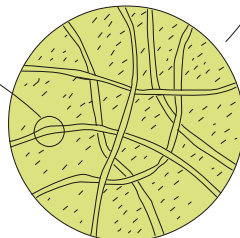
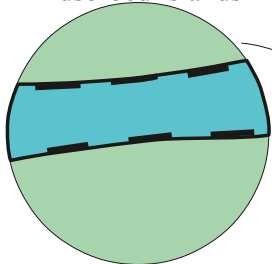
Porous Medium

Macro Pores

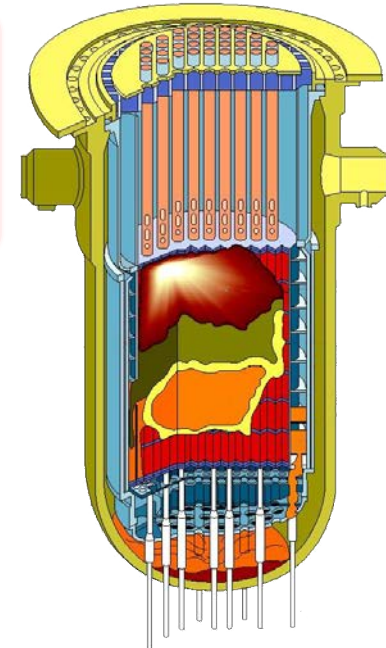


Adsorbed Islands

Micro Pores



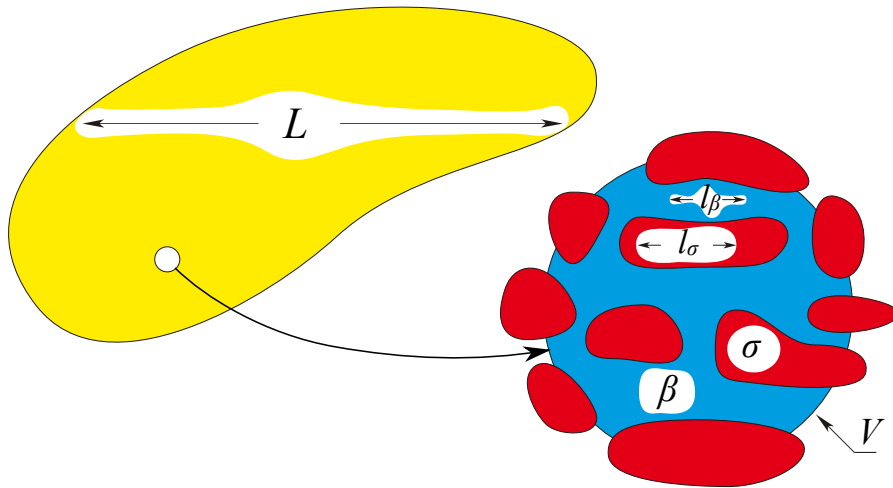
$$Re = \frac{\rho_r U_r \sqrt{k}}{\mu_r} > 1$$



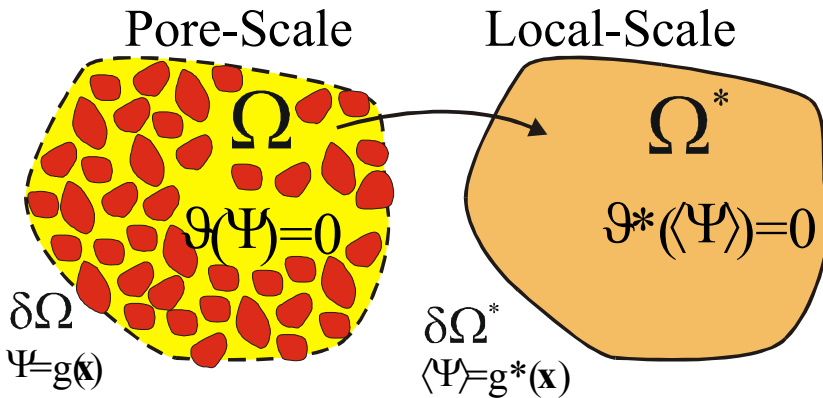
Financial support from Air Liquide, IFPen, TOTAL, ....!



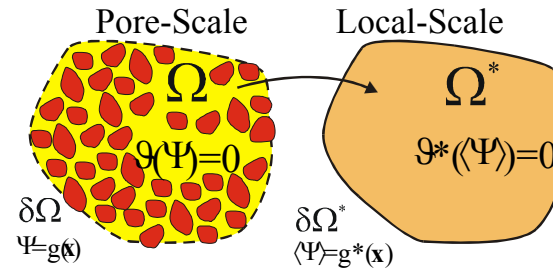
# Upscaling



- Separation of scales:  
 $l_\beta, l_\sigma \ll L$
- Objectives of a macro-scale theory?
  - Smoothing operator  $\rightarrow$  macro-scale equations and BCs
  - Link between micro- and macro-scale
  - Effective properties



# Upscaling: different point of views



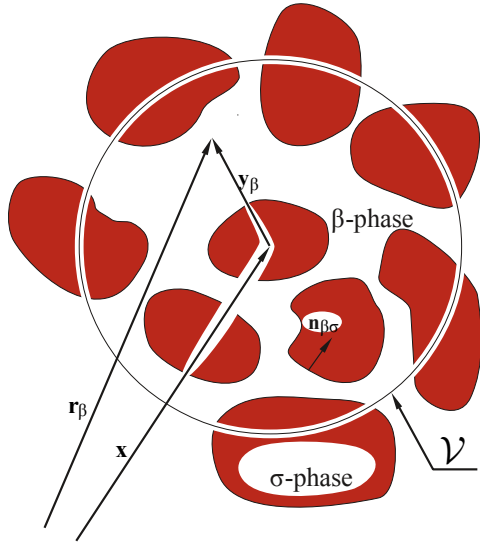
- **Heuristic: Darcy's law (1856)!**
- **Upscaling with *closure*: homogenization theory (Sanchez-Palencia, Bensoussans et al.,...), volume averaging (Whitaker, ... and variants),**

$$p_\beta = \langle p_\beta \rangle^\beta + \tilde{p}_\beta \quad \tilde{p}_\beta \stackrel{?}{=} f \left( \langle p_\beta \rangle^\beta, \nabla \langle p_\beta \rangle^\beta, \dots \right)$$
- **Stochastic theories (Matheron, Dagan, Gelhar, ...), ...**
- **Mixture theories: volume averaging + irreversible thermodynamics (Marle, Hassanizadeh and Gray, Bowen, ...)**
- **Other point of views...: dual-phase-lagging heat conduction (Wang et al., 2008; Vadasz, 2005...); mixed models; fractional derivatives (Néel,...); CTRW; ...**

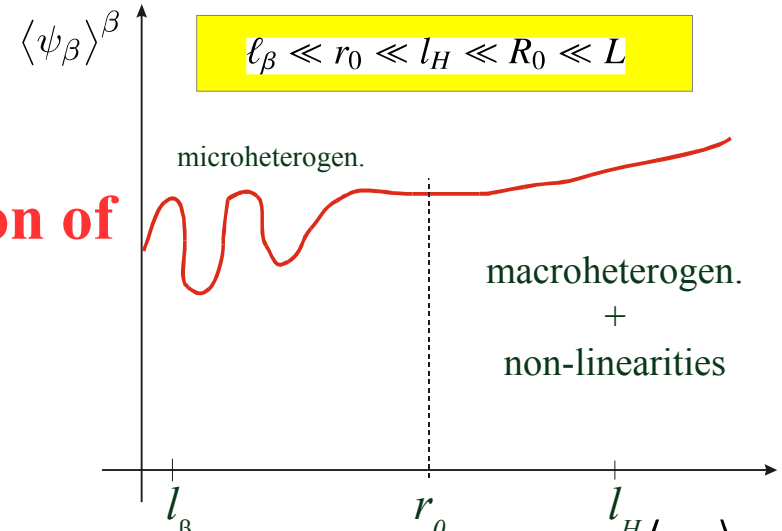
today emphasis



# Macro-scale variables: Example of volume averaging



**Need separation of scales!**



- $\beta$ -phase Volume Fraction  $\varepsilon_\beta = \langle \gamma_\beta \rangle$   
with  $\gamma_\beta =$  phase indicator function

- $\langle \psi_\beta \rangle = \varepsilon_\beta \langle \psi_\beta \rangle^\beta$  Intrinsic phase average

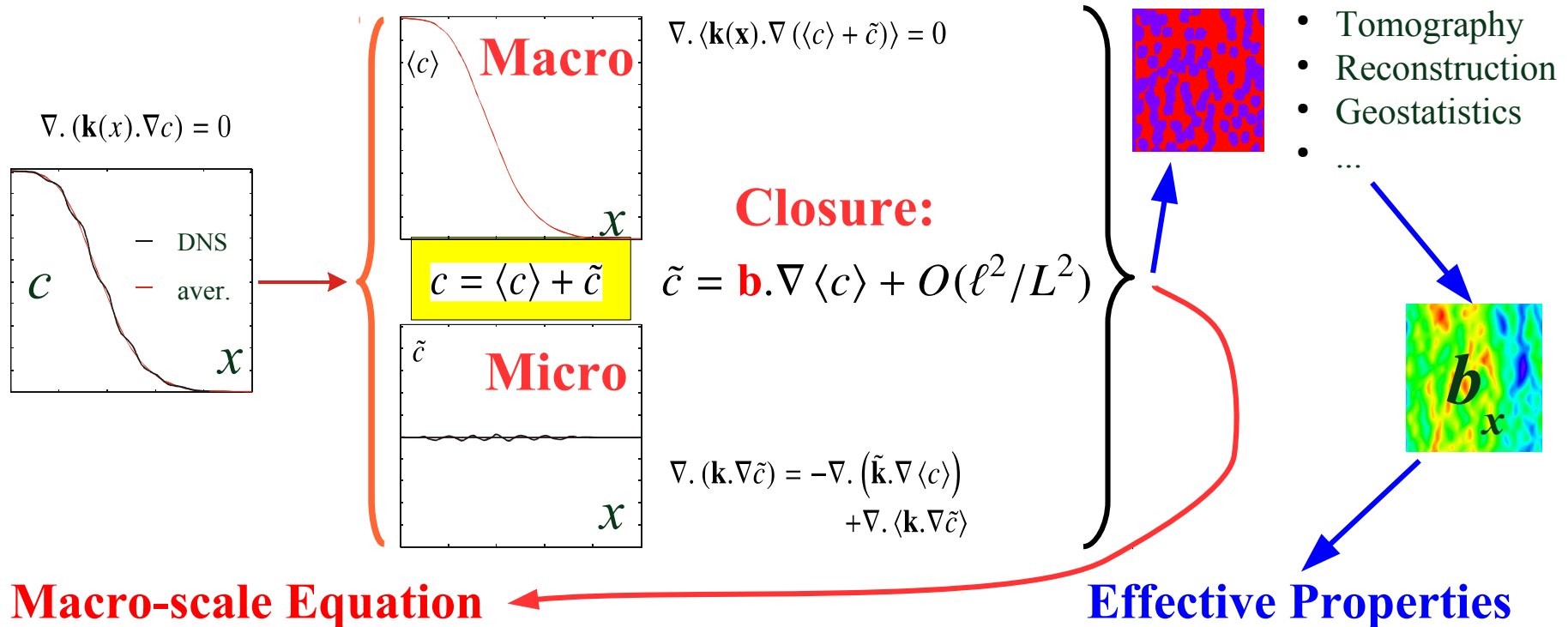
$$\gamma_\beta = \begin{cases} 1 & M \in V_\beta \\ 0 & M \notin V_\beta \end{cases}$$

$$\langle \psi_\beta \rangle \Big|_x = m * \psi_\beta \approx \frac{1}{V} \int_{V_\beta} \psi_\beta(\mathbf{x} + \mathbf{y}) dV$$





# Upscaling with *closure*: a schematic view for diffusion in a heterogeneous medium



**Macro-scale Equation**

$$\nabla \cdot (\mathbf{K}_{eff} \cdot \nabla \langle c \rangle) = 0$$

Note: high performance computing  $\rightarrow$  up to  $2048^3$  voxels



# One-Phase Flow: Phenomenology

- **Pore-scale equation**  $\nabla \cdot \mathbf{v}_\beta = 0$

$$\rho_\beta \frac{\partial \mathbf{v}_\beta}{\partial t} + \rho_\beta \mathbf{v}_\beta \cdot \nabla \mathbf{v}_\beta = -\nabla p_\beta + \rho_\beta \mathbf{g} + \mu_\beta \nabla^2 \mathbf{v}_\beta \quad \text{B.C.1 } \mathbf{v}_\beta = 0 \quad \text{at } A_{\beta\sigma}$$

$$\text{Re} = \frac{\rho_\beta U_r \ell_\beta}{\mu_\beta}; p_\beta^c = \frac{p_\beta \ell_\beta}{\mu_\beta U_r}; \text{Fr} = \frac{U_r}{\sqrt{\ell_\beta g}} \Rightarrow \text{Re} \left( \frac{\partial \mathbf{v}'_l}{\partial t'} + \mathbf{v}'_l \cdot \nabla' \mathbf{v}'_l \right) = -\nabla' p_l^c + \frac{\text{Re}}{\text{Fr}^2} \frac{\mathbf{g}}{g} + \nabla'^2 \mathbf{v}'_l$$

- **Pore-Scale regimes**

- **Creeping (Re  $\rightarrow$  0): leads to Darcy's law**

- **Laminar**

- **Turbulent**



Various regimes and transitions



# Upscaling One-Phase Flow

- Averaging**

$$p_\beta = \langle p_\beta \rangle^\beta + \tilde{p}_\beta ; \quad \mathbf{v}_\beta = \langle \mathbf{v}_\beta \rangle^\beta + \tilde{\mathbf{v}}_\beta$$

$$\nabla \cdot \langle \mathbf{v}_\beta \rangle = 0$$

$$\text{B.C.1 } \tilde{\mathbf{v}}_\beta = -\langle \mathbf{v}_\beta \rangle^\beta \quad \text{at } A_{\beta\sigma}$$

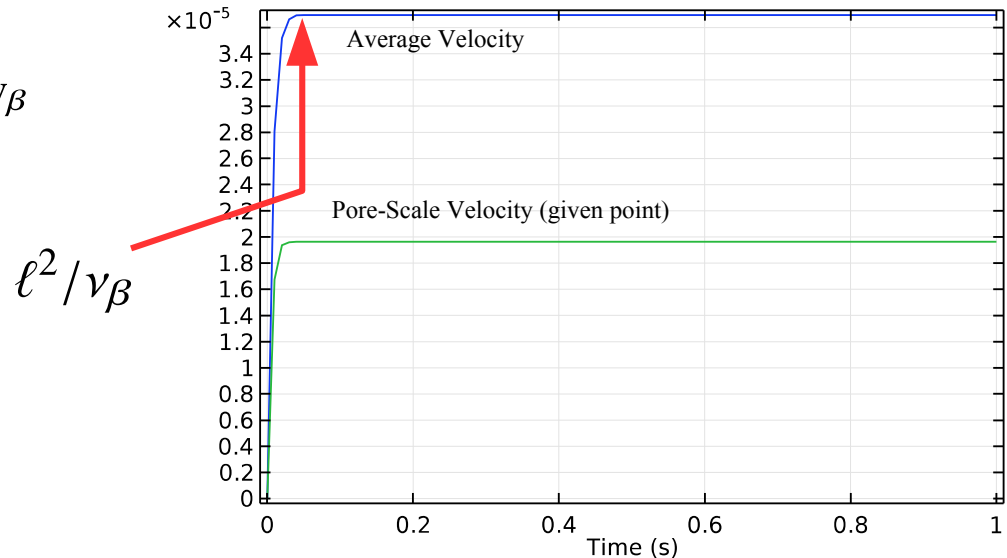
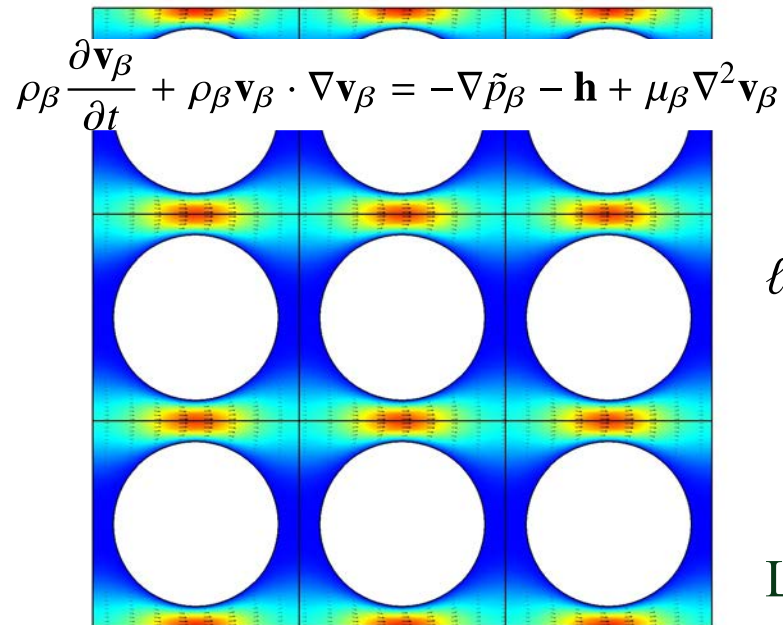
$$\begin{aligned} & \rho_\beta \frac{\partial \langle \mathbf{v}_\beta \rangle}{\partial t} + \rho_\beta \langle \mathbf{v}_\beta \rangle \cdot \nabla \langle \mathbf{v}_\beta \rangle^\beta + \nabla \cdot (\rho_\beta \langle \tilde{\mathbf{v}}_\beta \tilde{\mathbf{v}}_\beta \rangle) = \\ & -\varepsilon_\beta \nabla \langle p_\beta \rangle^\beta + \varepsilon_\beta \rho_\beta \mathbf{g} + \underbrace{\mu_\beta \nabla^2 \langle \mathbf{v}_\beta \rangle - \mu_\beta \nabla \varepsilon_\beta \cdot \nabla \langle \mathbf{v}_\beta \rangle^\beta}_{\text{Brinkman terms}} + \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\tilde{p}_\beta \mathbf{I} + \mu_\beta \nabla \tilde{\mathbf{v}}_\beta) dA \end{aligned}$$

Remark:  $\tilde{\mathbf{v}}_\beta \sim \langle \mathbf{v}_\beta \rangle^\beta$



# Various Regimes: Creeping

Re~0



Linear relationship between average velocity and pressure drop!

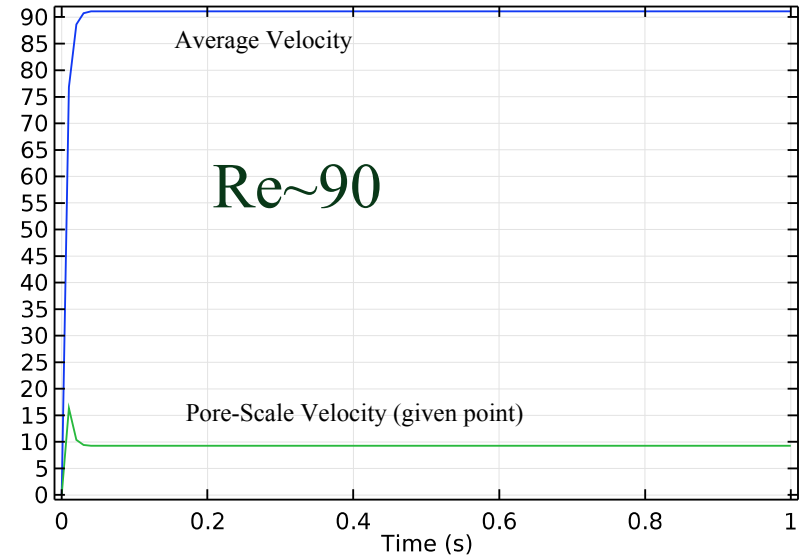
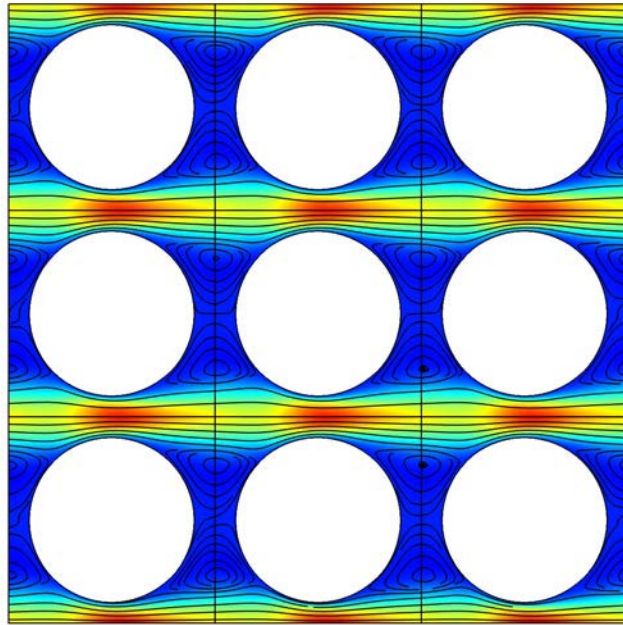
$$0 = -\varepsilon_\beta \nabla \langle p_\beta \rangle^\beta + \varepsilon_\beta \rho_\beta \mathbf{g} + \mu_\beta \nabla^2 \langle \mathbf{v}_\beta \rangle - \mu_\beta \nabla \varepsilon_\beta \nabla \langle \mathbf{v}_\beta \rangle^\beta$$

$$+ \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\tilde{p}_\beta \mathbf{I} + \mu_\beta \nabla \tilde{\mathbf{v}}_\beta) dA$$

$O\left(\frac{\langle v_\beta \rangle^\beta}{\ell^2}\right)$        $O\left(\frac{\langle v_\beta \rangle^\beta}{L^2}\right)$



# Various Regimes: Laminar Inertial



Still

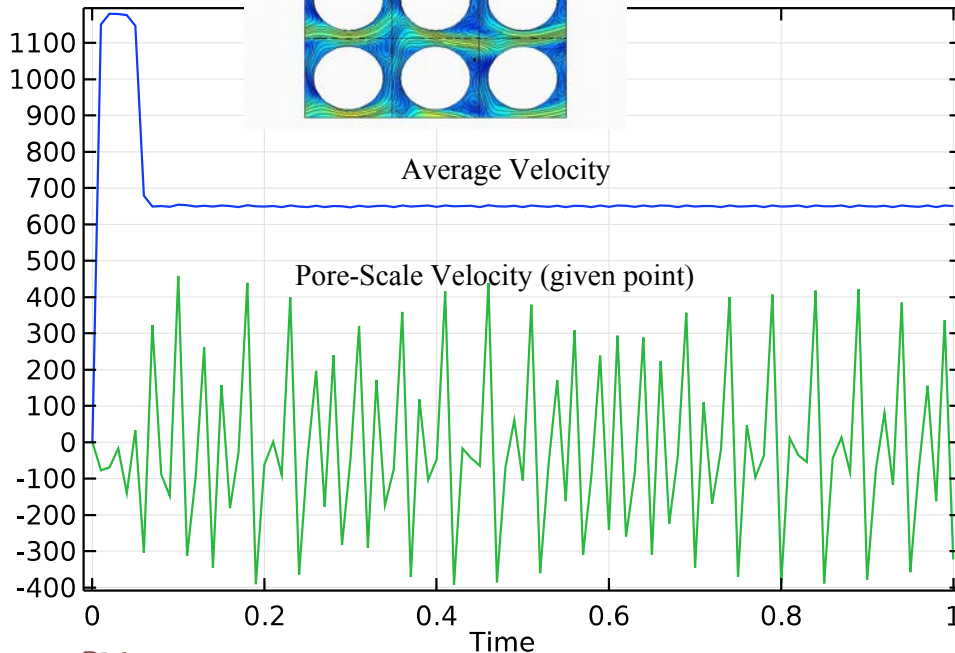
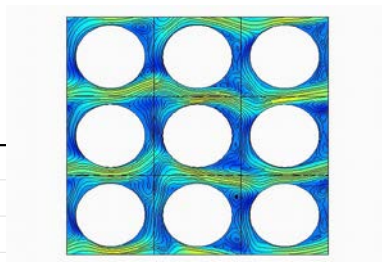
$$0 = -\varepsilon_\beta \nabla \langle p_\beta \rangle^\beta + \varepsilon_\beta \rho_\beta \mathbf{g} + \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\tilde{p}_\beta \mathbf{I} + \mu_\beta \nabla \tilde{\mathbf{v}}_\beta) dA$$

But non-linear relationship between average velocity and pressure drop!



# Various Regimes: towards turbulence

- $\langle v \rangle = \text{steady-state}$



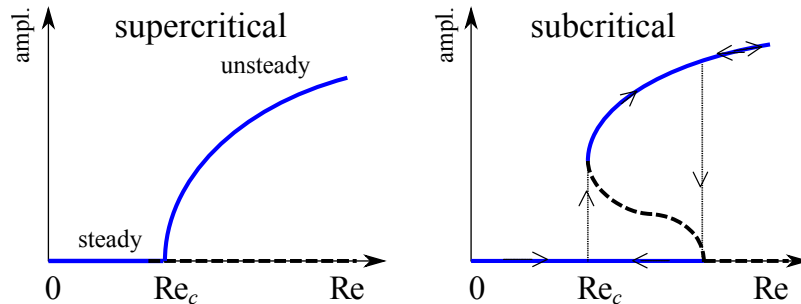
- Macro-Scale Turbulence? (see Jin et al., 2015, DNS)

- Localized turbulence (nearly periodic over  $N$ -UC) need  $\overline{\langle v_\beta \rangle}$
- Macro-scale turbulence? Ex.: entrance regions (D'Hueppe et al., ...)



# More...

- **Various types of bifurcations:**



Ex.: array of cylinders – not same  $Re_c$  for flow // or  $\perp$  to axis

- Bifurcations depends on  $Re$  and number of UC (Aagnaou et al., 2016)...and topology!  $\rightarrow REV \text{ size} = f(\langle \mathbf{v}_\beta \rangle^\beta)$
- Difficult to estimate order of magnitude when fully non-linear  $\rightarrow$  rely on DNS

- **RESULT:**  $(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g})$  and  $\langle \mathbf{v}_\beta \rangle$  nearly constant over REV!

and up to localized turbulence one has

$\overline{(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g})}$  and  $\overline{\langle \mathbf{v}_\beta \rangle}$  nearly constant over REV!



# Closure: Darcy regime (Stokes problem=creeping flow)

Re~0

- PDEs for deviations

$$0 = -\nabla \tilde{p}_\beta + \mu_\beta \nabla^2 \tilde{\mathbf{v}} - \varepsilon_\beta^{-1} \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\tilde{p}_\beta \mathbf{I} + \mu_\beta \nabla \tilde{\mathbf{v}}_\beta) dA$$

$$\nabla \cdot (\tilde{\mathbf{v}}_\beta) = -\nabla \cdot \langle \mathbf{v}_\beta \rangle^\beta \approx 0 \quad \text{B.C. 1} \quad \tilde{\mathbf{v}} = -\langle \mathbf{v}_\beta \rangle^\beta \text{ at } A_{\beta\sigma}$$

- Closure in the linear case (3 elementary solutions for  $\mathbf{e}_i$ )

$$\tilde{p}_\beta = \mu_\beta \mathbf{b} \cdot \langle \mathbf{v}_\beta \rangle^\beta + \dots; \quad \tilde{\mathbf{v}} = \mathbf{B} \cdot \langle \mathbf{v}_\beta \rangle^\beta + \dots$$





# Closure and Macro-Scale Equation (Darcy regime)

$$\nabla \cdot \mathbf{B} = 0$$

$$-\nabla \mathbf{b} + \nabla^2 \mathbf{B} = \varepsilon_\beta^{-1} \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\mathbf{I} \mathbf{b} + \nabla \mathbf{B}) \, dA$$

$$\mathbf{B} = -\mathbf{I} \quad \text{at } A_{\beta\sigma}, \quad \langle \mathbf{b} \rangle^\beta = 0$$

$$\mathbf{b}(\mathbf{r} + \mathbf{l}_i) = \mathbf{b}(\mathbf{r})$$

$$\mathbf{B}(\mathbf{r} + \mathbf{l}_i) = \mathbf{B}(\mathbf{r})$$

$$\langle \mathbf{v}_\beta \rangle = -\frac{1}{\mu_\beta} \mathbf{K} \cdot \left( \nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \right)$$

$$\mathbf{K}^{-1} = -\varepsilon_\beta^{-2} \frac{1}{\mathcal{V}} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot (-\mathbf{I} \mathbf{b} + \nabla \mathbf{B}) \, dA$$

$\mathbf{K}$  intrinsic permeability

See Sanchez-Palencia (homogenization), Whitaker, ...



# Other version of closure: permeameter-like problems

*Note: also suitable for non-linear problems (non-newtonian flows, etc...)*

$$\underbrace{\rho_\beta \mathbf{v}_\beta \cdot \nabla \tilde{\mathbf{v}}_\beta}_{=0, \text{ Darcy regime}} = -\nabla \tilde{p}_\beta + \nabla \cdot \left( \mu_\beta \left( \nabla \tilde{\mathbf{v}}_\beta + \nabla \tilde{\mathbf{v}}_\beta^T \right) \right) - \left( \nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \right)$$

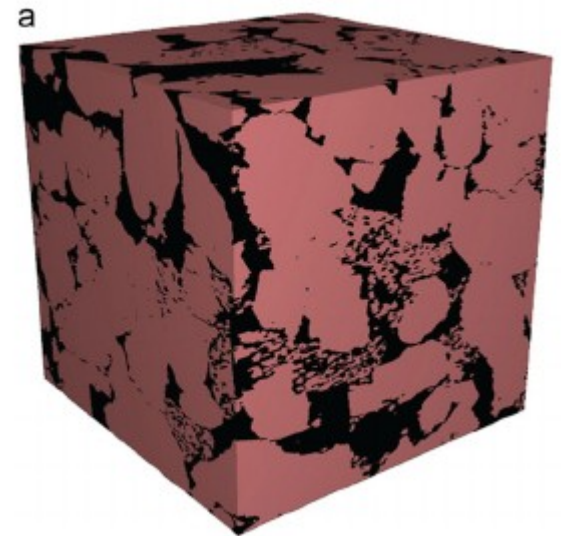
=0, Darcy regime

$$\nabla \cdot (\tilde{\mathbf{v}}_\beta) = 0$$

$$\tilde{\mathbf{v}} = -\langle \mathbf{v}_\beta \rangle^\beta \text{ at } A_{\beta\sigma}$$

**BCs** (see Guibert et al., 2016)?

- Periodicity conditions: good for anisotropic media, but percolation problem if “periodized” media
- Permeameter (various types: classical, Bamberger, ...)
- Bordering media: fluid or porous layer, ...



# Non Darcy regimes (Re>1)

- **Heuristic: Forchheimer, Ergun, ...**

$$0 = -\nabla P_\beta + \rho_\beta \mathbf{g} - \mu_\beta K^{-1} \mathbf{V}_\beta - \rho_\beta \eta^{-1} \|\mathbf{V}_\beta\| \mathbf{V}_\beta \quad \text{with } \mathbf{V}_\beta = \langle \mathbf{v}_\beta \rangle ; P_\beta = \langle p_\beta \rangle^\beta$$

$$K = \frac{\varepsilon^3 d^2}{h_K (1 - \varepsilon)^2} \quad ; \quad \eta = \frac{\varepsilon^3 d}{h_\eta (1 - \varepsilon)}$$

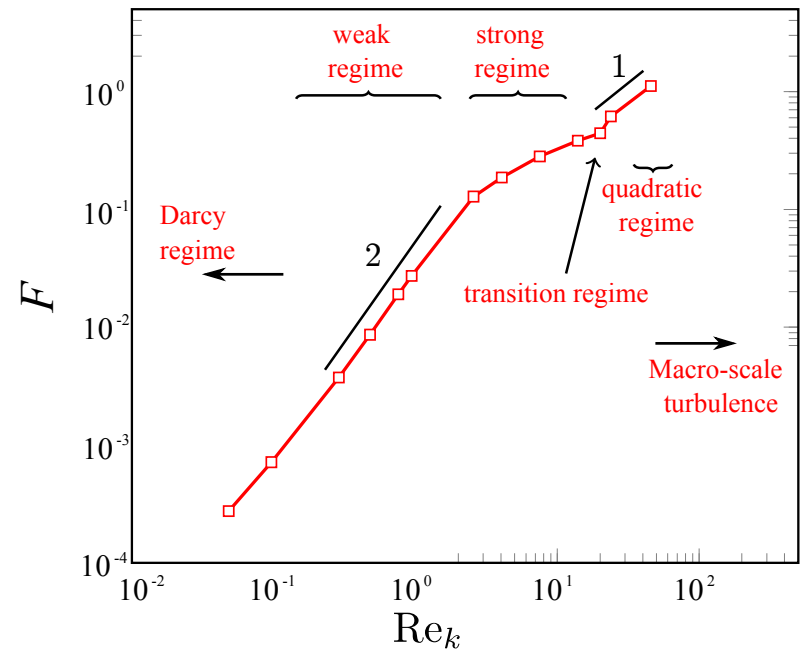
(passability)

- **Upscaling?**

$$0 = -\nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{g} - \mu_\beta \mathbf{K}_{app}^{-1} \cdot \langle \mathbf{v}_\beta \rangle$$

⇓

$$0 = -\nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{g} - \mu_\beta \mathbf{K}^{-1} \cdot \langle \mathbf{v}_\beta \rangle - \mathbf{F}(\langle \mathbf{v}_\beta \rangle) \cdot \langle \mathbf{v}_\beta \rangle$$



# Non Darcy regimes ( $Re > 1$ )

- laminar inertia effects: theory  $\rightarrow$  generalized Forchheimer equation

$$0 = -\nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{g} - \mu_\beta \mathbf{K}^{-1} \cdot \langle \mathbf{v}_\beta \rangle - \mathbf{F} (\langle \mathbf{v}_\beta \rangle) \cdot \langle \mathbf{v}_\beta \rangle$$

- $Re \rightarrow 0$ : Darcy,  $\mathbf{F}=0$
- $Re \sim 0$ : weak inertia,  $\mathbf{F} \cdot \langle \mathbf{v}_\beta \rangle \sim \langle \mathbf{v}_\beta \rangle^3$  (Levy, Mei & Auriault, Firdaouss, ...)
- $Re > 0$ : strong inertia,  $\mathbf{F} \cdot \langle \mathbf{v}_\beta \rangle \sim \langle \mathbf{v}_\beta \rangle^2$  (Whitaker, 1996; Lasseux et al., 2011;...)



# Weak Inertia

- **Solution following asymptotic expansions in  $\text{Re}$**

$$\delta = \frac{\ell}{\nu_\beta} \nu = \text{Re}$$

$$\mathbf{v}^* = \frac{\tilde{\mathbf{v}}_\beta}{\nu} + \lambda, \quad p^* = \ell_\beta \frac{\tilde{p}_\beta}{\mu_\beta \nu}$$

$$\mathbf{v}^* = \mathbf{v}_0^* + \delta \mathbf{v}_1^* + \delta^2 \mathbf{v}_2^* + O(\delta^3)$$

$$p^* = p_0^* + \delta p_1^* + \delta^2 p_2^* + O(\delta^3)$$

- **1<sup>st</sup> correction?**

– **Periodic media:**  $F_{\parallel} \sim \nu^2$

– **Periodic media+reversibility:**  $F_{\parallel}^{(1)} = F_{\perp}^{(1)} = 0$

Wodié et Lévy, 1991; Mei et Auriault, 1991; Firdaouss et al., 1997  
..... revisited by Pauthenet et al., 2017



# Strong Inertia

• **Solution for**  $\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \Rightarrow (\tilde{p}_\beta, \mathbf{v}_\beta) \Rightarrow \langle \mathbf{v}_\beta \rangle^\beta = v \boldsymbol{\lambda}$

$$0 = -\nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{g} - \mu_\beta \mathbf{K}_{app}^{-1} \cdot \langle \mathbf{v}_\beta \rangle$$



$$0 = -\nabla \langle p_\beta \rangle^\beta + \rho_\beta \mathbf{g} - \mu_\beta (\mathbf{I} + F\mathbf{P}) \cdot \mathbf{K}^{-1} \cdot \langle \mathbf{v}_\beta \rangle$$

$$\text{or } \nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} = -\varepsilon_\beta \mu_\beta (\mathbf{K}_D^{-1} \cdot \boldsymbol{\lambda} + \mathbf{f}) v$$

**P**: rotation

**F**: non-linear scaling

$$\mathbf{f} = K_\lambda^{-1} (F_\parallel \boldsymbol{\lambda} + F_\perp (\cos(\omega) \mathbf{n}_1 + \sin(\omega) \mathbf{n}_2))$$

$$\mathbf{n}_1 \times \mathbf{n}_2 = \boldsymbol{\lambda} \quad F_\parallel = \frac{\boldsymbol{\lambda} \cdot \mathbf{f}}{K_\lambda^{-1}} \quad K_\lambda^{-1} = \|\mathbf{K}_D^{-1} \cdot \boldsymbol{\lambda}\|$$

$$F_\perp = \|\mathbf{f} - \boldsymbol{\lambda} (\boldsymbol{\lambda} \cdot \mathbf{f})\| \quad \omega = \left( (\mathbf{I} - \boldsymbol{\lambda} \boldsymbol{\lambda}) \cdot \frac{\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g}}{\varepsilon_\beta \mu_\beta K_\lambda^{-1} v}, \mathbf{n}_1 \right)$$



# Transition to inertia regimes: which Re number?

$$\text{Re} = \frac{\langle v_{\beta} \rangle^{\beta} \ell}{\nu_{\beta}}$$

- Literature discussion (see polemic in Lage et Antohe, 2000; etc...)

- $\ell = \sqrt{K}$  ?

–  $K = \varepsilon d^2 / 32$  for a bundle of tubes (transition would change with porosity! → better

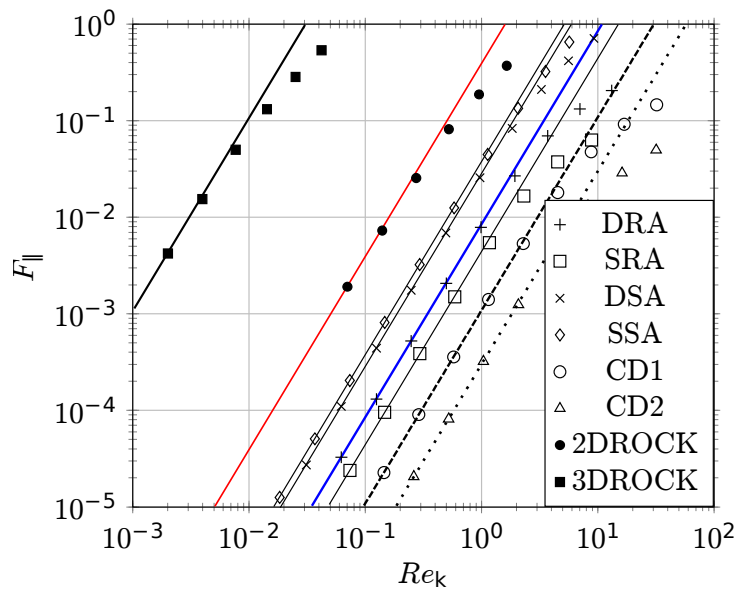
$$\ell = \sqrt{\frac{K}{\varepsilon}} \rightarrow \text{Re}_k$$

- Topological problem: array of cylinders → transition depends on the orientation → need a topological information!



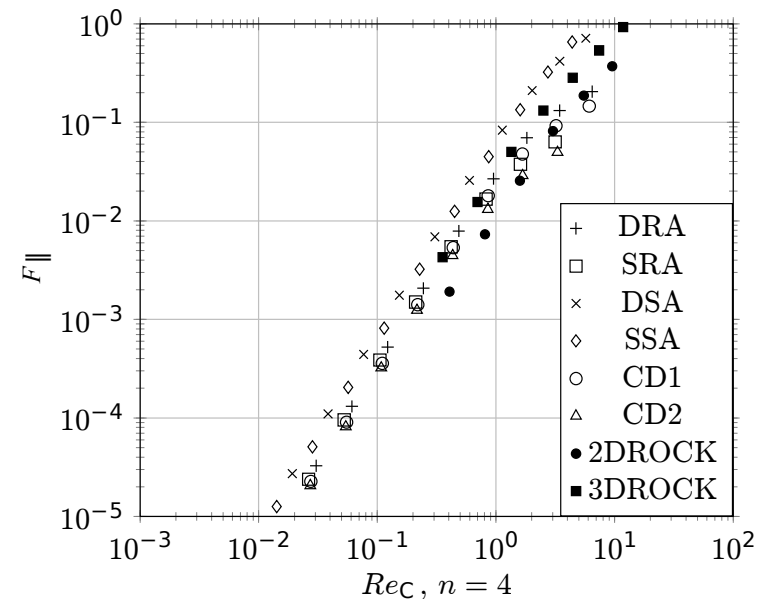
# Transition to inertia effect: effect of topology (Pauthenet et al., 2017)

## ● If $Re_k$



(various media, rocks, ...:  
Muljadi et al., 2016; etc...)

## ● $Re_C$



$$Re_C = \frac{v}{v_{\beta}} \sqrt{\frac{K_{\lambda}}{\varepsilon_{\beta}}} C_{\lambda}$$





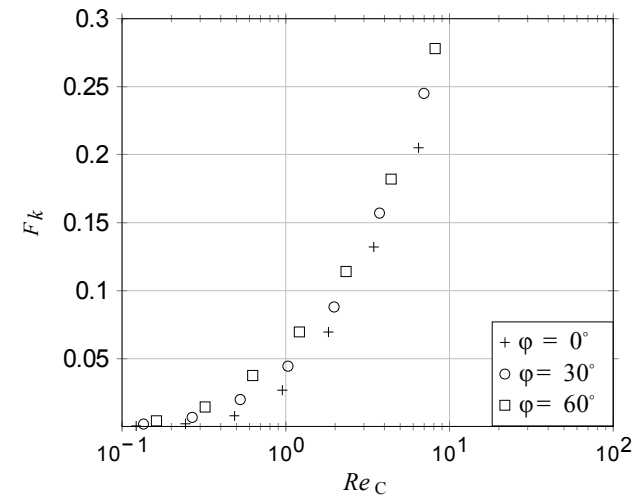
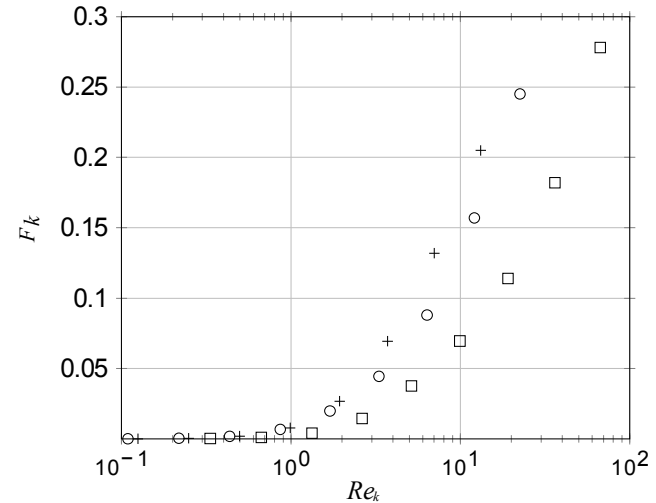
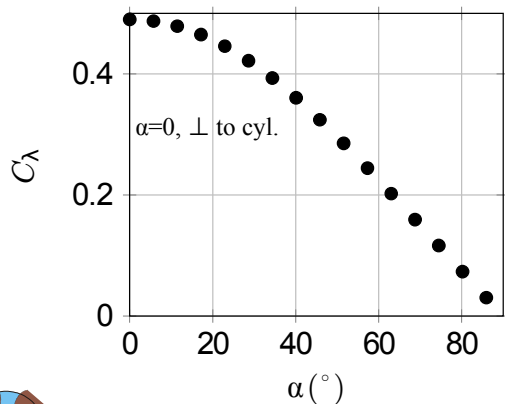
# Topology effect (Ex.: array of Cylinder)

- Tentative (open question)

from Darcy closure

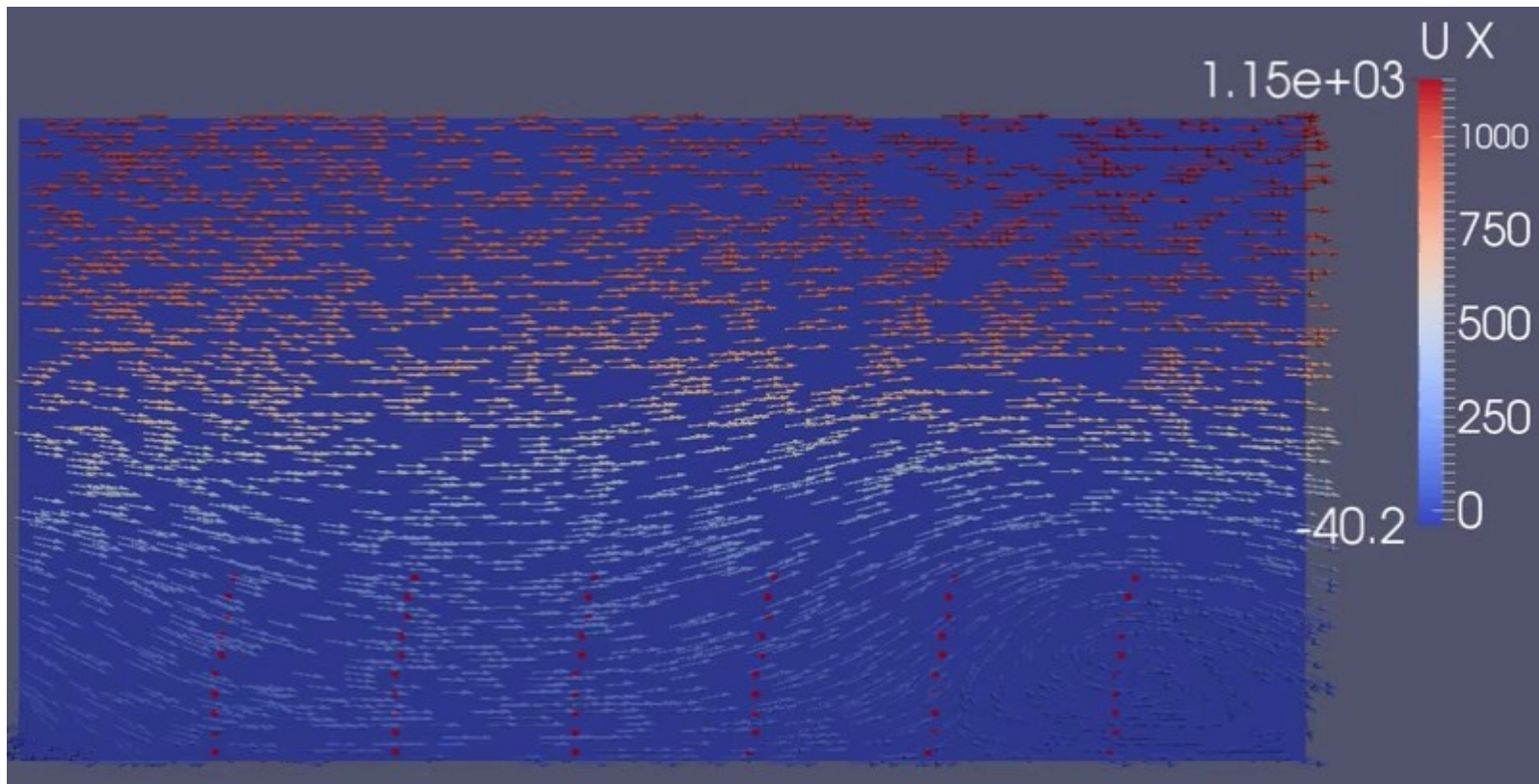
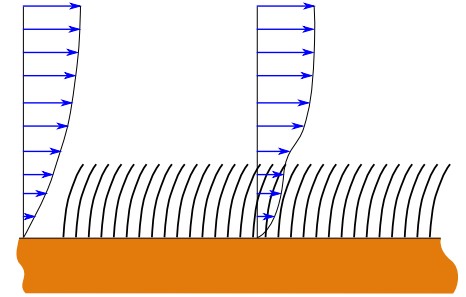
$$C_\lambda = \frac{\zeta(\mathbf{v}_0^* \cdot \nabla^* \mathbf{v}_0^*)}{\sqrt{\langle \nabla^* \mathbf{v}_0^* : T \nabla^* \mathbf{v}_0^* \rangle}^\beta} \sqrt{\frac{\lambda \cdot \mathbf{K}_D^{-1} \cdot \lambda}{K_\lambda^{-1}}}$$

$$\zeta(\mathbf{v}_0^* \cdot \nabla^* \mathbf{v}_0^*) = \langle \|\mathbf{v}_0^* \cdot \nabla^* \mathbf{v}_0^*\|^n \rangle \beta^{1/n}$$



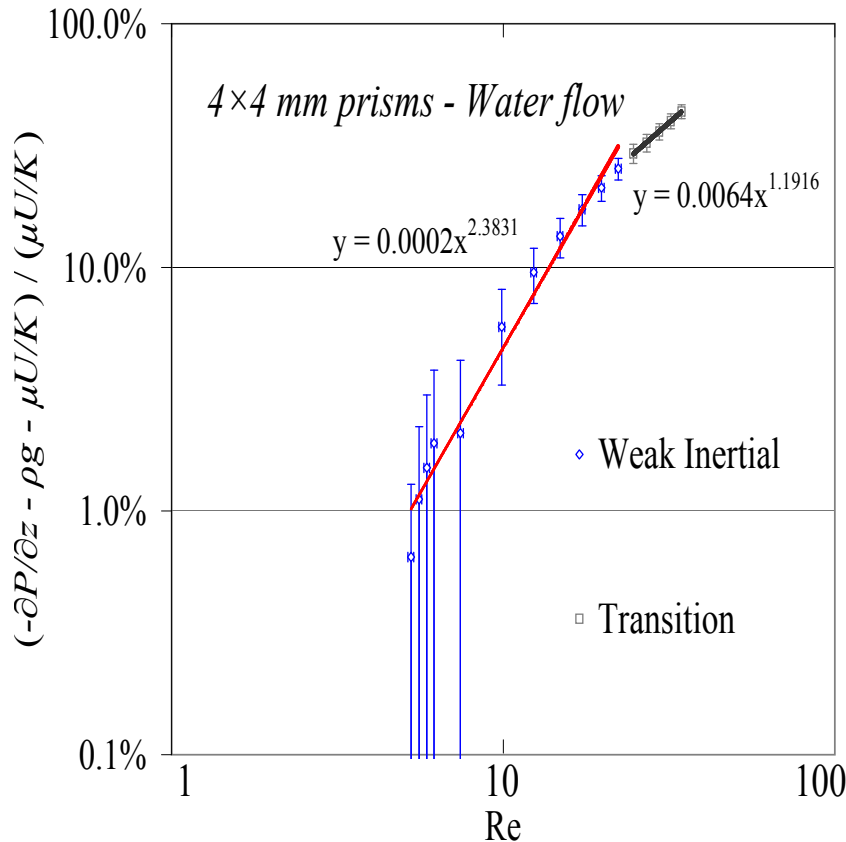
# Example: canopy flow

Honami generation (Pauthenet, 2017)  
...need  $\mathbf{K}_{\text{app}}$  for all fiber directions and  $\mathbf{V}_{\beta}$

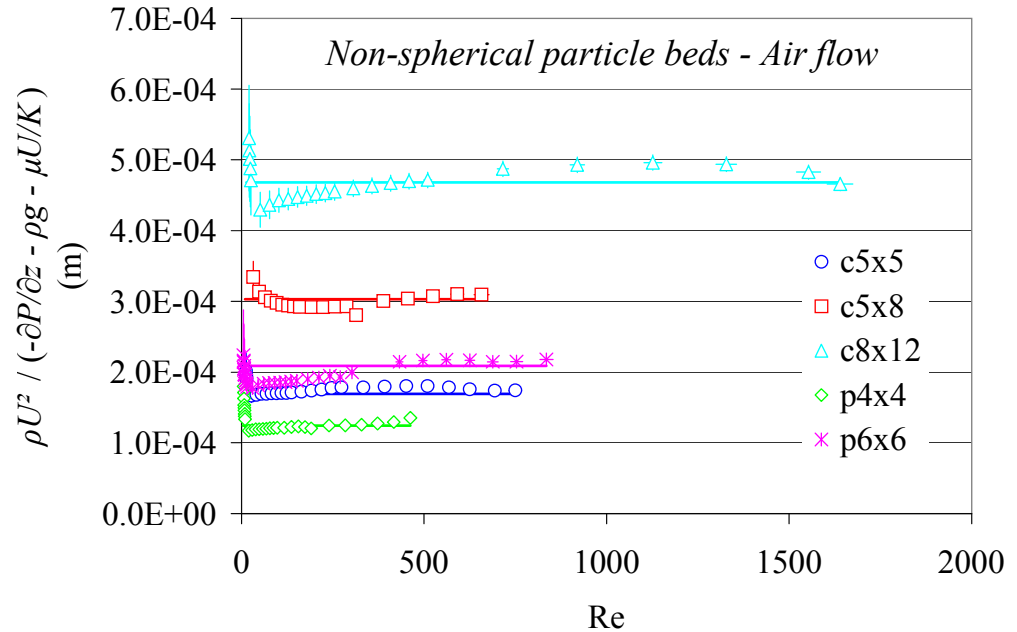


# Experimental Evidence: inertia regime

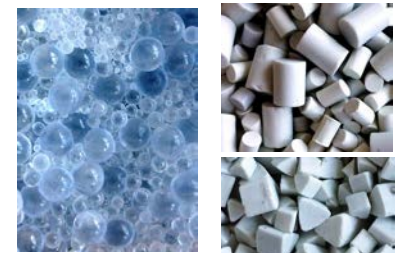
## Weak inertia



## Strong inertia



Clavier et al., 2014



Note the importance of transition regimes!



# Turbulent flows in porous media ( $Re \gg 1$ )

- **Turbulence: time and spatial averaging (see book De Lemos, 2006; ...)**
  - Time and spatial averaging commute!
  - However: not necessarily the same result if sequential closure!?

$$\text{I. } \mathbf{v}_\beta \rightarrow \langle \mathbf{v}_\beta \rangle \rightarrow \overline{\langle \mathbf{v}_\beta \rangle}$$

$$\text{II. } \mathbf{v}_\beta \rightarrow \overline{\mathbf{v}_\beta} \text{ (RANS, ...)} \rightarrow \langle \overline{\mathbf{v}_\beta} \rangle$$

- For one-phase flow: scheme “II” seems preferable: contrary to Antohe & Lage (1997), Getachew et al. (2000), see discussion in Nakayama & Kuwahara (1999), Pedras and de Lemos (2001), etc...
- **Open: Simultaneous closure over  $[R^3 \times R^t]$ ? More complex sequential closures ( $t \rightarrow x \rightarrow t \rightarrow \dots$ ) depending on the hierarchy of scales?**
- **Open question for multiphase flow?**

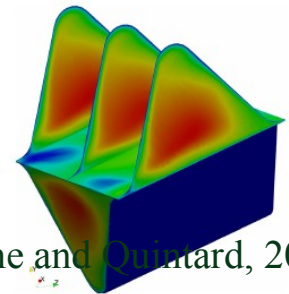


# Turbulent flows in porous media (continued)

- Localized Turbulence: i.e., nearly periodic (see Jin et al., 2015 for DNS results)
  - Slow unsteady flows → generalized Forchheimer for time averaged pressure and velocity  $\langle \mathbf{v}_\beta \rangle$  or  $\langle \overline{\mathbf{v}_\beta} \rangle$
  - Spatial averaging of RANS models → generalized Forchheimer equation,  $F$  not necessarily  $\sim \langle \mathbf{v}_\beta \rangle$  or  $\langle \mathbf{v}_\beta \rangle^2$

Example: structured packings

$$K_{yy}^* = \frac{K_0}{1 + \gamma_t \sqrt{Re}} \text{ (laminar)} \quad ; \quad K_{yy}^* = \frac{K_1}{1 + \beta_t Re} \text{ (turbulent)}$$

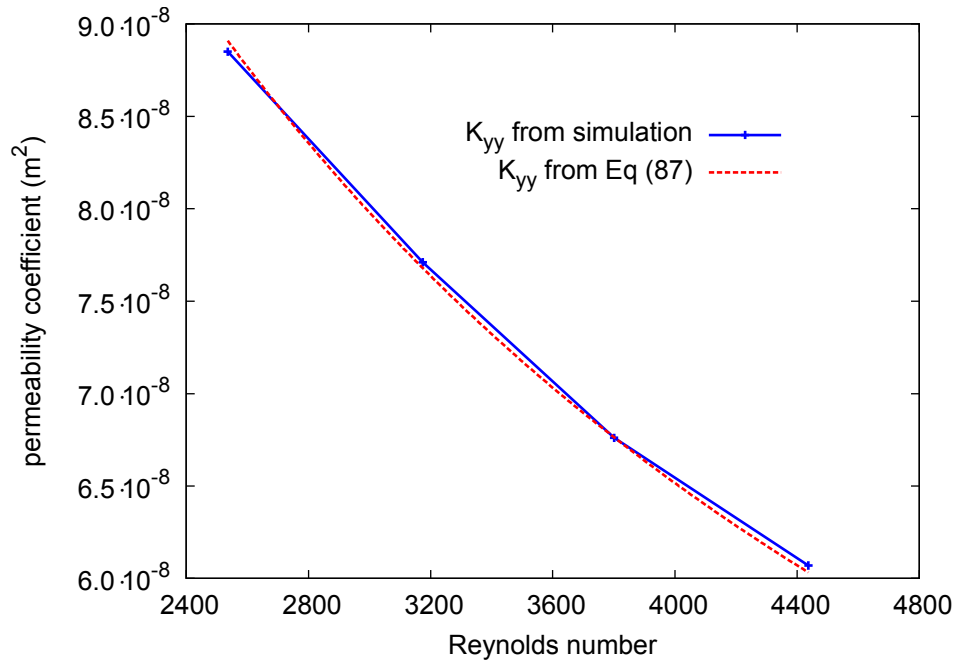
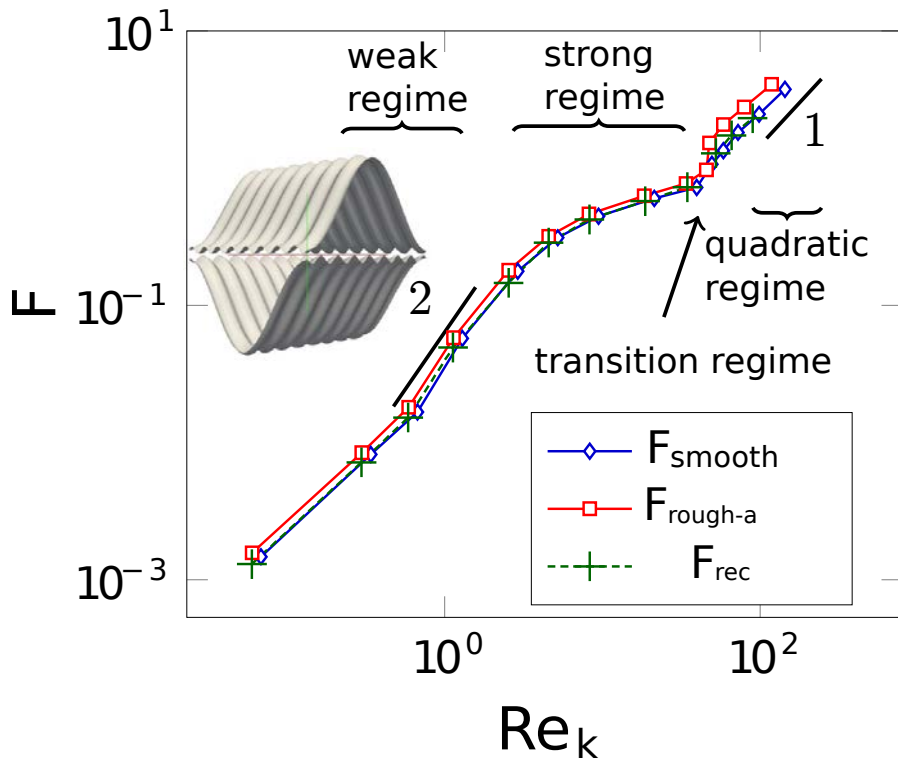


Soulaine and Quintard, 2014

- Porous media turbulence models (**beyond the scope of the lecture**)?
  - Pedras & De Lemos, Masuoka & Takatsu (1996), Nakayama & Kuwahara (1999), ...
  - note: useful for fluid/porous medium interface (D'Hueppe et al., 2012), deep inside the p.m. one may recover Darcy-Forchheimer regimes



# Example for Structured Packings



↑ Averaging of NS; Pasquier et al., 2016

Averaging of RANS model; Soulaïne & Quintard, 2014 ↗

$$\mathbf{K}^*(Re, \theta) \approx \begin{pmatrix} \frac{K_0}{1 + \beta_l Re} & 0 & 0 \\ 0 & \frac{K_0}{1 + \beta_l Re} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$



# Two-Phase Flow

## ● Pore-scale

$$\nabla \cdot \mathbf{v}_\alpha = 0$$

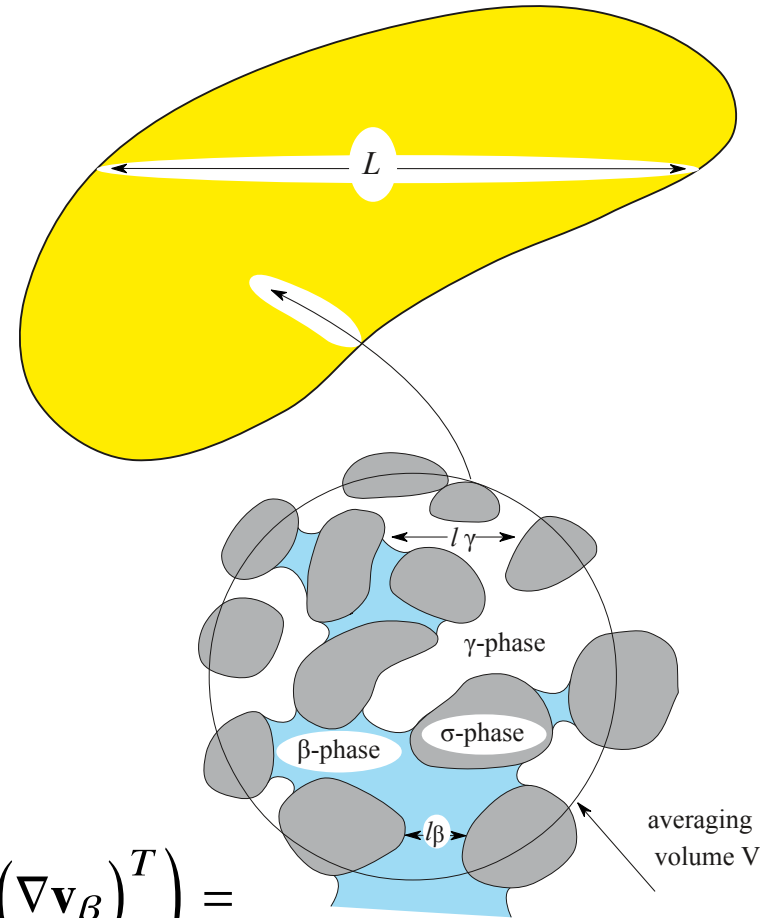
$$\frac{\partial \rho_\alpha \mathbf{v}_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{v}_\alpha \mathbf{v}_\alpha) = -\nabla p_\alpha + \rho_\alpha \mathbf{g} + \mu_\alpha \nabla^2 \mathbf{v}_\alpha \text{ in } V_\alpha \quad \alpha = \beta, \gamma$$

$$\text{B.C. 1 } \mathbf{v}_\beta = 0 \text{ at } A_{\beta\sigma}(t)$$

$$\text{B.C. 2 } \mathbf{v}_\gamma = 0 \text{ at } A_{\gamma\sigma}(t)$$

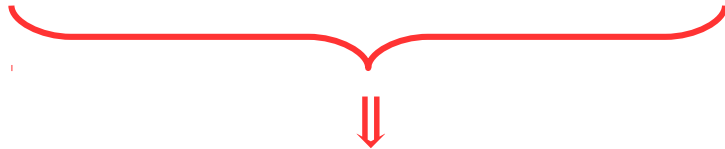
$$\text{B.C. 3 } \mathbf{v}_\beta = \mathbf{v}_\gamma \text{ at } A_{\beta\gamma}(t)$$

$$\begin{aligned} \text{B.C. 4} \quad & -\mathbf{n}_{\beta\gamma} p_\beta + \mu_\beta \mathbf{n}_{\beta\gamma} \cdot \left( \nabla \mathbf{v}_\beta + (\nabla \mathbf{v}_\beta)^T \right) = \\ & -\mathbf{n}_{\beta\gamma} p_\gamma + \mu_\gamma \mathbf{n}_{\beta\gamma} \cdot \left( \nabla \mathbf{v}_\gamma + (\nabla \mathbf{v}_\gamma)^T \right) + 2\sigma H_{\beta\gamma} \mathbf{n}_{\beta\gamma} \text{ at } A_{\beta\gamma}(t) \end{aligned}$$

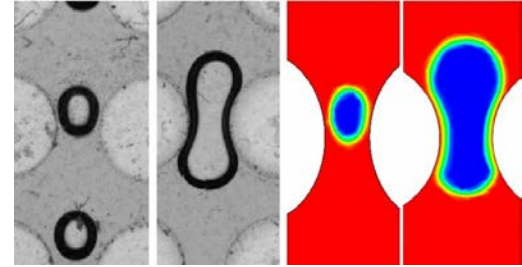


# Note: potentially unsteady flows

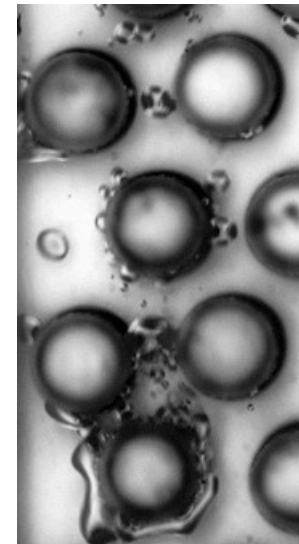
- Front propagation is in general time dependent!
- Specific features: Haines jumps, snap-off, ...
- Boiling, ...



- Common practice: quasi-static approximation...  $\langle \phi \rangle$  and not  $\overline{\langle \phi \rangle}$   
Justification: Ergodicity (not validated)?



Horgue et al.



See paper  
Gourbil et al.

Sapin et al., 2016





# DNS

- VOF, LBM, Cahn-Hilliard, SPH, ....: spurious velocities & convergence for high density and viscosity ratios, problems with wettability and Capillary number.
- Penalization (work with a phase indicator for the solid phase also; Raeni et al., 2012; Horgue et al., 2012): work in progress
- Achieving numerical convergence is very difficult, though averages (e.g. effective properties) may be good for statistical reasons (one phase flow: Romeu & Noetinger, 1995; may contribute to some success stories for  $k_r, P_c$  estimations)
- Need large computational resources
- Alternative to DNS?
  - Macro-scale models (various types)
  - PNM (pore network models), dynamic PNM, Hybrid PNM

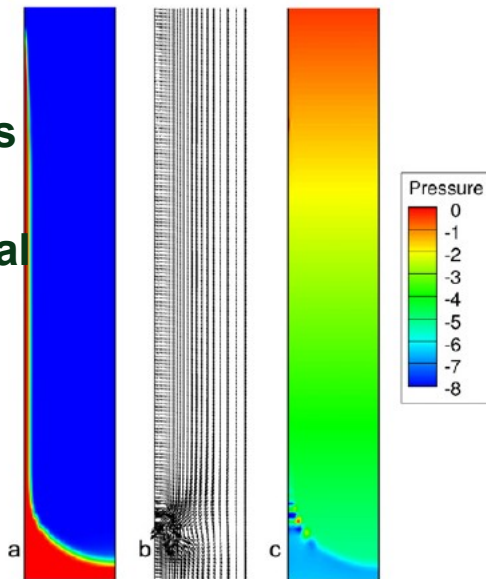
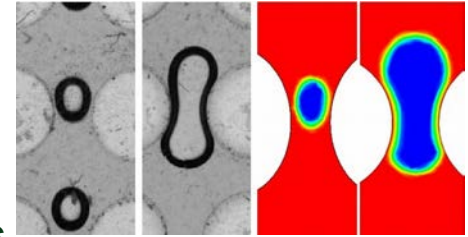


Fig. 6. Simulation of drainage with spurious currents along the meniscus curvature (HRIC,  $Co = 1.00$ ,  $Ca = 0.03$ ), (a) volume fraction, (b) velocity profile, (c) pressure distribution.

Horgue et al., 2012



# Upscaling: quasi-static theory

Whitaker, 1986; Auriault, 1987; Lasseux et al., 1996; ...

## ● Example: Case of B.C. 4

$$\begin{aligned}
 & \mathbf{n}_{\beta\gamma} \left( P_\gamma|_{\mathbf{x}} - P_\beta|_{\mathbf{x}} \right) + \mathbf{n}_{\beta\gamma} \mathbf{y} \cdot \left( \left( \nabla P_\gamma|_{\mathbf{x}} - \rho_\gamma \mathbf{g} \right) - \left( \nabla P_\beta|_{\mathbf{x}} - \rho_\beta \mathbf{g} \right) \right) \\
 & + \mathbf{n}_{\beta\gamma} \mathbf{y} \cdot \left( \rho_\gamma - \rho_\beta \right) \mathbf{g} + \dots = -\mathbf{n}_{\beta\gamma} \left( \tilde{p}_\gamma - \tilde{p}_\beta \right) \\
 & + \mu_\gamma \mathbf{n}_{\beta\gamma} \cdot \left( \nabla \mathbf{v}_\gamma + \left( \nabla \mathbf{v}_\gamma \right)^T \right) - \mu_\beta \mathbf{n}_{\beta\gamma} \cdot \left( \nabla \mathbf{v}_\beta + \left( \nabla \mathbf{v}_\beta \right)^T \right) \\
 & + 2\sigma H_{\beta\gamma} \mathbf{n}_{\beta\gamma} \text{ at } A_{\beta\gamma}
 \end{aligned}$$

$$Ca = \frac{\mu_r U_r}{\sigma}$$

$$Bo = \frac{\left| \left( \rho_\beta - \rho_\gamma \right) g \right| r_0^2}{\sigma}$$

+ Re, (We=Re×Ca) numbers

+ *Dynamic* Bond number

} If  $\approx 0 \Rightarrow$

$$P_\gamma|_{\mathbf{x}} - P_\beta|_{\mathbf{x}} = 2\sigma H_{\beta\gamma}$$

$H_{\beta\gamma} = \text{Constant over REV}$

i.e., the classical capillary pressure theory!



# Macro-Scale Models : Quasi-Static

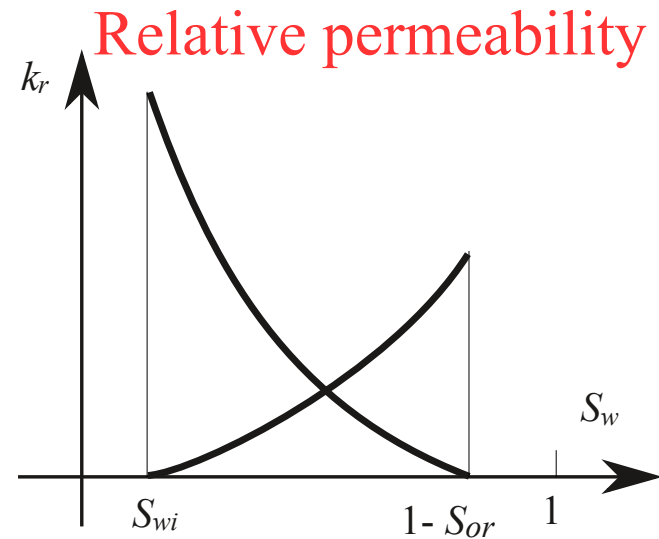
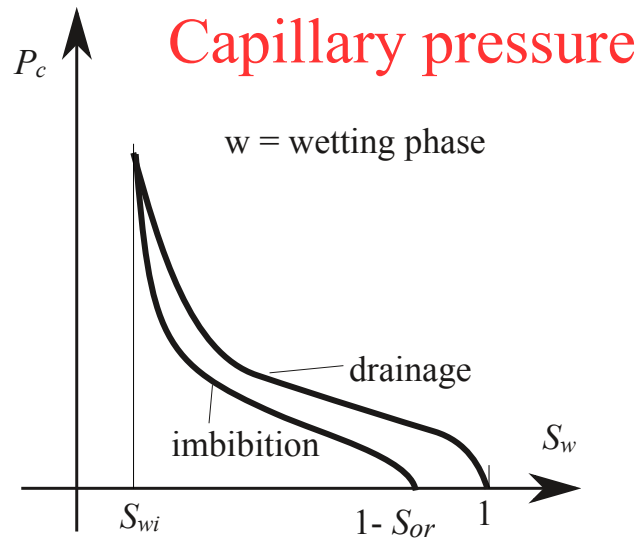
$$\mathbf{V}_\alpha = \langle \mathbf{v}_\alpha \rangle ; P_\alpha = \langle p_\alpha \rangle^\alpha ; \varepsilon_\alpha = \varepsilon S_\alpha \quad \alpha = \beta, \gamma$$

- **Heuristic (Muskat): generalized Darcy's laws**

$$\frac{\partial \varepsilon S_\alpha}{\partial t} + \nabla \cdot \mathbf{V}_\alpha = 0 ; \mathbf{V}_\alpha = -\frac{1}{\mu_\alpha} \mathbf{K}_\alpha \cdot (\nabla P_\alpha - \rho_\alpha \mathbf{g})$$

$$P_\gamma - P_\beta = P_c(S_\beta)$$

$$\mathbf{K}_\gamma = \mathbf{K} k_{r\gamma}(S_\gamma)$$



# Macro-Scale Models : Quasi-Static, viscous coupling

- Model with viscous coupling (upscaling, also heuristic models,...)

(Ex. : Whitaker, 1986)

$$\left\{ \begin{array}{l} \tilde{\mathbf{v}}_\alpha = \mathbf{A}_{\alpha\beta} \cdot \langle \mathbf{v}_\beta \rangle^\beta + \mathbf{A}_{\alpha\gamma} \cdot \langle \mathbf{v}_\gamma \rangle^\gamma \\ \tilde{p}_\alpha = \left( \mathbf{a}_\alpha \cdot \langle \mathbf{v}_\beta \rangle^\beta + \mathbf{a}_\alpha \cdot \langle \mathbf{v}_\gamma \rangle^\gamma \right) \end{array} \right. \quad \alpha = \beta, \gamma$$

$$\mathbf{V}_\alpha = -\frac{1}{\mu_\alpha} \mathbf{K}_\alpha \cdot (\nabla P_\alpha - \rho_\alpha \mathbf{g}) + \mathbf{K}_{\alpha\kappa} \cdot \mathbf{V}_\kappa \quad \alpha, \kappa = \beta, \gamma \quad \alpha \neq \kappa$$

Phase interaction 

Ex. : Two-Phase Poiseuille Flow

$$K_{ll} \approx K \alpha^2$$

$$K_{gg} \approx K \alpha^2$$

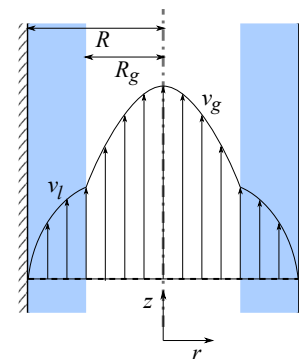
$$K_{lg} \approx \frac{\mu_g}{\mu_l} \frac{(1-\alpha)^2}{\alpha}$$

$$K_{gl} \approx \frac{\alpha}{1-\alpha}$$

$$\frac{\mu_g}{\mu_l} \rightarrow 0$$

$$\mu_l$$

$$\alpha = S_g$$



# Inertia Effects

- Ergun (Heuristic): no-phase interaction terms

$$0 = -\nabla P_\beta + \rho_\beta \mathbf{g} - \mu_\beta \frac{1}{K k_{r\beta}} \mathbf{V}_\beta - \frac{\rho_\beta}{\eta \eta_\beta} \|\mathbf{V}_\beta\| \mathbf{V}_\beta$$

- Schulenberg and Muler (1987) (Heuristic and )

Similar to theory for fluidized beds

$$0 = -\nabla P_\beta + \rho_\beta \mathbf{g} - \mu_\beta \frac{1}{K k_{r\beta}} \mathbf{V}_\beta - \frac{\rho_\beta}{\eta \eta_\beta} \|\mathbf{V}_\beta\| \mathbf{V}_\beta + \frac{\mathbf{F}_{\beta\gamma}^S}{S_\beta}$$

- Upscaling (Lasseux et al., 2008; ...)

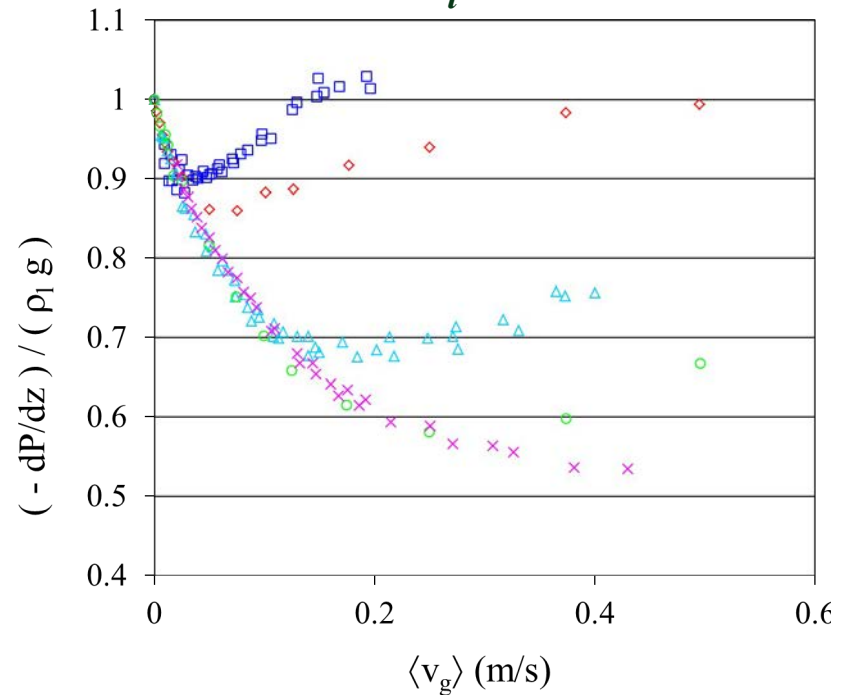
$$\mathbf{V}_\alpha = -\frac{1}{\mu_\alpha} \mathbf{K}_\alpha \cdot (\nabla P_\alpha - \rho_\alpha \mathbf{g}) - \mathbf{F}_{\alpha\alpha} \cdot \mathbf{V}_\alpha \\ + \mathbf{K}_{\alpha\kappa} \cdot \mathbf{V}_\kappa - \mathbf{F}_{\alpha\kappa} \cdot \mathbf{V}_\kappa \quad \alpha, \kappa = \beta, \gamma \quad \alpha \neq \kappa$$



# Importance of Cross-Terms, and Non-Linear Effects



Case  $V_l=0$



from Clavier et al. (2015)

(IRSN: context of nuclear reactor severe accident)

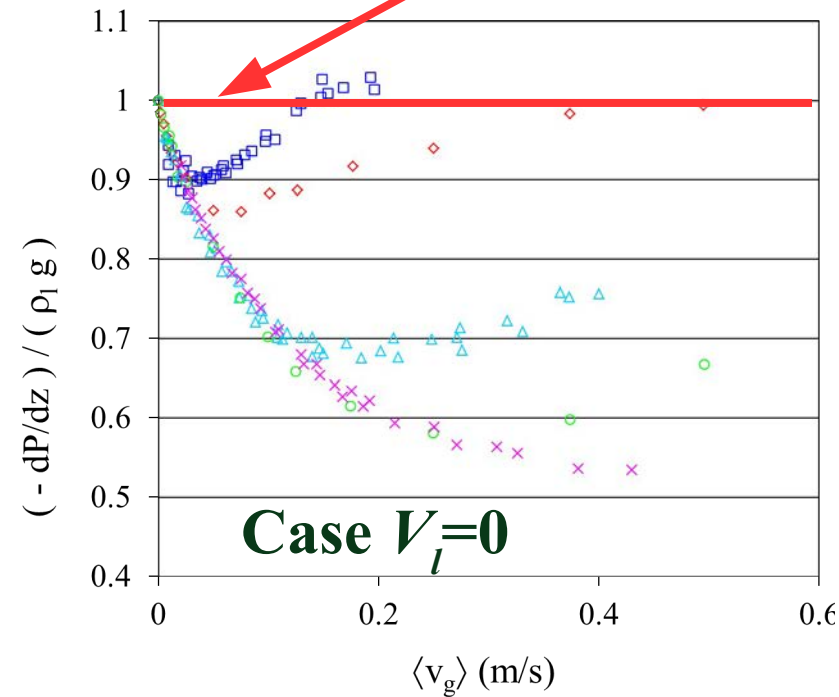
See also Taherzadeh & Saidi (2015)



# Importance of Cross-Terms, and Non-Linear Effects

Models without cross-terms

$t \rightarrow \infty$



$$\left( \frac{\partial P_\beta}{\partial z} - \rho_\beta \mathbf{g} \cdot \mathbf{e}_z \right) = -\mu_\beta K_\beta^{-1} (1 + F_{\beta\beta}) V_\beta$$

$$+ \mu_\beta K_\beta^{-1} K_{\beta\gamma} (1 - K_{\beta\gamma}^{-1} F_{\beta\gamma}) V_\gamma$$

$$\left( \frac{\partial P_\gamma}{\partial z} - \rho_\gamma \mathbf{g} \cdot \mathbf{e}_z \right) = -\mu_\gamma K_\gamma^{-1} (1 + F_{\gamma\gamma}) V_\gamma$$

$$+ \mu_\gamma K_\gamma^{-1} K_{\gamma\beta} (1 - K_{\gamma\beta}^{-1} F_{\gamma\beta}) V_\beta$$

Case  $V_\beta=0$  :

$$\left( \frac{\partial P_\beta}{\partial z} - \rho_\beta \mathbf{g} \cdot \mathbf{e}_z \right) = +\mu_\beta K_\beta^{-1} K_{\beta\gamma} (1 - K_{\beta\gamma}^{-1} F_{\beta\gamma}) V_\gamma$$

from Clavier et al. (2015), Chikhi et al. (2016)

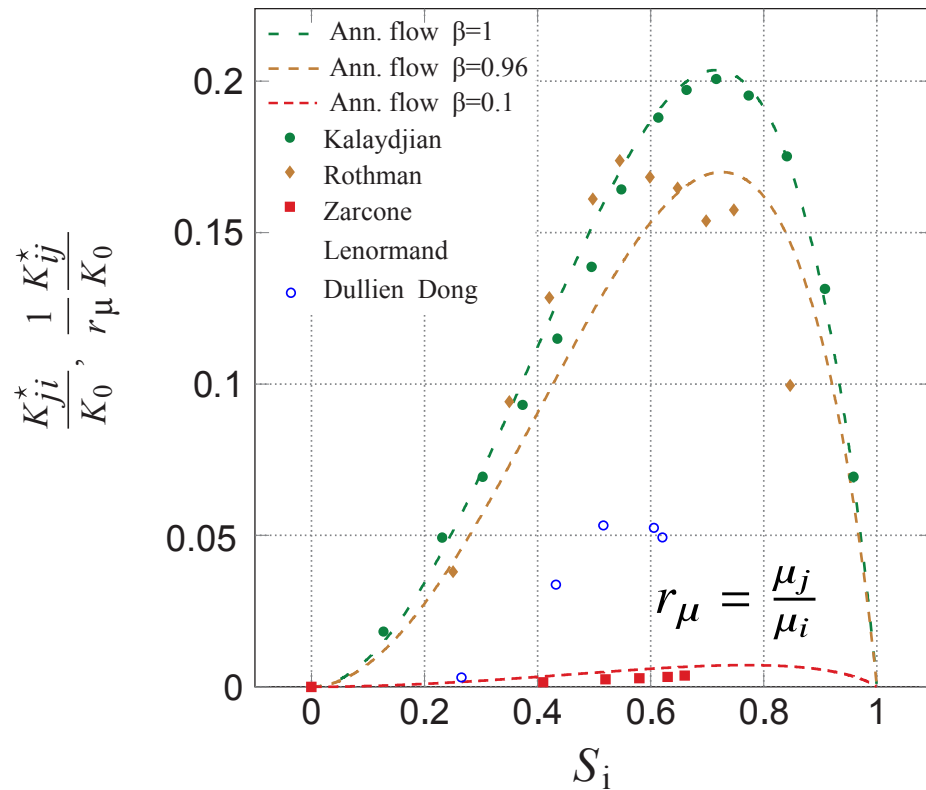
see also Taherzadeh & Saidi (2015) using Tutu et al. experiments



# Impact of $K_0$ on cross-term effect

$$K_{ij} = \beta K_{ij} \text{ (annular 2-phase flow)}$$

(Pasquier et al., 2017)



$$\begin{pmatrix} U_i \\ U_j \end{pmatrix} + \mathcal{K}^* \begin{pmatrix} \frac{\partial P}{\partial x} - \rho_i g \\ \frac{\partial P}{\partial x} - \rho_j g \end{pmatrix} = 0$$

$$\begin{aligned} \mathcal{K}^* &= \begin{pmatrix} K_{ii}^*/\mu_i & K_{ij}^*/\mu_j \\ K_{ji}^*/\mu_i & K_{jj}^*/\mu_j \end{pmatrix} \\ &= \frac{1}{1 - K_{ij}K_{ji}} \begin{pmatrix} K_{ii}/\mu_i & K_{jj}K_{ij}/\mu_j \\ K_{ii}K_{ji}/\mu_i & K_{jj}/\mu_j \end{pmatrix} \end{aligned}$$

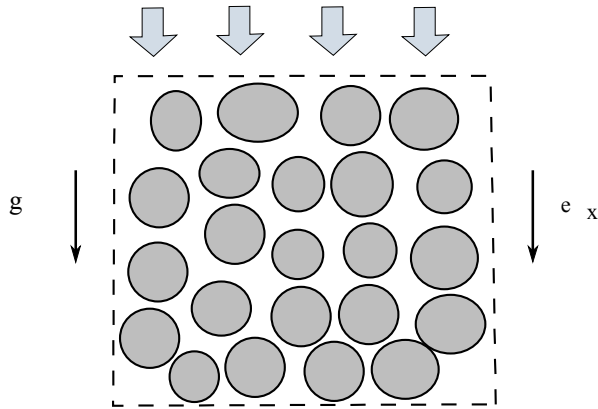
$$\mathcal{N}_g = \frac{K_0 (\rho_i - \rho_j) g}{U \mu_i} \quad \text{Gravity number}$$





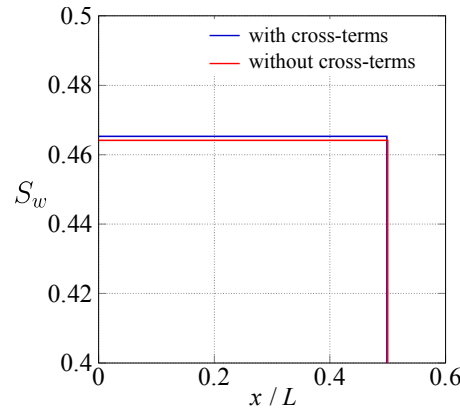
# Effect of cross terms on the flow dynamic

$$\frac{\partial S_l}{\partial t} + \frac{\partial \mathcal{F}_l}{\partial S_l} \frac{\partial S_l}{\partial x} = 0$$

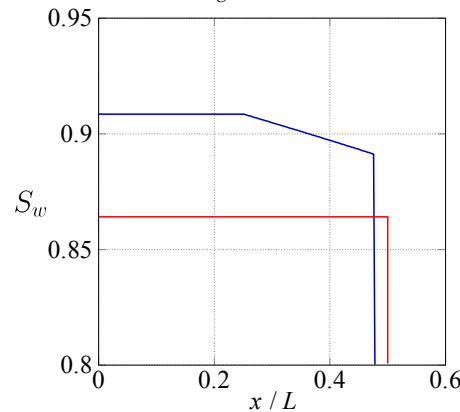


(a) Configuration 1: Imbibition

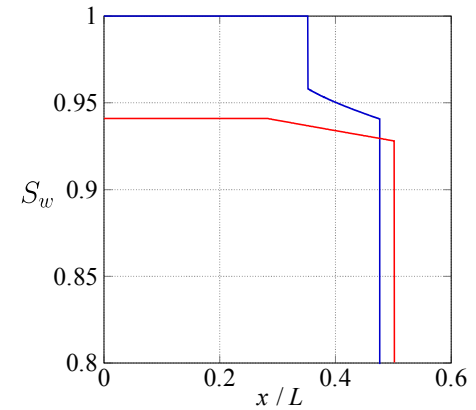
adapted Buckley-Leverett theory  
(Pasquier et al., 2017)



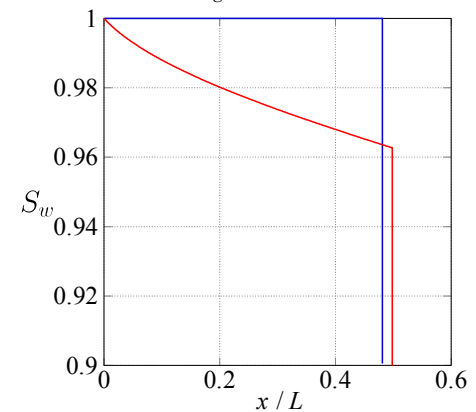
(b)  $\frac{1}{N_g} = 0$



(d)  $\frac{1}{N_g} = 0$



(f)  $\frac{1}{N_g} = 0$



(h)  $\frac{1}{N_g} = 1.1$



# More: Dynamic Models

- impact of  $\partial S/\partial t$ ,  $V_\alpha$ ,  $a_v$ ...:
  - Quintard & Whitaker (1990, from large-scale heterogeneity effects and multi-zone)
  - Hilfer (1998, multi-zone)
  - Panfilov & Panfilova (2005, meniscus)
  - Hassanizadeh and Gray (Irr. Therm.,  $a_v$  as state variable, 1993), also Kalaydjian (1987)...
  - Phase field, Cahn-Hilliard (Cueto-Felgueroso & Juanes, 2009)...



# Examples of dynamic equations

- Quintard & Whitaker, 1990

$$\mathbf{V}_\beta = -\frac{1}{\mu_\beta} \mathbf{K}_\beta^* \cdot (\nabla P_\beta - \rho_\beta \mathbf{g}) - \mathbf{u}_\beta \frac{\partial \varepsilon S_\beta}{\partial t}$$
$$-\mathbf{U}_\beta \cdot \nabla \frac{\partial \varepsilon S_\beta}{\partial t} - \frac{1}{\mu_\beta} \mathbf{M}_\beta : \nabla \nabla P_\beta$$
$$-\frac{1}{\mu_\beta} \Phi_\beta - \frac{1}{\mu_\beta} \mathbf{R}_\beta : \nabla \Phi_\beta$$
$$p_c = \mathcal{F} \left( S_\beta, (\rho_\gamma - \rho_\beta) \mathbf{g}, \nabla P_\beta, \frac{\partial \varepsilon S_\beta}{\partial t}, \dots \right)$$

- Kalaydjian, Hassanizadeh & Gray, ...

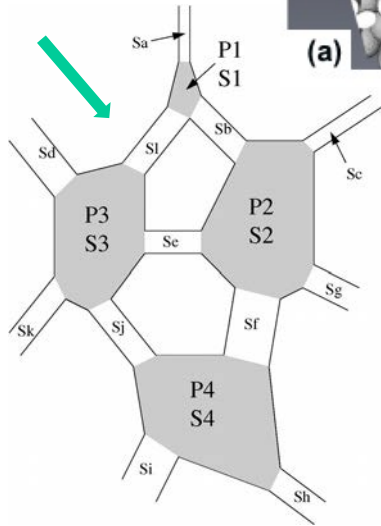
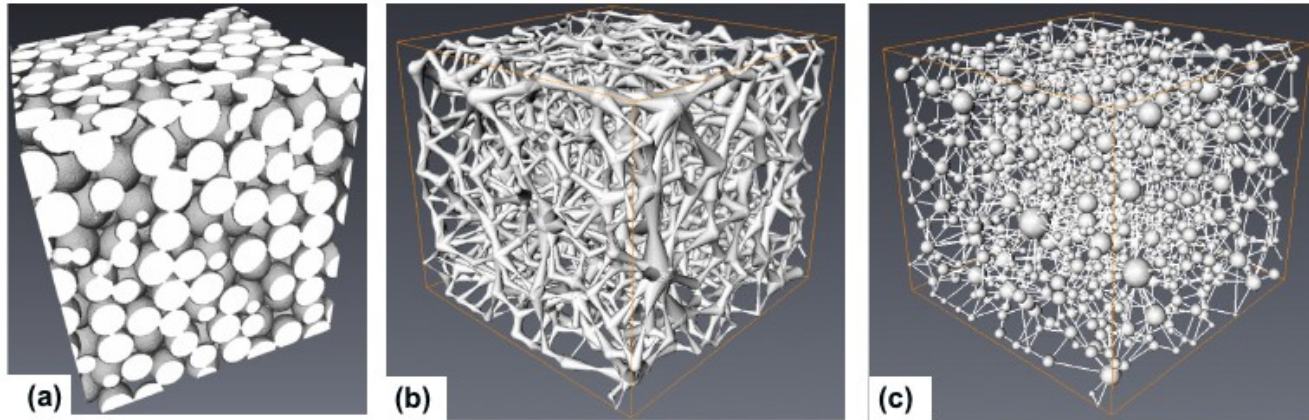
$$P_\gamma - P_\beta = p_c - L_1 \frac{\partial \varepsilon S_\beta}{\partial t}$$

- ...see also Petroleum engng literature on pseudo-functions!
- Usefulness for highly permeable media?



# PNM (Pore Network Model)

(reviews: Dullien, 1992;...; Jockar-Niasar & Hassanizadeh, 2012)



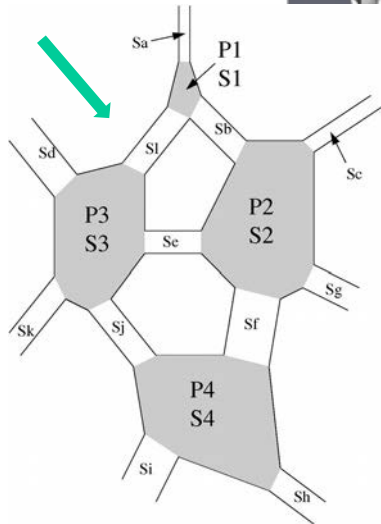
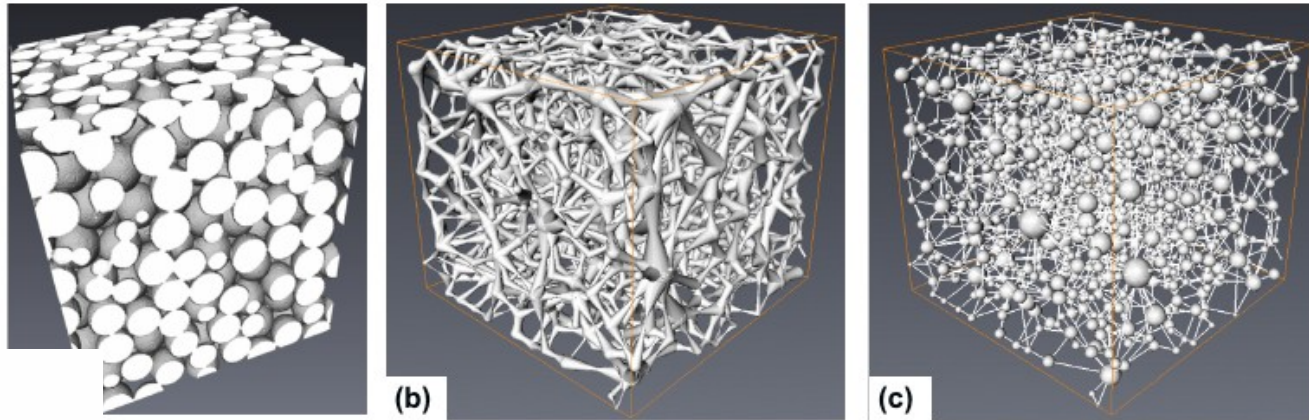
Phase repartition according to capillary equilibrium

- Quasi-static rules → **Percolation theory**: vast literature and important results about flow patterns, etc.
- Used also to estimate  $k_r$ ,  $P_c$
- Drawback: structural properties are lost for other phases (e.g. solid phase)



# PNM - Dynamic

(See review: Jockar-Niasar & Hassanizadeh, 2012)



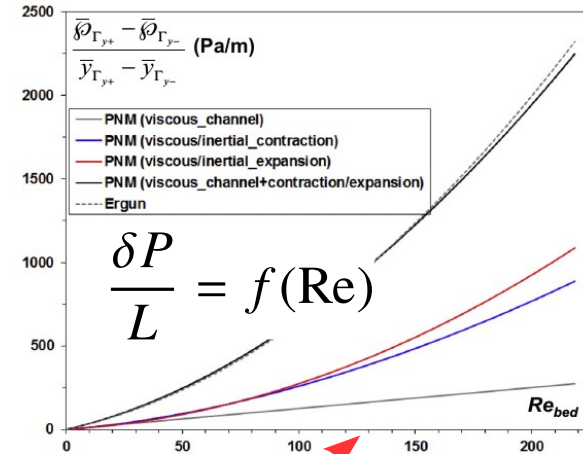
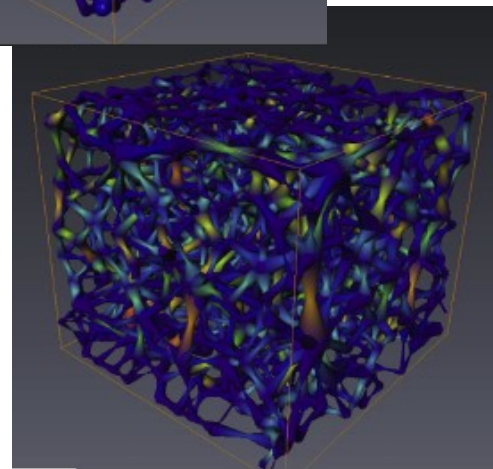
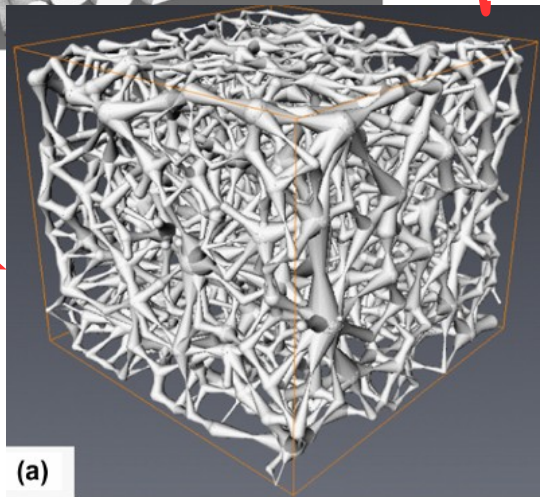
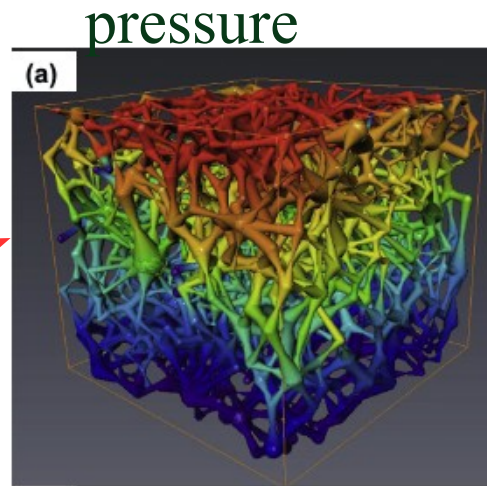
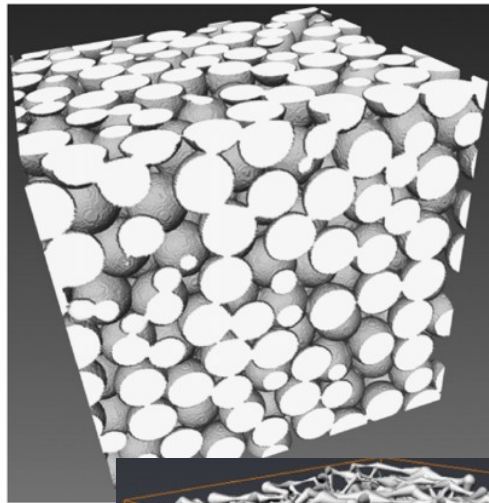
- Time dependent solution, solve for pressure field with approximate solutions for describing flow in connections
- Used to estimate  $k_r$ ,  $P_c$  and other relations (hysteresis)

Pore-scale rules

(analytical: e.g. Poiseuille)



# Example: application to the interpretation of tomographic images in the inertia regime



Larachi et al., 2014

velocity

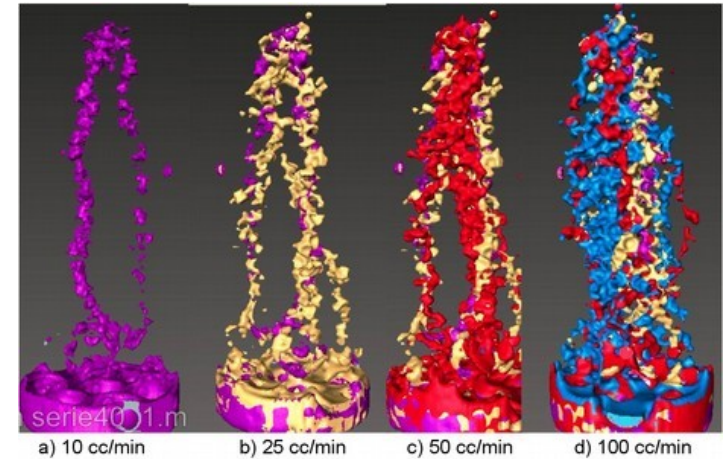


# Mixed or Hybrid Models

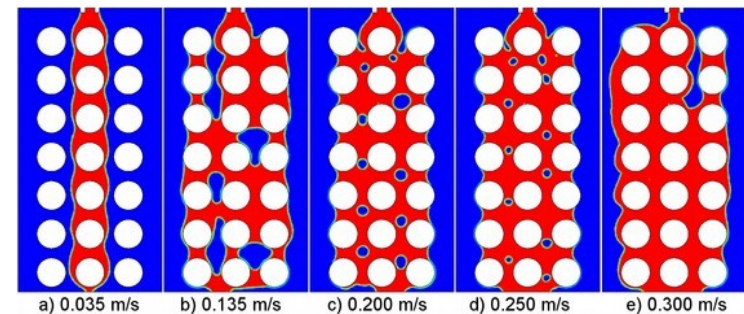
- Network models → a meso-scale representation!
- If low  $Re$ ,  $Ca$ ,  $Bo$  → percolation theory
- Otherwise: Coupling network model and Dynamic rules (which may come from local VOF simulations)

Melli & Scriven, 1991; Horgue et al. (PhD CIFRE/IFP/IMFT), 2012

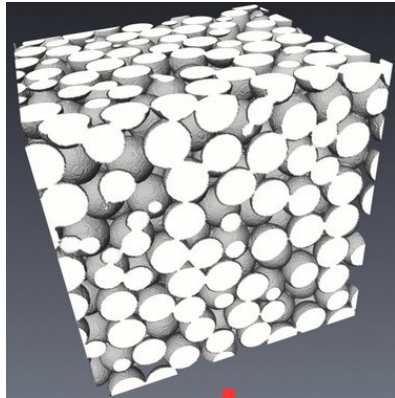
Trickle Bed (X-ray, IFP)



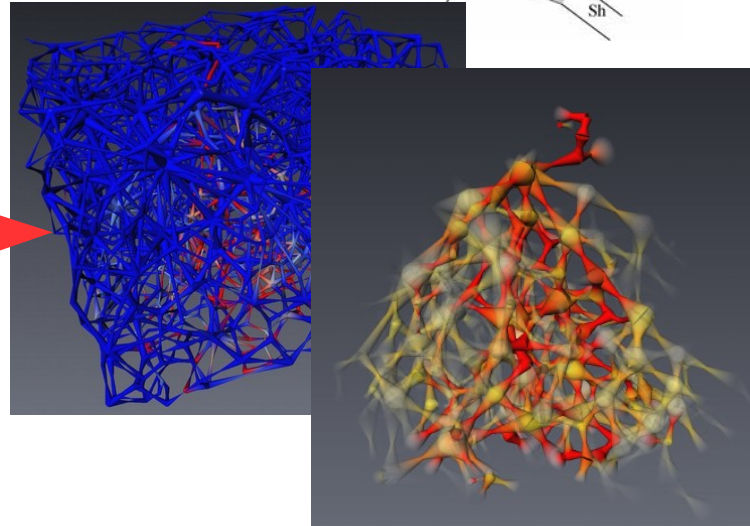
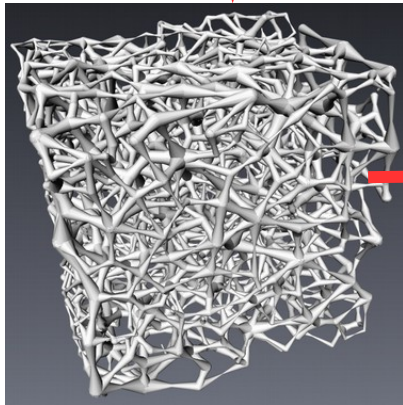
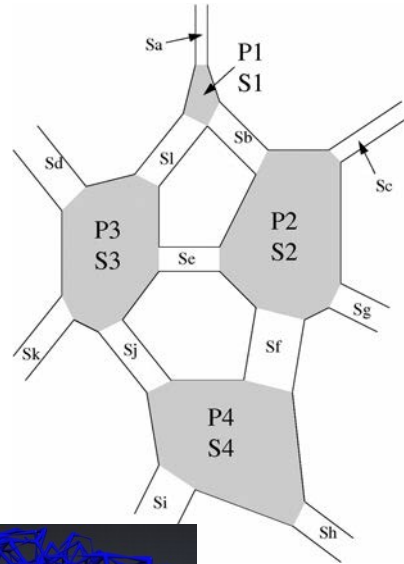
Micromodel experiments



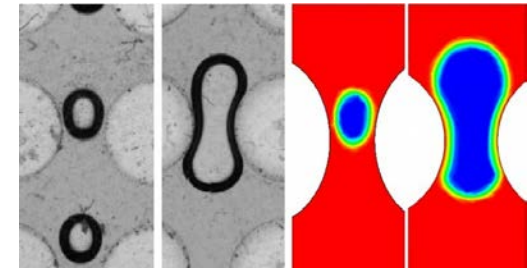
# Mixed or Hybrid Models



Horgue  
et al.,  
2012



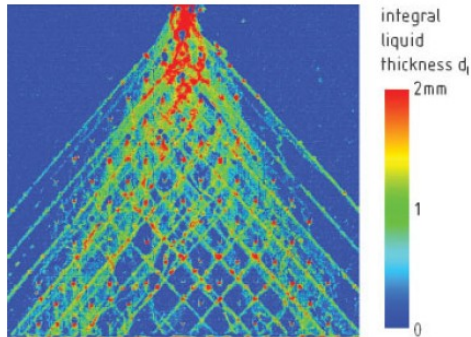
- 1.) Mass and momentum balance for the network
- 2.) Dynamic rules coming from local VOF simulations (or from experiments)





# Macro-Scale Models with Phase “Splitting”

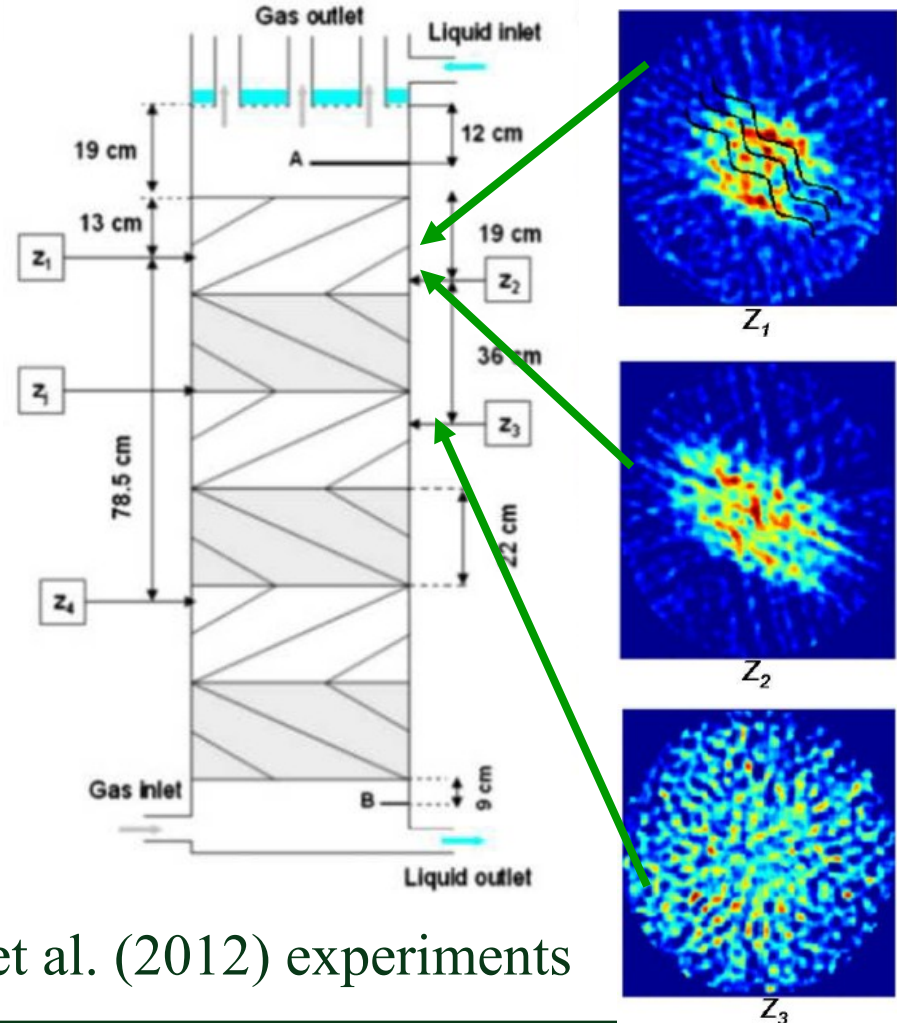
- Example: Flow through Structured Media



$V_l = 100 \text{ l/h}$ ,  $F\text{-factor} = 0 \text{ Pa}^{0.5}$

Mahr and Mewes (2007)

MellaPak (Sulzer Chemtech)



Fourati et al. (2012) experiments



# Macro-Scale Models with Phase “Splitting”

- $$\varepsilon \frac{\partial S_\gamma}{\partial t} + \nabla \cdot \mathbf{U}_\gamma = 0,$$

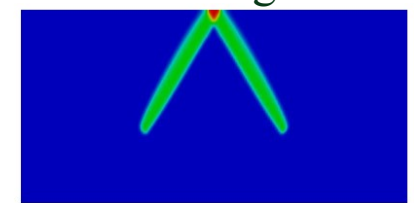
$$\varepsilon \frac{\partial S_{\beta_1}}{\partial t} + \nabla \cdot \mathbf{U}_{\beta_1} = \dot{m},$$

$$\varepsilon \frac{\partial S_{\beta_2}}{\partial t} + \nabla \cdot \mathbf{U}_{\beta_2} = -\dot{m}.$$

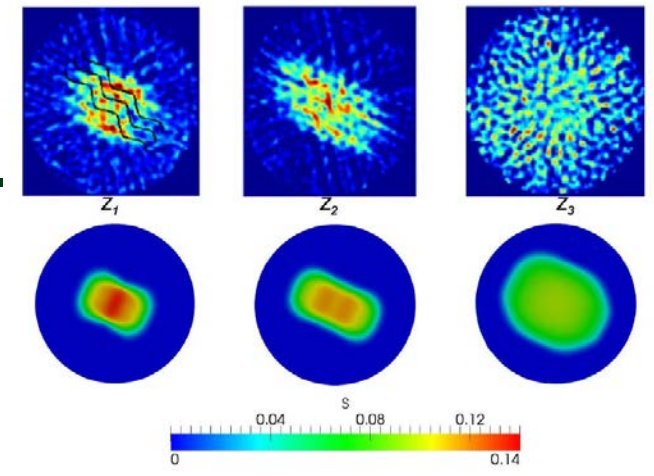
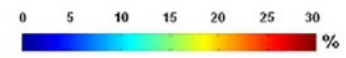
+ 3  
momentum  
equations

- Comparison with Fourati et al. (2012) experiments (Soullaine et al. 2014) → calibration of exchange term on the 1<sup>st</sup> stack**

Model with liquid phase splitting, no-exchange



(b) t = 1 s



# Conclusions

- **One-Phase Flow:**
  - Generalized Forchheimer equations → a practical model for laminar inertia regimes or localized turbulence
  - Porous Media Turbulence models
- **Two-Phase Flow:**
  - Importance of cross-terms, Experimental determination?
  - Extensions to more dynamic flows:
    - Extended generalized Darcy's laws
    - Models with phase splitting
    - Hybrid models
  - Highly transient effects, complex time and space averaging?
- **Coupling with other transport mechanisms (dispersion, heat transfer, ...)?**

