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# *Application de la théorie des jeux à l'optimisation du placement et chaînage des VNFs*

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La technologie Network Function Virtualization (VNF) vise à simplifier le déploiement des services réseau de manière flexible, automatisée et indépendante du fournisseur. Le déploiement de ces services nécessite l'allocation des Virtual Network Function - Forwarding Graphs (VNF-FGs), ce qui implique de placer et d'enchaîner les VNFs en fonction des requêtes des VNF-FGs. Dans ce travail, nous considérons le problème d'allocation hors-ligne des VNF-FGs pour améliorer l'utilisation des ressources du réseau afin de répondre à un maximum de demandes, et réduire les coûts. Etant donné une fonction coût non linéaire associée à chaque ressource réseau, nous formulons le problème comme un problème de routage non linéaire à chemin unique dans un graphe étendu. Ensuite, nous proposons un algorithme heuristique de routage inspiré de la théorie des jeux pour le résoudre. Nous montrons que cet algorithme converge et établissons son rapport d'approximation dans un certain nombre de cas. Les résultats expérimentaux obtenus pour différentes topologies de réseau et différentes fonctions de coût montrent que cet algorithme fournit des solutions de très bonne qualité avec des temps de calcul assez modestes.

**Mots-clefs :** Service Function Chain, Virtual Network Function, Slice as a Service, Game Theory, Optimization.

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## 1 Introduction

According to ETSI, a VNF Forwarding Graph (VNF-FG) is a directed graph of an ordered set of Virtual Network Functions (VNFs) that compose an end-to-end service [1]. The logical service graph described by a VNF-FG is decoupled from the network and has to be mapped to existing network capabilities. This implies not only fulfilling of the service requirements in terms of Quality of Service (QoS), but also considering the constraints of the underlying infrastructure. Two types of decisions have to be made: (a) where to run the VNFs, and (b) how to interconnect them in the physical network, taking into account specific VNFs ordering requirements. This problem is known as the VNF Forwarding Graph Embedding (VNF-FGE) problem and has been studied in the online context where requests to create network services are submitted one after the other [2, 3], and in the offline context where multiple VNF-FGs have to be embedded in the physical network at the same time [4, 5].

Despite the previous efforts on the VNF-FGE problem, little considered the re-optimization of resources utilization on the physical infrastructure with non-linear cost functions. Thus, in this work, we study the offline joint routing and VNF placement problem with non-linear cost functions to reduce total costs and improve resource utilization under a physical infrastructure capacity constraints. We formulate the problem as a non-linear single-path routing problem in an extended graph. To solve this problem, we adapt a single-path routing algorithm inspired by Game Theory which was proposed in [6]. Experimental results obtained for different network topologies and different cost functions show that this algorithm provides very good quality solutions with a rather modest computation time.

## 2 Problem Statement

We are given a physical network represented by a directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V}$  is the set of nodes and  $\mathcal{E}$  is the set of links. A subset  $\mathcal{D} \subseteq \mathcal{V}$  of the nodes have the required compute and storage

resources to host VNF. These nodes will be called the *function nodes* in the following. They are used to execute one or more VNFs out of a set  $\mathcal{F}$  of  $K$  VNFs, where a function  $f \in \mathcal{F}$  is available at function nodes  $v \in \mathcal{D}(f) \subseteq \mathcal{D}$ . A function node corresponds to a logical entity representing reserved compute and storage capacity in a data center. Multiple VNFs may be deployed at the same function node, and a VNF may be replicated at different function nodes. The transmission links of the network and the function nodes are the critical resources in our problem and, in the following, we shall denote by  $\mathcal{R} = \mathcal{E} \cup \mathcal{D}$  the set of these resources.

We are also given a set  $\mathcal{W}$  of traffic demands. Each traffic demand  $w \in \mathcal{W}$  is characterized by its source node  $s_w$ , its destination node  $t_w$  and the sequence  $\mathcal{F}_w = (f_1^w, f_2^w, \dots, f_{K_w}^w)$  of network functions it must flow through in that prescribed order. The sequence is called the *Service Chain* associated with the traffic demand. The processing path of demand  $w$  may therefore be decomposed into different stages, where stage 0 corresponds to the transmission of the original traffic volume from the source to VNF  $f_1^w$  (excluded), stage 1 represents the processing at VNF  $f_1^w$  followed by the transmission to VNF  $f_2^w$ , etc. We define  $b_{k,r}^w$  as the capacity required from resource  $r$  for processing demand  $w$  at stage  $k$ . Here,  $r$  could be either a transmission link  $e \in \mathcal{E}$  or a function node  $v \in \mathcal{D}(f_k^w)$ .

The problem at hand amounts to choosing a single path through the network for each traffic demand such that the function nodes are visited in the prescribed order and that a certain network cost function is minimized. We shall shortly discuss the network cost function to be optimized, but we would first like to emphasize that a first difficulty is related to the modeling of precedence constraints on the order in which the function nodes must be visited by each traffic request. We handle this constraint by constructing an expanded network. Due to the lack of space, we omit the discussion about expanded networks that can be found in [7].

### 3 Penalized Best-Response Algorithm

Suppose that we are given a set  $\Pi_w$  of candidate feasible paths through the expanded network for each demand  $w \in \mathcal{W}$ , a feasible solution for the joint routing and VNF placement problem is then defined as a vector  $\boldsymbol{\pi} = (\pi_w) \in \Pi$ , where  $\pi_w$  is the path assigned to traffic demand  $w$  and  $\Pi = \bigotimes_{w \in \mathcal{W}} \Pi_w$ . The goal is to find a feasible solution  $\boldsymbol{\pi} \in \Pi$  that minimizes the network cost  $F(\boldsymbol{\pi}) = \sum_{r \in \mathcal{R}} y_r(\boldsymbol{\pi}) \phi_r(y_r(\boldsymbol{\pi}))$ . Formally, the problem is as follows:

$$\text{minimize } F(\boldsymbol{\pi}) = \sum_{r \in \mathcal{R}} y_r(\boldsymbol{\pi}) \phi_r(y_r(\boldsymbol{\pi})) \quad (\text{OPT})$$

subject to:

$$\pi_w \in \Pi_w, \quad w \in \mathcal{W}. \quad (1)$$

where  $y_r(\boldsymbol{\pi})$  is the total amount of traffic flowing through resource  $r$  and  $\phi_r(\cdot)$  is the cost per unit of capacity of resource  $r$  that may depend on the total traffic flowing on that resource.

In order to solve the above problem, we propose a heuristic algorithm based on game theory. The idea is to view the traffic demands as the players of a non-cooperative game in which each player independently optimizes its own objective function. Starting from an arbitrary initial feasible solution, the algorithm then mimics the best-response dynamics of the game, that is, the players take turns in some order to adapt their strategy based on the most recent known strategy of the others.

Let us think of the traffic demands as the players of the game. The strategy of player  $w \in \mathcal{W}$  is the path  $\pi_w$  it chooses in the set  $\Pi_w$ , and a strategy profile of the game is a feasible solution to problem (OPT), that is, a vector  $\boldsymbol{\pi} \in \Pi$ . Given the strategy of the other players  $\boldsymbol{\pi}_{-w} = (\pi_u)_{u \neq w}$ , we shall assume that the player  $w$  seeks to solve the following problem:

$$\text{minimize}_{\pi \in \Pi_w} c_w(\boldsymbol{\pi}, \boldsymbol{\pi}_{-w}) = f_w(\boldsymbol{\pi}, \boldsymbol{\pi}_{-w}) + p_w(\boldsymbol{\pi}, \boldsymbol{\pi}_{-w}), \quad (\text{OPT-}w)$$

where the value  $f_w(\boldsymbol{\pi}, \boldsymbol{\pi}_{-w})$  associated to path  $\pi$  by player  $w$  reflects the cost of this path, whereas the term

Topology	# Nodes	# Links	PWL	Quadratic	M/M/1
AARNET	19	46	0.71 ±0.65	1.81 ±0.85	1.43 ±1.76
ARPANET	25	56	1.15 ±0.5	3.725 ±1.55	1.5 ±0.79
NSFNET	13	30	0.42 ±0.49	1.67 ±1.12	0.31 ±0.32
IBM	18	48	1.7 ±0.8	2.85 ±0.98	0.53 ±0.71
CESNET	10	18	0.051 ±0.003	0.047 ±0.008	0.070 ±0.056

**TABLE 1:** Relative gap (% ± std) to the optimal solution for different topologies.

Function Costs	AARNET		ARPANET		NSFNET		IBM		CESNET	
	BR	Gurobi	BR	Gurobi	BR	Gurobi	BR	Gurobi	BR	Gurobi
PWL	7.1	1.2	8.09	1.72	4.93	0.72	6.83	0.9	1.96	0.07
Quadratic	7.89	126	8.89	124	6.3	40	8.27	109	2.33	0.3
M/M/1	8.17	300	9.84	300	7.52	300	9.17	300	2.79	300

**TABLE 2:** Average computing times (seconds) for the 100 problem instances.

$p_w(\pi, \pi_{-w})$  is a penalty term measuring the impact of player  $w$ 's choice on other players. More precisely, we shall assume that the selfish and penalty objective functions of player  $w$  are respectively as follows:

$$f_w(\pi, \pi_{-w}) = \sum_{r \in \pi} y_r^w(\pi) \phi_r(y_r^{-w} + y_r^w(\pi)), \quad (2)$$

$$p_w(\pi, \pi_{-w}) = \sum_{r \in \mathcal{R}} \sum_{u \neq w} y_r^u(\pi_u) [\phi_r(y_r^{-w} + y_r^w(\pi)) - \phi_r(y_r^{-w})] \quad (3)$$

Given the strategies  $\pi_{-w}$  of the other players, the path  $\pi$  minimizing  $c_w(\pi, \pi_{-w})$  in problem (OPT- $w$ ) is known as the best response of player  $w$ . The heuristic algorithm that we propose is an iterative algorithm that starts from an initial feasible solution  $\pi^{(0)}$ . At each iteration, the players update their strategies in a given order by computing their best responses to the strategies of the others. This algorithm has the following characteristics: (a) it converges in a finite number of steps, (b) If it ever reaches a global optimum of problem (OPT), it returns this optimal solution, (c) it computes an optimal solution of problem (OPT) for linear resource costs, that is, when the functions  $\phi_r(\cdot)$  are constant functions  $\phi_r(y) = a_r$  and (d) its approximation ratio is  $(2^{\frac{1}{d+1}} - 1)^{-(d+1)}$  when the functions  $\phi_r(\cdot)$  are polynomials of degree  $d$ , that is,  $\phi_r(y) = \sum_{n=0}^d a_{r,n} y^n$ . Due to lack of space, all proofs are omitted and can be found in [7].

## 4 Performance Evaluation

In this section, we experimentally evaluate the performance of the penalized best-response algorithm. We first describe the different types of objective functions used to solve the penalized best-response algorithm, then we present the numerical results for the different topologies evaluated under the different types of objective functions.

Given the cost per-unit of capacity of resource  $r$  function  $\phi_r$ , we denote by  $\Phi_r = y_r \phi_r(y_r)$  the cost function of resource  $r$ . We consider four types of cost functions: linear ( $\Phi_r = a \times y_r$ ), increasing piecewise linear, quadratic ( $\Phi_r = y_r^2/c_r^2$ ) and the Kleinrock Function  $M/M/1$  ( $\Phi_r = y_r/(c_r - y_r)$ ). In contrast to linear cost functions that assume constant costs per unit capacity, non-linear cost functions allow the costs per unit capacity to increase as the traffic  $y_r$  increases. This allows to achieve a better load distribution.

In order to evaluate the performance of the penalized best-response algorithm, we use five network topologies (see Table 1) collected from the IEEE literature and from *The Internet Topology Zoo* [8]. We expand the original topology with three function nodes that can host two types of VNFs out of three. Then, for each topology, we generate 100 random instances with 25 traffic demands each. We report in Tables 1 and 2 the performance of the penalized Best Response (BR) algorithm for different cost functions under

distinct topologies. We evaluate the algorithm performance by calculating the relative gap between the penalized BR solution and the optimal solution from Gurobi. Following the obtained results, the penalized BR algorithm provides very good quality solutions for the different cost functions with impressive computation time. Its average relative gap to the optimal solution is under 5%, and it is almost negligible for small topologies such as CESNET. The average execution time of the algorithm depends on the size of the topology and on the cost function used. Nevertheless except for the PWL cost function where the MILP solver is known to be efficient and quick at solving, the solver is significantly slower for the other more complex cost functions. In the case of the quadratic cost function, the maximum relative error of the BR algorithm over the 100 instances is 3.725%. However, if we look at the computing times in Table 2, we observe that the BR algorithm is at most 16 times faster and at worst 6.3 times faster than the solver. Here, the standard deviation has been omitted because the runtime values for the different instances for each topology were similar. It is interesting to note that for the  $M/M/1$  cost function, the solver has reached its time limit that has been set to 5 minutes, while the BR algorithm takes less than 10 sec to execute with a solution as close as 1.43% to the solver. While PWL approximations can sometimes be considered for non-linear functions, it is not evident how to choose the intervals to build an approximation for non-linear cost functions.

## 5 Conclusion

In this work, we have considered the offline VNF placement and chaining problem, assuming that a non-linear cost function is associated to each network resources. With respect to previous works, the main originality of the considered model is that the cost per unit capacity of a resource is not constant, instead it grows with its utilization rate, which is an essential feature to achieve a better load distribution. We have formulated the problem as a single-path routing problem in an extended network and adapted an existing game-theoretic algorithm to solve it. Our numerical results suggest that the algorithm provides near-optimal solutions in substantially lower computing times than the solver for non-linear cost functions.

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