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Ariel Medero, Marcelo Menezes Morato, Vicenc Puig, Olivier Sename. MPC-based optimal parameter scheduling of LPV controllers: Application to Lateral ADAS Control. MED 2022 - 30th Mediterranean Conference on Control and Automation, Jun 2022, Athènes, Greece. 10.1109/MED54222.2022.9837184 . hal-03654127

**HAL Id: hal-03654127**

**<https://hal.science/hal-03654127v1>**

Submitted on 28 Apr 2022

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# MPC-based optimal parameter scheduling of LPV controllers: Application to Lateral ADAS Control\*

Ariel Medero<sup>1,2</sup>, Marcelo M. Morato<sup>2,3</sup>, Vicenc Puig<sup>1</sup> and Olivier Sename<sup>2</sup>

**Abstract**—In this paper, we propose an optimal online tuning scheme for design-related scheduling parameters of adaptive Linear Parameter Varying (LPV) control systems. Specifically, the method is conceived within the Model Predictive Control (MPC) framework, which we demonstrate to ensure an input-to-state stable closed-loop and a recursively feasible optimisation program. The major advantage of the proposed solution is that it automatically determines the LPV scheduling parameters online, without the need to for the designer to develop any scheduling function (which is often a repetitive and obscure task). Moreover, it offers a direct and simple tuning procedure, able to directly incorporate multi-objective performance goals into a single quadratic cost. The proposed method is tested for an LPV Advanced Driver Assistance System, showing enhanced performances when compared to state-of-the-art methods based on nonlinear scheduling functions.

## I. INTRODUCTION

Linear Parameter Varying (LPV) approaches are nowadays a mature topic in the control systems literature [1], with results seen for a wide range of applications, e.g. [2], [3]. Through the LPV toolkit, important results obtained for the control and observation of Linear Time-Invariant (LTI) systems can be extended to the scope Linear Time Varying (LTV) systems. The same is also true for many nonlinear systems, where the LPV approach is enabled by means of the so-called “linear embeddings” [4].

The key idea when using LPV models is that the non-linear/TV is neatly encompassed within bounded scheduling parameters  $\rho$ . There are many ways to classify LPV systems, however, we focus henceforth in their sorting w.r.t. the nature of these varying parameters, as follows: (i) dynamics-related parameters, or (ii) design-related parameter.

In the case of dynamics-related scheduling variables, as the name indicates, their variations are given according to the dynamics of the system itself, therefore requiring them to be either measured or estimated. As for design-related varying parameters, these are additional variables introduced by the control designer, usually chosen in order to formulate adaptive control laws using the LPV framework. This latter

class of LPV systems is often related to robust control methods, for which a non exhaustive list of examples is [5], [6], [7].

In the case of design-related parameters, an important aspect to be mentioned is that the online scheduling is defined by the control designer. Therefore, a key issue that arises is how to define a proper scheduling function in accordance with performance objectives. The most common approach, as seen in the prior references, is to develop some function that maps the varying parameter according to measured system variables (such as states and inputs). Even though these scheduling laws are independent of the LPV controller synthesis itself, the choice of the scheduling function and the tuning of its parameters becomes an additional design step. Usually, developing a coherent scheduling function is quite tedious, inherently empirical, and often offers very little insight w.r.t. to control objectives. Moreover, small variations in the scheduling of the design varying parameter may cause important changes in the performance of the closed-loop system, and thus this task is of uttermost importance.

Taking into account the previous discussion, the main contribution of this work is to propose a new method to automatically determine design-related scheduling parameters. Specifically, the proposed method is based on Model Predictive Control (MPC<sup>1</sup>), which makes use of the closed-loop dynamics and the (previously designed) LPV control law to make an optimal choice of the scheduling variable (online). By taking into account the system dynamics along a prediction horizon and by using a quadratic cost, the proposed strategy exhibits a significant advantage over the standard function-development methods, requiring only simple tuning of performance-related weights, which are able to encompass multi-objective goals.

In this work, the proposed MPC-based approach is used to extend the preliminary result obtained by the Authors in [12], regarding an Advanced Driver Assistance System (ADAS) for the lateral control of vehicles. Specifically, we consider an LPV ADAS ( $H_\infty$ ) controller, for which a hyperbolic function was previously used to map the design-related scheduling parameter w.r.t. a residual signal related to driver errors. Herein, the proposed MPC is implemented instead, which optimally maps the ADAS varying parameter (of the LPV

\*This work has been supported by the French National Research Agency (CNRS, “Investissements d’Avenir”, ANR-15-IDEX-02). It has also been partially funded by the Spanish State Research Agency (AEI) and the European Regional Development Fund (ERFD) through the project SCAV (ref. MINECO DPI2017-88403-R) and by FPI UPC grant 2020FPIUPC-008.

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<sup>1</sup>MPC is a very wide-spread technique for the regulation of constrained processes [8]. With wide applicability, including results for nonlinear and LPV systems [9], [10], the MPC framework offers a very direct method to optimally determine decision variables along a rolling horizon. We stress that the focus of this paper is not the synthesis of an LPV MPC algorithm, as in recent works [10], [11], but rather the exploitation of the MPC framework in the context of defining design-related LPV scheduling parameters.

state feedback control  $K(\rho)$ ) automatically, as illustrated in the scheme presented in Fig. 1.

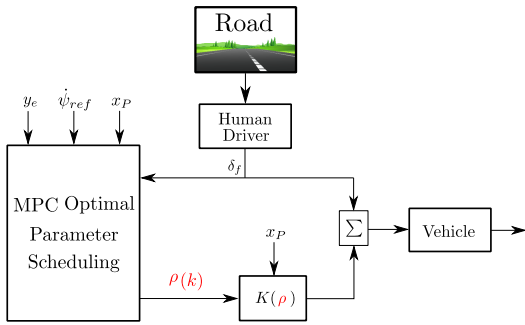


Fig. 1. Driver plus ADAS Closed-Loop System

**Paper organisation.** Sec. II covers the MPC-based optimal scheduling method for the selection of the varying design parameter. Sec. III briefly explains how the LPV state feedback  $H_\infty$  ADAS control  $K(\rho)$  was designed as in [12]. In Sec. IV, simulation results are presented in order to illustrate the performances obtained with the proposed MPC-based scheduling parameter tuning; moreover, advantages of the proposed optimal parameter scheduling approach are discussed. Finally, Sec. V presents some conclusions.

**Notation.** The index set  $\mathbb{N}_{[a,b]}$  represents  $\{i \in \mathbb{N} | a \leq i \leq b\}$ , with  $0 \leq a \leq b$ . The predicted value of a given variable  $v(k)$  at time instant  $k+i$ , computed based on the information available at instant  $k$ , is denoted as  $v(k+i|k)$ .  $\mathcal{K}$  refers to the class of positive and strictly increasing scalar functions that pass through the origin.  $\|\cdot\|$  denotes the 2-norm.

## II. THE PROPOSED MPC STRATEGY

### A. Preliminaries

As previously stated, the main contribution of this work is to propose an MPC-based setup for the automatic definition of design-related scheduling parameters of adaptive LPV control schemes. Specifically, we consider the problem of rendering coherent scheduling parameters for an LPV lateral ADAS steering control strategy. In this system, the MPC framework should enable a direct tuning of the envisioned trade-off between conflicting driving performances objectives.

Lateral ADAS control systems are subjected to constraints. Moreover, the dynamics of the driver of the vehicle also affect the lateral motions as explained in section III. Moreover, as illustrated in Fig. 1, we point out that the ADAS closed-loop system, whose dynamics are given by Eq. (1), is assumed to operate under an adaptive LPV  $H_\infty$  control system, for which  $\rho$  is a design-related scheduling parameter.

Therefore, we consider the following closed-loop discrete-time model, which includes nominal driver, vehicle dynamics, performance weighting functions and ADAS state feedback gain:

$$x_P(k+1) = A_{CL}(\rho(k))x_P(k) + B_w w(k), \quad (1)$$

where  $x_P$  are the states of the generalised plant  $P(\rho)$ , which describes the ADAS  $H_\infty$  control problem, and  $w(k)$  is the

vector of exogenous inputs. In Sec. III (and in particular Fig 3), more details are given, including parameters and synthesis choices of the corresponding LPV control layer.

Note that  $A_{CL}(\rho)$  is a closed-loop matrix within which the control input of the system is embedded; thus, we use:

$$A_{CL}(\rho) := A + B_u K(\rho), \quad (2)$$

where  $A$  and  $B_u$  are open-loop matrices related to the vehicle and driver dynamics, and  $K(\rho)$  is a feedback gain related to the known LPV controller. Specifically, we use an affine parameter feedback representation, for which:

$$K(\rho) := K_0 + K_1 \rho, \quad (3)$$

where  $\rho \in \Omega \subset \mathbb{R}$  is the scheduling variable related to the LPV controller, to be defined via the proposed MPC framework, and  $K_0$  and  $K_1$  are known control matrices.

*Remark 1:* We stress that this state-feedback LPV controller is assumed to be already known and previously synthesised. The goal here is not control design itself, but rather to focus on the problem of correctly scheduling this adaptive LPV control scheme online. Full details on the ADAS controller synthesis are given in [12], and a summary is presented in section III.

For the correct regulation of this system, we use the following constraints in order to represent admissibility:  $x_P(k) \in \mathcal{X}$ ,  $u(k) = K(\rho(k))x_P(k) \in \mathcal{U}$  and  $\rho(k) \in \Omega$ ,  $\delta\rho(k+1) := (\rho(k+1) - \rho(k)) \in \delta\Omega$ ,  $\forall k \geq 0$ , where:

$$\begin{cases} \mathcal{X} & := \{x_P \in \mathbb{R}^{n_x} : \|x_j\| \leq \bar{x}_j, \forall j \in \mathbb{N}_{[1, n_x]}\}, \\ \mathcal{U} & := \{u \in \mathbb{R}^{n_u} : \|u_i\| \leq \bar{u}_i, \forall i \in \mathbb{N}_{[1, n_u]}\}, \\ \Omega & := \{\rho \in \mathbb{R} : \|\rho\| \leq \bar{\rho}\}, \\ \delta\Omega & := \{\delta\rho \in \mathbb{R} : \|\delta\rho\| \leq \bar{\delta\rho}\}. \end{cases} \quad (4)$$

*Assumption 1:* The dynamics of the closed-loop model in Eq. (1) are asymptotically stable, in the sense that all eigenvalues of  $A_{CL}(\rho)$  are found within the unitary circle for frozen values of  $\rho \in \Omega$ .

### B. The MPC Setup

The goal of the proposed MPC is to determine the scheduling parameter  $\rho$  that tunes the LPV controller such that the following objectives are obtained: (i) the closed-loop states  $x_P$  are steered to some steady-state target  $x_r$ , despite the presence of the load disturbances  $w$ ; (ii) the scheduling parameter is maximised<sup>2</sup>, so that the lateral ADAS is used as little as possible; (iii) all system constraints, as given in Eq. (4), are respected. Therefore, we propose the following multi-objective cost function:

$$\begin{aligned} J(x_P(k)) &= \sum_{i=1}^{N_p} \ell(x_P(k+i|k) - x_r, \rho(k+i-1|k)) \quad (5) \\ &= \sum_{i=1}^{N_p} (\|x_P(k+i|k) - x_r\|_Q - \|\rho(k+i-1|k)\|_R), \end{aligned}$$

<sup>2</sup>In the considered ADAS control strategy, a maximisation of  $\rho$  corresponds to a less aggressive steering correction upon the driver's steering. Further discussion is given in Sec. III.

being  $\ell(x_P - x_r, \rho)$  a quadratic stage cost which weights the performance along the horizon. We consider  $Q$  and  $R$  as positive definite weighting matrices, used to imply the envisioned trade-off between state regulation (objective (i)) and scheduling parameter maximisation (objective (ii)). The prediction horizon  $N_p$  determines the size of the (rolling) future window of closed-loop states which are taken into account.

Now, let consider that Assumption 1 holds. Then, the proposed MPC is operated as follows, during the implementation: at each sampling instant  $k$ , the closed-loop states  $x_P(k)$  are measured, and the following optimisation problem, which embeds the performance objectives (i)-(ii), as well as the operational constraints (iii), is solved:

$$\begin{aligned} & \min_{\Theta_k} J(x_P(k)), & (6) \\ \text{s.t.} \quad & x_P(k+j+1|k) = A_{\text{CL}}(\rho(k+j|k))x_P(k+j|k) \\ & \quad + B_w w(k+j|k), \forall j \in \mathbb{N}_{[0, N_p-1]}, \\ & x_P(k+j+1|k) \in \mathcal{X}, \forall j \in \mathbb{N}_{[0, N_p-1]}, \\ & (K(\rho(k+j|k))x_P(k+j|k)) \in \mathcal{U}, \forall j \in \mathbb{N}_{[0, N_p-1]}, \\ & \rho(k+j|k) \in \Omega, \forall j \in \mathbb{N}_{[0, N_p-1]}, \\ & (\rho(k+j|k) - \rho(k+j-1|k)) \in \delta\Omega, \forall j \in \mathbb{N}_{[0, N_p]}, \end{aligned}$$

where  $\Theta_k = \text{col}\{\rho(k+j|k)\}, \forall j \in \mathbb{N}_{[0, N_p-1]}$  gives the sequence of scheduling parameters actions along the prediction horizon. Note that  $x_P(k|k) = x_P(k)$  and  $\rho(k-1|k) = \rho(k-1)$  are the known initial conditions of this optimisation.

Let  $J^*(x_P(k))$  be the optimal solution of the procedure in Eq. (6) at instant  $k$ , from which  $\Theta_k^*$  is the optimal sequence of control inputs. Then, the corresponding MPC-generated scheduling parameter law at time instant  $k$  is the first entry of  $\Theta_k^*$ , i.e.  $\rho^*(k|k)$ , which is passed to the LPV control loop.

*Remark 2:* In the proposed formulation, we consider that the future load disturbance variables  $w(k+j|k), \forall j \in \mathbb{N}_{[0, N_p-1]}$  are constant, for simplicity. This is a reasonable approximation since the rate of change for  $w$  is slower than the rate of the MPC operation. In any case, the proposed formulation can also be used for the case of rapidly changing disturbances, where their future values (or estimates)  $w(k+j|k), \forall j \in \mathbb{N}_{[0, N_p-1]}$  can be plugged directly to the optimisation.

*Remark 3:* The optimisation in Eq. (6) is nonlinear by definition, since the term  $A_{\text{CL}}(\rho)x_P$  in the prediction model yields  $(A+B(K_0+K_1\rho))x_P$ , where both  $\rho$  and  $x_P$  are decision variables. Nevertheless, this nonlinearity can be readily suppressed by solving the optimisation using a bisection mechanism over the bi-linear plane  $\rho x_P$ .

*Remark 4:* In many cases, the rate of decision of the scheduling variable does not necessarily have to match the sampling rate of the process itself. Under these conditions, the MPC can operate less often, thus allowing more time for the optimisation to be solved. For the discrete-time samples when the MPC does not operate, the scheduling variable are simply kept constant.

### C. MPC Properties

From Assumption 1, we already establish that the closed-loop is stable. Nonetheless, we detail next how the proposed MPC renders a recursively feasible optimisation problem, for any feasible starting condition  $x_P(0)$ . Moreover, we demonstrate how the stability of the closed-loop system is maintained and thus not affected by any possible value of  $\rho$  generated by the proposed MPC scheme.

*Definition 1:* Positive Invariant Set

Assume that there exists a set  $\mathbf{X}_f$ .  $\mathbf{X}_f$  is a positively invariant set for the LPV system in Eq. (1) iff, for any  $x_P \in \mathbf{X}_f$  and  $\rho \in \Omega$ , it follows that  $x_P^+ \in \mathbf{X}_f$ , where the successor state  $x_P^+$  is given by  $A_{\text{CL}}(\rho)x$ .

*Assumption 2:* It is implied that: (i) The state set  $\mathcal{X}$  is closed, contains the origin, and represents an admissible positive invariant set; and that (ii) the MPC cost  $J(x_P)$  is continuous and positive for all  $x_P \in \mathcal{X}$ . Moreover  $J(\cdot)$  represents a control Lyapunov function for the LPV system in Eq. (1), meaning that there exist constants  $b > 0$  and  $\sigma > 1$  such that  $J(x_P) \leq b|x_P|^\sigma$ . It is implied, thus, that  $J(A_{\text{CL}}(\rho)x_P) - J(x_P) \leq \ell(x_P - x_r, \rho)$ , for all  $\rho \in \Omega$  and  $J(x_1 - x_r) - J(x_2 - x_r) \leq \alpha_r(|x_1 - x_2|)$

The following Theorem gives sufficient conditions for the closed-loop to be stable and for the optimisation to be recursively feasible.

*Theorem 1:* Stability and Recursive Feasibility

Consider that Assumptions 2 holds. Consider that the LPV system in Eq. (1) has the scheduling parameter  $\rho$  defined through an MPC, as rendered through the optimisation Eq. (6). Then, asymptotic stability is ensured if the following conditions hold  $\forall \rho \in \Omega$ :

- (C1) The origin  $x_P - x_r = 0$  lies in the interior of  $\mathcal{X}$ ;
- (C2)  $\mathcal{X}$  is a positively invariant set;
- (C3) The discrete Lyapunov equation is verified within this invariant set, this is,  $\forall x_P - x_r \in \mathcal{X}$  and  $\forall \rho \in \Omega$ :  $J(x_P^+ - x_r) - J(x_P - x_r) \leq -\ell(x_P - x_r, \rho)$ .
- (C4) The terminal scheduling parameter is admissible, i.e.  $\rho \in \Omega$ .

Assuming that the initial solution of the MPC problem  $P^*(k)$  is feasible, then, the MPC is recursively feasible, steering  $e(k) = x_P(k) - x_r$  to the origin.

*Proof:* This proof is standard; refer to [13]. ■

*Proposition 1 (Stability):* Let there exist a solution positive cost  $J(x_P)$ . Then, the closed-loop LPV system (1) is stable for any MPC-generated input  $\rho \in \Omega$ . That is, for any feasible initial condition  $x_P(0)$  and constant steady-state reference  $x_r$ , such that  $x_P - x_r \in \mathcal{X}$ , it is implied that  $\|x_P(k) - x_r\| \leq \beta(\|x_P(0)\|, k)$ , where  $\beta$  is a  $\mathcal{K}$ -function which passes through the origin.

*Proof:* Assume that  $\mathcal{X}$  is a positive invariant set for the closed-loop dynamics in Eq. (1). Let there be an MPC cost  $J(\cdot)$  such that Assumption 2 holds. Let the LPV controller ensure closed-loop stability for all  $\rho \in \Omega$  (Assumption 1). Since  $\ell(\cdot)$  is a quadratic stage cost, (C3) of Theorem 1 is satisfied, thus establishing stability. ■

*Proposition 2 (Recursive Feasibility):* Consider that  $\mathcal{X}$  is indeed a positively invariant set. Consider an initial con-

dition  $x_P(0) \in \mathcal{X}$ . Consider an optimal sequence  $\Theta_0^* = \{\rho(0)^*, \rho(1)^*, \dots, \rho(N_p - 1)^*\}$ . Then, given  $x_P(0)$ ,  $\Theta_1^c = \{\rho(1)^*, \dots, \rho(N_p - 1)^*, \rho(N_p - 1)\}$  define feasible (candidate) solution of the MPC problem in Eq. (6), which means that the optimisation is recursively feasible.

*Proof:* This proof is standard. Let Assumption 2 hold. Then, from stability and the conditions from Theorem 1, we can infer that the optimisation in (6) is recursively feasible. ■

#### D. Major Advantages

Before giving further details of LPV lateral ADAS control strategy (Sec. III), we stress the major advantages of the proposed MPC-based scheduling parameter generation scheme (main result of this work):

- It provides a direct and simple cost function (see (6)), able to provide an easy-to-tune trade-off between conflicting objectives ((i) and (ii)). The designer must only choose the tuning matrices  $Q$  and  $R$ .
- It automatically generates a coherent scheduling parameter, online. So, the designer do not need to determine any nonlinear function  $\rho(k) = f_\rho(x_P(k))$  to correctly map the scheduling parameter according to objectives, which could be tedious and often repetitive task.
- The resulting optimisation ensures that the closed-loop remains stable, as long as Assumption 1 is satisfied (i.e. the LPV control is properly working). Moreover, for any feasible initial condition  $x_P(0)$ , the following solutions will also be feasible.

### III. LPV $H_\infty$ LATERAL ADAS CONTROL STRATEGY

In this Section, we summarise the main contribution from the preliminary work [12]. The main goal of such a strategy is to determine an auxiliary steering command that will not be felt invasive by the driver, and thus shall only act when needed to maintain vehicle and passenger safety. We note that most of the details from the preliminary work have been here omitted for the sake of brevity.

The closed-loop formed by the ADAS-Driver-Vehicle system is illustrated in Fig. 2. The nominal driver is modeled using a first-order LTI driver model, with:

- *Inputs:* lateral error of the vehicle  $y_e \in \mathbb{R}$ , curvature of the path ahead in terms of  $\dot{\psi}_{ref} \in \mathbb{R}$ .
- *States:*  $x_d \in \mathbb{R}$
- *Output:* driver steering action  $\delta_0 \in \mathbb{R}$

and the vehicle dynamics considered are the lateral vehicle dynamics in the form of the well known ‘‘bicycle model’’ [5], [14] with:

- *Inputs:* steering action  $\delta \in \mathbb{R}$ .
- *States:*  $x_v = [v_y, \psi]^T \in \mathbb{R}^2$ , the vehicle lateral velocity and yaw rate respectively
- *Output:*  $x_v$

Notice in Fig. 2 that the LPV state-feedback controller  $K(\rho)$  acts in parallel to the driver. The generalized plant interconnection  $P(\rho)$  used to describe the LPV  $H_\infty$  control design problem is given in Fig. 3. Two weights are introduced

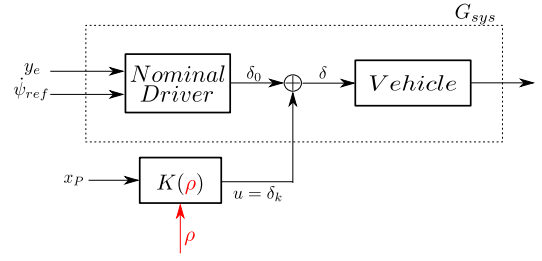


Fig. 2. Driver plus ADAS Closed-Loop System

to determine the performance of the controller. The tracking performance weight  $W_e$  is given by a first-order LTI system with:

- *Inputs:* yaw rate error  $e_\psi \in \mathbb{R}$ .
- *States:*  $x_e \in \mathbb{R}$
- *Output:* tracking performance output  $z_1 \in \mathbb{R}$

The actuator performance weight  $W_\delta$  is a second-order bandpass LTI system with:

- *Inputs:* ADAS steering action  $\delta_k$ .
- *States:*  $x_u \in \mathbb{R}^2$
- *Output:* actuator performance output  $z_2 \in \mathbb{R}$

Two main features of  $W_\delta$  allow it to minimize the intrusiveness of  $K(\rho)$  to the driver:

- 1)  $W_\delta$  is multiplied by the design-related varying parameter  $\rho$ . With this parameter, for large values of  $\rho$ , the control action is heavily penalized, whereas for small  $\rho$ , the assistance steering will gain in control authority. As detailed in Sec. II, it is the objective of scheduling the appropriate choice of  $\rho(k)$  in order to avoid unnecessary intrusions (objective (ii)).
- 2)  $W_\delta$  is shaped as a bandpass LTI system, extended from [15], [5], to restrict the controller to act only in the frequency range between  $f_1 = 1H_z$  and  $f_2 = 10H_z$ . ADAS steering actions outside this range can be felt as invasive and/or cause bothersome vibrations on the steering wheel.

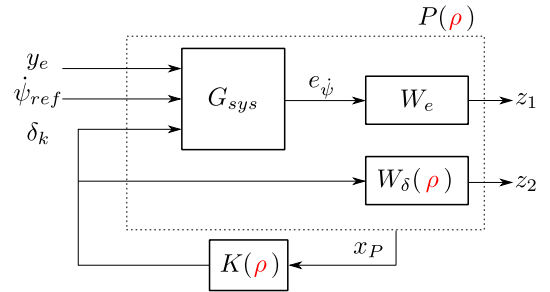


Fig. 3. Control Loop Interconnection for LPV/ $H_\infty$  Design

Notice, the state-feedback law,  $\delta_k = K(\rho)x_P$ , makes use of the full state  $x_P$  from the generalized plant  $P(\rho)$ , with  $x_P$  as:

$$x_P := [x_v^T \ x_e^T \ x_u^T \ x_d^T]^T \in \mathbb{R}^6, \quad (7)$$

Finally, the vector of exogenous inputs is given by:

$$w := [y_e \ \dot{\psi}_{ref}]^T \in \mathbb{R}^2. \quad (8)$$

For the synthesis of the controller  $K(\rho)$ , the so-called gridding approach is used, based on the use of parameter-dependent Lyapunov functions [16]. In this work, a discrete-time synthesis method was used to obtain a parameter dependent state-feedback controller under the affine structure presented in Eq. (3). Accordingly, the vector of selected grid points for synthesis is  $\rho \in [0.1, 1, 100]$ , with a parameter varying rate upper bound of  $\delta\rho = 400T_s$ . The sampling time used for discretisation of the generalised plant at each grid-point is  $T_s = 0.01$  s.

#### IV. SIMULATION RESULTS

Simulations of the proposed ADAS strategy in Fig. 1 using the novel MPC optimal scheduling strategy combined with the LPV  $H_\infty$  controller, are here presented. Note that the state variables  $x_P$  are assumed to be available. In practice, the states regarding the nominal driver and control performance weights can be computed directly online, while the vehicle dynamic states can be either directly measured or estimated. Regarding the vector of exogenous inputs  $w(k)$ , it is assumed to be provided by the higher layer of the ADAS stack which includes perception and planning strategies.

The simulation environment is the same as in [12]: a realistic, nonlinear full-model of a Renault Megane car [5], [14] is used to mimic the vehicles dynamics. The driver, simulated using a driver model, must perform an emergency double-lane change (DLC) maneuver at high speeds  $v_x = 40$  m/s. However the full method is designed with  $v_x = 35$  m/s for robustness assessment.

In order to thoroughly evaluate the effectiveness of the proposed MPC/ADAS strategy, we consider four scenarios:

- No MPC [12]: the ADAS assistance is enacted with the scheduling strategy in [12], derived empirically from a mathematical function.
- MPC (i), “lateral velocity” tuning, for which the MPC penalises only the lateral velocity, thus using  $Q = Q_i = \text{diag}\{1, 0, 0, 0, 0, 0\}$  and  $R = 1$  in (6).
- MPC (ii), “tracking error” tuning, for which the MPC penalises only the tracking error performance state  $x_e$  from  $W_e$ , thus using  $Q = Q_{iv} = \text{diag}\{0, 0, 1, 0, 0, 0\}$  and  $R = 1$  in (6).
- MPC (iii), “trade-off”, for which the MPC is set to penalise both  $v_y$  and  $x_e$ , thus using  $Q = Q_i + Q_{ii}$  and  $R = 1$  in (6).

The proposed MPC scheduling strategy is synthesised with a prediction horizon of  $N_p = 8$  samples, which, considering the sampling time  $T_s = 0.01$  s, amounts to 80 ms of previews system dynamics. The constraints in  $\mathcal{X}$  and  $\mathcal{U}$ , from (4), are chosen according to limitations in the actuator and vehicle dynamics.

Bearing in mind the previous discussion, Fig. 4 presents the scheduling parameter  $\rho$  obtained for the different simulation scenarios. Furthermore, the additional steering generated

by the ADAS scheme (under the LPV control law) is shown in Fig. 5.

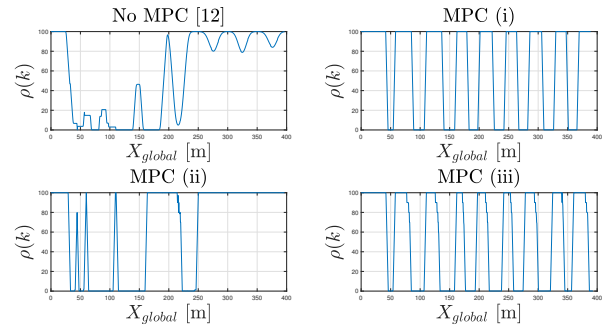


Fig. 4. Scheduling Signal  $\rho(k)$

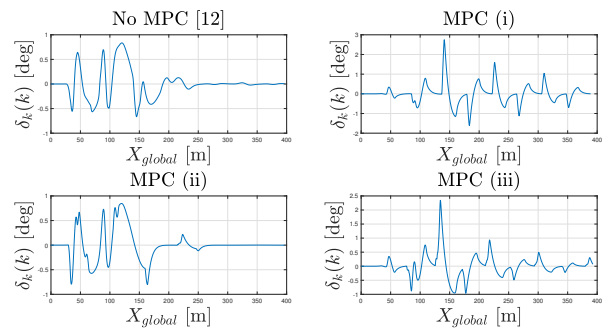


Fig. 5. LPV  $H_\infty$  Controller Command  $\delta_k$ , Under  $\rho(k)$  Scheduling

Meanwhile, Fig. 6 offers a comparison between all simulation scenarios in terms of the vehicle trajectory and the lateral velocity  $v_y$  experienced by the passengers. In this figure a new scenario is added showcasing the open-loop case without ADAS assistance:

- No ADAS, “only driver” case, for which only the simulated driver steers the vehicle, without ADAS assistance.

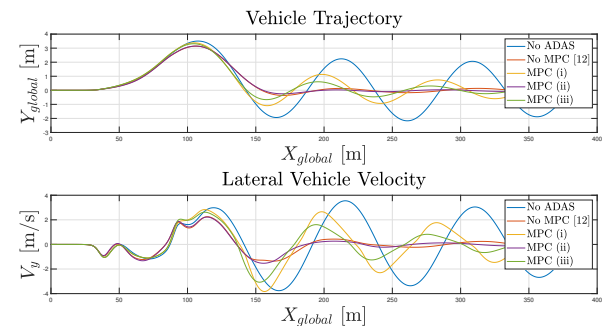


Fig. 6. Global Vehicle Trajectory (Top), Vehicle Lateral Velocity (Bottom)

From these results, we first note that the maneuver poses a great challenge to the driver. From the No ADAS trajectories in Fig. 6, we can see that the vehicle is not stable under the DLC situation, which thus causes a large lateral velocity, something that could cause a dangerous accident due to

under-steering in a real scenario. In the No MPC [12] ADAS case, this situation no longer occurs, even with a small steering assistance as seen in Fig. 5, which already shows how necessary an ADAS could be.

Then, when taking into account the results obtained with the proposed MPC scheduling, we mention that the first tuning MPC (i) (i.e. “lateral velocity”) exhibits the worse performance of the four tested ADAS cases, in terms of the vehicle trajectory and lateral velocity. This issue can be, in parts, attributed to the mismatch between the design and simulation longitudinal velocity  $v_x$ , rendering the MPC system model not accurate enough for the online scenario. Therefore, when using the proposed strategy, special attention should be given to the robustness of the MPC prediction with respect model uncertainties, meaning that a constraint tightening/tube technique may be necessary [10].

We also note that the results obtained in cases No MPC [12] and MPC (ii) are quite similar, as seen in the scheduling of  $\rho$  in Fig. 4 and specially the controller steering in Fig. 5. This means that the scheduling strategy used in [12] induces the LPV  $H_\infty$  controller to prioritise the minimisation of the error signal  $e_{\dot{y}}$ . This issue is not at all evident, since the scheduling strategy in [12] consist of a nonlinear function mapped by an estimation of the driver’s error with respect a nominal driver model, thus the connection with  $e_{\dot{y}}$  is not immediately obvious. On the other hand, the results from the “tracking error” MPC tuning are able to recover the (already good) performances from [12], but with a clear intention on the end result by the control designer, plus, the choice on how much or less importance is given to minimising  $e_{\dot{y}}$  is straightforward by modifying the  $Q$  matrix weight in the MPC cost  $J(\cdot)$ .

Finally, we note the results for case MPC (iii), even though they do not improve on the results from [12], showcases how simple it is with the proposed optimal MPC scheduling strategy to mix different performances by simply modifying the MPC cost  $J(\cdot)$ . It can be easily seen in Fig. 5 and Fig. 6, the results for the case “trade-off” are a middle ground between the absolute strategies in the MPC (i), “lateral velocity” and MPC (ii), “tracking error” cases.

## V. CONCLUSIONS

In this work, we introduced an optimal scheduling scheme for design-related varying parameters, to be used in an LPV control framework. This scheme uses the knowledge from the previously designed LPV control law and the system dynamics as the prediction system model of the MPC-based scheduler, with the tuning being made by means of a quadratic cost minimisation. The scheme was tested for a lateral ADAS problem, improving the previous results obtained by the Authors [12], under adequate tuning. Most importantly, we demonstrate that the MPC scheme is able to enact a wide range of closed-loop behaviours, under different tuning approaches, which is a feature that the designer can intuitively exploit to achieve desired trade-off between conflicting performance goals. This represents the major advantage of the proposed strategy, which allows to combine

multiple performance objectives under a single cost function. This is a significant advantage over most works present in the literature dealing with adaptive LPV controllers, which rely on the ad-hoc development of scheduling functions, where the tuning becomes increasingly difficult when combining multiple variables. Additionally, due to the flexibility of the MPC-based optimal scheduling strategy, it can be extended to most LPV control problems making use of design-related varying parameter.

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