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▶ **To cite this version:**

Pietro Lami, Ivan Lanese, Jean-Bernard Stefani, Claudio Sacerdoti Coen, Giovanni Fabbretti. Reversibility in Erlang: Imperative Constructs -Technical Report. [Research Report] Inria - Research Centre Grenoble – Rhône-Alpes. 2022, pp.1-28. hal-03655372

HAL Id: hal-03655372
https://hal.science/hal-03655372v1

Submitted on 29 Apr 2022

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Reversibility in Erlang: Imperative Constructs - Technical Report*

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Abstract. A relevant application of reversibility is causal-consistent reversible debugging, which allows one to explore concurrent computations backward and forward to find a bug. This approach has been put into practice in CauDEr, a causal-consistent reversible debugger for the Erlang programming language. CauDEr supports the functional, concurrent and distributed fragment of Erlang. However, Erlang also includes imperative features to manage a map (shared among all the processes of a same node) associating process identifiers to names. Here we extend CauDEr and the related theory to support such imperative features. From a theoretical point of view, the added primitives create different causal structures than those derived from the concurrent Erlang fragment previously handled in CauDEr, yet we show that the main results proved for CauDEr are still valid.

Keywords: Debugging · Erlang · Reversible computing · Causality

1 Introduction

Reversible computing is a programming paradigm in which programs run both forwards (the standard computation) and backwards. Any forward computation in a reversible language can be undone with a finite number of backward steps. Reversible computing has applications in many areas, such as low-power computing [12], simulation [1], robotics [18], biological modeling [19] and others. We are particularly interested in applying reversible computing to debugging [2].

In a sequential system, undoing forward actions in reverse order of completion starting from the last one produces a backward computation. Undoing a forward action can be seen as a backward action. In a concurrent environment, one cannot easily decide which is the last action since many actions can be executed at the same time, and a total order of actions may not be available. Even if a total order exists, undoing actions in reverse order may be too restrictive since the

* This work has been partially supported by French ANR project DCore ANR-18-CE25-0007.

order of execution of concurrent actions may depend on the relative speed of the processors executing them and has no impact on the final state. For instance, when looking for a bug causing a visible misbehavior in a concurrent system, independent actions may be disregarded since they cannot contain the bug.

The first definition of reversibility in a concurrent setting has been proposed by Danos and Krivine [4]: *causal-consistent reversibility*. In short, it states that any action can be undone provided that all its effects (if any) have been undone.

The idea of a causal-consistent reversible debugger was introduced in [7]. The main concept of [7] is to use causal-consistent reversibility to explore backward a concurrent execution starting from a visible misbehavior looking for the bug causing it. The CauDEr debugger [2], described in [15,21,6], applies these ideas to provide a reversible debugger for the functional, concurrent and distributed fragment of the Erlang programming language [5].

Here, we extend CauDEr and its underlying theory by adding the support for some primitives that are not considered in the previous versions. These primitives, namely `register`, `unregister`, `whereis` and `registered`, provide imperative behaviors inside the Erlang language whose core is functional. More precisely, they define a map linking process identifiers (pids) to names. They make it possible to add, delete and read elements from the map. From the technical point of view, supporting these primitives is not trivial since they introduce causal dependencies that are different from those originating from the functional and concurrent fragment of Erlang considered in [15,16,21]. In particular, read actions commute, but do not commute with add and delete actions. Such causal dependencies cannot be reliably represented in the general approach to derive reversible semantics for a given language presented in [13], because the approach in [13] considers a causal relation based on resources consumed and produced only, and does not support read operations. Similar dependencies are considered in [6], to model the set of nodes in an Erlang network, but this model does not include a delete operation, while we consider one. Similar dependencies are also used in [8] to study operations on shared tuple spaces in the framework of the coordination language Klaim, however they only access single tuples, while we also access multiple tuples or check for the absence of a given tuple. Also, their work is in the context of an abstract calculus and has never been implemented.

The paper is structured as follows. Section 2 briefly recalls the reversible semantics on which CauDEr is based [21]. Then, in Section 3, we extend the reversible semantics of Erlang to support imperative features. In Section 4 we describe our extension to CauDEr. Finally, in Section 5 we discuss related work and conclude the paper with hints for future work. Proofs and further technical details are available in the Appendix.

2 Background

We build our technical development on the reversible semantics for Erlang in [21]. We give below a quick overview of it, while referring to [21] for further details.

$$
\begin{array}{rcl}
\mathit{program} & ::= & \mathit{mod}_1 \ldots \mathit{mod}_n \\
\mathit{mod} & ::= & \mathit{fun\_def}_1 \ldots \mathit{fun\_def}_n \\
\mathit{fun\_def} & ::= & \mathit{fun\_rule}\{';' \mathit{fun\_rule}\}'.' \\
\mathit{fun\_rule} & ::= & \mathit{Atom}\ \mathit{fun} \\
\mathit{fun} & ::= & (\lbrack \mathit{exprs} \rbrack)\ [\mathsf{when}\ \mathit{expr}] \rightarrow \mathit{exprs} \\
\mathit{exprs} & ::= & \mathit{expr}\ \{',' \mathit{expr}\} \\
\mathit{expr} & ::= & \mathit{atomic} \mid \mathit{Var} \mid\ '\{'[\mathit{exprs}]'\}' \mid\ '['[\mathit{exprs}|\mathit{exprs}]']' \mid \mathsf{if}\ \mathit{if\_clauses}\ \mathsf{end} \\
 & \mid & \mathsf{case}\ \mathit{expr}\ \mathsf{of}\ \mathit{cr\_clauses}\ \mathsf{end} \mid \mathsf{receive}\ \mathit{cr\_clauses}\ \mathsf{end} \mid \mathit{expr}\ !\ \mathit{expr} \\
 & \mid & \mathit{pattern} = \mathit{expr} \mid [\mathit{Mod}:]\mathit{expr}(\lbrack \mathit{exprs} \rbrack) \mid \mathit{fun\_expr} \mid \mathit{Opexprs} \\
\mathit{atomic} & ::= & \mathit{Atom} \mid \mathit{Char} \mid \mathit{Float} \mid \mathit{Integer} \mid \mathit{String} \\
\mathit{if\_clauses} & ::= & \mathit{expr} \rightarrow \mathit{exprs}\ \{';' \mathit{expr} \rightarrow \mathit{exprs}\} \\
\mathit{cr\_clause} & ::= & \mathit{pattern}\ [\mathsf{when}\ \mathit{expr}] \rightarrow \mathit{exprs}\ \{';' \mathit{pattern}\ [\mathsf{when}\ \mathit{expr}] \rightarrow \mathit{exprs}\} \\
\mathit{fun\_expr} & ::= & \mathsf{fun}\ \mathit{fun}\ \{';' \mathit{fun}\}\ \mathsf{end} \\
\mathit{patterns} & ::= & \mathit{pattern}\ \{',' \mathit{pattern}\} \\
\mathit{pattern} & ::= & \mathit{atomic} \mid \mathit{Var} \mid\ '\{'[\mathit{patterns}]'\}' \mid\ '['[\mathit{patterns}|\mathit{pattern}]']'
\end{array}
$$

Fig. 1. Language syntax

The language syntax. Erlang is a functional, concurrent and distributed programming language based on the actor paradigm [10] (concurrency based on asynchronous *message-passing*).

The syntax of the language is shown in Fig. 1. A program is a collection of module definitions, a module is a collection of function definitions, a function is a mapping between the function name and the function expression. An expression can be a variable, an atom, a list, a tuple, a call to a function, a case expression, an if expression, or a pattern matching equation. We distinguish expressions and patterns. Here, patterns are built from atomic values, variables, tuples and lists. When we have a case *expr* of *cr_clauses* end expression we first evaluate *expr* to a value, say v, then we search for a clause that matches v and such that the guard when *expr* is satisfied. If one is found then the case construct evaluates to the clause expression. The *if* expression is very similar to the evaluation of the *case* expression just described. Pattern matching is written as $\mathit{pattern} = \mathit{expr}$. Then, $\mathit{expr}_1 ! \mathit{expr}_2$ allows a process to send a message to another one. Expression expr_1 must evaluate either to a pid or to an atom (identifying the receiver process) and expr_2 evaluates to the message payload, indicated with v. The whole function evaluates to v and, as a side-effect, the message will be sent to the target process. The complementary operation of message sending is receive *cr_clauses* end. This construct takes a message targeting the process that matches one of the clauses. If no message is found then the process suspends.

Erlang includes a number of built-in functions (BIFs). In [21], they only consider self, which returns the process identifier of the current process, and spawn, that creates a new process. BIFs supporting distribution are considered in [6]. For a deeper discussion we refer to [21,6].

The language semantics. Here we describe the semantics of the language. We begin by providing the definitions of *process* and *system*.

$$(Op)\ \frac{\mathsf{eval}(op, v_1, \ldots, v_n) = v}{\theta, C[op\ (v_1, \ldots, v_n)], S \xrightarrow{\tau} \theta, C[v], S}$$

Fig. 2. A sample rule belonging to the expression level.

Definition 1 (Process). *A process is a tuple $\langle p, \theta, e, S\rangle$, where p is the process pid, θ is the process environment, e is the expression under evaluation and S is a stack of process environments.*

Stack S is used to store away the process state to start a sub-computation of the expression under evaluation and then to restore it, once the sub-computation ends. We refer to [21] for a discussion on why it is needed.

Definition 2 (System). *A system is a tuple $\Gamma; \Pi$. Γ is the global mailbox, that is a set of messages of the form $(sender\_pid, receiver\_pid, payload)$. Π is the pool of running processes, denoted by an expression of the form*

$$\langle p_1, \theta_1, e_1, S_1\rangle \mid \ldots \mid \langle p_n, \theta_n, e_n, S_n\rangle$$

where "$\mid$" is an associative and commutative parallel operator.

The semantics in [21] is defined in a modular way, similarly to the one presented in [15,6]: there is a semantics for the expression level and one for the system level. This approach simplifies the design of the reversible semantics since only the system one needs to be updated. The expression semantics is defined as a labeled transition relation, where the label describes side-effects (e.g., creation of a message) or requests of information to the system level. The semantics, described in Appendix A.1 due to space constraints, is a classical call-by-value semantics for a higher-order language. Fig. 2 shows a sample rule of the expression level: the Op rule, used to evaluate arithmetic and relational operators. This rule uses the auxiliary function **eval** to evaluate the expression and an evaluation context C to find the redex in a larger term.

The system semantics uses the label from the expression level to execute the associated side-effect or to provide the necessary information. Below we list the labels used in the expression semantics:

- τ, denoting the evaluation of a (sequential) expression without side-effects;
- $\mathsf{send}(v_1, v_2)$, where v_1 and v_2 represent, respectively, the pid of the sender and the value of the message;
- $\mathsf{rec}(\kappa, \overline{cl_n})$, where $\overline{cl_n}$ denotes the n clauses of a $\mathsf{receive}$ expression;
- $\mathsf{spawn}(\kappa, a/n, [\overline{v_n}])$, where a/n represents the name and arity of the function executed by the spawned process, while $[\overline{v_n}]$ is the list of its parameters.

Symbol κ is a placeholder for the result of the evaluation, not known at the expression level, that the system rules will replace with the correct value.

We do not show here the system rules, they are available in Appendix A.2. We show instead below how sample rules are extended to support reversibility.

$$(Send)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(p',v)} \theta', e', S' \quad \lambda \text{ is a fresh identifier}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi \rightharpoonup \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, \mathsf{send}(\theta, e, S, \{v, \lambda\}){:}h, \theta', e', S'\rangle \mid \Pi}$$

$$(\overline{Send})\ \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, \mathsf{send}(\theta, e, S, \{v, \lambda\}){:}h, \theta', e', S'\rangle \mid \Pi \leftharpoondown \Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi$$

Fig. 3. A sample rule belonging to the forward semantics and its counterpart.

A reversible semantics. Two relations describe the reversible semantics: one forward ($\rightharpoonup$) and one backward ($\leftharpoondown$). The former extends the system semantics using a *Landauer embedding* [12]. The latter proceeds in the opposite direction and allows us to undo an action by ensuring causal consistency, thus before undoing an action we ensure that all its consequences have been undone.

Syntactically, every process is extended with a history, denoted with h, which stores the information needed in the backward semantics to *undo* an action. In the semantic rules we highlight the history in red. The history is composed of *history items*, to distinguish the last rule executed by a process and track the related information. The history items introduced in [21] are:

$$\{\tau(\theta, e, S), \mathsf{send}(\theta, e, S, \{v, \lambda\}), \mathsf{rec}(\theta, e, S, p, \{v, \lambda\}), \mathsf{spawn}(\theta, e, S, p), \mathsf{self}(\theta, e, S)\}$$

Fig. 3 shows a sample rule from the forward semantics and its counterpart from the backward semantics. W.r.t. the standard semantics, here messages also carry a unique identifier λ, without which messages with the same value could not be distinguished. This choice is discussed in [15].

In the premises of rule *Send*, we can see the expression-level semantics in action, transitioning from configuration (θ, e, S) to (θ', e', S'). The forward semantics uses the corresponding label to determine the associated side-effect: the message $(p, p', \{v, \lambda\})$ is added to the set of messages Γ. Also, the history of process p is enriched with the corresponding history item.

The reverse rule, $\overline{Send}$, can be applied only when all the consequences of the *Send*, in particular the reception of the sent message, have been undone. Such constraint is enforced by requiring the message to be in Γ. Then we can remove the message $(p, p', \{v, \lambda\})$ from Γ and restore p to the previous state.

3 Reversible Erlang with Imperative Primitives

Syntax of imperative primitives. In our extension, atoms and pids are central. An atom is a literal constant. Pid is an abbreviation for process identifier: each process is identified by a pid. In Erlang, a pid can be associated to an atom. Thus, one can refer the process, e.g., when specifying the target of a message, using the associated atom instead of the pid. On the one hand, an atom is more meaningful than a pid for a human. On the other hand, this allows one to decide which process plays a given role. E.g., if a process crashes another one can be registered under the same atom so that the replacement is transparent to other

processes (provided that they use the atom to interact). All pairs $\langle atom, pid \rangle$ form a map, shared among the processes of the same node (we consider here a single node, we discuss in Section 4 how to deal with multiple nodes).

Our extension is based on the syntax in Fig. 1, but we add the following built-in functions (BIFs):

- register/2 (where /2 denotes the arity): given an atom a and a pid p, it inserts the pair $\langle a, p \rangle$ in the map and returns the atom **true**. If either the atom a or the pid p is already registered, an exception is raised;
- unregister/1: given an atom a, it removes the (unique) pair $\langle a, p \rangle$ from the map and returns **true** if the atom a is found, raises an exception otherwise;
- whereis/1: given an atom, it returns the associated pid if it exists, the atom **undefined** otherwise;
- registered/0: returns a list (possibly empty) of all the atoms in the map.

3.1 Semantics of imperative features

Standard semantics of imperative features. According to the official documentation [5], the BIFs above are implemented in Erlang using request and reply signals between the process and the manager of the map. To simplify the modelization, we opted to implement these BIFs as synchronous actions. This choice does not alter the possible behaviors since the behavior visible to Erlang users is determined by the order in which the request messages are processed at the manager. We begin by providing the updated definition of *system* (the definition of process is unchanged).

Definition 3 (System). *A system is a tuple $\Gamma; \Pi; \mathsf{M}$. Γ and Π are as in Def. 2.* M *is a set of registered pairs atom-pid of the form $\{\langle a_1, p_1 \rangle; \ldots; \langle a_n, p_n \rangle\}$, where a_i are atoms and p_i pids. Given an atom a, M_a is the set $\{\langle a, p \rangle | \langle a, p \rangle \in \mathsf{M}\}$; given a pid p, M_p is the set $\{\langle a, p \rangle | \langle a, p \rangle \in \mathsf{M}\}$.*

Sets M_a and M_p contain at most one element.

As in the previous section, we have a double-layered semantics: one level for expressions ($\rightarrow$) and one for systems ($\hookrightarrow$).

To simplify the presentation w.r.t. [21], we extend rule *Op* (Fig. 4) to deal also with built-in functions. To this end, we extend the operator **eval** to produce also the label for functions with side-effects. We define **eval** on them as:

- $\mathsf{eval}(\mathsf{self}) = (\kappa, \mathsf{self}(\kappa))$;
- $\mathsf{eval}(\mathsf{spawn}, \mathsf{fun}() \rightarrow exprs\ \mathsf{end}) = (\kappa, \mathsf{spawn}(\kappa, exprs))$;
- $\mathsf{eval}(\mathsf{register}, atom, pid) = (\kappa, \mathsf{register}(\kappa, atom, pid))$;
- $\mathsf{eval}(\mathsf{unregister}, atom) = (\kappa, \mathsf{unregister}(\kappa, atom))$;
- $\mathsf{eval}(\mathsf{whereis}, atom) = (\kappa, \mathsf{whereis}(\kappa, atom))$;
- $\mathsf{eval}(\mathsf{registered}) = (\kappa, \mathsf{registered}(\kappa))$.

On sequential expressions **eval** returns (v, τ), with v the result of the evaluation.

Thanks to our extension, rule *Op* in Fig. 4 covers all function invocations, including BIFs with side effects, while in [15,21] each such BIF requires a dedicated

$$(Op)\ \frac{\mathsf{eval}(op, v_1, \ldots, v_n) = (v, label)}{\theta, C[op\ (v_1, \ldots, v_n)], S \xrightarrow{label} \theta, C[v], S}$$

Fig. 4. Standard semantics: evaluation of function applications, revised.

rule. Furthermore, new BIFs with side effects can be added without changing the expression level (function **eval** needs to be updated though).

The other rules for evaluating expressions are collected in Appendix A.1.

The semantics of the system level can be found in Appendix A.2 (Figures 11 and 10). Equivalently, the rules describing the imperative primitives can be obtained from the ones in Fig. 5, which describes the forward semantics, by dropping the red part. Rules are divided into *write rules* (above the line), which modify the map, and *read rules* (below the line), that only read it. This has an impact on their concurrent behavior, as described later on. We highlight in blue the parts related to the map.

In all the rules, the tuple representing the system includes the map M, where we store all the registered pairs atom-pid.

Rule *RegisterS* defines the success case of the register BIF, which adds the tuple $\langle a, p' \rangle$ to the map. The register fails either when the atom a or the pid p' are already used, or when the pid p' refers to a dead process (this is checked by predicate isAlive), as described by rule *RegisterF*. Similarly, for the unregister, the success case corresponds to rule *UnregisterS*, which removes from the map the (unique) pair atom-pid for a given atom a. The failure case, when there is no pid registered under atom a, corresponds to rule *UnregisterF*. Both failure cases replace the current expression with ϵ and the current stack with []. This denotes an uncaught exception (in this paper we do not consider exception handling). The predicate isAlive takes a pid p and the pool of running processes and controls that the process with pid p is alive ($\langle p, \theta, e, S \rangle$ with $e \neq \bot$).

Rules *SendS* and *SendF* define the behavior of send actions when the receiver is identified with an atom. The former is fired when the receiver is registered in the map, resulting in the addition of the message to Γ, the latter when it is not, resulting in an uncaught exception.

Rules *Whereis1*, *Whereis2* and *Registered* define the behavior of the respective primitives; these rules read M without modifying it. Rule *Registered* uses the auxiliary function registered. We define it as: $\mathsf{registered}(\mathsf{M}) = [a_1, \ldots, a_n]$ where $\mathsf{M} = \{\langle a_1, p_1 \rangle, \ldots, \langle a_n, p_n \rangle\}$.

Finally, we have two rules dealing with process termination. If the pid of the process is not registered on the map, rule *End* simply changes the expression to $\bot$, denoting a terminated process. Otherwise, rule *EndUn* applies, additionally removing the pid from the map.

Reversible semantics. The definition of the forward semantics poses a number of challenges, due to the need of balancing two conflicting requirements when defining the history information to be stored. On the one hand, we need to keep enough information to be able to define a corresponding backward semantics.

$$(RegisterS)\ \frac{\theta, e, S \xrightarrow{\mathsf{register}(\kappa,a,p')} \theta', e', S' \qquad t \text{ fresh} \qquad \mathsf{M}_a = \emptyset \qquad \mathsf{M}_{p'} = \emptyset \qquad \mathsf{isAlive}(p', \Pi)}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{regS}(\theta, e, S, \{\langle a, p', t, \top\rangle\}){:}h, \theta', e'\{\kappa \to \mathsf{true}\}, S'\rangle \mid \Pi; \mathsf{M} \cup \{\langle a, p', t, \top\rangle\}}$$

$$(UnregisterS)\ \frac{\theta, e, S \xrightarrow{\mathsf{unregister}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \{\langle a, p', t, \top\rangle\}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{del}(\theta, e, S, \mathsf{M}_a, \mathsf{M}^a \cup M^{p'}){:}h, \theta', e'\{\kappa \to \mathsf{true}\}, S'\rangle \mid \Pi; \mathsf{M} \setminus \mathsf{M}_a \cup \mathrm{kill}(\mathsf{M}_a)}$$

$$(EndUn)\ \frac{e \text{ is a value } \vee\, e = \epsilon \qquad \mathsf{M}_p = \{\langle a, p, t, \top\rangle\}}{\Gamma; \langle p, h, \theta, e, [\,]\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{del}(\theta, e, [\,], \mathsf{M}_p, \mathsf{M}^a \cup \mathsf{M}^p){:}h, \theta, \bot, [\,]\rangle \mid \Pi; \mathsf{M} \setminus \mathsf{M}_p \cup \mathrm{kill}(\mathsf{M}_p)}$$

$$(RegisterF)\ \frac{\theta, e, S \xrightarrow{\mathsf{register}(\kappa,a,p')} \theta', e', S' \qquad \mathsf{M}_a \neq \emptyset \vee \mathsf{M}_{p'} \neq \emptyset \vee \neg\ \mathsf{isAlive}(p', \Pi)}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readS}(\theta, e, S, \mathsf{M}_a \cup \mathsf{M}_{p'}){:}h, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(UnregisterF)\ \frac{\theta, e, S \xrightarrow{\mathsf{unregister}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readF}(\theta, e, S, a, \mathsf{M}^a){:}h, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(SendS)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(a,v)} \theta', e', S' \qquad \lambda \text{ fresh} \qquad \mathsf{M}_a = \{\langle a, p', t, \top\rangle\}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, \mathsf{sendS}(\theta, e, S, \{v, \lambda\}, \mathsf{M}_a){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

$$(SendF)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(a,v)} \theta', e', S' \qquad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readF}(\theta, e, S, a, \mathsf{M}^a){:}h, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(Whereis1)\ \frac{\theta, e, S \xrightarrow{\mathsf{whereis}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \{\langle a, p', t, \top\rangle\}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readS}(\theta, e, S, \mathsf{M}_a){:}h, \theta', e'\{\kappa \to p'\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Whereis2)\ \frac{\theta, e, S \xrightarrow{\mathsf{whereis}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readF}(\theta, e, S, a, \mathsf{M}^a){:}h, \theta', e'\{\kappa \to \mathsf{undefined}\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Registered)\ \frac{\theta, e, S \xrightarrow{\mathsf{registered}(\kappa)} \theta', e', S' \qquad \mathsf{registered}(\mathsf{M}) = atoms}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readM}(\theta, e, S, \mathsf{M}){:}h, \theta', e'\{\kappa \to atoms\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(End)\ \frac{e \text{ is a value } \vee\, e = \epsilon \qquad \mathsf{M}_p = \emptyset}{\Gamma; \langle p, h, \theta, e, [\,]\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{readF}(\theta, e, [\,], p, \mathsf{M}^p){:}h, \theta, \bot, [\,]\rangle \mid \Pi; \mathsf{M}}$$

Fig. 5. Forward reversible semantics (standard semantics by dropping the red part).

This requires to understand when all the consequences of an action have been undone, and to restore the state prior to its execution. On the other hand, we need to avoid storing information allowing one to distinguish computations obtained by only swapping independent actions (this would invalidate Lemma 2, as discussed in Example 3).

We first extend the definition of system.

Definition 4 (System). *A system is a tuple* $\Gamma; \Pi; \mathsf{M}$. *Γ and Π are as in Def. 3. Now each element of* M *is a quadruple* $\langle a, p, t, s\rangle$ *where a and p are as in Def. 3, t is a unique identifier for the tuple and s can be either $\top$ or $\bot$.*

Unique identifiers t are used to distinguish identical tuples existing at different times. For example, if we have two successful pairs of register and unregister operations of the same tuple, without a unique identifier we would not know which unregister operation is connected to which register. This information is relevant since the tuple generates a causal link between a register and the corresponding unregister. This justification is similar to the one for unique identifiers λ for messages, discussed in [16].

Tuples whose last field is $\top$ match the ones in the standard semantics, we call them *alive* tuples. Those with $\bot$ are *ghost* tuples, namely alive tuples that have been removed from the map in a past forward action. We will discuss their need in Example 2.

Given an atom a, M^a is the set $\{\langle a, p, t, \bot\rangle | \langle a, p, t, \bot\rangle \in \mathsf{M}\}$; similarly, given a pid p, $\mathsf{M}^p = \{\langle a, p, t, \bot\rangle | \langle a, p, t, \bot\rangle \in \mathsf{M}\}$. Dually, from now on, sets M_a and M_p include only alive tuples. We define function kill, which takes a map and sets to $\bot$ the last field of all its tuples.

We describe below the forward and backward semantics of the imperative primitives. The semantics of other constructs is as in the original work [21], but for the introduction of the global map M, and can be found in Appendix A.3.

The forward semantics is defined in Fig. 5. The following history items have been added to describe the imperative features: regS, readS, readF, sendS, readM, and del. Notably, readS is created by both rules *RegisterF* and *Whereis*1 (which both read some alive tuples), readF is created by rules *UnregisterF*, *SendF*, *Whereis*2 and *End* (which all require the absence of some alive tuple), del is created by both rules *UnregisterS* and *EndUn* (which both turn an alive a tuple $\langle \_, \_, \_, \top\rangle$ into a ghost $\langle \_, \_, \_, \bot\rangle$).

All the new history items, like the old ones, carry the old state θ, e, S, thus allowing the backward computation to restore it. Furthermore, they carry some additional information to enable us to understand their causal dependencies:

- regS carries the tuple inserted in the map;
- readS carries the read tuple(s);
- sendS carries the read tuple as well, but also the sent message;
- readF carries the atom or the pid which the rule tried to read and the ghost tuples for such atom or pid, if any;
- readM carries the whole map read by the rule;
- del carries the removed tuple and the ghost tuples on the same atom or pid.

Fig. 6 presents the backward semantics. In previous works [6,21,16] there is one backward rule for each forward rule. Here, we were able to define one backward rule for each kind of history item, thus some backward rule covers more than one forward rule. This is possible because the history item contains enough information to correctly reverse forward rules with similar effects. E.g., both rules *RegisterF* and *Whereis*1 read information from the map, and the

$$(\overline{RegisterS})\quad \Gamma;\langle p,\mathsf{regS}(\theta,e,S,\{\langle a,p',t,\top\rangle\}){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M}\cup\{\langle a,p',t,\top\rangle\} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M} \quad \text{if } \mathsf{readop}(t,\Pi)=\emptyset$$

$$(\overline{Del})\quad \Gamma;\langle p,\mathsf{del}(\theta,e,S,\{\langle a,p',t,\top\rangle\},\mathsf{M}_1){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M}\cup\{\langle a,p',t,\bot\rangle\} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M}\cup\{\langle a,p',t,\top\rangle\} \quad \text{if } \mathsf{M}_a=\emptyset\wedge\mathsf{M}_{p'}=\emptyset\wedge\mathsf{readmap}(\mathsf{M}\cup\{\langle a,p',t,\bot\rangle\},\Pi)=\emptyset\wedge\mathsf{readfail}(t,\Pi)=\emptyset\wedge\mathsf{M}_1=\mathsf{M}^a\cup\mathsf{M}^{p'}$$

$$(\overline{ReadS})\quad \Gamma;\langle p,\mathsf{readS}(\theta,e,S,\mathsf{M}_1){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M} \quad \text{if } \mathsf{M}_1\subseteq\mathsf{M}$$

$$(\overline{SendS})\quad \Gamma\cup\{(p,p',\{v,\lambda\})\};\langle p,\mathsf{sendS}(\theta,e,S,\{v,\lambda\},\mathsf{M}_1){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M} \quad \text{if } \mathsf{M}_1\subseteq\mathsf{M}$$

$$(\overline{ReadF})\quad \Gamma;\langle p,\mathsf{readF}(\theta,e,S,\iota,\mathsf{M}_1){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M} \quad \text{if } \mathsf{M}_\iota=\emptyset\wedge\mathsf{M}_1=\mathsf{M}^\iota$$

$$(\overline{ReadM})\quad \Gamma;\langle p,\mathsf{readM}(\theta,e,S,\mathsf{M}_1){:}h,\theta',e',S'\rangle \mid \Pi;\mathsf{M} \leftharpoonup \Gamma;\langle p,h,\theta,e,S\rangle \mid \Pi;\mathsf{M} \quad \text{if } \mathsf{M}_1=\mathsf{M}$$

Fig. 6. Backward reversible semantics.

history item tracks the read information. Hence, a single rule can exploit this information to check that the same read information is still available in the map.

Rule $\overline{RegisterS}$ undoes the corresponding forward action, removing the element that was added by it. To this end, rule $\overline{RegisterS}$ requires that the element added from the corresponding forward rule is still in the map (ensuring that possible deletions of the same tuple have been undone) and, as a side condition, that no process performed a read operation on a tuple with unique identifier t. This last condition is checked by the predicate $\mathsf{readop}(t,\Pi)$, which scans the histories of processes in Π looking for such reads.

Rule $\overline{Del}$ undoes either rule $UnregisterS$ or rule $EndUn$, turning a ghost tuple back into an alive one. Let us discuss its side conditions. The first two conditions require that in M there is no alive tuple on the same atom a or process p'. The third one ensures that no process performed a registered getting M, while the fourth that no process read a ghost tuple with identifier t. Finally, we require ghost tuples on both a and p' to be the same as when the corresponding forward action has been performed. The last condition ensures that rule $\overline{Del}$ will not commute with pairs of operations that add and then delete tuples on the same atom or pid, e.g., a pair register-unregister. This is needed to satisfy the properties described in Section 3.2, such as causal consistency.

Rule $\overline{ReadS}$ reverses rules $Whereis1$ and $RegisterF$. The only side conditions requires that the element(s) read by the forward rule must be alive. Rule $\overline{SendS}$ is analogous, but it also requires that the sent message is in Γ.

Rule $\overline{ReadF}$ undoes actions from rules $UnregisterF$, $SendF$, $Whereis2$, and End. As a side condition, we require that no alive tuple matching ι - which is either a pid or an atom - exists and that the ghost tuples related to ι are the same as when the corresponding forward action triggered.

Rule $\overline{ReadM}$ is used to undo rule $Registered$. It requires that the map M_1 stored in the history is exactly the current map M.

3.2 Properties

Here we discuss some properties of the reversible semantics introduced in the previous section. Since most of the properties are related to causality, we need to study the concurrency model of the imperative primitives. Notably, this is not specific to reversibility and the same notion can be useful in other contexts, e.g., to find races [9].

To study concurrency for the imperative primitives we define for each history item k the set of resources (atoms and pids) read or written by the corresponding transition. The idea is that two transitions (including at least a forward one) are in conflict on the map if they both access the same resource and at least one of the accesses is a write ($RegisterS$, $UnregisterS$, $EndUn$). To obtain k, we indicate with $t = (s \rightleftharpoons_{p,r,k} s')$ a (forward or backward) transition from system s to system s', where p is the pid of the process performing the action, r is the applied rule and k the item added or removed to/from the history. We call computation a sequence of consecutive transitions, and denote with ϵ the empty computation. Two transitions are co-initial if they start from the same state, co-final if they end in the same state.

Definition 5 (Resources read or written). *We define functions* read(k) *and* write(k) *as follows:*

k	read(k)	write(k)
$\mathsf{regS}(\theta, \mathsf{e}, \mathsf{S}, \{\langle \mathsf{a}, \mathsf{p}, \mathsf{t}, \top\rangle\})$	$\emptyset$	$\{a, p\}$
$\mathsf{del}(\theta, \mathsf{e}, \mathsf{S}, \{\langle \mathsf{a}, \mathsf{p}, \mathsf{t}, \top\rangle\}, \mathsf{M})$	$\emptyset$	$\{a, p\}$
$\mathsf{readS}(\theta, \mathsf{e}, \mathsf{S}, \mathsf{M})$	$\{a \mid \mathsf{M}_a \neq \emptyset\} \cup \{p \mid \mathsf{M}_p \neq \emptyset\}$	$\emptyset$
$\mathsf{sendS}(\theta, \mathsf{e}, \mathsf{S}, \{\mathsf{v}, \lambda\}, \{\langle \mathsf{a}, \mathsf{p}, \mathsf{t}, \top\rangle\})$	$\{a, p\}$	$\emptyset$
$\mathsf{readF}(\theta, \mathsf{e}, \mathsf{S}, \iota, \mathsf{M})$	$\{\iota\}$	$\emptyset$
$\mathsf{readM}(\theta, \mathsf{e}, \mathsf{S}, \mathsf{M})$	$\{a \mid a \textit{ is an atom}\}$	$\emptyset$

Intuitively, items regS and del write on the resources a and p of the tuple added or removed. Item readS reads one or two tuples, and accesses in read modality all the involved pids and atoms. Item sendS just reads the atom and pid of the accessed tuple. Item readF accesses in read modality either an atom or a pid, as tracked in the history item. Finally, item readM exactly stores the current map, and needs to be in conflict with any transition writing on the map, even if it writes a tuple with atom and pid not previously used. Hence, we have chosen as read resources the set of all possible atoms, independently on whether they are currently used or not. We could also store all possible pids, but this will not impact the semantics, since each write access touches on an atom.

Definition 6 (Concurrent transitions). *Two co-initial transitions,* $t_1 = (s \rightleftharpoons_{p_1,r_1,k_1} s_1)$ *and* $t_2 = (s \rightleftharpoons_{p_2,r_2,k_2} s_2)$*, are* in conflict *if one of these conditions hold:*

- *if no transition is on the map, we refer to [16, Definition 12];*
- *if exactly one transition is on the map, they are in conflict if they are taken by the same process, namely* $p_1 = p_2$*, and a* $\overline{SendS}$ *is in conflict with a receive of the same message;*

– *if both transitions are on the map, and at least one is forward, then they are in conflict iff* $\mathrm{read}(k_1) \cap \mathrm{write}(k_2) \neq \emptyset$, $\mathrm{read}(k_2) \cap \mathrm{write}(k_1) \neq \emptyset$ *or* $\mathrm{write}(k_1) \cap \mathrm{write}(k_2) \neq \emptyset$*;*

Two co-initial transitions are concurrent if they are not in conflict.

Intuitively, concurrent transitions can be executed in any order (we will formalize this in Lemma 2). Notably, co-initial backward transitions are never in conflict.

Example 1 (Conflicting register*).* Consider a system S where two processes, say p_1 and p_2, try to register two different pids under the same atom a, and a is not already present in M (recall that an atom can be associated to one pid only). In this scenario the order in which the two actions are performed matters, because the first process to perform the action succeeds, while the second is doomed to fail. The two possibilities lead us to two states of the system, one where p_1 has succeeded and p_2 failed, say S', and the other where p_2 succeeded and p_1 failed, say S''. Clearly $S' \neq S''$, hence the two operations are in conflict. Indeed, $\mathrm{write}(k_1) \cap \mathrm{write}(k_2) = \{a\} \neq \emptyset$. ◊

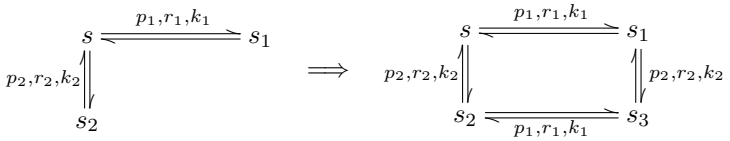
Example 2 (Register followed by delete). Consider a system S where a process, say p_1, can do a registered operation. Another process, say p_2, performs a (successful) register followed by a delete operation (e.g., unregister) of a same tuple. In the standard semantics, executing first p_1 and then p_2 or vice versa would lead to the same state. If we were not using ghost tuples, the histories of p_1 and p_2 would be the same as well. However, we want to distinguish these two computations, since undoing the unregister would change the result of the registered, hence they cannot commute (cfr. Lemma 2). Ghost tuples are our solution to this problem. We get a similar behavior also if we consider, instead of the registered operation, any other read operation involving the added tuple. ◊

We can now discuss some relevant properties of the reversible semantics. As standard (see, e.g., [16] and the notion of consistency in [14]) we restrict to reachable systems, namely systems obtained from a single process with empty history (and empty Γ and M) via some computation. First, each transition can be undone.

Lemma 1 (Loop Lemma). *For every pair of reachable systems, s_1 and s_2, we have $s_1 \rightharpoonup s_2$ iff $s_2 \rightharpoonup s_1$.*

Let us denote with $\underline{t}$ the transitions undoing t, which exists thanks to the Loop Lemma. Next lemma shows that concurrent transitions can be executed in any order. It can be seen as a safety check on the notion of concurrency.

Lemma 2 (Square lemma). *Given two co-initial concurrent transitions $t_1 = (s \rightleftharpoons_{p_1,r_1,k_1} s_1)$ and $t_2 = (s \rightleftharpoons_{p_2,r_2,k_2} s_2)$, there exist two transitions $t_2/t_1 = (s_1 \rightleftharpoons_{p_2,r_2,k_2} s_3)$, $t_1/t_2 = (s_2 \rightleftharpoons_{p_1,r_1,k_1} s_3)$. Graphically:*



Next example shows that in order to ensure that the Square Lemma holds the semantics needs to be carefully crafted, in particular one should avoid to store information allowing to distinguish the order of execution of concurrent transitions.

Example 3 (Information carried by the register *history item).* If the history item of the register would contain the whole map, it would be impossible to swap the register action with an unregister action even if on a tuple with different pid and atom, because of the Square Lemma (Lemma 2). Indeed, the Square Lemma requires to reach the same state after two concurrent transitions are executed, regardless of their order. If we save the whole map in the history item of the register, we would reach two different states:

- if we execute the register operation first, the saved map would include the tuple that the unregister operation will delete;
- if we execute the unregister operation first, the map saved by the register will not contain the deleted tuple. ◊

We now want to prove causal-consistency [4,17], which essentially states that we store the correct amount of causal and history information.

Definition 7 (Causal Equivalence). *Let $\asymp$ be the smallest equivalence on computations closed under composition and satisfying:*

1. *if $t_1 = (s \rightleftharpoons_{p_1,r_1,k_1} s_1)$ and $t_2 = (s \rightleftharpoons_{p_2,r_2,k_2} s_2)$ are concurrent and $t_3 = (s_1 \rightleftharpoons_{p_2,r_2,k_2} s_3)$, $t_4 = (s_2 \rightleftharpoons_{p_1,r_1,k_1} s_3)$ then $t_1 t_3 \asymp t_2 t_4$;*
2. *$t\underline{t} \asymp \epsilon$ and $\underline{t}t \asymp \epsilon$*

Intuitively, computations are causal equivalent if they differ only for swapping concurrent transitions and for adding do-undo or undo-redo pairs of transitions.

Definition 8 (Causal Consistency). *Two co-initial computations are co-final iff they are causal equivalent.*

Intuitively, if co-initial computations are co-final then they have the same causal information and can reverse in the same ways: we want computations to reverse in the same ways iff they are causal equivalent.

In order to prove causal consistency, we rely on the theory developed in [17]. It considers a transition system with forward and backward transitions which satisfies the Loop Lemma and has a notion of independence. The latter is concurrency in our case. The theory allows one to reduce the proof of causal consistency and of other relevant properties to the validity of five axioms: Square Property (SP), Backward Transitions are Independent (BTI), Well-Foundedness (WF), Co-initial Propagation of Independence (CPI) and Co-initial Independence Respects Event (CIRE). SP is proved in Lemma 2, BTI corresponds to the observation that two backward transitions are always concurrent (see Def. 6), and WF requires backward computations to be finite. WF holds since each backward transition consumes an history item, which are in a finite number. CPI and

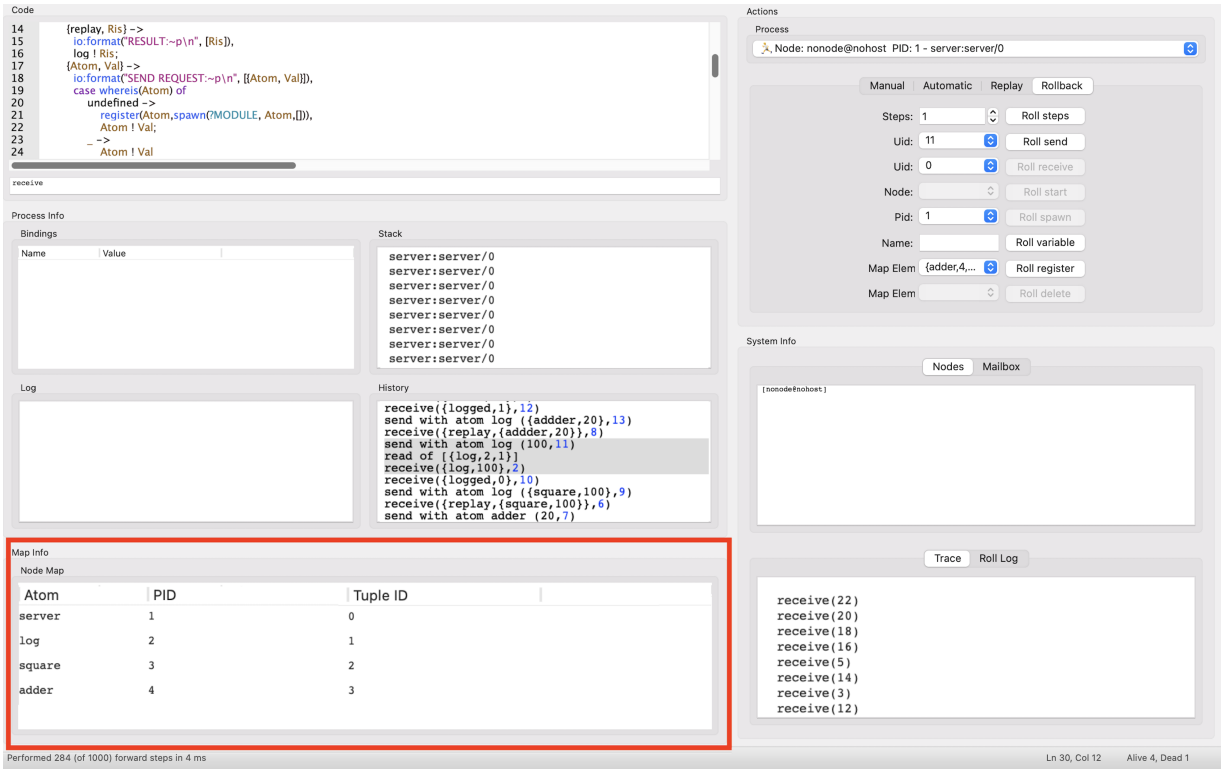


Fig. 7. A screenshot of CauDEr.

CIRE hold thanks to [17, Prop. 5.4] because the notion of concurrency is defined in terms of transition labels only. Hence, causal consistency follows from [17, Prop. 3.6]. We obtain as well a number of other properties (a list can be found in [17, Table 1]), including various forms of causal safety and causal liveness, that intuitively say that a transition can be undone iff its consequences have been undone. We refer to [17] for precise definitions and further discussion.

4 CauDEr with Imperative Primitives

We exploited the theory presented above to extend CauDEr [2,15,21,6], a Causal-consistent reversible Debugger for Erlang. CauDEr is written in Erlang and provides a graphical interface for user interaction. Previously CauDEr supported only the sequential, concurrent and distributed fragment of Erlang, and we added support for the imperative primitives. The updated code can be found at [3].

While the theory discussed so far does not consider distribution, we extended the version of CauDEr supporting distribution [6], where systems can be composed of multiple nodes. As far as the imperative primitives are concerned, the

only difference is that each node has its own map, shared only among its processes.

Fig. 7 shows a snapshot of the new version of CauDEr. The interface is organized as follows. On the left, from top to bottom, we can see (i) the program under debugging, (ii) the state, history and log (log is not discussed in this paper) of the selected process, (iii) the map of the node of the current process (the main novelty in the interface due to our extension, highlighted with a red square). On the top-right we can find execution controls (they are divided in multiple tabs, here we see the tab about rollback, described below), and on the bottom-right information on the system structure and on the execution.

CauDEr works as follows: the user selects the Erlang source file, then CauDEr loads the program and shows the source code to the user. Then, the user can choose the function that will act as an entry point, specify its arguments, and select the identifier of the node where the first process should run. The user can perform single steps on some process (both forward and backward), n steps in the chosen direction in automatic (a scheduler decides which process will execute each step), or use the rollback operator.

The rollback operator allows one to undo a selected action (e.g., the send of a given message) far in the past, including all and only its consequences. This is convenient to look for a bug causing a visible misbehavior, as described below. The semantics of the rollback operator roughly explores the graph of consequences of the target action, and undoes them in a causal-consistent order using the backward semantics. The rollback operator is formalized in Appendix A.5.

Case study. We consider as a case study a simple server dispatching requests to various mathematical services, and logging the results of the evaluation on a logger. Services can be stateful, and are spawned only when there is a first request for them. Our example includes two stateless services, computing the square and the logarithm, respectively, and a stateful service adding all the numbers it receives. The logger keeps track of the values it receives, and answers each request with the sequential number of the element in the log. The code of our case study is depicted in Fig. 16 in Appendix, and is also available in the repository [3].

In our sample scenario, we invoke the program with the list of requests [{square, 10}, {adder, 20}, {log, 100}, {adder, 30}, {adder, 100}].

The two first requests are successfully answered, while the request to compute the logarithm of 100 is not. By checking the history of the server (this is exactly the one shown in Fig. 7, relevant items are grayed, most recent items are on top) we notice that the request has been sent by the server as message 11. By using CauDEr rollback facilities to undo the send of message 11 (including all and only its consequences), one notice that the send has been performed at line 24 (also visible in the screenshot, upon rollback the line becomes highlighted), which is used for already spawned services. This is wrong since this is the first request for a logarithm. One can now require to rollback the register of atom log (used as target of the send). We can now see that the system logger has been registered under this atom in the main function:

register (log,spawn(?MODULE, logger,[0,[]])),
This is wrong. The bug is that the same atom has been used both for the system logger and for logarithm service.

Finding such a bug without the support of reversibility, and rollback in particular, would not be easy. Also, rollback allows us to go directly to points of interest (e.g., where atom log has been registered), even if we do not know which process performed the action. Hence, debugging via rollback scales better than standard techniques to larger programs, where finding the bug without reversibility would be even more difficult.

5 Conclusion, Related and Future Work

We have extended CauDEr and the underlying reversible semantics of Erlang to support imperative primitives used to associate names to pids. This required to distinguish write accesses from read accesses to the map, since the latter commute while the former do not. Also, the interplay between delete and read operations required us to keep track of removed tuples. Notably, a similar approach needs to be used to define the reversible semantics of imperative languages, such as C or Java.

While we discussed most related work, in particular work on reversibility in Erlang, in the Introduction, we mention here some related approaches. Indeed, reversibility of imperative languages with concurrency has been considered, e.g., in [11]. There however actions are undone (mostly) in reverse order of completion, hence their approach does not fit causal-consistent reversibility. Generation of reversible code is also studied in the area of parallel simulation, see, e.g., [20], but there reversed code is sequential, and concurrency is added on top of it by the simulation algorithm. Also, this thread of research lacks theoretical results.

The current approach, as well as the theory in [17] on which we rely to prove properties, defines independence as a binary relation on transitions. We plan to extend this approach in future work by defining independence as a binary relation on sequences of transitions, since we found cases where single transitions do not commute, while sequences can.

For instance, a registered() does not commute with either register(a, _) or unregister(a), but it can commute with their composition since the set of registered tuples is the same before and after. Notably, covering this case would require to extend the theory in [17] as well.

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A Semantics

In this section we present the rules of the semantics that we could not include in the paper due to space reasons. Explanations are quite terse, we refer to [21] for a deeper discussion.

A.1 Expression level semantics

We divide the rules of the expression level semantics in two sets: the set of sequential expressions, depicted in Fig. 8, and the set of concurrent expressions, depicted in Fig. 9

$$(Var)\ \frac{}{\theta, C[X], S \xrightarrow{\tau} \theta, C[\theta(X)], S}$$

$$(Seq1)\ \frac{}{\theta, C[v, e], S \xrightarrow{\tau} \theta, C[e], S} \quad (Seq2)\ \frac{}{\theta, v, \mathsf{seq}(C[\_]) : S \xrightarrow{\tau} \theta, C[v], S}$$

$$(If)\ \frac{\mathsf{eval\_guard}(g_1\theta, \ldots, g_n\theta) = i}{\theta, C[\mathsf{if}\ g_1\ \rightarrow\ e_1; \ldots; g_n\ \rightarrow\ e_n\ \mathsf{end}], S \xrightarrow{\tau} \theta, e_i, \mathsf{seq}(C[\_]) : S}$$

$$(Case)\ \frac{\mathsf{match\_case}(\theta, v, cl_1, \ldots, cl_n) = \langle \theta_i, e_i \rangle}{\theta, C[\mathsf{case}\ v\ \mathsf{of}\ cl_1; \ldots; cl_n\ \mathsf{end}], S \xrightarrow{\tau} \theta\theta_i, e_i, \mathsf{seq}(C[\_]) : S}$$

$$(Match)\ \frac{\mathsf{match}(pat\theta, v) = \sigma}{\theta, C[pat = v], S \xrightarrow{\tau} \theta\sigma, C[v], S}$$

$$(Fun)\ \frac{}{\theta, C[\mathsf{fun} fun_1; \ldots; fun_m \mathsf{end}], S \xrightarrow{\tau} \theta, C[\langle \theta, \mathsf{fun} fun_1; \ldots; fun_m \mathsf{end} \rangle], S}$$

$$(Call1)\ \frac{\mathsf{match\_fun}((v_1, \ldots, v_n), \mathsf{def}(f/n, \mathsf{P})) = (\sigma, e)}{\theta, C[f(v_1, \ldots, v_n)], S \xrightarrow{\tau} \sigma, e, (\theta, C[\_]) : S}$$

$$(Call2)\ \frac{\mathsf{match\_fun}((v_1, \ldots, v_n), \langle \theta', \mathsf{fun} fun_1; \ldots; fun_m \mathsf{end} \rangle) = (\sigma, e)}{\theta, C[\langle \theta', \mathsf{fun} fun_1; \ldots; fun_m \mathsf{end} \rangle (v_1, \ldots, v_n)], S \xrightarrow{\tau} \sigma, e, (\theta, C[\_]) : S}$$

$$(Return)\ \frac{}{\sigma, v, (\theta, C[\_]) : S \xrightarrow{\tau} \theta, C[v], S}$$

Fig. 8. Standard semantics: evaluation of sequential expressions.

The sequential expressions (in Fig. 8) define the behavior of some constructs of the language without side-effects, like the case construct or the call of a function; these rules define also the evaluation of an expression inside the data structures of the language. We label the evaluation of sequential expressions with τ since we do not need to distinguish them in the system semantics.

$$(Send)\ \frac{}{\theta, C[v_1\,!\,v_2], S \xrightarrow{\mathsf{send}(v_1,v_2)} \theta, C[v_2], S}$$

$$(Receive)\ \frac{}{\theta, C[\mathsf{receive}\ cl_1; \ldots; cl_n\ \mathsf{end}], S \xrightarrow{\mathsf{rec}(\kappa,\overline{cl_n})} \theta, \kappa, \mathsf{seq}(C[\_]) : S}$$

Fig. 9. Concurrent semantics: evaluation of concurrent expressions.

Fig. 9 shows the semantics of concurrent expressions. Rule *Send*, used to send a message to a process, reduces an expression $v_1!v_2$ to v_2. The side-effect is that the message v_2 is sent, i.e., the message is added to Γ. We label the step with $send(v_1, v_2)$ in this way the system rule *Send* can add the message to the global mailbox Γ. Rule *Receive*, used to receive a message, returns a fresh variable, κ, since the receive expression cannot be reduced at this level without accessing to the global mailbox. Here, κ can be seen as a placeholder that the system rules will replace with the correct value. Like before, we label the step with enough information for rule *Receive* to complete the reduction.

A.2 System level semantics

Fig. 10 contains the same rules as Fig. 5, without all the red parts referring to history and causal information. In Fig. 11 we show the rules of the system semantics not related to the map, hence not depicted in Fig. 5 and in Fig. 10. Let us describe rule *Send* as an example. We apply rule *Send* when a process performs a send and, as side-effect, we update Π by adding the triple (p, p', m), where p is the pid of the sender, p' of the receiver and m is the message.

Rule *Receive* searches in the queue of messages, from the oldest to the newest, the first message that matches one of the n clauses (thanks to the auxiliary function matchrec). Then, the system semantics updates the process' environment with the new variables introduced by the selected branch, replaces κ with the corresponding expression, and removes v from the process' queue.

Now, let us discuss more in detail rule *Spawn*; this rule evaluates the spawn expression, it chooses a fresh identifier p as the pid of the new process, replaces κ with p and finally adds to Π the new process.

Finally, rule *Seq* is used for the expression without side-effects while rule *Self* is used for the self function, which evaluates to the current process pid.

A.3 A reversible semantics

The forward reversible semantics, showed in Fig. 12 and defined by the relation $\rightharpoonup$, is the extension of the system semantics where each process has been updated with a history of its previous configurations. More precisely, when a process performs a forward step, the current configuration and if necessary additional pieces of information are saved in the history and the process evolves in a new state. The history is needed - while going backward - to check that all the consequences of an action, if any, have been undone.

$$(RegisterS)\ \frac{\theta, e, S \xrightarrow{\mathsf{register}(\kappa,a,p')} \theta', e', S' \qquad \mathsf{M}_a = \emptyset \qquad \mathsf{M}_{p'} = \emptyset \qquad \mathsf{isAlive}(p', \Pi)}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta', e'\{\kappa \to \mathsf{true}\}, S'\rangle \mid \Pi; \mathsf{M} \cup \{\langle a, p'\rangle\}}$$

$$(UnregisterS)\ \frac{\theta, e, S \xrightarrow{\mathsf{unregister}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \{\langle a, p'\rangle\}}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta', e'\{\kappa \to \mathsf{true}\}, S'\rangle \mid \Pi; \mathsf{M} \setminus \mathsf{M}_a}$$

$$(EndUn)\ \frac{e \text{ is a value } \vee\, e = \epsilon \qquad \mathsf{M}_p = \{\langle a, p\rangle\}}{\Gamma; \langle p, \theta, e, [\,]\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta, \bot, [\,]\rangle \mid \Pi; \mathsf{M} \setminus \mathsf{M}_p}$$

$$(RegisterF)\ \frac{\theta, e, S \xrightarrow{\mathsf{register}(\kappa,a,p')} \theta', e', S' \qquad \mathsf{M}_a \neq \emptyset \vee \mathsf{M}_{p'} \neq \emptyset \vee \neg\ \mathsf{isAlive}(p', \Pi)}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(UnregisterF)\ \frac{\theta, e, S \xrightarrow{\mathsf{unregister}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(SendS)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(a,v)} \theta', e', S' \qquad \mathsf{M}_a = \{\langle a, p'\rangle\}}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma \cup \{(p, p', \{v\})\}; \langle p, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

$$(SendF)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(a,v)} \theta', e', S' \quad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta, \epsilon, [\,]\rangle \mid \Pi; \mathsf{M}}$$

$$(Whereis1)\ \frac{\theta, e, S \xrightarrow{\mathsf{whereis}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \{\langle a, p'\rangle\}}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta', e'\{\kappa \to p'\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Whereis2)\ \frac{\theta, e, S \xrightarrow{\mathsf{whereis}(\kappa,a)} \theta', e', S' \qquad \mathsf{M}_a = \emptyset}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta', e'\{\kappa \to \mathsf{undefined}\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Registered)\ \frac{\theta, e, S \xrightarrow{\mathsf{registered}(\kappa)} \theta', e', S' \qquad \mathsf{registered}(\mathsf{M}) = atoms}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta', e'\{\kappa \to atoms\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(End)\ \frac{e \text{ is a value } \vee\, e = \epsilon \qquad \mathsf{M}_p = \emptyset}{\Gamma; \langle p, \theta, e, [\,]\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \theta, \bot, [\,]\rangle \mid \Pi; \mathsf{M}}$$

Fig. 10. Standard semantics: read and write system rules.

Fig. 13 depicts the uncontrolled backward semantics. The backward semantics restores previous states of a process' computation if all of the consequences of the target action have been undone. Rule $\overline{Receive}$ puts back the received message in the set of sent message (Γ) and can always be applied. Rule $\overline{Send}$ can be applied when the message is in Γ because in this case we are sure that each of its consequences has been undone already. Rule $\overline{Spawn}$ can be applied when the child has an empty history. Finally, rules $\overline{Seq}$ and $\overline{Self}$ can always be applied since they never have consequences.

$$(Seq)\ \frac{\theta, e, S \xrightarrow{\tau} \theta', e', S'}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \hookrightarrow \Gamma; \langle p, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Receive)\ \frac{\theta, e, S \xrightarrow{\mathsf{rec}(\kappa, \overline{cl_n})} \theta', e', S' \quad \mathsf{matchrec}(\theta, \overline{cl_n}, v) = (\theta_i, e_i)}{\Gamma \cup \{(p, p', v)\}; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \hookrightarrow \Gamma; \langle p, (\theta'\theta_i, e'\{\kappa \mapsto e_i\}), S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Spawn)\ \frac{\theta, e, S \xrightarrow{\mathsf{spawn}(\kappa, exprs)} \theta', e', S' \quad p' \text{ is a fresh pid}}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \hookrightarrow \Gamma; \langle p, \theta', e'\{\kappa \mapsto p'\}, S'\rangle \mid \langle p', id, exprs, ()\rangle \mid \Pi; \mathsf{M}}$$

$$(Self)\ \frac{\theta, e, S \xrightarrow{\mathsf{self}(\kappa)} \theta', e', S'}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \hookrightarrow \Gamma; \langle p, \theta', e'\{\kappa \mapsto p\}, S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Send)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(p', v)} \theta', e', S'}{\Gamma; \langle p, \theta, e, S\rangle \mid \Pi; \mathsf{M} \hookrightarrow \Gamma \cup \{(p, p', v)\}; \langle p, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

Fig. 11. Standard semantics: system rules.

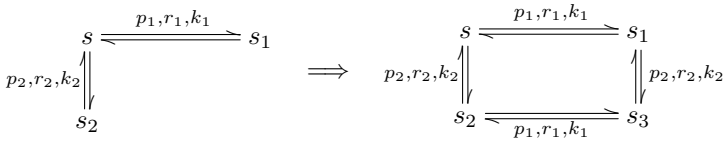
A.4 Proofs of reversibility properties.

Lemma 1 (Loop Lemma). *For every pair of reachable systems, s_1 and s_2, we have $s_1 \rightharpoonup s_2$ iff $s_2 \leftharpoonup s_1$.*

Proof. The proof that a forward transition can be undone follows by rule inspection. The other direction relies on the restriction to reachable systems: consider the process undoing the action. Since the system is reachable, restoring the memory item would put us back in a state where the undone action can be performed again (if the system would not be reachable the memory item would be arbitrary, hence there would not be such a guarantee), as desired. Again, this can be proved by rule inspection. □

We indicate with $t = (s \rightleftharpoons_{p,r,k} s')$ the transition from system s to system s', where p is the pid of the process performing the action, r is the rule applied to perform the transition and k is the item added or removed to/from the history by the applied rule.

Lemma 2 (Square lemma). *Given two co-initial concurrent transitions $t_1 = (s \rightleftharpoons_{p_1,r_1,k_1} s_1)$ and $t_2 = (s \rightleftharpoons_{p_2,r_2,k_2} s_2)$, there exist two transitions $t_2/t_1 = (s_1 \rightleftharpoons_{p_2,r_2,k_2} s_3)$, $t_1/t_2 = (s_2 \rightleftharpoons_{p_1,r_1,k_1} s_3)$. Graphically:*



Proof. We distinguish the following cases depending on the applied rules:

$$(Seq)\ \frac{\theta, e, S \xrightarrow{\tau} \theta', e', S'}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \tau(\theta, e, S){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Receive)\ \frac{\theta, e, S \xrightarrow{\mathsf{rec}(\kappa, \overline{cl_n})} \theta', e', S' \quad \mathsf{matchrec}(\overline{cl_n}\theta, v) = (\theta_i, e_i)}{\Gamma \cup (\{p, p', \{v, \lambda\}\}); \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{rec}(\theta, e, S, p', \{v, \lambda\}){:}h, (\theta'\theta_i, e'\{\kappa \mapsto e_i\}), S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Spawn)\ \frac{\theta, e, S \xrightarrow{\mathsf{spawn}(\kappa, exprs)} \theta', e', S' \quad p' \text{ is a fresh pid}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{spawn}(\theta, e, S, p'){:}h, (\theta', e'\{\kappa \mapsto p'\}), S'\rangle \mid \langle p', (), (id, exprs), ()\rangle \mid \Pi; \mathsf{M}}$$

$$(Self)\ \frac{\theta, e, S \xrightarrow{\mathsf{self}(\kappa)} \theta', e', S'}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma; \langle p, \mathsf{self}(\theta, e, S){:}h, (\theta', e'\{\kappa \mapsto p\}), S'\rangle \mid \Pi; \mathsf{M}}$$

$$(Send)\ \frac{\theta, e, S \xrightarrow{\mathsf{send}(p', v)} \theta', e', S' \quad \lambda \text{ is a fresh identifier}}{\Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M} \rightharpoonup \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, \mathsf{send}(\theta, e, S, \{v, \lambda\}){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M}}$$

Fig. 12. Forward reversible semantics.

$$(\overline{Seq})\ \ \Gamma; \langle p, \tau(\theta, e, S){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M} \leftharpoonup \Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M}$$

$$(\overline{Receive})\ \ \Gamma; \langle p, \mathsf{rec}(\theta, e, S, p', \{v, \lambda\}){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M} \leftharpoonup \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M}$$

$$(\overline{Spawn})\ \ \Gamma; \langle p, \mathsf{spawn}(\theta, e, S, p'){:}h, \theta', e', S'\rangle \mid \langle p', (), (id, e''), ()\rangle \mid \Pi; \mathsf{M} \leftharpoonup \Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M}$$

$$(\overline{Self})\ \ \Gamma; \langle p, \mathsf{self}(\theta, e, S){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M} \leftharpoonup \Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M}$$

$$(\overline{Send})\ \ \Gamma \cup \{(p, p', \{v, \lambda\})\}; \langle p, \mathsf{send}(\theta, e, S, \{v, \lambda\}){:}h, \theta', e', S'\rangle \mid \Pi; \mathsf{M} \leftharpoonup \Gamma; \langle p, h, \theta, e, S\rangle \mid \Pi; \mathsf{M}$$

Fig. 13. Backward reversible semantics.

1. Two forward transitions. We have the following cases:
   - the two transitions are not on the map then, we can easily prove that by applying rule r_2 to p_1 in s_1 and rule r_1 to p_2 in s_2 we have two transitions t_1/t_2 and t_2/t_1 which are co-final;
   - one of the two transitions is on the map and the other one is not on the map we can apply the same reasoning here;
   - both transitions are on the map, we have a case analysis on the applied rules. We show a few examples, the others are similar.
     - if $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\theta_1, \mathsf{e}_1, \mathsf{S}_1, \mathsf{M}_1, \mathsf{M})$ and $r_2 \in \{UnregisterS, EndUn\}$, $k_2 = \mathsf{del}(\theta_2, \mathsf{e}_2, \mathsf{S}_2, \mathsf{M}_2, \mathsf{M}')$ we have that both $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ and $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$ are in the system map of s and by Definition 6 that $a_1 \neq a_2$ and $p_1 \neq p_2$. Then applying k_1 and after k_2 or vice-versa leads us to the same state because

       $$(\mathsf{M} \setminus \mathsf{M}_1 \cup \mathsf{kill}(\mathsf{M}_1)) \setminus \mathsf{M}_2 \cup \mathsf{kill}(\mathsf{M}_2) = (\mathsf{M} \setminus \mathsf{M}_2 \cup \mathsf{kill}(\mathsf{M}_2)) \setminus \mathsf{M}_1 \cup \mathsf{kill}(\mathsf{M}_1)$$

The only side effect of the rule is to deactivate of a tuple from the system map.

- if $r_1 = RegisterS$ and $r_2 \in \{UnregisterS, EndUn\}$, $k_2 = \mathsf{del}(\_, \_, \_, \mathsf{M}_1, \mathsf{M}_2)$ we know thanks to rule r_1 that $\langle a_1, p_1, t_1, \top\rangle$ is not in the system map. Thanks to rule r_2 we know that the tuple $\mathsf{M}_1 = \langle a_2, p_2, t_2, \top\rangle$ is in the system map. By Definition 6 we have that $a_1 \neq a_2$, $p_1 \neq p_2$ and so for this reason M_2 is not affected by r_1. We can see that adding the tuple $\langle a_1, p_1, t_1, \top\rangle$ and then deactivating the tuple $\langle a_2, p_2, t_2, \top\rangle$, or vice-versa, leads us to the same state because

  $$(\mathsf{M} \cup \{\langle a_1, p_1, t_1, \top\rangle\}) \backslash \mathsf{M}_1 \cup \mathrm{kill}(M_1) = (\mathsf{M} \backslash \mathsf{M}_1 \cup \mathrm{kill}(M_1)) \cup \{\langle a_1, p_1, t_1, \top\rangle\}$$

  The only side effects of the two rules are respectively an insertion and a deactivation of a tuple from the system map.
- if $r_1 = RegisterS$ and $r_2 \in \{UnregisterF, SendF, WhereIs2, End\}$, $k_2 = \mathsf{readF}(\_, \_, \_, \iota, \mathsf{M}_2)$ we know thanks to rule r_1 that $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ is not in the system map. Thanks to rule r_2 we know that the M_2 is a subset of the system map. By Definition 6 we have that $a_1 \neq \iota \wedge \iota \neq p_1$ so M_2 is not affected by r_1, hence we can see that adding the tuple $\langle a_1, p_1, t_1, \top\rangle$ and then reading a ghost atom, or vice-versa, leads us to the same state.
- if $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\theta_1, \mathsf{e}_1, \mathsf{S}_1, \mathsf{M}_1, \mathsf{M})$ and $r_2 \in \{UnregisterF, SendF, WhereIs2, End\}$, $k_2 = \mathsf{readF}(\_, \_, \_, \iota, \mathsf{M}_2)$ we know thanks to rule r_1 that $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ is in the system map. By Definition 6 we have that $a_1 \neq \iota \wedge \iota \neq p_1$ so M_2 is not affected by r_1, hence we can see that adding the tuple $\langle a_1, p_1, t_1, \top\rangle$ and then reading a ghost atom, or vice-versa, leads us to the same state.
- if $k_1 = \mathsf{del}(\theta_1, \mathsf{e}_1, \mathsf{S}_1, \mathsf{M}_1, \mathsf{M})$ and $k_2 = \mathsf{readS}(\_, \_, \_, \mathsf{M}_2)$ we know thanks to rule r_1 that $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ is in the system map. Thanks to rule r_2 we know that the M_2 is a subset of the system map. By Definition 6 we have that $\mathsf{M}_1 \cap \mathsf{M}_2 = \emptyset$ and so for this reason M_2 is not affected by r_1. We can see that killing the tuple $\langle a_1, p_1, t_1, \top\rangle$ and then reading a tuple (or two), or vice-versa, leads us to the same state.
- if $r_1 = RegisterS$ and $r_2 \in \{RegisterF, WhereIs1\}$, $k_2 = \mathsf{readS}(\_, \_, \_, \mathsf{M}_2)$ we know thanks to rule r_1 that $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ is not in the system map. Thanks to rule r_2 we know that the M_2 is a subset of the system map. By Definition 6 we have that $\langle \_, p_1, \_, \_\rangle \notin \mathsf{M}_2$ and $\langle a_1, \_, \_, \_\rangle \notin \mathsf{M}_2$ and so for this reason M_2 is not affected by r_1. We can see that adding the tuple $\langle a_1, p_1, t_1, \top\rangle$ and then reading a tuple (or two), or vice-versa, leads us to the same state.
- if $r_1 = r_2 = RegisterS$ we know thanks to rule r_1 that $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$ is not in the system map and thanks to rule r_2 that $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$ is not in the system map. By Definition 6 we have that $a_1 \neq a_2$ and $p_1 \neq p_2$. We can see that adding the tuple M_1

and then adding the tuple M_2, or vice-versa, leads us to the same state.

$$(\mathsf{M} \setminus \mathsf{M}_1) \cup \mathsf{M}_2 = (\mathsf{M} \setminus \mathsf{M}_2) \cup \mathsf{M}_1$$

2. One forward transition and one backward transition. We have the following cases:
   - the two operations are on the map then we could have the following cases:
     - $r_1 = RegisterS$, where M_1 in k_1 is $\{\langle a_1, p_1, t_1, \top\rangle\}$, and $r_2 = \overline{Del}$, where $k_2 = \mathsf{del}(\_, \_, \_, \mathsf{M}_2, \_)$ and $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$. By Definition 6, we know $a_1 \neq a_2$ and $p_1 \neq p_2$ (because otherwise they would be in conflict), then the system map contains the tuple $\langle a_2, p_2, t_2, \bot\rangle$ (otherwise it would not possible to apply the rule r_2) and we can see that r_1 and r_2 commute because

$$(\mathsf{M} \cup \mathsf{M}_1) \setminus \mathrm{kill}(\mathsf{M}_2) \cup \mathsf{M}_2 = (\mathsf{M} \setminus \mathrm{kill}(\mathsf{M}_2) \cup \mathsf{M}_2) \cup \mathsf{M}_1$$

     - $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\_, \_, \_, \mathsf{M}_1, \mathsf{M}'_1)$, where $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$, and $r_2 = \overline{RegisterS}$, where $k_2 = \mathsf{regS}(\_, \_, \_, \mathsf{M}_2)$ and $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$. By Definition 6 we have that $a_1 \neq a_2$ and $p_1 \neq p_2$, then the system map contains the tuple $\langle a_1, p_1, t_1, \top\rangle$ and moreover that system map contains $\langle a_2, p_2, t_2, \top\rangle$ as well. Hence we can see that

$$(\mathsf{M} \setminus \mathsf{M}_1 \cup \mathrm{kill}(M_1)) \setminus \mathsf{M}_2 = (\mathsf{M} \setminus \mathsf{M}_2) \setminus \mathsf{M}_1 \cup \mathrm{kill}(M_1)$$

     - $r_1 \in \{UnregisterF, SendF, WhereIs2, End\}$, $k_1 = \mathsf{readF}(\_, \_, \_, \iota, \mathsf{M})$ and $r_2 = \overline{RegisterS}$, where $k_2 = \mathsf{regS}(\_, \_, \_, \mathsf{M}_2)$ and $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$. By Definition 6 we have that $a_2 \neq \iota \wedge \iota \neq p_2$ and so we can see that the two operations commute.
     - $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\_, \_, \_, \mathsf{M}_1, \mathsf{M}'_1)$, where $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$, and $r_1 = \overline{ReadS}$ where $k_2 = \mathsf{readS}(\_, \_, \_, \mathsf{M}_2)$. By Definition 6 we have that $\langle \_, \_, t_1, \_\rangle \notin \mathsf{M}_2$, also we know that r_2 does not affect M'_1 so we can conclude that the two operations commute.
     - $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\_, \_, \_, \mathsf{M}_1, \mathsf{M}'_1)$, where $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$, and $r_1 = \overline{ReadF}$ where $k_2 = \mathsf{readF}(\_, \_, \_, \iota, \mathsf{M}_2)$. By Definition 6 we have that $a_1 \neq \iota \wedge \iota \neq p_1$, so r_1 does not affect M_2, also r_2 does not affect M'_1 since it is a read operation, so we can conclude that the two operations commute.
     - $r_1 \in \{UnregisterS, EndUn\}$, $k_1 = \mathsf{del}(\_, \_, \_, \mathsf{M}_1, \mathsf{M}'_1)$, where $\mathsf{M}_1 = \{\langle a_1, p_1, t_1, \top\rangle\}$, $r_2 = \overline{Del}$ and $k_2 = \mathsf{del}(\_, \_, \_, \mathsf{M}_2, \mathsf{M}'_2)$, where $\mathsf{M}_2 = \{\langle a_2, p_2, t_2, \top\rangle\}$. By Definition 6 we have that $a_1 \neq a_2 \wedge p_1 \neq p_2$, so r_1 does not affect M'_2, and similarly r_2 does not affect M'_1, so we can see that

$$(\mathsf{M} \setminus \mathsf{M}_1 \cup \mathrm{kill}(M_1)) \setminus \mathrm{kill}(\mathsf{M}_2) \cup M_2 = (\mathsf{M} \setminus \mathrm{kill}(\mathsf{M}_2) \cup M_2) \setminus \mathsf{M}_1 \cup \mathrm{kill}(M_1)$$

     - we can apply the same reasoning in the other cases.

  - the claim of the other cases follows easily (see [15, Lemma 13] and [6, Lemma 3.1]);
3. Two backward transitions. The claim follows easily (for more examples see [15, Lemma 13] and [6, Lemma 3.1]):
  - if the rules are co-initial then the side-conditions of both rules are respected and it is equivalent apply before r_1 or r_2, then is easy to see that $t_1/t_2 = t_2/t_1$. □

A.5 Rollback semantics.

Now we introduce a rollback semantics that reverts the system back to a previous state, specified in input by the user, by undoing several steps in an automatic manner.

We denote a system in rollback mode with $[\![\mathcal{S}]\!]_{\{p,\psi\}}$, where we want to start a backward derivation until the action ψ performed by the process p; we want to undo all the actions that depend on it.

More generally, given $[\![\mathcal{S}]\!]_{\Phi}$, then Φ is the sequence of undo requests that need to be satisfied; Φ can be seen as a stack where the first element is the most recent request and once the stack is empty, the system has reached the state desired by the user.

In this semantics we consider requests $\{p, \psi\}$, asking process p to undo a specific action, namely:

- $\{p, s\}$: a single step back;
- $\{p, \lambda^{\Downarrow}\}$: the receive of the message uniquely identified by λ;
- $\{p, \lambda^{\Uparrow}\}$: the send of the message uniquely identified by λ;
- $\{p, sp_{p'}\}$: the spawn of process p'.
- $\{p, \mathsf{regS}(t)\}$: the operation $\mathsf{regS}(\_, \_, \_, \{\langle \_, \_, t, \top\rangle\})$.
- $\{p, \mathsf{del}(t)\}$: the operation $\mathsf{del}(\_, \_, \_, \{\langle \_, \_, t, \top\rangle\}, \_)$.
- $\{p, \mathsf{read}(t)\}$: the operations $\mathsf{readS}(\_, \_, \_, \mathsf{M} \cup \{\_, \_, t, \_\})$, $\mathsf{sendS}(\_, \_, \_, \_, \mathsf{M} \cup \{\_, \_, t, \_\})$ or $\mathsf{readM}(\_, \_, \_, \mathsf{M} \cup \{\langle \_, \_, t, \_\rangle\})$.
- $\{p, \mathsf{notread}(t)\}$: the operations $\mathsf{readF}(\_, \_, \_, \_, \mathsf{M} \cup \{\_, \_, t, \_\})$, or $\mathsf{readM}(\_, \_, \_, \mathsf{M} \cup \{\langle \_, \_, t, \bot\rangle\})$.

Example 4. For example, if we have a request $\{p, \mathsf{regS}(t)\}$ we want to undo the register made by the process with pid p that inserted the tuple $\langle \_, \_, t, \top\rangle$ in the global map. ◊

Fig. 14 depicts the rollback semantics and relation $\rightsquigarrow$ indicates which backward rule shall we apply and when. Rule $U - Satisfy$ performs a single step back using the backward semantics and removes the corresponding request.

Rule $U - Act$ performs a single step back using the backward semantics when the action that we require to undo ($\{p, \psi\}$) is not the most recent action in the process history.

Rule *Request* is fired when in the system there is no backward transition enabled for the process targeted by the first request on Ψ. This means that the action is blocked by some operation in another process and with the help of the

$$(U-Satisfy)\ \frac{\mathcal{S} \leftharpoondown_{p,r,\Psi'} \mathcal{S}' \ \wedge\ \psi \in \Psi'}{[\![\mathcal{S}]\!]_{\{p,\psi\}:\Psi} \rightsquigarrow [\![\mathcal{S}']\!]_{\Psi}} \qquad (U-Act)\ \frac{\mathcal{S} \leftharpoondown_{p,r,\Psi'} \mathcal{S}' \ \wedge\ \{p,r\} \notin \Psi'}{[\![\mathcal{S}]\!]_{\{p,\psi\}:\Psi} \rightsquigarrow [\![\mathcal{S}']\!]_{\{p,\psi\}:\Psi}}$$

$$(Request)\ \frac{\mathcal{S} = \Gamma; \langle p,h,\theta,e,S\rangle \mid \Pi; \mathsf{M} \ \wedge\ \mathcal{S} \not\leftharpoondown_{p,r,\Psi'} \ \wedge \{p',\psi'\} = dep(\langle p,h,\theta,e,S\rangle, \mathcal{S})}{[\![\mathcal{S}]\!]_{\{p,\psi\}:\Psi} \rightsquigarrow [\![\mathcal{S}']\!]_{\{p',\psi'\}:\{p,\psi\}:\Psi}}$$

Fig. 14. Rollback semantics

operator dep (in Fig. 15) the rule computes a new request, aimed at solving the dependency, and pushes it on Ψ.

In Fig. 15, we show the dep operator, where we added the dependencies generated by the imperative primitives. Let us discuss them in detail.

In the first case, a send cannot be undone since the message sent is not in the global mailbox, so a request has to be made to the receiver p' of undoing the receipt of the message identified by λ.

If there are multiple dependencies to solve, we add them one by one. This happens, for example, in the case of the registered primitive, where we need to undo the regS of all the pairs which are accessible in the system map (M') but are not in the map read (M) and of all the pairs which are accessible in the map read but are not accessible in the system map. We also want to undo all the del operations of the pairs that are not in the map read but that are not accessible in the system map.

If we were to add all the dependencies at once, it would be more complex, since by resolving one dependency, we could also resolve some deeper ones; in this way, we would need an additional check to avoid starting a computation to cancel a dependency that no longer exists.

Adding dependencies one by one solves the problem, so the dep operator non-deterministically selects one of them. The order in which dependencies are resolved is not relevant.

$$\begin{array}{lll}
\mathsf{dep}(< _, \mathsf{send}(_,_,_,\{v,\lambda\}):h,_,_,_>, \Gamma \cup \{(p,p',\{v,\lambda\})\}; _; _) & = \{p', \lambda^{\Downarrow}\} & \\
\mathsf{dep}(< p, \mathsf{sendS}(_,_,_,\{v,\lambda\},_):h,_,_,_>, \Gamma \cup \{(p,p',\{v,\lambda\})\}; _; _) & = \{p', \lambda^{\Downarrow}\} & \\
\mathsf{dep}(< _, \mathsf{sendS}(_,_,_,_,\{\langle a,p,t,\top\rangle\}):h,_,_,_>,_;_;\mathsf{M}') & = \{p', \mathsf{del}(t)\} & \text{if } \langle a,p,t,\top\rangle \notin \mathsf{M}' \\
\mathsf{dep}(< _, \mathsf{spawn}(_,_,_,p'):h,_,_,_>,_;\Pi;_) & = \{p', s\} & \text{if } p' \in \Pi \\
\mathsf{dep}(< _, \mathsf{readS}(_,_,_,\mathsf{M} \cup \{\langle a,_,t,\top\rangle\}):h,_,_,_>,_;_;\mathsf{M}') & = \{p', \mathsf{del}(t)\} & \text{if } \langle a,_,t,\top\rangle \notin \mathsf{M}' \\
\mathsf{dep}(< _, \mathsf{readS}(_,_,_,\mathsf{M} \cup \{\langle _,p,t,\top\rangle\}):h,_,_,_>,_;_;\mathsf{M}') & = \{p', \mathsf{del}(t)\} & \text{if } \langle _,p,t,\top\rangle \notin \mathsf{M}' \\
\mathsf{dep}(< _, \mathsf{readF}(_,_,_,a,\mathsf{M}):h,_,_,_>,_;_;\mathsf{M}' \cup \{\langle a,_,t,_\rangle\}) & = \{p', \mathsf{regS}(t)\} & \text{if } \langle a,_,t,_\rangle \notin \mathsf{M} \\
\mathsf{dep}(< _, \mathsf{readM}(_,_,_,\mathsf{M}):h,_,_,_>,_;_;\mathsf{M}' \cup \{\langle _,_,t,\top\rangle\}) & = \{p', \mathsf{regS}(t)\} & \text{if } \langle _,_,t,\top\rangle \notin \mathsf{M} \\
\mathsf{dep}(< _, \mathsf{readM}(_,_,_,\mathsf{M}):h,_,_,_>,_;_;\mathsf{M}' \cup \{\langle _,_,t,\bot\rangle\}) & = \{p', \mathsf{del}(t)\} & \text{if } \langle _,_,t,\bot\rangle \notin \mathsf{M} \\
\mathsf{dep}(< _, \mathsf{regS}(_,_,_,\{\langle _,_,t,\top\rangle\}):h,_,_,_>,_;_;\mathsf{M}') & = \{p', \mathsf{del}(t)\} & \text{if } \langle _,_,t,\top\rangle \notin \mathsf{M}' \\
\mathsf{dep}(< _, \mathsf{regS}(_,_,_,\{\langle _,_,t,\top\rangle\}):h,_,_,_>,_;_;_) & = \{p', \mathsf{read}(t)\} & \\
\mathsf{dep}(< _, \mathsf{del}(_,_,_,\{\langle a,_,t,\top\rangle\},\mathsf{M}):h,_,_,_>,_;_;\mathsf{M}' \cup \{\langle a,_,t_a,_\rangle\}) & = \{p', \mathsf{regS}(t_a)\} & \text{if } \langle a,_,t_a,_\rangle \notin \mathsf{M} \\
\mathsf{dep}(< _, \mathsf{del}(_,_,_,\{\langle _,p,t,\top\rangle\},\mathsf{M}):h,_,_,_>,_;_;\mathsf{M}' \cup \{\langle _,p,t_p,_\rangle\}) & = \{p', \mathsf{regS}(t_p)\} & \text{if } \langle _,p,t_p,_\rangle \notin \mathsf{M} \\
\mathsf{dep}(< _, \mathsf{del}(_,_,_,\{\langle a,p,t,\top\rangle\},_):h,_,_,_>,_;_;\mathsf{M}') & = \{p', \mathsf{readfail}(t)\} &
\end{array}$$

Fig. 15. Dependencies operator

Example 5. For example the case of the del is:

$$\mathsf{dep}(< \_, \mathsf{del}(\_, \_, \_, \{\langle a, p, t, \top\rangle\}, \mathsf{M}) : h, \_, \_, \_ >, \_; \_; \mathsf{M}')$$

This operation request will be to undo:

- if a is in the map of the system (M') and not in the map of the del operator (M), then the tuple $\langle a, \_, k_a, \_\rangle$ is in the map and there is a process that has in its history the element $\mathsf{regS}(\_, \_, \_, \{\langle a, \_, k_a, \top\rangle\})$; this process has to undo the register operation;
- the same reasoning could be applied with the pid p;
- undo all the registered and readF operations that have read a or p. $\Diamond$

A.6 Code of the case study.

The code of the case study can be found in Fig. 16.

```
1  -module(server).
2  -export([main/1]).
3
4  main(A)->
5      register(server,spawn(?MODULE, server,[])),
6      register(log,spawn(?MODULE, logger,[0,[]])),
7      sendRequest(A).
8
9  server() ->
10     receive
11         {logged,Log} ->
12           io:format("LOGGED TIME:~p\n", [Log]);
13         {replay, Ris} ->
14           io:format("RESULT:~p\n", [Ris]),
15           log ! Ris;
16         {Atom, Val} ->
17           io:format("SEND REQUEST:~p\n", [{Atom, Val}]),
18           case whereis(Atom) of
19               undefined ->
20                   register(Atom,spawn(?MODULE, Atom,[])),
21                   Atom ! Val;
22               _ ->
23                   Atom ! Val
24           end
25     end,
26     server().
27
28 logger(N,L)->
29     receive
30       Val ->
31         server ! {logged,N}, logger(N+1,L++[Val])
32     end.
33
34 square() ->
35   receive
36     N ->
37       server ! {replay,{square,N*N}}, square()
38   end.
39
40 log() ->
41   receive
42     N ->
43       server ! {replay,{log,math:log10(N)}}, log()
44   end.
45
46 adder() -> adder(0).
47 adder(N)->
48   receive
49     Val ->
50       server ! {replay,{addder,Val + N}}, adder(Val + N)
51   end.
52
53 sendRequest([]) -> ok;
54 sendRequest([El | T]) ->
55   server ! El, sendRequest(T).
```

Fig. 16. Case study