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Pierre Gaillard

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From particular polynomials to rational solutions to the mKdV equation

Pierre Gaillard,

Université de Bourgogne Franche Comté,
Institut de mathématiques de Bourgogne,
9 avenue Alain Savary BP 47870
21078 Dijon Cedex, France :
E-mail : Pierre.Gaillard@u-bourgogne.fr

Abstract

Rational solutions to the modified Korteweg-de Vries (mKdV) equation are given in terms of a quotient of determinants involving certain particular polynomials. This gives a very efficient method to construct solutions. We construct very easily explicit expressions of these rational solutions for the first orders $n = 1$ until 10.

Key Words : mKdV equation, rational solutions.

PACS numbers :

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

We consider the modified Korteweg-de Vries (mKdV) equation in the following normalization

$$4u_t + 6u^2u_x - u_{xxx} = 0, \quad (1)$$

with as usual the subscript meaning the partial derivatives.

The mKdV equation has many applications in various fields as in nonlinear optics for example to model supercontinuum generation in optical fibres [1] or to describe pulses consisting of a few optical cycles [2].

A lot of methods have been used to construct solutions to the mKdV equation. We can recall some works about this topic. In 1972, Hirota [4] constructed the exact soliton for the mKdV equation. Tanaka [5] was the first to solve the mKdV equation by using the inverse scattering technique. In 1976, Ono constructed rational solutions to the mKdV equation [6] by using Bäcklund transformation. Tanaka constructed rational solutions to this equation using recurrence relations in [7].

In 2012, solutions to the mKdV equation has been constructed via bilinear

Bäcklund transformation in [8] and rational solutions in terms of wronskians were obtained.

Periodic solutions and rational solution of first and second order were presented in [9] in 2016, by using a Darboux transformation.

Here, we construct rational solutions by using particular types of polynomials. To prove the results, we use the bilinear Hirota method. The solution can be expressed as the derivative in x of the logarithm of the quotient of two determinants. So rational solutions as a quotient of polynomials in x and t are given and we construct explicit rational solutions in the simplest cases $n = 1$ until 10.

2 Rational solutions to the mKdV equation

We consider the following polynomials $p_n(x, t)$ defined by

$$\begin{cases} p_n(x, t) = \sum_{l=0}^n \frac{x^k t^{\binom{n-k}{3}}}{k! \left(\frac{n-k}{3}\right)!} \left(1 - \left[\frac{1}{2}(n-k+1-3\left[\frac{n-k}{3}\right])\right]\right), & k \geq 0, \\ p_n(x, t) = 0, & n < 0, \end{cases} \quad (2)$$

with $[x]$ denoting the largest integer less or equal to x .

We denote $A_n(x, t)$ the determinant defined by

$$A_n(x, t) = \det(p_{n+1-2i+j}(x, t))_{\{1 \leq i \leq n, 1 \leq j \leq n\}} \quad (3)$$

With these notations we have the following result

Theorem 2.1 *The function $v_n(x, t)$ defined by*

$$v_n(x, t) = \partial_x \left(\ln \frac{A_{n+1}(x, t)}{A_n(x, t)} \right) \quad (4)$$

is a rational to the mKdV equation (1)

$$4u_t + 6u^2 u_x - u_{xxx} = 0.$$

3 Explicit rational solutions to the mKdV equation for the first orders

In the following, we will call

$$v_k(x, t) = \partial_x \left(\ln \frac{A_{k+1}(x, t)}{A_k(x, t)} \right)$$

a rational solution to the mKdV equation of order k .

In this section we will give some explicit examples of rational solutions to the mKdV equation.

3.1 Rational solutions of order 1 to the mKdV equation

Example 3.1 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{-(2x^3 + 3t)x}{(-x^3 + 3t)x^2} \quad (5)$$

is a rational to the KdV equation.

3.2 Rational solutions of order 2 to the mKdV equation

Example 3.2 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{3(x^6 - 6tx^3 + 90t^2)(-x^3 + 3t)x^2}{(-x^3 + 3t)^2(-x^6 + 15tx^3 + 45t^2)} \quad (6)$$

is a rational to the mKdV equation.

3.3 Rational solutions of order 3 to the mKdV equation

Example 3.3 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{(4x^{15} - 150tx^{12} + 2250t^2x^9 + 37800t^3x^6 - 141750t^4x^3 + 212625t^5)(-x^6 + 15tx^3 + 45t^2)}{(-x^6 + 15tx^3 + 45t^2)^2(-x^9 + 45tx^6 + 4725t^3)x} \quad (7)$$

is a rational to the mKdV equation.

3.4 Rational solutions of order 4 to the mKdV equation

Example 3.4 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (8)$$

with

$$n(x, t) = -5(x^{24} - 114tx^{21} + 4410t^2x^{18} - 15120t^3x^{15} + 2182950t^4x^{12} - 50604750t^5x^9 + 250047000t^6x^6 + 1875352500t^7x^3 + 1406514375t^8)(-x^9 + 45tx^6 + 4725t^3)x$$

and

$$d(x, t) = (-x^9 + 45tx^6 + 4725t^3)^2(x^{15} - 105tx^{12} + 1575t^2x^9 - 33075t^3x^6 - 992250t^4x^3 + 1488375t^5)x^2$$

is a rational to the mKdV equation.

3.5 Rational solutions of order 5 to the mKdV equation

Example 3.5 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (9)$$

with

$$n(x, t) = 3(2x^{33} - 525tx^{30} + 50400t^2x^{27} - 1984500t^3x^{24} + 65488500t^4x^{21} + 62511750t^5x^{18} - 111583473750t^6x^{15} + 2644247025000t^7x^{12} + 590736037500t^8x^9 + 406131025781250t^9x^6 - 682300123312500t^{10}x^3 + 10234501849687500t^{11})(x^{15} - 105tx^{12} + 1575t^2x^9 - 33075t^3x^6 - 992250t^4x^3 + 1488375t^5)x^2$$

and

$$d(x, t) = (x^{15} - 105tx^{12} + 1575t^2x^9 - 33075t^3x^6 - 992250t^4x^3 + 1488375t^5)^2(x^{21} - 210tx^{18} + 10395t^2x^{15} - 264600t^3x^{12} - 5457375t^4x^9 - 343814625t^5x^6 + 3438146250t^6x^3 + 5157219375t^7)$$

is a rational to the mKdV equation.

3.6 Rational solutions of order 6 to the mKdV equation

Example 3.6 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (10)$$

with

$$n(x, t) = 7(x^{48} - 516tx^{45} + 104760t^2x^{42} - 10697400t^3x^{39} + 664438950t^4x^{36} - 20691389250t^5x^{33} + 308450835000t^6x^{30} - 62918076375000t^7x^{27} + 5078958570984375t^8x^{24} - 107131794804168750t^9x^{21} + 2646690914052281250t^{10}x^{18} + 19530938501266500000t^{11}x^{15} - 766159925718531093750t^{12}x^{12} + 13402310448457399218750t^{13}x^9 + 91280600892196340625000t^{14}x^6 - 228201502230490851562500t^{15}x^3 + 171151126672868138671875t^{16})(x^{21} - 210tx^{18} + 10395t^2x^{15} - 264600t^3x^{12} - 5457375t^4x^9 - 343814625t^5x^6 + 3438146250t^6x^3 + 5157219375t^7)$$

and

$$d(x, t) = (x^{21} - 210tx^{18} + 10395t^2x^{15} - 264600t^3x^{12} - 5457375t^4x^9 - 343814625t^5x^6 + 3438146250t^6x^3 + 5157219375t^7)^2(x^{27} - 378tx^{24} + 42525t^2x^{21} - 2182950t^3x^{18} + 19646550t^4x^{15} - 3094331625t^5x^{12} - 147496474125t^6x^9 + 6637341335625t^7x^6 + 232306946746875t^9)x$$

is a rational to the mKdV equation.

3.7 Rational solutions of order 7 to the mKdV equation

Example 3.7 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (11)$$

with

$$n(x, t) = -(8x^{63} - 7308tx^{60} + 2770740t^2x^{57} - 573388200t^3x^{54} + 73517390100t^4x^{51} - 6018268721850t^5x^{48} + 335265468060600t^6x^{45} - 15896126159991000t^7x^{42} + 112751070091852500t^8x^{39} + 109151834921265386250t^9x^{36} - 9193362965791861106250t^{10}x^{33} + 421668079507718418375000t^{11}x^{30} - 6155017110530839175625000t^{12}x^{27} + 48652560275540649553125000t^{13}x^{24} - 7698733672089088443189375000t^{14}x^{21} + 432691544810520766800717187500t^{15}x^{18} + 275075049234943040535014062500t^{16}x^{15} + 49670782713309725940554613281250t^{17}x^{12} - 1126712952508127399160038261718750t^{18}x^9 + 3889474849754083624497666328125000t^{19}x^6)$$

$$+14585530686577813591866248730468750 t^{20}x^3 + 7292765343288906795933124365234375 t^{21})(x^{27} - 378 tx^{24} + 42525 t^2x^{21} - 2182950 t^3x^{18} + 19646550 t^4x^{15} - 3094331625 t^5x^{12} - 147496474125 t^6x^9 + 6637341335625 t^7x^6 + 232306946746875 t^9)x$$

and

$$d(x, t) = (x^{27} - 378 tx^{24} + 42525 t^2x^{21} - 2182950 t^3x^{18} + 19646550 t^4x^{15} - 3094331625 t^5x^{12} - 147496474125 t^6x^9 + 6637341335625 t^7x^6 + 232306946746875 t^9)^2(-x^{36} + 630 tx^{33} - 135135 t^2x^{30} + 13825350 t^3x^{27} - 638512875 t^4x^{24} + 29499294825 t^5x^{21} + 1179971793000 t^6x^{18} + 54426198952125 t^7x^{15} - 12683959292379375 t^8x^{12} + 132879573539212500 t^9x^9 - 2092853283242596875 t^{10}x^6 - 31392799248638953125 t^{11}x^3 + 31392799248638953125 t^{12})x^2$$

is a rational to the mKdV equation.

3.8 Rational solutions of order 8 to the mKdV equation

Example 3.8 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (12)$$

with

$$\begin{aligned} n(x, t) = & 9(x^{78} - 1500 tx^{75} + 970200 t^2x^{72} - 358835400 t^3x^{69} + 85386089250 t^4x^{66} - 13839887719350 t^5x^{63} + \\ & 1590995300895000 t^6x^{60} - 133885499492745000 t^7x^{57} + 7772156345589260625 t^8x^{54} - 299229467351821256250 t^9x^{51} + \\ & 40195977979910303801250 t^{10}x^{48} - 7885034029944807986250000 t^{11}x^{45} + 860331027602278517691656250 t^{12}x^{42} - \\ & 49042436051036490839057531250 t^{13}x^{39} + 1751863511574639624252341250000 t^{14}x^{36} \\ & - 60341491647562717548686917687500 t^{15}x^{33} + 728485213859962379636955201562500 t^{16}x^{30} \\ & + 205090979865932365476475041337500000 t^{17}x^{27} - 6110692198752612939851069319445312500 t^{18}x^{24} + \\ & 183798097133187256020153764614828125000 t^{19}x^{21} - 2884871725550740774926862886632781250000 t^{20}x^{18} - \\ & 98745174585313076455269232726174921875000 t^{21}x^{15} + 2341892012943419271472979832640650000000000 t^{22}x^{12} + \\ & 232317049249110996043649525549017968750000 t^{23}x^9 + 131767326370980143068507465272333629150390625 t^{24}x^6 - \\ & 100622321956021200161405700753418407714843750 t^{25}x^3 + 1509334829340318002421085511301276115722656250 t^{26})(-x^{36} + \\ & 630 tx^{33} - 135135 t^2x^{30} + 13825350 t^3x^{27} - 638512875 t^4x^{24} + 29499294825 t^5x^{21} + 1179971793000 t^6x^{18} + \\ & 54426198952125 t^7x^{15} - 12683959292379375 t^8x^{12} + 132879573539212500 t^9x^9 - 2092853283242596875 t^{10}x^6 - \\ & 31392799248638953125 t^{11}x^3 + 31392799248638953125 t^{12})x^2 \end{aligned}$$

and

$$\begin{aligned} d(x, t) = & (-x^{36} + 630 tx^{33} - 135135 t^2x^{30} + 13825350 t^3x^{27} - 638512875 t^4x^{24} + 29499294825 t^5x^{21} + \\ & 1179971793000 t^6x^{18} + 54426198952125 t^7x^{15} - 12683959292379375 t^8x^{12} + 132879573539212500 t^9x^9 - \\ & 2092853283242596875 t^{10}x^6 - 31392799248638953125 t^{11}x^3 + 31392799248638953125 t^{12})^2(-x^{45} + \\ & 990 tx^{42} - 363825 t^2x^{39} + 67567500 t^3x^{36} - 6810804000 t^4x^{33} + 445298879025 t^5x^{30} - 8210637059625 t^6x^{27} + \\ & 1121710685720625 t^7x^{24} - 48406130360713125 t^8x^{21} - 23885103343673446875 t^9x^{18} + 1037875841064050113125 t^{10}x^{15} - \\ & 23588549355427309378125 t^{11}x^{12} - 305263579893765180187500 t^{12}x^9 - 9615802766653603175906250 t^{13}x^6 + \\ & 72118520749902023819296875 t^{14}x^3 + 72118520749902023819296875 t^{15}) \end{aligned}$$

is a rational to the mKdV equation.

3.9 Rational solutions of order 9 to the mKdV equation

Example 3.9 The function $v_k(x, t)$ defined by

$$v_k(x, t) = \frac{n(x, t)}{d(x, t)} \quad (13)$$

with

$$\begin{aligned}
n(x, t) = & 5(2x^{99} - 4653tx^{96} + 4799520t^2x^{93} - 2919456540t^3x^{90} + 1177028552700t^4x^{87} - 334613109981150t^5x^{84} + \\
& 69733689267592350t^6x^{81} - 10932392980379871000t^7x^{78} + 1306220634815585989500t^8x^{75} - 121146984881525086233750t^9x^{72} + \\
& 9320789025408230752882500t^{10}x^{69} - 500705890189599166577505000t^{11}x^{66} - 41832040420750126783836600000t^{12}x^{63} + \\
& 18338410130868093522520964700000t^{13}x^{60} - 2730856493733147315608983911000000t^{14}x^{57} + 242934266578687370437331166460875000t^{15}x^{54} - \\
& 14969624200681230581860585801011937500t^{16}x^{51} + 584715350204300649895694060606585156250t^{17}x^{48} \\
& - 4798448149052652504054108653314125000000t^{18}x^{45} + 1269212187595244032943536914536686017187500t^{19}x^{42} - \\
& 179122900491707815151177546562032102442187500t^{20}x^{39} + 12565064668809514621695666757366189442561718750t^{21}x^{36} - \\
& 449301113343170109970511345442173092137152343750t^{22}x^{33} + 573586202683635891089283164387903227397156250000t^{23}x^{30} - \\
& 78877145899749961380668818349383506289020937500000t^{24}x^{27} + 22816724570611143207434688336826913771144789531250000t^{25}x^{24} - \\
& 375334689633655774507281362220239230191992461640625000t^{26}x^{21} + 7735165716727020590083875357398512721527415017382812500t^{27}x^{18} + \\
& 13634147508744289200140961784353357911649986469140625000t^{28}x^{15} \\
& - 62547867595763406255434077653018720492489568895507812500t^{29}x^{12} \\
& + 11338984947224872923457450178681819821218595816010742187500t^{30}x^9 \\
& + 48430918757638440283241990593691840592323493993808593750000t^{31}x^6 \\
& - 90807972670572075531078732363172201110606551238391113281250t^{32}x^3 \\
& + 45403986335286037765539366181586100555303275619195556640625t^{33})(-x^{45} + 990tx^{42} - 363825t^2x^{39} + 67567500t^3x^{36} - \\
& 6810804000t^4x^{33} + 445298879025t^5x^{30} - 8210637059625t^6x^{27} + 1121710685720625t^7x^{24} - 48406130360713125t^8x^{21} - \\
& 23885103343673446875t^9x^{18} + 1037875841064050113125t^{10}x^{15} - 23588549355427309378125t^{11}x^{12} - 305263579893765180187500t^{12}x^9 - \\
& 9615802766653603175906250t^{13}x^6 + 72118520749902023819296875t^{14}x^3 + 72118520749902023819296875t^{15})
\end{aligned}$$

and

$$\begin{aligned}
d(x, t) = & (-x^{45} + 990tx^{42} - 363825t^2x^{39} + 67567500t^3x^{36} - 6810804000t^4x^{33} + 445298879025t^5x^{30} - 8210637059625t^6x^{27} + \\
& 1121710685720625t^7x^{24} - 48406130360713125t^8x^{21} - 23885103343673446875t^9x^{18} + 1037875841064050113125t^{10}x^{15} - \\
& 23588549355427309378125t^{11}x^{12} - 305263579893765180187500t^{12}x^9 - 9615802766653603175906250t^{13}x^6 + 72118520749902023819296875t^{14}x^3 + \\
& 72118520749902023819296875t^{15})^2(-x^{54} + 1485tx^{51} - 868725t^2x^{48} + 269594325t^3x^{45} - 49165491375t^4x^{42} + \\
& 5714434826100t^5x^{39} - 396365167823625t^6x^{36} + 24418778773764375t^7x^{33} - 435655173246418125t^8x^{30} - 194269936514328675000t^9x^{27} - \\
& 40358223939201682263750t^{10}x^{24} + 5744505548909944754437500t^{11}x^{21} - 290076716794050362473171875t^{12}x^{18} + \\
& 1331101840126763068207593750t^{13}x^{15} - 209648539819965183242696015625t^{14}x^{12} - 3997298825900669493827404031250t^{15}x^9 + \\
& 179878447165530127222233181406250t^{16}x^6 + 3147872825396777226389080674609375t^{18})x
\end{aligned}$$

is a rational to the mKdV equation.

We could go on and present more explicit rational solutions, but they become very complicated. For example, in the case of order 10 the numerator includes 59 terms and the denominator 59 terms with big coefficients. It will be relevant to study in detail the structure of these polynomials.

4 Conclusion

Rational solutions to the modified Korteweg de Vries equation have been given in terms of determinants involving particular polynomials. This gives a very efficient method to construct rational solutions of this equation. We have constructed some examples of these rational solutions for the first orders.

We can mention some other recent works about this equation. For example, in [10], first and second-order rational solutions are given as limiting cases of periodic solutions. Explicit periodic and rational solutions of first and second

order are given in [11]. The first four exact rational solutions of the set of rational solutions of the mKdV equation are presented in [12]. Multiple periodic solutions of the mKdV equation are given in [13] and in particular first to third-order rational solutions.

The solutions presented in this work are different from these given in [59] to It would be relevant to better understand the structure of polynomials defining the rational solutions of this equation.

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