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► **To cite this version:**

Shalini Sharma, Émilie Chouzenoux, Víctor Elvira, Angshul Majumdar. A dynamical model for stock forecasting via deep recurrent dictionary learning. 2023. hal-03654152v2

**HAL Id: hal-03654152**

**<https://hal.science/hal-03654152v2>**

Preprint submitted on 9 Jan 2024

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1        **A dynamical model for stock forecasting via deep recurrent**  
2                                **dictionary learning**

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20 **Abstract**

State-space models (SSM) and recurrent neural networks (RNN) are widely used approaches for dynamical system modeling. In the case of SSMs, they include explicit modeling of all components, including the noise characterization, and thus allow for interpretability and uncertainty quantification. However, the underlying dynamical model parameters must be specified, and closed-form inference is possible only in a few simple cases. RNNs, on the other hand, can learn, through supervised training, rather complex nonlinearities from the data but lack the aforementioned advantages of SSMs. In this work, we combine the benefits of both approaches by introducing a Gaussian SSM whose state and evolution operators can be learnt from the data. In order to deal with the ill-posedness of this parameter estimation problem, we propose an innovative factorized form of both the state and observation operators reminiscent of deep nonnegative matrix factorization models. An expectation-maximization method combined with a block alternating strategy is introduced to estimate each of the involved positive latent factors, while jointly performing the probabilistic state inference. Our resulting formulation and inference tool is deep recurrent dictionary learning (DRDL). We then specialize DRDL for the problem of stock forecasting by proposing an online training strategy and a probabilistic assessment of the trading decision. Numerical experiments on a problem of stock market data inference show its superiority among several state-of-the-art dynamic modeling tools.

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21 *Keywords:* Time series analysis; State space models; Deep nonnegative  
22 matrix factorization; Kalman filtering; Bayesian smoothing; EM algorithm;  
23 Stock forecasting; Stock trading.

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## 24 **1. Introduction**

25 Modeling dynamical systems has been a topic of interest to signal processing,  
26 machine learning and control engineering researchers for more than five decades.  
27 Applications range in areas as diverse as financial market analysis to electric  
28 demand forecasting. We propose a new dynamical recurrent modeling technique  
29 that combines the advantages of state-of-the-art deep learning tools with those  
30 of traditional state-space models. The proposed tool is then particularized to  
31 processing stock market time series. In the following, we review the literature  
32 of dynamic modeling around this particular application and we describe the  
33 contributions of the paper.

### 34 *1.1. State-of-the-art review*

35 Modeling the stock market is a well-known challenging problem Yang &  
36 Wu (2006). The difficulty lies in the non-stationary and nonlinearity of the  
37 underlying dynamical process. Moreover, financial markets are not only in-  
38 fluenced by consumer behavior but also by a myriad of external factors like  
39 natural disasters, administrative policies, political decisions, international rela-  
40 tions, etc., to name a few. Therefore developing reliable algorithmic models for  
41 stock trading still remains a challenging yet interesting topic from the point of  
42 view of both finance and machine learning/signal processing Fama (2021); Shah  
43 et al. (2019). Auto-regressive moving average (ARMA) models have been used  
44 to model stock market Zhao-yang (2010); Atsalakis & Valavanis (2010); Moon  
45 et al. (2021); Nowicka-Zagrajek & Weron (2002). ARMA assumes the stochastic  
46 process to be stationary; this turns out to be too simplistic and consequently  
47 unrealistic for the stock market. This limitation was partially overcome by au-  
48 toregressive integrated moving average (ARIMA) Ariyo et al. (2014) models

49 (also referred as Box-Jenkins model). ARIMA has been used in the past for  
50 stock forecasting and trading Devi et al. (2013), Petrica et al. (2016). However,  
51 Box-Jenkins/ARIMA methods could not model non-smooth variations in time  
52 series Makridakis & Hibon (1997); O'Donovan (1983). ARIMA with regressors  
53 were introduced to overcome the limitations, leading to ARIMAX Chadsuthi  
54 et al. (2012); Ababio (2012). Unfortunately, ARIMAX introduced other prob-  
55 lems such as over/under-fitting because of the handling of extra predictors and  
56 variables.

57 SSM is another powerful approach for modeling and analysing time-series  
58 Särkkä (2013); Newman et al. (2023). Many studies used SSMs for stock fore-  
59 casting, and analysis Östermark (1991); Saini et al. (2014); Elvira & Chouzenoux  
60 (2022). The celebrated Kalman filter is a solution to the inference of a linear  
61 SSM where the noise is assumed to be Gaussian Kalman (1960). The literature  
62 illustrates its minimal use in stock forecasting Choudhry & Wu (2009) but found  
63 its application in other financial analyses, see for example Wells (2013). To over-  
64 come the restrictive linearity assumption, extended Kalman filter (EKF) Ljung  
65 (1979), unscented Kalman filter (UKF) Wan & Van Der Merwe (2000) were in-  
66 troduced. Particle filters Djuric et al. (2003); Doucet & Johansen (2009); Elvira  
67 et al. (2019); Ntemi & Kotropoulos (2021) further relaxed the Gaussianity as-  
68 sumption. The advantage of the SSM approach is that it can model uncertainty  
69 in the estimate Doucet & Johansen (2009); Bach & Jordan (2004). Uncertainty  
70 is crucial for financial markets since it gives a measure of the associated risk  
71 Rigotti & Shannon (2005).

72 The main drawback of the aforesaid signal processing-based forecasting ap-  
73 proaches is that they need the model's specification. Unfortunately, specifying  
74 an underlying model for the stock market is difficult, if not impossible. Several  
75 works in the literature have thus investigated the learning of model parameters  
76 in SSMs. In the case of linear-Gaussian state-space models (LG-SSMs), see for  
77 instance the methods in Sharma et al. (2020); Shumway & Stoffer (1982); Khan  
78 & Dutt (2007), and (Särkkä, 2013, Chapter 12). All these works consider the  
79 observation and state operators to be unknown and estimated from data using

80 expectation-maximization (EM) methods. However, the aforementioned works  
81 can only consider linear models and do not account for any prior knowledge  
82 on the involved operators. Inferring model parameters for non-linear SSMs has  
83 been explored in more generic algorithms, e.g., particle MCMC methods An-  
84 drieu et al. (2010), SMC<sup>2</sup> Chopin et al. (2013), and nested PFs Crisan & Miguez  
85 (2018). In all these cases, the inference is costly, as they use Monte-Carlo sam-  
86 pling methods and thus do not generally scale well. The problem of scalability  
87 in SSM model inference has been mildly explored. Let us mention our two re-  
88 cent works Chouzenoux & Elvira (2020); Sharma et al. (2021), both focusing  
89 on LG-SSMs. In Chouzenoux & Elvira (2020), a sparsity prior is introduced on  
90 the linear matrices to infer, providing an interpretable and compressible model.  
91 Though this method is promising, it does not allow explicit control of the final  
92 dimension of the model easily, and as such, still requires an increased compu-  
93 tational time at inference. In Sharma et al. (2021), we proposed an online (still  
94 EM-based) estimation approach in the context of stock market time series pro-  
95 cessing. The online processing allows a reduced complexity and memory burden  
96 while being beneficial to capturing non-linear phenomena in such volatile time  
97 series. However, the parametric estimation step lacked of robustness and lack  
98 of sufficient imposed structure on the estimated factors.

99 Neural network (NN) models represent another family of approaches for time  
100 series modeling. By construction, these methods excel when the model spec-  
101 ification is missing, as they implicitly learn the model from the data through  
102 the training phase. In particular, the approximation capability of recurrent  
103 neural networks (RNNs) for dynamical systems allows to learn the underlying  
104 phenomena given enough training data Gonzalez-Olvera & Tang (2010); Won  
105 et al. (2010); Yin et al. (2022). RNN and its subsequent versions are used in  
106 several studies for stock price forecasting Saad et al. (1998); Tino et al. (2001).  
107 Deeper neural network architectures are known to yield better results than their  
108 shallow counterparts Bengio et al. (2007); Shao et al. (2014). They are engi-  
109 neered to approximate highly non-linear function in high-dimensional spaces  
110 and are supposed to be more suitable for challenging problems Cheridito et al.

111 (2021); Bianchini & Scarselli (2014). 1-D CNN performs better when compared  
112 to LSTM and RNN owing to their ease of training. There are studies, such as  
113 Sezer & Ozbayoglu (2018), that use them as financial forecasting. However, 1-D  
114 CNNs cannot process streaming data. Hence some works have recently proposed  
115 the combination of RNN with 1-D CNN in order to model time-series signals  
116 Long et al. (2019). It must be noted that deep neural networks are computa-  
117 tionally intensive Abbe & Sandon (2018). Furthermore, deep neural networks  
118 only provide data estimates for each time step and do not provide uncertainty  
119 quantification, while such information would be necessary for stock forecasting  
120 to assess risks. Therefore, recent works have been dedicated to combine proba-  
121 bilistic forecasting and deep learning techniques, so as to predict the probability  
122 distribution of future events in the time series given its past/historical recordings  
123 Salinas et al. (2020); Jiang (2021). Deep factor model based on dropout-based  
124 heuristic and complex semantics have also been considered in Chauhan et al.  
125 (2020). These techniques provide probability distributions as outputs, thanks  
126 to specific learning strategies inherited from the Bayesian NN literature. How-  
127 ever, these works, up to our knowledge, do not mention any explicit strategy  
128 to estimate uncertainty/confidence score on future predictions/decisions that  
129 would help to assess their reliability. Standard (non deep) machine learning  
130 models have also been combined to statistical time series modeling tools. For  
131 instance, the work Pai & Lin (2005) uses ARIMA and support vector machine.  
132 Another work uses a combination of ARIMA and random forests for the same  
133 task Kumar & Thenmozhi (2014). Finally, several works, such as Ding et al.  
134 (2015); Chong et al. (2017); Fischer & Krauss (2018), use information mined  
135 from news articles and blogs via natural language processing for stock forecast-  
136 ing Cheng et al. (2022); Li et al. (2023). Strictly speaking, this is not artificial  
137 intelligence since these are dependent on human cognizance.

### 138 *1.2. Contributions compared to existing literature*

139 As a summary, SSMs are valuable tools for probabilistic time series model-  
140 ing and inference. But there is a crucial need for new strategies to cope with

141 the curse of dimensionality in learning SSM model parameters. In this work,  
 142 we propose to impose a structured prior on the observation/state operators in-  
 143 volved in an LG-SSM. We introduce deep nonnegative matrix factorized (deep  
 144 NMF) models for both operators. Deep NMF De Handschutter et al. (2021) is  
 145 a generalized form of NMF Cichocki et al. (2009) that models latent representa-  
 146 tions from complex data through a product of a (usually small) number of linear  
 147 operators (called latent factors) satisfying positivity constraints. Deep NMF has  
 148 been employed with success on various unsupervised machine learning tasks Yu  
 149 et al. (2015); Trigeorgis et al. (2016); Xue et al. (2017); Chen et al. (2021);  
 150 Flenner & Hunter (2017). When embedded into an NN structure, it leads to  
 151 the so-called deep ReLU networks Liu & Liang (2021); Daubechies et al. (2022);  
 152 Chen et al. (2019). Deep NMF shares connections with the recently introduced  
 153 deep dictionary learning (deep DL) Tariyal et al. (2016); Mahdizadehghadam  
 154 et al. (2019), the main difference being in the priors imposed in the latent fac-  
 155 tors (positivity, in the case of deep NMF, low-rank/sparsity in the case of deep  
 156 DL). In our work, deep NMF is neither used for unsupervised representation  
 157 learning nor in an NN framework. In contrast, it is embedded into a Gaussian  
 158 SSM to model, allowing to track and predict complex latent phenomena in time  
 159 series. A novel algorithm is proposed, that learns the positive latent factors  
 160 jointly with the probabilistic state inferential task induced by our SSM. We call  
 161 this modelling and inference tool *Deep Recurrent Dictionary Learning (DRDL)*.  
 162 We further specialize this tool, to make it practically efficient in the context of  
 163 large and volatile time series arising in stock market data. In particular, we  
 164 perused the online training strategy we previously introduced in Sharma et al.  
 165 (2021).

166 **Contributions in a nutshell.** In this work, we:

- 167 • Introduce an LG-SSM model involving deep positive latent factors;
- 168 • Propose a new EM-based inference method to jointly perform the time  
 169 series prediction task and the deep linear positive factors estimation;
- 170 • Devise efficient implementation strategies for practical use of the method



171 in the context of stock market time series analysis;

- 172 • Investigate through experiments and benchmark comparisons on real fi-  
173 nancial data the performance of the novel DRDL approach.

### 174 1.3. Paper organization

175 The rest of the paper is organized as follows. The proposed DRDL model  
176 and its inference is presented in the following Section 2. The practical imple-  
177 mentation of DRDL in the context of stock market data analysis is discussed in  
178 Section 3. The experimental results and their analysis are described in Section  
179 4. The conclusions of this work are discussed in section 5.

### 180 1.4. Paper organization

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## 186 2. Proposed Work

### 187 2.1. Considered model

188 Let us consider an observed sequence  $(\mathbf{x}_k)_{1 \leq k \leq K}$  of vectors of size  $N_x \geq 1$ .  
189 We aim to estimate  $(\mathbf{z}_k)_{1 \leq k \leq K}$ , a sequence of unknown hidden/latent vectors  
190 of size  $N_z \geq 1$ . The DRDL approach relies on the following re-parametrized  
191 LG-SSM:

192 For every  $k \in \{1, \dots, K\}$ ,

$$\begin{cases} \mathbf{z}_k &= \mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2 \mathbf{z}_{k-1} + \mathbf{v}_{1,k}, \\ \mathbf{x}_k &= \mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2 \mathbf{z}_k + \mathbf{v}_{2,k}. \end{cases} \quad (1)$$

193 where  $\mathbf{D}_0 \in [0, +\infty)^{N_z \times N_z}$ ,  $\mathbf{D}_1 \in [0, +\infty)^{N_z \times N_z}$ , and  $\mathbf{D}_2 \in [0, +\infty)^{N_z \times N_z}$   
194 are three positive-valued linear factors leading to a multi-linear state oper-  
195 ator  $\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2$ . Similarly,  $\mathbf{H}_0 \in [0, +\infty)^{N_x \times N_z}$ ,  $\mathbf{H}_1 \in [0, +\infty)^{N_x \times N_z}$ ,  $\mathbf{H}_2 \in$

196  $[0, +\infty)^{N_z \times N_z}$  are three positive-valued linear factors yielding the multi-linear  
 197 observation model  $\mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2$ .<sup>1</sup> The process noise  $(\mathbf{v}_{1,k})_{1 \leq k \leq K}$  is assumed to  
 198 have a Gaussian distribution with zero-mean and symmetric definite positive  
 199 covariance matrix  $\mathbf{Q} \in \mathbb{R}^{N_z \times N_z}$ . The observation noise  $(\mathbf{v}_{2,k})_{1 \leq k \leq K}$ , is also as-  
 200 sumed zero-mean Gaussian with symmetric definite positive covariance matrix  
 201  $\mathbf{R} \in \mathbb{R}^{N_x \times N_x}$ . We consider  $\mathbf{z}_0 \sim \mathcal{N}(\bar{\mathbf{z}}_0, \mathbf{P}_0)$  as the initial state, with  $\bar{\mathbf{z}}_0 \in \mathbb{R}$   
 202 and  $\mathbf{P}_0 \in \mathbb{R}^{N_z \times N_z}$  defined as definite symmetric positive matrix. The model  
 203 in Eq. (1) can be interpreted as a multi-linear Gaussian model involving a se-  
 204 quence of  $K$  hidden states represented by  $(\mathbf{z}_k)_{1 \leq k \leq K}$ . As discussed earlier, clas-  
 205 sical inference approaches for SSM require specifying every model parameters.  
 206 In the model above, this would mean setting the positive latent factor matri-  
 207 ces matrices  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  involved in both state and observation  
 208 models. In practical applications such as stock market analysis, these param-  
 209 eters are unknown and must be learnt from the observed data. The objective  
 210 is thus to provide a point-wise estimate of the positive latent factor matrices  
 211  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  and a probabilistic estimate of sequence  $(\mathbf{z}_k)_{1 \leq k \leq K}$ ,  
 212 given the observed sequence  $(\mathbf{x}_k)_{1 \leq k \leq K}$ . This can be seen as solving jointly (i)  
 213 two deep NMF problems, (ii) a filtering/smoothing problem.

## 214 2.2. Discussion on the model

215 We now discuss the main characteristics of the proposed DRDL method.  
 216 The model is mathematically described in Eq. (1) and displayed in Fig. 1. The  
 217 top equation describes the hidden state evolution, assuming Markovianity be-  
 218 tween two consecutive hidden states. The second equation links the hidden and  
 219 observed states. A first interesting aspect, inherited from the SSM paradigm,  
 220 is that two Gaussian noise terms are explicitly introduced in DRDL to cope  
 221 with model uncertainty, which is in contrast with most deep learning models for

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<sup>1</sup>Throughout the paper, we consider three-terms factorizations, for the sake of readability.  
 The 3-layers modeling and inference methodology has the great advantage of being generic  
 enough to be straightforwardly extended to any number, greater or equals to one, of factors.

222 time series processing (e.g., LSTM). A second novel feature of (1) lies in using  
 223 deep NMF models instead of generic matrices (in the linear case) or functions  
 224 (in the non-linear case), as it is usually the case in SSMs Andrieu et al. (2010);  
 225 Chopin et al. (2013); Crisan & Miguez (2018), taking advantage on the acknowl-  
 226 edged representation power of deep NMF De Handschutter et al. (2021). One  
 227 important benefit of the proposed approach w.r.t. most existing methods in the  
 228 literature is that we avoid Monte Carlo simulation or complex optimization pro-  
 229 cedure, which is known to suffer more severely the curse of dimensionality. In  
 230 our method, each latent factor can be understood as representations in abstract  
 231 spaces of the phenomena occurring between both pairs of variables. Third, in  
 232 contrast with the typical usage of deep NMF in machine learning, relying on  
 233 backpropagation for their model training Chen et al. (2021); Flenner & Hunter  
 234 (2017), DRDL model allows the construction of an handcrafted training strat-  
 235 egy (see the next section), which benefits from a low computational cost, sound  
 236 optimality guarantees (in terms of Bayesian estimator), and enables uncertainty  
 quantification.

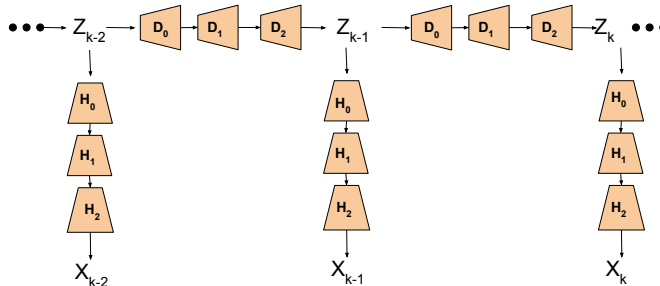


Figure 1: Schematic Diagram for Deep Recurrent Dictionary Learning

237

### 238 2.3. DRDL inference algorithm

239 Using SSM models for time series processing (e.g., for a prediction task)  
 240 amounts to solving the so-called smoothing/filtering problem, i.e., the proba-  
 241 bilistic estimation of the hidden state  $(\mathbf{z}_k)_{1 \leq k \leq K}$ . In our context, as the deep  
 242 NMF factors  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  involved in the construction of the state

243 transition and the observation transition models are most often unknown, we  
 244 must also infer them from the observed data, jointly with the hidden states  
 245 (through the aforementioned filtering/smoothing procedure). To do so, we propose  
 246 an expectation-maximization (EM) approach (see (Särkkä, 2013, chap.12)  
 247 and Shumway & Stoffer (1982)). The EM method alternates iteratively between  
 248 the probabilistic inference of the state  $(\mathbf{z}_k)_{1 \leq k \leq K}$ , while the positive  
 249 factors  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  are fixed (E-step), and the update of these  
 250 factors, assuming fixed state (M-step). More precisely, the E-step consists in  
 251 fixing the linear operators obtained in the previous M-step and applying the  
 252 classical Kalman/RTS recursions, for obtaining the filtered/smooth distributions  
 253  $p(\mathbf{z}_k | \mathbf{x}_{1:k})$  and  $p(\mathbf{z}_k, \mathbf{x}_{1:K})$ , respectively. Then, the M-step updates the  
 254 operators  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  by maximizing an upper bound of:

$$\begin{aligned}
 \varphi_K(\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2) \\
 = \log p(\mathbf{x}_{1:K} | \mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2). \quad (2)
 \end{aligned}$$

255 We explicit hereafter the construction of the  $i+1$ -th EM update, given estimates  
 256 from the previous iteration  $i$ .

### 257 2.3.1. E-step: Kalman/RTS inference

258 At this step, we considered the factors  $\mathbf{D}_0^{[i]}, \mathbf{D}_1^{[i]}, \mathbf{D}_2^{[i]}, \mathbf{H}_0^{[i]}, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}$  to be  
 259 fixed (either from the previous M-step or from the initialization at the first  
 260 iteration), and the goal is the probabilistic estimation of the latent states. As we  
 261 aforementioned, (1) is a multi-linear Gaussian model whose observation operator  
 262  $\mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2$ , evolution/state operator  $\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2$ , and hidden state  $(\mathbf{z}_k)_{1 \leq k \leq K}$  must  
 263 be estimated. For each  $k \in \{1, \dots, K\}$ , the probabilistic estimate of the latter  
 264 conditioned to all available data up to time  $k$ , is provided by the Kalman filter  
 265 through the following Gaussian filtering distribution:

$$p(\mathbf{z}_k | \mathbf{x}_{1:k}) = \mathcal{N}(\mathbf{z}_k; \bar{\mathbf{z}}_k, \mathbf{P}_k). \quad (3)$$

266 For every  $k$ , the mean  $\bar{\mathbf{z}}_k$  and the covariance  $\mathbf{P}_k$  are given by the Kalman  
 267 iterations:

268 For  $k = 1, \dots, K$ :

269 *Predict state:*

$$\begin{cases} \mathbf{z}_{k|k-1} &= \mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]} \bar{\mathbf{z}}_{k-1}, \\ \mathbf{P}_{k|k-1} &= \mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]} \mathbf{P}_{k-1} (\mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]})^\top + \mathbf{Q}. \end{cases} \quad (4)$$

270 *Update state:*

$$\begin{cases} \mathbf{y}_k &= \mathbf{x}_k - \mathbf{H}_0^{[i]} \mathbf{H}_1^{[i]} \mathbf{H}_2^{[i]} \mathbf{z}_{k|k-1}, \\ \mathbf{S}_k &= \mathbf{H}_0^{[i]} \mathbf{H}_1^{[i]} \mathbf{H}_2^{[i]} \mathbf{P}_{k|k-1} (\mathbf{H}_0^{[i]} \mathbf{H}_1^{[i]} \mathbf{H}_2^{[i]})^\top + \mathbf{R}, \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1} (\mathbf{H}_0^{[i]} \mathbf{H}_1^{[i]} \mathbf{H}_2^{[i]})^\top \mathbf{S}_k^{-1}, \\ \bar{\mathbf{z}}_k &= \mathbf{z}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k, \\ \mathbf{P}_k &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^\top. \end{cases} \quad (5)$$

271 Hereabove,  $\mathbf{y}_k$  represents the measurement pre-fit residual,  $\mathbf{S}_k$  represents the  
 272 pre-fit covariance,  $\mathbf{K}_k$  represents Kalman gain,  $\bar{\mathbf{z}}_k$  represents the updated (a  
 273 posteriori) state estimate,  $\mathbf{P}_k$  represents the updated (a posteriori) covariance  
 274 estimate. The backward recursion from the RTS smoother allow to build the  
 275 smoothing distribution  $p(\mathbf{z}_k | \mathbf{x}_{1:K})$ .

276 For  $k = K, \dots, 1$

277 *Backward Recursion:*

$$\begin{cases} \mathbf{z}_{k+1}^- &= \mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]} \bar{\mathbf{z}}_k, \\ \mathbf{P}_{k+1}^- &= \mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]} \mathbf{P}_k (\mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]})^\top + \mathbf{Q}, \\ \mathbf{G}_k &= \mathbf{P}_k (\mathbf{D}_0^{[i]} \mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]})^\top [\mathbf{P}_{k+1}^-]^{-1}, \\ \mathbf{z}_k^s &= \bar{\mathbf{z}}_k + \mathbf{G}_k [\mathbf{z}_{k+1}^s - \mathbf{z}_{k+1}^-], \\ \mathbf{P}_k^s &= \mathbf{P}_k - \mathbf{G}_k [\mathbf{P}_{k+1}^s - \mathbf{P}_{k+1}^-] \mathbf{G}_k^\top. \end{cases} \quad (6)$$

278 Consequently, for every time step  $k \in \{1, \dots, K\}$ , the RTS smoother provides:

$$p(\mathbf{z}_k | \mathbf{x}_{1:K}) = \mathcal{N}(\mathbf{z}_k; \mathbf{z}_k^s, \mathbf{P}_k^s). \quad (7)$$

279 *2.3.2. M-step: Evolution operators update*

280 The M-step performs an optimization step to increase the likelihood of the  
 281 positive latent factors  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$ , given the smoothing distribu-  
 282 tion obtained in the E-step. It proceeds by building the upper-bound:

$$\begin{aligned} \varphi_k(\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2) \\ \geq \mathcal{Q}(\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2; \Theta^{[i]}). \end{aligned} \quad (8)$$

283 Hereabove,  $\Theta^{[i]} = \{\Sigma^{[i]}, \Phi^{[i]}, \mathbf{B}^{[i]}, \mathbf{C}^{[i]}, \Delta^{[i]}\}$  gathers five quantities defined from  
284 the outputs of the E-step described in Sec. 2.3.1):

$$\begin{aligned} \mathcal{Q}(\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2; \Theta^{[i]}) = \\ - \frac{K}{2} \text{tr} \left( \mathbf{Q}^{-1} \Sigma^{[i]} - \mathbf{C}^{[i]} (\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2)^\top - \mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2 (\mathbf{C}^{[i]})^\top \right. \\ \left. + \mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2 \Phi^{[i]} (\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2)^\top \right) \\ - \frac{K}{2} \text{tr} \left( \mathbf{R}^{-1} \Delta^{[i]} - \mathbf{B}^{[i]} (\mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2)^\top - \mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2 (\mathbf{B}^{[i]})^\top \right. \\ \left. + \mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2 \Sigma^{[i]} (\mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2)^\top \right), \end{aligned} \quad (9)$$

285 with:

$$\begin{aligned} \Sigma^{[i]} &= \frac{1}{K} \sum_{k=1}^K \mathbf{P}_k^s + \mathbf{z}_k^s (\mathbf{z}_k^s)^\top, \\ \Phi^{[i]} &= \frac{1}{K} \sum_{k=1}^K \mathbf{P}_{k-1}^s + \mathbf{z}_{k-1}^s (\mathbf{z}_{k-1}^s)^\top, \\ \mathbf{B}^{[i]} &= \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k (\mathbf{z}_k^s)^\top, \\ \mathbf{C}^{[i]} &= \frac{1}{K} \sum_{k=1}^K \mathbf{P}_k^s \mathbf{G}_{k-1}^\top + \mathbf{z}_k^s (\mathbf{z}_{k-1}^s)^\top, \\ \Delta^{[i]} &= \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^\top. \end{aligned} \quad (10)$$

286 The updates  $\{\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2^{[i+1]}, \mathbf{H}_0^{[i+1]}, \mathbf{H}_1^{[i+1]}, \mathbf{H}_2^{[i+1]}\}$  given the knowledge  
287 of  $\Theta^{[i]}$ , amounts to maximizing  $\mathcal{Q}(\cdot; \Theta^{[i]})$  under positivity constraints on the  
288 factors. In contrast with the linear unconstrained model case studied in (Särkkä,  
289 2013, Chapter 12), the maximization problem here does not have a closed-form  
290 solution. It is highly non-convex due to the multi-linearity of our model. Luckily,  
291 it happens to be convex with respect to each of the factors. We thus propose

292 to resort to the following alternating maximization step:

$$\begin{aligned}
\mathbf{D}_0^{[i+1]} &= \operatorname{argmax}_{\mathbf{D}_0 \geq 0} \mathcal{Q}(\mathbf{D}_0, \mathbf{D}_1^{[i]}, \mathbf{D}_2^{[i]}, \mathbf{H}_0^{[i]}, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}; \Theta^{[i]}) \\
\mathbf{D}_1^{[i+1]} &= \operatorname{argmax}_{\mathbf{D}_1 \geq 0} \mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1, \mathbf{D}_2^{[i]}, \mathbf{H}_0^{[i]}, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}; \Theta^{[i]}) \\
\mathbf{D}_2^{[i+1]} &= \operatorname{argmax}_{\mathbf{D}_2 \geq 0} \mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2, \mathbf{H}_0^{[i]}, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}; \Theta^{[i]}) \\
\mathbf{H}_0^{[i+1]} &= \operatorname{argmax}_{\mathbf{H}_0 \geq 0} \mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2^{[i+1]}, \mathbf{H}_0, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}; \Theta^{[i]}) \\
\mathbf{H}_1^{[i+1]} &= \operatorname{argmax}_{\mathbf{H}_1 \geq 0} \mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2^{[i+1]}, \mathbf{H}_0^{[i+1]}, \mathbf{H}_1, \mathbf{H}_2^{[i]}; \Theta^{[i]}) \\
\mathbf{H}_2^{[i+1]} &= \operatorname{argmax}_{\mathbf{H}_2 \geq 0} \mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2^{[i+1]}, \mathbf{H}_0^{[i+1]}, \mathbf{H}_1^{[i+1]}, \mathbf{H}_2; \Theta^{[i]})
\end{aligned}$$

293 This approach ensures by construction the following inequality:

$$\begin{aligned}
\mathcal{Q}(\mathbf{D}_0^{[i+1]}, \mathbf{D}_1^{[i+1]}, \mathbf{D}_2^{[i+1]}, \mathbf{H}_0^{[i+1]}, \mathbf{H}_1^{[i+1]}, \mathbf{H}_2^{[i+1]}; \Theta^{[i]}) \\
\geq \mathcal{Q}(\mathbf{D}_0^{[i]}, \mathbf{D}_1^{[i]}, \mathbf{D}_2^{[i]}, \mathbf{H}_0^{[i]}, \mathbf{H}_1^{[i]}, \mathbf{H}_2^{[i]}; \Theta^{[i]}), \quad (11)
\end{aligned}$$

294 which is key to guarantee the convergence properties for the EM iteration. In-  
295 deed, the proposed updates yield an increase of the lower bound of the marginal  
296 likelihood, so as a consequence, an increase of the marginal log-likelihood itself.  
297 The overall procedure is thus guaranteed to yield a monotonic increase of the  
298 marginal log-likelihood function  $\varphi_K$  and classical results about majorization-  
299 minimization methods allow to ensure convergence guarantees to a stationary  
300 point of  $\varphi_K$  Jacobson & Fessler (2007). The six sub-problems are quadratic pro-  
301 gramming (convex) problems and can be solved through several available solvers.  
302 We decided to use the simple and fast alternating least squares approach Ci-  
303 chocki et al. (2009), reminiscent from the literature of deep nonnegative matrix  
304 factorization Chen et al. (2021), and the deep ReLU neural networks models  
305 Daubechies et al. (2022), both showing a satisfactory behavior in preliminary  
306 experiments. We start by computing each subproblem solution ignoring the pos-  
307 itivity constraints, and then capped the negative entries of the obtained factors.  
308 This yields the following analytic updates:

$$\begin{aligned}
\mathbf{D}_0^{[i+1]} &= \text{ReLu} \left( \mathbf{C}^{[i]} (\mathbf{D}_2^{[i]})^\top (\mathbf{D}_1^{[i]})^\top (\mathbf{D}_1^{[i]} \mathbf{D}_2^{[i]} \boldsymbol{\Phi}^{[i]} (\mathbf{D}_2^{[i]})^\top (\mathbf{D}_1^{[i]})^\top)^\dagger \right), \\
\mathbf{D}_1^{[i+1]} &= \text{ReLu} \left( ((\mathbf{D}_0^{[i+1]})^\top \mathbf{Q}^{-1} \mathbf{D}_0^{[i+1]})^\dagger (\mathbf{D}_0^{[i+1]})^\top \mathbf{Q}^{-1} \mathbf{C}^{[i]} (\mathbf{D}_2^{[i]})^\top \right. \\
&\quad \left. \times (\mathbf{D}_2^{[i]} \boldsymbol{\Phi}^{[i]} (\mathbf{D}_2^{[i]})^\top)^\dagger \right), \\
\mathbf{D}_2^{[i+1]} &= \text{ReLu} \left( ((\mathbf{D}_1^{[i+1]})^\top (\mathbf{D}_0^{[i+1]})^\top \mathbf{Q}^{-1} \mathbf{D}_0^{[i+1]} \mathbf{D}_1^{[i+1]})^\dagger (\mathbf{D}_1^{[i+1]})^\top \right. \\
&\quad \left. \times (\mathbf{D}_0^{[i+1]})^\top \mathbf{Q}^{-1} \mathbf{C}^{[i]} (\boldsymbol{\Phi}^{[i]})^{-1} \right), \\
\mathbf{H}_0^{[i+1]} &= \text{ReLu} \left( \mathbf{B}^{[i]} (\mathbf{H}_2^{[i]})^\top (\mathbf{H}_1^{[i]})^\top (\mathbf{H}_1^{[i]} \mathbf{H}_2^{[i]} \boldsymbol{\Sigma}^{[i]} (\mathbf{H}_2^{[i]})^\top (\mathbf{H}_1^{[i]})^\top)^\dagger \right), \\
\mathbf{H}_1^{[i+1]} &= \text{ReLu} \left( ((\mathbf{H}_0^{[i+1]})^\top \mathbf{R}^{-1} \mathbf{H}_0^{[i+1]})^\dagger (\mathbf{H}_0^{[i+1]})^\top \mathbf{R}^{-1} \mathbf{B}^{[i]} (\mathbf{H}_2^{[i]})^\top \right. \\
&\quad \left. \times (\mathbf{H}_2^{[i]} \boldsymbol{\Sigma}^{[i]} (\mathbf{H}_2^{[i]})^\top)^\dagger \right), \\
\mathbf{H}_2^{[i+1]} &= \text{ReLu} \left( ((\mathbf{H}_1^{[i+1]})^\top (\mathbf{H}_0^{[i+1]})^\top \mathbf{R}^{-1} \mathbf{H}_0^{[i+1]} \mathbf{H}_1^{[i+1]})^\dagger \right. \\
&\quad \left. \times (\mathbf{H}_1^{[i+1]})^\top (\mathbf{H}_0^{[i+1]})^\top \mathbf{R}^{-1} \mathbf{B}^{[i]} (\boldsymbol{\Sigma}^{[i]})^{-1} \right). \tag{12}
\end{aligned}$$

310

311 Hereabove,  $(\cdot)^\dagger$  denotes the pseudo-inverse operator. Moreover,  $\text{ReLu}(\cdot)$   
312 states for the rectified linear unit function, that projects each entry of its input  
313 to the positive orthant.

#### 314 2.4. The DRDL algorithm summarized

315 We summarize in Alg. 1 the DRDL algorithm, for the probabilistic inference  
316 of the sequence of hidden state  $(\mathbf{z}_k)_{1 \leq k \leq K}$ , jointly with the point-wise estimation  
317 of the latent factors  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$ , assuming the data follows the  
318 DRDL model (1). In practice, DRDL algorithm is ran for a maximum number  
319 of iterations  $i_{\max}$ , set so as to reach stabilisation of the latent factors.

320

### 321 3. Application to Stock Trading

322 We now particularize the DRDL inference algorithm to the stock trading  
323 applications. In particular, we address the forecasting/trading tasks given a set  
324 of  $K$  daily (i.e.,  $k$  is a day index) observations of stock market data.



**Algorithm 1. DRDL (3 layers) inference algorithm.**

**Inputs.** Prior parameters  $(\bar{\mathbf{z}}_0, \mathbf{P}_0)$ ; model noise covariance matrices  $\mathbf{Q}, \mathbf{R}$ ; set of observations  $\{\mathbf{x}_k\}_{1 \leq k \leq K}$ .

**Initialization.** Set positive latent factors

$$\{\mathbf{D}_0^{(0)}, \mathbf{D}_1^{(0)}, \mathbf{D}_2^{(0)}, \mathbf{H}_0^{(0)}, \mathbf{H}_1^{(0)}, \mathbf{H}_2^{(0)}\}.$$

**Recursive step.** For  $i = 0, 1, \dots, i_{\max}$ :

(E step) Run the Kalman filter (4)-(5) and RTS smoother (6) using latent factors  $\{\mathbf{D}_0^{(i)}, \mathbf{D}_1^{(i)}, \mathbf{D}_2^{(i)}, \mathbf{H}_0^{(i)}, \mathbf{H}_1^{(i)}, \mathbf{H}_2^{(i)}\}$ .

Calculate  $(\boldsymbol{\Sigma}^{(i)}, \boldsymbol{\Phi}^{(i)}, \mathbf{B}^{(i)}, \mathbf{C}^{(i)}, \boldsymbol{\Delta}^{(i)})$  using (10).

(M step) Compute  $\{\mathbf{D}_0^{(i+1)}, \mathbf{D}_1^{(i+1)}, \mathbf{D}_2^{(i+1)}, \mathbf{H}_0^{(i+1)}, \mathbf{H}_1^{(i+1)}, \mathbf{H}_2^{(i+1)}\}$  using (12).

**Output.** State filtering/smoothing pdfs (3) and (7) along with point-wise estimates of the latent factor from (12).

325 **3.1. Online implementation**

326 First, in order to better cope with high volatility of stock market quantities  
327 and allow immediate feedback to the users for on-the-fly trading, we propose  
328 here an online implementation of our DRDL approach. We make use of sliding  
329 windows of size of  $\tau \in \{1, \dots, K\}$  time steps. The model parameters are inferred  
330 for every  $k \in \{0, \dots, K - \tau\}$  using the last  $\tau$  data points observed in the window,  
331 i.e.  $\mathcal{X}_k = \{\mathbf{x}_j\}_{j=k+1}^{k+\tau}$ , then followed by the EM approach described in detail  
332 above. The sliding window approach leverages two advantages. First, it helps  
333 in faster processing of the sequence as one can choose a small number  $\tau$  of  
334 data points in the window. Second, it might provide better modeling, since the

335 constant linear factors assumption is likely to better model the time series if  $\tau$   
336 is small. However, a too small  $\tau$  might also degrades the inference capabilities.  
337 Hence, it is essential to find a tradeoff in finding an optimal  $\tau$ , as we will  
338 show in our experiments. When implementing the online strategy, a warm start  
339 approach is employed for the Kalman filter initialization. The observation/state  
340 factors are set to their most recent values, and the mean/covariance of the state  
341 for processing  $\mathcal{X}_{k+1}$  are initialized using the results of the processing of  $\mathcal{X}_k$ . Let  
342 us note that, when we set  $\tau = K$ , we retrieve the offline version of the algorithm,  
343 where the EM inference tool is ran only once.

### 344 3.2. Modeling and post-processing for stock market analysis tasks

345 Stock market data processing typically amounts to solving two distinct ap-  
346 plicative problems, namely daily stock price forecasting and stock trading deci-  
347 sion (among 3 options: buy/hold/sell) estimation. We hereafter explain how to  
348 post-process DRDL results to tackle both above-stated problems.

#### 349 3.2.1. Stock forecasting

350 Let us first specify the observation model in stock forecasting. For a given  
351 window size  $\tau > 0$ , for each  $k \in \{0, \dots, K - \tau\}$ , we observe  $(\mathbf{x}_j)_{k+1 \leq j \leq k+\tau} \in \mathbb{R}^{15}$ ,  
352 gathering 14 technical indicators <sup>2</sup> as well as the adjusted close price. Running  
353 our DRDL on the considered window yields the following mean estimate of the  
354 15 quantities for the next time step indexed as  $k + \tau + 1$ :

$$\hat{\mathbf{x}}_{k+\tau+1} = \mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2 \mathbf{z}_{k+\tau|k+\tau-1}, \quad (13)$$

---

<sup>2</sup>We retained the relative strength index (RSI), the William percentage range, the absolute price oscillator (APO), the commodity channel index, the Chande momentum oscillator (CMO), the directional movement Indicator (DMI), the ultimator oscillator, the WMA, the exponential moving average (EMA), the Simple Moving Average (SMA), the triple EMA, the moving average convergence (MAC), the percentage price oscillator, the rate of change (ROC). Detailed definitions can be found in <https://www.investopedia.com/terms/t/technicalindicator.asp>

355 with the covariance matrix

$$\mathbf{S}_{k+\tau+1} = \mathbf{H}_0 \mathbf{H}_1 \mathbf{H}_2 (\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2 \mathbf{P}_{k+\tau} (\mathbf{D}_0 \mathbf{D}_1 \mathbf{D}_2)^\top + \mathbf{Q}) + \mathbf{R}. \quad (14)$$

356 Hereabove,  $\{\mathbf{D}_0, \mathbf{D}_1, \mathbf{D}_2, \mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  are the factors estimated during the M-  
 357 step of our EM-based inference method, and  $(\mathbf{z}_{k+\tau|k+\tau-1}, \mathbf{P}_{k+\tau})$  are byproducts  
 358 of the Kalman prediction step (4)-(5), computed during the E-step of the EM.  
 359 The proposed methodology aims at predicting the entire 15-dimensional vector.  
 360 However, stock forecasting is typically focused on the prediction of a single  
 361 quantity such as the adjusted close price.

### 362 3.2.2. Stock trading

363 In stock trading, a different set of inputs are passed to the model. For each  
 364 window index  $k \in \{0, \dots, K - \tau\}$ , the observed data points  $(\mathbf{x}_j)_{k+1 \leq j \leq k+\tau} \in$   
 365  $\mathbb{R}^{17}$ ,  $x_j[i]$ , for  $i \in \{1, \dots, 14\}$ , are the same 14 technical indicators as in stock  
 366 forecasting. Additionally,  $[x_j[15], x_j[16], x_j[17]] \in \{0, 1\}^3$  gathers the decisions  
 367 “hold”, “buy”, or “sell”, which are calculated for each stocks for every day so as  
 368 to maximize the annualized returns. The labels are further turned into soft hot  
 369 encoded vectors as explained in (Sharma et al., 2021, Sec. 3.3.2). The mean  
 370 and covariance of these 17 quantities can be estimated for next day following  
 371 (13) and (14). We then define our class label for next time step as

$$\ell_{k+\tau+1} = \operatorname{argmax}_{i \in \{1,2,3\}} \hat{x}_{k+\tau+1}[i + 14]. \quad (15)$$

### 372 3.3. Probabilistic assessment of stock trading decision

373 We now describe the procedure to assess the uncertainty quantification as-  
 374 sociated to the DRDL predictions. Let  $k \in \{0, \dots, K - \tau\}$  be the window index  
 375 on which Algorithm 1 has been run. The probabilistic estimation of the quan-  
 376 tities of interest for the next time step (i.e., one-day ahead prediction)  $\mathbf{x}_{k+\tau+1}$   
 377 conditioned to the data observed in the window  $\mathbf{x}_{k:k+\tau}$ , reads as a multivariate  
 378 Gaussian distribution

$$p(\mathbf{x}_{k+\tau+1} | \mathbf{x}_{k:k+\tau}) = \mathcal{N}(\mathbf{x}_{k+\tau+1}; \hat{\mathbf{x}}_{k+\tau+1}, \mathbf{S}_{k+\tau+1}), \quad (16)$$

379 with mean and covariance ( $\hat{\mathbf{x}}_{k+\tau+1}, \mathbf{S}_{k+\tau+1}$ ), given by (13) and (14), respec-  
380 tively. Equation (16) assigns a probability score to any decision (e.g., trading)  
381 based on the prediction output of the DRDL method. Let us focus on the  
382 particular example of assessing the uncertainty of the stock trading decision at  
383 time index  $k + \tau + 1$ , given observations at indexes  $j \in \{k + 1, \dots, k + \tau\}$ . The  
384 trading decision relies on the discrete maximization step (15). Let us express  
385 the probability mass function (pmf) of this decision, from the gaussian predic-  
386 tive probability density function (pdf) of the observed data points in Eq. (16).  
387 The pmf can here be summarized as  $\mathbf{p}_{k+\tau+1} \in [0, 1]^3$  where each  $p_{k+\tau+1}[i]$ ,  
388  $i \in \{1, 2, 3\}$  is a probability, and  $\sum_{i=1}^3 p_{k+\tau+1}[i] = 1$ . Each  $p_{k+\tau+1}[i]$  represents  
389 the probability inferred by DRDL that the true value  $x_{k+\tau+1}[i + 14]$  is greater  
390 than  $x_{k+\tau+1}[j + 14]$ , for  $j = \{1, 2, 3\} \setminus i$ . According to Eqs. (16) and (15),  
391  $\mathbf{p}_{k+\tau+1}$  can be obtained through

$$(\forall i \in \{1, 2, 3\}) \quad p_{k+\tau+1}[i] = \int_{\mathcal{Y}_i} \mathcal{N}(\mathbf{y}; \hat{\mathbf{x}}_{k+\tau+1}[15 : 17], \mathbf{S}_{k+\tau+1}[15 : 17, 15 : 17]) d\mathbf{y}, \quad (17)$$

392 with

$$\mathcal{Y}_i = \{\mathbf{y} \in \mathbb{R}^3 \mid \mathbf{y}[i] \geq \mathbf{y}[j], j = \{1, 2, 3\} \setminus i\}. \quad (18)$$

393 Due to the intricate form of the constrained set in (18), the integral in (17)  
394 is intractable. It can be easily approximated with high precision by direct  
395 simulation. In practice, we sampled  $10^4$  three-dimensional sample from a normal  
396 standard distribution. The samples can be re-used for all time steps using  
397 coloring and shifting according to the covariance and mean, respectively. Thanks  
398 to this procedure, we can infer  $\mathbf{p}_{k+\tau+1}$  for every  $k$ , and then assess the next day  
399 stock trading outcome by using the standard cross-entropy loss:

$$\text{log-loss} = \frac{1}{K - \tau + 1} \sum_{k=0}^{K-\tau} \sum_{i=1}^3 -(L_{k+\tau+1}[i] \log(p_{k+\tau+1}[i])), \quad (19)$$

400 where the true labels are denoted  $\mathbf{L}_{k+\tau+1} \in \{0, 1\}^3$  for each time  $k + \tau + 1$   
401 (hereagain, we use soft hot encoding representation).

402 3.4. Summarized pipeline

403 We provide in Alg. 2 the summary of our proposed pipeline for applying  
 404 DRDL, in Algorithm 1, in the context of stock forecasting (steps a-b with  $N_x =$   
 405 15) and trading (steps a-b-c-d with  $N_x = 17$ ).

**Algorithm 2. DRDL (3 layers) method for stock forecasting and trading.**

**Inputs.** Prior parameters  $(\bar{\mathbf{z}}_0, \mathbf{P}_0)$ ; model noise covariance matrices  $\mathbf{Q}, \mathbf{R}$ ; set of observations  $\{\mathbf{x}_k\}_{1 \leq k \leq K}$ ; windows size  $\tau$ .

**Initialization.** Set positive latent factors

$\{\mathbf{D}_0^{(0)}, \mathbf{D}_1^{(0)}, \mathbf{D}_2^{(0)}, \mathbf{H}_0^{(0)}, \mathbf{H}_1^{(0)}, \mathbf{H}_2^{(0)}\}$ .

**Window processing.** For  $k = 0, 1, \dots, K - \tau$ :

- a. Run DRDL algorithm 1 on sequence  $(\mathbf{x}_j)_{k+1 \leq j \leq k+\tau}$ , initialized with estimates from  $k - 1$ th window (warm start).
- b. Calculate one-step ahead predicted mean  $\hat{\mathbf{x}}_{k+\tau+1}$  and its covariance  $\mathbf{S}_{k+\tau+1}$  using (13)-(14).
- c. Compute one-step ahead predicted label  $\ell_{k+\tau+1}$  using (15).
- d. Compute  $\mathbf{p}_{k+\tau+1}$  using (17)-(18).

**Output.** Forecasting/trading predictions and log-loss value (19).

406

## 407 **4. Experimental Results**

### 408 *4.1. Dataset*

409 The finance dataset used for experiments is curated from Yahoo finance  
410 repository.<sup>3</sup> We curated data for 180 stocks which comprises stocks from USA,  
411 UK, India and China. The data is prepared by scrapping daily adjusted close  
412 prices, open price, volume, high price, low price for a span of twenty years (i.e.,  
413 01/01/1998 to 01/10/2019) using yahoo finance API for Python. Having stocks  
414 from different market cap is always advisable by the traders, as it gives them  
415 breadth while investing in advanced as well as emerging markets Fawaz et al.  
416 (2019). The diversification also allows to assess the model robustness to various  
417 trends Kumar & Shah (2009). From the knowledge of the close prices, we build  
418 two observation sequences associated to the resolution of two specific problems,  
419 namely stock forecasting and stock trading, as described in Sec. 3.2. The data  
420 is scaled by normalising the 14 technical indicator values. For both problems,  
421 we will compare DRDL and several state-of-the-art methods arising from signal  
422 processing and machine learning literature. In all experiments, each of the 180  
423 observed time series is split into two parts, namely a train phase made of the  
424 first recorded 2546 days, and a test phase made of the next 2882 days. The train  
425 phase is used to learn the models parameters (for instance, the linear factors  
426 involved in DRDL), while the test phase is used to evaluate the performance of  
427 the learnt models, their parameters being fixed. More details about DRDL and  
428 the retained benchmark methods setting are provided in the next subsection.  
429 Figure 2 displays the evolution of 4 of the 14 technical indicators used as input  
430 of the inference tools, during the test phase. One can notice the high volatility  
431 in the observed data.

---

<sup>3</sup><https://yahoo.finance.com>

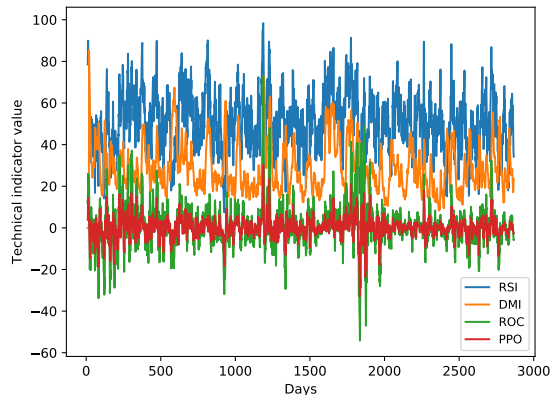


Figure 2: Evolution of four (among 14) observed technical indicators during the test phase.

432 *4.2. Practical Settings*

433 *4.2.1. DRDL settings*

434 As described in Sec. 3.2, DRDL can be specified to tackle both stock fore-  
 435 casting problem, in which case  $N_x = 15$ , and stock trading problem where  
 436  $N_x = 17$ . Three variants of DRDL will be compared, depending on the number  
 437 of linear factors (i.e., layers) in the multi-linear model. More specifically, we  
 438 will distinguish in our experiments:

439 **DRDL (1 layer):**  $\mathbf{D}_0 = \mathbf{D}_1 = \mathbf{D}_2 = \mathbf{Id}$  and  $\mathbf{H}_1 = \mathbf{H}_2 = \mathbf{Id}$  fixed and  $\{\mathbf{H}_0\}$  is  
 440 estimated;

441 **DRDL (2 layers):**  $\mathbf{D}_0 = \mathbf{D}_1 = \mathbf{D}_2 = \mathbf{Id}$  and  $\mathbf{H}_2 = \mathbf{Id}$  fixed and  $\{\mathbf{H}_0, \mathbf{H}_1\}$   
 442 are estimated;

443 **DRDL (3 layers):**  $\mathbf{D}_0 = \mathbf{D}_1 = \mathbf{D}_2 = \mathbf{Id}$  and  $\{\mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2\}$  are estimated.

444 Note that, ignoring the positivity constraint, DRDL (1 layer) would identify with  
 445 our previously published method RDL Sharma et al. (2021). We implement the  
 446 sliding window approach described in Sec. 3.1, for various choices of  $\tau$  described  
 447 hereafter. In all experiments, the initial value are set as ;  $\bar{\mathbf{z}}_0$  is an all zero

448 vector;  $\mathbf{P}_0=10^{-7}\text{Id}$ ,  $\mathbf{Q}=10^{-2}\text{Id}$  and  $\mathbf{R}=10^{-2}\text{Id}$ , where Id states for the identity  
 449 matrix. Moreover, we set the dimension of the state as  $N_z = 14$ , which also  
 450 corresponds to the number of measured technical indicators, we observed better  
 451 performance of the model. The entries of the linear factors to estimate are  
 452 initialized at time 0 using independent realizations of a uniform distribution on  
 453  $[0, 10^{-1}]$ . As mentioned in Sec. 3.1, to initialize the next processed windows, we  
 454 used warm start strategy. The estimation of the linear factors (i.e. M-step of  
 455 the EM method) is only conducted during the training phase. A maximum of  
 456 50 iterations of the EM loop are used in Alg. 1, which was observed sufficient to  
 457 reach stability of the estimated factors. During the test phase, the linear latent  
 458 factors are fixed, and only the Kalman/RTS inference is ran (i.e., we inhibit  
 459 M-step in Alg. 1). The scores shown are arithmetic means of 10 random trials,  
 460 and are computed only during the test phase.

#### 461 4.2.2. Compared methods

462 The proposed DRDL approach is analyzed by comparing with state-of-the-  
 463 art-methods deep learning namely Multi-filter neural network Long et al. (2019),  
 464 Long short term memory Fischer & Krauss (2018), 2-D deep CNN (CNN-TA)  
 465 Sezer & Ozbayoglu (2018) and ARIMA Ariyo et al. (2014). We select the state-  
 466 of-the-art methods for each task for a fair comparison. For stock forecasting, the  
 467 comparison is done with ARIMA and LSTM. The ARIMA parameter value are  
 468 set to  $(p, d, q) = (5, 1, 5)$ . The LSTM is customized from its original version to  
 469 carry out regression tasks by replacing the softmax output layer with an affine  
 470 layer. The Adam optimizer is used with learning rate  $10^{-4}$  and 200 epochs.  
 471 We used a mini-batch strategy where batch-size is fixed to 16 to reduce the  
 472 objective function’s mean square error (MSE). The evaluation of the methods  
 473 is done using metrics like mean absolute error (MAE), root means square error  
 474 (RMSE), SMAPE (Symmetric mean absolute percentage error), and Pearson  
 475 correlation factor.

476 For the stock trading task, the comparison is made with CNN-TA, Multi-  
 477 filter neural network, and LSTM implemented with their original architecture



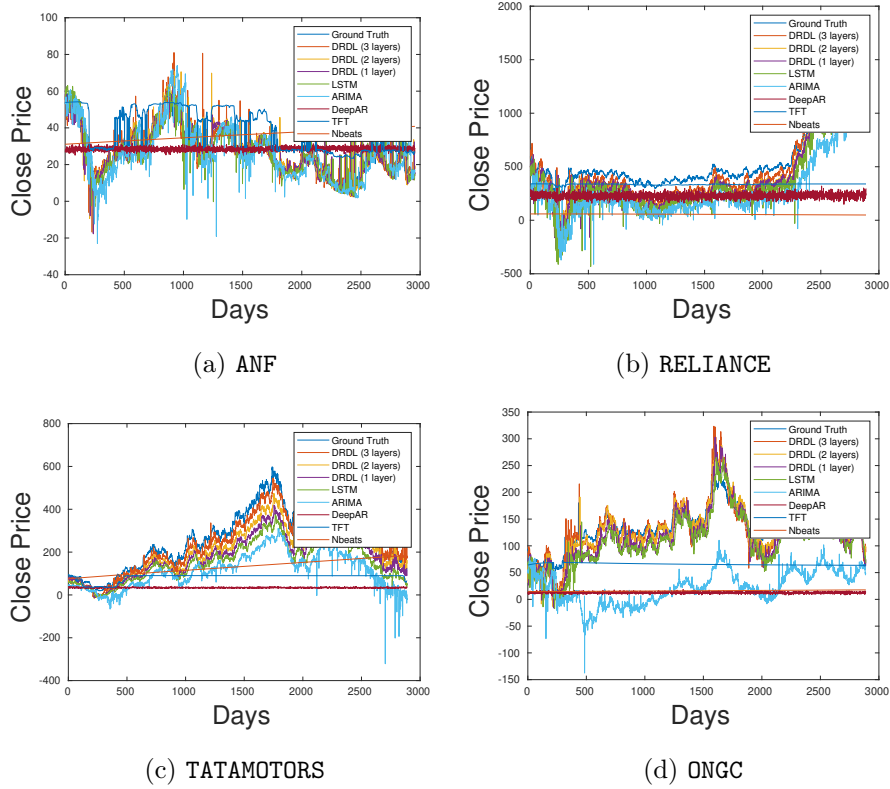


Figure 3: Ground truth and inferred adjusted close price on the test phase for four different stocks, using DRDL with 1 to 3 layers, LSTM or ARIMA.

478 and parameter values set to the specified ones in the respective paper Sezer  
 479 & Ozbayoglu (2018), Long et al. (2019) and Fischer & Krauss (2018). The  
 480 evaluation of the methods is done using classification metrics such as the F1-  
 481 score, recall, precision for each class. We also performed trading simulation  
 482 experiments in terms of annualized returns. We additionally present the log-  
 483 loss values provided by DRDL (see Sec. 3.3) to illustrate how the probabilistic  
 484 assessment can be used to let the researchers analyze market sentiments and  
 485 diversify a balanced portfolio.

486 *4.2.3. Hardware and Software descriptions*

487 We curated the data using the python API. The data curation and experi-  
488 mental results for the DRDL model are computed using Python 3.6 code. The  
489 implementation is done using the potential python libraries like NumPy, scikit-  
490 learn and pandas. In contrast, CNN-TA is implemented in its original version  
491 using the keras. MFNN and LSTM are implemented with PyTorch. The tech-  
492 nical indicators are evaluated using the libraries Ta-lib<sup>4</sup> and Ta4j<sup>5</sup>. Provided  
493 computational times correspond to codes running on Xeon E3-1225V5 clocked  
494 at 3.3GHz, with a 4Gb GPU (GeForce GT 730), 16GB RAM, 200GB HDD and  
495 Ubuntu OS.

496 *4.3. Numerical results for stock forecasting problem*

497 *4.3.1. Influence of window size*

498 The choice of window size is an essential aspect as it can enhance and limit  
499 the methodology’s potential. To understand the model behavior, we present  
500 Table 1 which provides detailed information on the performance of the model on  
501 varying window sizes. The table offers an analysis of various metrics like Pearson  
502 correlation (r), RMSE (Root Mean Square Error), MAE (Mean Absolute Error),  
503 and SMAPE (Symmetric mean absolute percentage error) for different window  
504 sizes  $\tau$ . The model’s performance improves as the window size increases till  
505 a stabilization point. We can see that a balanced choice is  $\tau = 650$  to reach  
506 stabilized performance on this particular task and dataset. We further use this  
507 value in upcoming experiments.

508 *4.3.2. Comparison with benchmark models*

509 To understand better, we present table 2 which provides comprehensive anal-  
510 ysis on performance estimation on the stock forecasting problem using DRDL,  
511 LSTM, ARIMA, DeepAR Salinas et al. (2020), Nbeats Oreshkin et al. (2019),  
512 and TFT Lim et al. (2019). Table 2 presents comparison in terms of Pearson

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<sup>4</sup><http://ta-lib.org>

<sup>5</sup><http://www.ta4j.org>

| Window size $\tau$ | $r$         | RMSE $\downarrow$ | MAE (%) $\downarrow$ | SMAPE (%) $\downarrow$ |
|--------------------|-------------|-------------------|----------------------|------------------------|
| 250                | 0.45        | 29.43             | 0.47                 | 31.8                   |
| 300                | 0.49        | 27.81             | 0.23                 | 28.6                   |
| 350                | 0.53        | 21.61             | 0.29                 | 25.5                   |
| 500                | 0.69        | 13.79             | 0.13                 | 23.4                   |
| 650                | <b>0.71</b> | <b>13.35</b>      | <b>0.11</b>          | <b>18.4</b>            |
| 700                | 0.72        | 13.62             | 0.10                 | 18.5                   |

Table 1: Results of DRDL (3 layers) on stock forecasting problem for different window size. Scores averaged on test phase, on all the 180 stocks.

| Model           | $r$         | RMSE $\downarrow$  | MAE(%)             | SMAPE(%)    |
|-----------------|-------------|--------------------|--------------------|-------------|
| ARIMA           | 0.13        | 78.6 (1.89)        | 1.23(0.56)         | 65.5        |
| LSTM            | 0.24        | 297.5 (2.64)       | 6.12 (0.65)        | 47          |
| DeepAR          | 0.40        | 73.89 (1.85)       | 0.43(0.34)         | 58.13       |
| Nbeats          | 0.38        | 95.52 (1.99)       | 0.57 (0.12)        | 63.75       |
| TFT             | 0.52        | 35.79(1.56)        | 0.35(0.019)        | 38.25       |
| DRDL (1 layer)  | 0.65        | 23.24 (1.64)       | 0.19(0.02)         | 35.3        |
| DRDL (2 layers) | 0.69        | 14.2 (1.43)        | 0.15(0.006)        | 23.2        |
| DRDL (3 layers) | <b>0.71</b> | <b>13.35(0.37)</b> | <b>0.11(0.003)</b> | <b>18.4</b> |

Table 2: Comparative analysis of DRDL against state-of-the-art methods for stock forecasting problem: Pearson correlation score ( $r$ ), MAE, RMSE, SMAPE scores and their respective std. deviation on the estimation of next time step adjusted close price, averaged over the data in the test set and the stocks.

513 correlation factor  $r$ , RMSE, MAE and SMAPE. We can see that DRDL (3 lay-  
514 ers) architecture outperforms DRDL (2 layers), DRDL (1 layer) as well as the  
515 other benchmarks models. We also notice that the average performance of TFT  
516 is comparable to DRDL (1 layer) architecture. Fig A.4 in appendix section

| Method          | Train Time cost (h.) | Test Time cost (min.) |
|-----------------|----------------------|-----------------------|
| DRDL (3 layers) | 2.37h                | 20 min                |
| DRDL (2 layers) | 2.12h                | 18.4 min              |
| DRDL (1 layer)  | 1.78h                | 15.8 min              |
| ARIMA           | 2.01h                | 36 min                |
| LSTM            | 8 days               | 45 min                |
| DeepAR          | 2.45h                | 20 min                |
| TFT             | 2.25h                | 27 min                |
| Nbeats          | 3.12h                | 25 min                |

Table 3: Averaged time over 10 random runs for processing the dataset (train(hrs) and test(min)), for DRDL and its competitors.

| Method          | Sharpe Ratio | T-test |
|-----------------|--------------|--------|
| DRDL (3 layers) | 2.14         | 0.63   |
| DRDL (2 layers) | 1.99         | 0.78   |
| DRDL (1 layer)  | 1.84         | 0.83   |
| ARIMA           | 1.03         | 0.89   |
| LSTM            | 0.88         | 1.34   |
| DeepAR          | 2.06         | 0.58   |
| TFT             | 1.22         | 0.67   |
| Nbeats          | 1.33         | 0.54   |

Table 4: Comparison of Sharpe ratio and T-test score, for DRDL and its competitors averaged over 180 stocks.

517 Appendix A displays the Pearson correlation analysis between ground truth  
518 daily adjusted close price time series and predicted ones along test phase, using  
519 DRDL (3 layers) for four representative stock cases. Table 4 presents the statisti-  
520 cal test (t-test) and stock market simulation (Sharpe Ratio) on the forecasting  
521 results. We observe that the proposed method with 3 layers architecture gives  
522 better performance as its average score for 185 stocks is smallest as compared to  
523 other state-of-the-art methods hence we can conclude that more similarity exists

524 between the actual closing prices and predicted closing prices when compared  
525 for different state-of-the art methods. We also present an average analysis of  
526 Sharpe ratio for proposed method and its competitors which gives more infor-  
527 mation on the risk- adjusted return on the investment. A higher Sharpe ratio  
528 indicates good investment returns, likewise we see that the forecast estimation  
529 from DRDL (3 layers) and DeepAR yields a higher Sharpe ratio as compared  
530 to other state-of-the-art method.

| Method          | F1 Score    |             |             | Precision   |             |             | Recall      |             |             | Training time<br>(in hrs) | Testing Time<br>(in min) |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|---------------------------|--------------------------|
|                 | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         |                           |                          |
| DRDL (3 layers) | <b>0.61</b> | <b>0.30</b> | <b>0.29</b> | <b>0.85</b> | <b>0.26</b> | <b>0.29</b> | <b>0.51</b> | <b>0.54</b> | <b>0.54</b> | 4.12                      | 10.17                    |
| DRDL (2 layers) | 0.61        | 0.32        | 0.34        | 0.83        | 0.23        | 0.26        | 0.48        | 0.51        | 0.53        | 3.56                      | 12.50                    |
| DRDL (1 layer)  | 0.59        | 0.19        | 0.23        | 0.88        | 0.15        | 0.12        | 0.45        | 0.52        | 0.51        | 2.00                      | 11.56                    |
| MFNN            | 0.58        | 0.11        | 0.06        | 0.79        | 0.11        | 0.04        | 0.47        | 0.37        | 0.16        | 5.34                      | 14.23                    |
| LSTM            | 0.86        | 0.05        | 0.05        | 0.84        | 0.07        | 0.06        | 0.89        | 0.05        | 0.05        | 12.53                     | 13.50                    |
| CNN-TA          | 0.85        | 0.08        | 0.09        | 0.84        | 0.11        | 0.09        | 0.85        | 0.07        | 0.10        | 4.57                      | 14.36                    |

Table 5: Comparison of classification scores of different methods on the stock trading problem. All scores are averaged over 180 stocks and over the days of the test phase.

| Window size ( $\tau$ ) | F1 Score    |             |             | Precision   |             |             | Recall      |             |             |
|------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                        | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         |
| 250                    | 0.61        | 0.23        | 0.19        | 0.91        | 0.15        | 0.12        | 0.46        | 0.51        | 0.50        |
| 300                    | 0.61        | 0.23        | 0.19        | 0.91        | 0.15        | 0.12        | 0.46        | 0.50        | 0.51        |
| 350                    | 0.61        | 0.26        | 0.27        | 0.89        | 0.18        | 0.19        | 0.47        | 0.53        | 0.52        |
| 500                    | 0.61        | 0.26        | 0.27        | 0.89        | 0.18        | 0.19        | 0.47        | 0.52        | 0.53        |
| 650                    | <b>0.61</b> | <b>0.30</b> | <b>0.29</b> | <b>0.85</b> | <b>0.26</b> | <b>0.29</b> | <b>0.51</b> | <b>0.54</b> | <b>0.54</b> |
| 700                    | 0.61        | 0.30        | 0.29        | 0.87        | 0.21        | 0.20        | 0.48        | 0.53        | 0.54        |

Table 6: Classification scores of DRDL (3 layers) for varying window size.

531 Table 3 presents the computational time for forecasting the next day closing  
532 price for our dataset. We distinguish the time required to train the methods  
533 (on the first ten years) and to test them (on the next ten years) using the walk-  
534 forward method described in (Sharma et al., 2021, Section 4.2.1). We observed

| Window size ( $\tau$ ) | F1 Score    |             |             | Precision   |             |             | Recall      |             |             |
|------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                        | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         |
| 250                    | 0.61        | 0.21        | 0.19        | 0.91        | 0.14        | 0.12        | 0.47        | 0.49        | 0.48        |
| 300                    | 0.61        | 0.24        | 0.20        | 0.90        | 0.16        | 0.13        | 0.47        | 0.50        | 0.49        |
| 350                    | 0.62        | 0.25        | 0.24        | 0.89        | 0.17        | 0.16        | 0.48        | 0.51        | 0.52        |
| 500                    | 0.62        | 0.25        | 0.24        | 0.89        | 0.17        | 0.16        | 0.48        | 0.52        | 0.51        |
| 650                    | <b>0.61</b> | <b>0.32</b> | <b>0.34</b> | <b>0.83</b> | <b>0.23</b> | <b>0.26</b> | <b>0.48</b> | <b>0.51</b> | <b>0.53</b> |
| 700                    | 0.59        | 0.33        | 0.32        | 0.84        | 0.20        | 0.23        | 0.46        | 0.53        | 0.51        |

Table 7: Classification scores of DRDL (2 layers) with varying window size.

| Window size ( $\tau$ ) | F1 Score    |             |             | Precision   |             |             | Recall      |             |             |
|------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                        | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         | Hold        | Sell        | Buy         |
| 250                    | 0.84        | 0.10        | 0.10        | 0.85        | 0.10        | 0.10        | 0.85        | 0.12        | 0.12        |
| 300                    | 0.80        | 0.10        | 0.12        | 0.85        | 0.10        | 0.10        | 0.74        | 0.17        | 0.14        |
| 350                    | 0.68        | 0.15        | 0.15        | 0.82        | 0.10        | 0.10        | 0.62        | 0.30        | 0.31        |
| 500                    | 0.59        | 0.18        | 0.22        | 0.86        | 0.15        | 0.14        | 0.46        | 0.51        | 0.51        |
| 650                    | <b>0.59</b> | <b>0.19</b> | <b>0.23</b> | <b>0.88</b> | <b>0.15</b> | <b>0.12</b> | <b>0.45</b> | <b>0.52</b> | <b>0.51</b> |
| 700                    | 0.59        | 0.24        | 0.22        | 0.90        | 0.16        | 0.14        | 0.46        | 0.51        | 0.52        |

Table 8: Classification scores of DRDL (1 layer) with varying window size.

535 the highest computational time with the LSTM approach. The other methods  
536 have rather similar computational time, DRDL (1 layer) being the fastest. The  
537 computational time of DRDL (3 layers), reaching the best performance metrics,  
538 stays reasonable, and is comparable with the one of DeepAR and TFT. Here, we  
539 must recall that, in contrast with most of its competitors (except ARIMA), our  
540 implementation of DRDL method (for both train/test phases) does not exploit  
541 GPU facilities such as PyTorch. Complexity reductions could certainly occur if  
542 this was the case.

543 The stock forecasting results are presented in Fig. 3. The comparison is  
544 carried out between the proposed DRDL for different layer number, LSTM,  
545 ARIMA, DeepAR, TFT, Nbeats method. We observed that in some cases (c-

| Stock symbols             | DRDL (3 layers) | DRDL (2 layers) | DRDL (1 layer) | CNN-TA       | MFNN         | LSTM   |
|---------------------------|-----------------|-----------------|----------------|--------------|--------------|--------|
| WIPRO.BO                  | <b>-13.89</b>   | -23.26          | -29.14         | -18.14       | -27.81       | -47.74 |
| AAPL                      | <b>19.12</b>    | 11.3            | 10.14          | 0            | 12.92        | 0      |
| AMZN                      | -13.23          | -11.92          | 21.23          | <b>30.64</b> | -20.85       | -0.15  |
| IOC.BO                    | -13.48          | -23.28          | <b>-2.68</b>   | -3.03        | -26.42       | -3.1   |
| TATACHEM.BO               | 1.23            | <b>3.83</b>     | 2.19           | -1.54        | -8.32        | 0      |
| SPICEJET.BO               | <b>11.92</b>    | 10.17           | -8.63          | -24.08       | -28.21       | 0      |
| ATML                      | <b>-4.13</b>    | -5.78           | -10.19         | -33.25       | -27.07       | -33.82 |
| DOM.L                     | 4.56            | <b>9.34</b>     | 2.83           | 0.11         | 8.22         | 0.47   |
| INDRAMEDCO.BO             | -5.78           | -10.34          | -3.65          | -14.22       | <b>-3.53</b> | -50.86 |
| Average on all 180 stocks | <b>3.87</b>     | 2.67            | 2.34           | -5.08        | -11.45       | -13.02 |

Table 9: Annualized returns resulting from the stock trading decisions of different methods during the test phase.

546 d), LSTM approach failed to reach satisfying results which might be due to  
547 vanishing gradient issues. In cases (a-b), ARIMA performs quite good when  
548 compared to its performance in other cases (c-d). In contrast, DRDL (3 layers)  
549 reaches stable and satisfactory outcomes. DRDL (2 layers) outperforms DRDL  
550 (1 layer) and both benchmark methods but remains lower quality than its 3-  
551 layers variant.

#### 552 4.4. Numerical results for the stock trading problem

##### 553 4.4.1. Influence of the window size

554 The challenging task with the DRDL approach is to preserve a balance be-  
555 tween the computational time and optimal predictions. We experimented with  
556 various window sizes to analyze and preserve the best parameter for our future  
557 experiments. We present Table 6 which depicts the experimental performance  
558 of DRDL (3 layers) architecture, Table 7 depicts the experimental performance  
559 of DRDL (2 layers) architecture, Table 8 depicts the experimental performance  
560 of DRDL (1 layer) architecture for different window sizes. The empirical results  
561 state that the performance of the approach increases as it feeds more data to the  
562 model for better understanding. We can preserve a balance parameter  $\tau = 650$ ,

563 which indicates stabilized performance. We further use this value in upcoming  
564 experiments.

#### 565 4.4.2. Classification metrics

566 To explain the empirical analyses of the trading process (classification), we  
567 present confusion matrices. The trading process involves classifying the signal  
568 into three classes, namely "Buy," "hold," and "sell" classes. The summarized  
569 performance for 180 stocks by DRDL and other state-of-the-art methods is pre-  
570 sented in Fig. A.5 in Appendix A. Among the three classes, we see the predic-  
571 tion of hold class is captured efficiently when compared to the other classes. The  
572 LSTM approach predicts the best score over the other state-of-the-art methods  
573 when compared to false negatives scores. However, in LSTM and CNN-TA ap-  
574 proaches are highlighted many false positives for the "hold" class. It can be  
575 noted that these deep learning techniques have labeled most signals as hold  
576 class, jeopardizing the model behavior for the other classes ("buy," "sell"). The  
577 nature of the finance market is highly volatile and non-linear; hence we get to  
578 see a highly imbalanced dataset. However, we noticed that the DRDL approach  
579 handles it by imposing an activation function on the operators. These opera-  
580 tors are expected to evolve continuously as we grow deeper with time sequence.  
581 Table. 5 and Fig. A.5 add more weight to the analysis. The results state that  
582 the DRDL approach managed to predict the highly unbalanced data. The sen-  
583 sitivity score (Recall) is presented well by the DRDL approach compared to the  
584 state-of-the-art methods. The diagonals of the confusion matrix of the DRDL  
585 approach also takes the maximum values, which a valid classifier should expect.  
586 When dealing with highly imbalanced dataset such as finance dataset, it is more  
587 important to study classification metrics like F1 Score, Precision and Recall for  
588 each class. This helps in analyzing the model behavior for each class. The Table  
589 5 presents analysis of these classification metrics for DRDL compared to other  
590 deep learning state-of-the-art methods. We conclude that DRDL outperforms  
591 the other methods stated.



592 In Table 5, we also present the computational times (train and test) for  
593 conducting trading simulations for our dataset. Hereagain, LSTM is the more  
594 demanding method at training. All methods have rather comparable test times,  
595 despite DRDL is implemented on CPU only. In particular, besides its proba-  
596 bilistic output, DRDL is not more costly than its competitors. Adding more  
597 layers to DRDL slightly increases its train time, but does not affect much the  
598 test time.

#### 599 *4.4.3. Annualized Returns*

600 Stock market aims to analyze and evaluate the return on investment for  
601 a given stock. Every trader is indeed interested in evaluating his investment  
602 returns and taking risks accordingly. We simulate market scenarios Sezer &  
603 Ozbayoglu (2018) by evaluating the annualized returns by the predicted stock  
604 trading decisions provided by DRDL using 1 to 3 layers as well as the decisions  
605 from from the benchmark models. Table 9 presents a detailed study of nine  
606 stocks for DRDL methodology and state-of-the-art methods. We display only  
607 empirical values for nine stocks and the average results over the 180 stocks. To  
608 make it easy for readers we have highlighted best annualized returns in bold.  
609 It is clearly evident that the DRDL approach yields higher returns when tested  
610 for a duration of 10 years when compared to annualized returns obtained from  
611 deep learning state-of-the-art methods predictions.

#### 612 *4.4.4. Portfolio diversification*

613 Many researchers and traders believe that it is essential to know the asso-  
614 ciated sentiments associated with each stock to understand the stock market.  
615 Traders support and recommend having a mix of stock sentiments in one’s  
616 portfolio. The market is very well divided into three types of stock sentiments:  
617 small-cap, mid-cap, and large-cap. To read about them in detail please refer  
618 to Sharma et al. (2021)[section 4.4.4]. To evaluate this sentiment using the  
619 predicted signals from the proposed approach, we calculated probabilistic quan-  
620 tification, as explained in sec. 3.3. The practitioner uses this quantification

|           | Stock symbols | DRDL<br>3 layers | DRDL<br>2 layers | DRDL<br>1 layer |
|-----------|---------------|------------------|------------------|-----------------|
| Small-cap | ALOKTEXT.BO   | <b>1.04</b>      | 1.20             | 1.09            |
|           | ALKYLAMINE.BO | <b>1.17</b>      | 1.19             | 1.34            |
|           | ZEEMEDIA6.BO  | 0.89             | 0.99             | <b>0.23</b>     |
|           | PVP.BO        | <b>1.34</b>      | 1.98             | 2.78            |
| Mid-cap   | IOC.BO        | 1.02             | 1.05             | <b>0.87</b>     |
|           | TATACHEM.BO   | 0.76             | <b>0.45</b>      | 0.94            |
|           | SPICEJET.BO   | <b>0.34</b>      | 0.65             | 1.20            |
|           | BHEL.BO       | <b>0.20</b>      | 0.51             | 1.15            |
| Large-cap | AAPL          | 1.13             | <b>0.98</b>      | 1.11            |
|           | AMZN          | <b>0.11</b>      | 0.41             | 0.43            |
|           | HINDZINC.BO   | <b>0.03</b>      | 0.65             | 0.45            |
|           | ONGC.BO       | 0.20             | 0.13             | <b>0.09</b>     |
|           | SIEMENS.NS    | 0.12             | <b>0.02</b>      | 0.11            |

Table 10: Comparative analysis of uncertainty quantification provided by DRDL using 1 to 3 layers. The quantification is listed for stocks with market capitalization categories. The log-loss is computed over the test phase.

621 score to have a well-diversified portfolio. The score provides a confidence score  
622 that helps the investor decide where to invest in the market to have a balance  
623 of market sentiments and maximize returns.

624 To understand further, we present Table 10 which provides a log-loss score.  
625 The log-loss score provides the confidence score in terms of its volatility nature,  
626 where the smaller value is considered, the better and less volatile. We evaluated  
627 the confidence score for the proposed approach for different configuration. The  
628 market capitalization of these stocks can be found <sup>6</sup>. The log-loss value provides  
629 the probabilistic inference for the predictions. The inference tries to penalize

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<sup>6</sup><https://finance.yahoo.com/screener>

630 the events for which the method assigns a low probability. We observed that the  
631 log-loss value reached a meager value which indicated good prediction accuracy  
632 in large-cap stocks, which are expected to be least volatile. In contrast, we  
633 achieved a higher log-loss value for predictions associated with small-cap stocks  
634 as they are highly unstable and new to the market.

### 635 **Acknowledgment**

636 The CNRS-CEFIPRA project supported this work under grant NextGenBP  
637 PRC2017. E.C. acknowledges support from the European Research Council  
638 Starting Grant MAJORIS ERC-2019-STG-850925.

### 639 **5. Conclusion**

640 In our approach, time-series sequences are modeled with a flexible Gaus-  
641 sian SSM. The transition matrices (state and observation models) are unknown,  
642 and are estimated thanks to an expectation-minimization strategy, assuming a  
643 particular deep NMF structure. The DRDL approach inherits advantages from  
644 sophisticated modeling techniques while quantifying the uncertainty in the pre-  
645 dictions. We have then adapted the DRDL approach to deal with a challenging  
646 large scale financial time series problem, to target stock forecasting and trading  
647 tasks. In particular, the method is able to successfully operate in an online  
648 processing manner, allowing to capture piece-wise linear characteristics in the  
649 data. The results show that the proposed method outperforms the state-of-the-  
650 art techniques. Given these promising results, we plan as future work to delve  
651 deeper into the area of financial forecasting, including the application of our  
652 technique in forecasting derivatives.

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872 **Appendix A. Results**

873 This section displays results from Confusion matrices and Pearson correlation  
874 graph. Due to space constraints we have attached additional results in appendix.

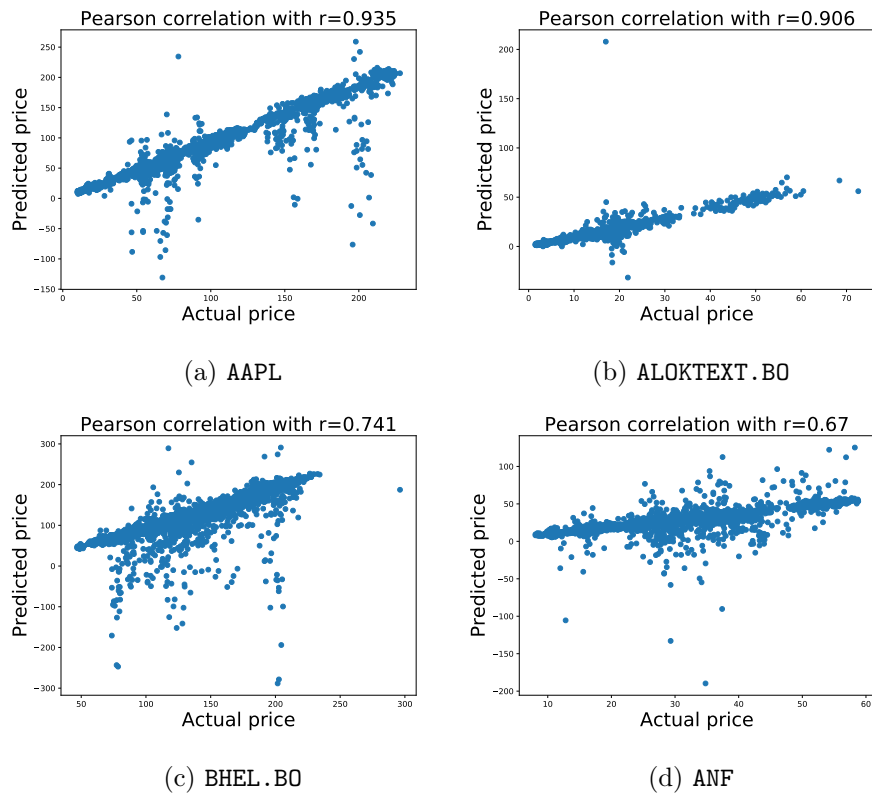
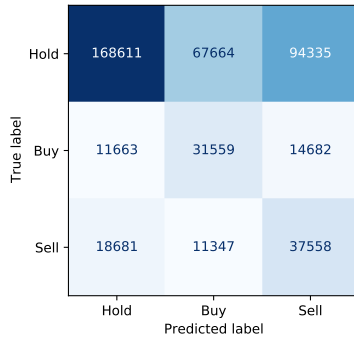
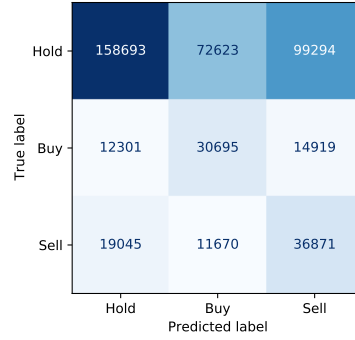


Figure A.4: Pearson correlation graph between ground truth adjusted close price and predicted one with DRDL (3 layers), during test phase, for four different stocks.

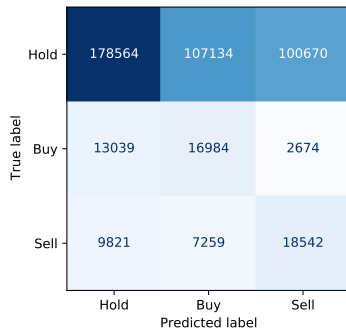
875



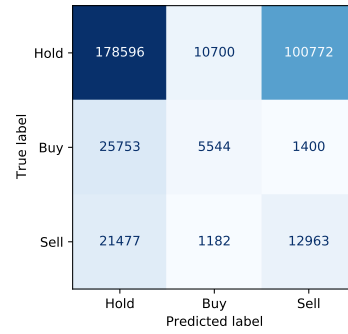
(a) DRDL (3 layers)



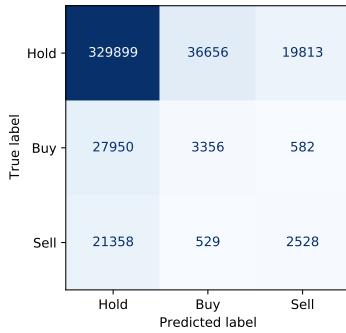
(b) DRDL (2 layers)



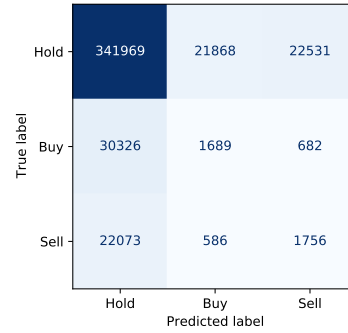
(c) DRDL (1 layer)



(d) MFNN



(e) CNN-TA



(f) LSTM

Figure A.5: Confusion matrices on stock trading classification task (averaged over days in test phase and over stocks) for DRDL with 1 to 3 layers, and deep learning techniques MFNN, CNN-TA and LSTM.