Modeling Expert Preference using the Qualitative Belief function Framework

Amel Ennaceur, Zied Elouedi, Eric Lefevre

Modeling Expert Preference using the Qualitative Belief function Framework

Amel Ennaceur, Zied Elouedi
LARODEC, University of Tunis, Institut Supérieur de Gestion Tunis, Tunisia
amel_naceur@yahoo.fr, zied.elouedi@gmx.fr

Eric Lefevre
Univ. Lille Nord of France, UArtois EA 3926 LGI2A France
eric.lefevre@univ-artois.fr

***Abstract*—This paper investigates the problem of preference modeling under multi-criteria decision making methods in which some assessments cannot be provided in the pair-wise comparison process. Therefore, we introduce a new method that will be able to elicitate preferences in an uncertain environment, where the expert may express incomplete and incomparable ones. Indeed, we suggest to transform these qualitative assessments into quantitative information based on belief function framework. Then, in order to illustrate our approach, we propose to compare our method to the existing approaches.**

***Keywords*-Multi-criteria decision making; Qualitative belief function theory; Expert judgments; Pair-wise comparison; Uncertain preferences; Uncertainty;**

I. Introduction

Most multi-criteria decision making (MCDM) methods have been developed using mainly preferences relations [1]. Therefore, the expert is usually required to provide qualitative assessments for determining the performance of each alternative with respect to each criterion and the relative importance of the evaluation criteria with respect to the overall objective of the problem. Therefore, one of the prevailing ways for eliciting expert opinions is pair-wise comparisons. The popularity of the paired comparison methods can perhaps be contributed to the observation that experts are more comfortable making comparisons rather than directly assessing a quantity of interest. There are various methods of pair-wise comparisons. One of the commonly used methods is the Analytic Hierarchy Process (AHP), which was developed by Saaty [2], [3], where experts use a scale to quantify their degree of preferences.

Though its popularity and efficiency, this method is often criticized. Sometimes, a decision maker could have many difficulties when expressing his assessments. Thus, in some cases he cannot either estimate his preference with a numerical value, or there are some situations in which the information may not be quantified. He may also be unable to express his opinions due to the lack of experience. Obviously, rejecting these difficulties in building the pair-wise comparison between criteria and also alternatives is not a good practice. As a result, this lack of consistency in decision making can lead to inconsistent conclusions.

To deal with this uncertainty, a more realistic approach should be proposed. Several AHP methods are combined within uncertain theories. One of these extensions is the Fuzzy AHP appeared in [4], and since then, several fuzzy AHP developments have been proposed [5]. There are also other ways to solve AHP problems, such as referenced AHP [6] etc. In particular in the belief function framework, several works have also been proposed: DS/AHP method [7], belief AHP [8], etc.

Besides, another group of studies attempts to use preference judgment that is closely related to the knowledge representation by fuzzification. To facilitate the pair-wise comparison process, Herrera-Viedma et al. [9] developed a new method, called fuzzy preference relations (Fuzzy PreRa), which focuses on avoiding the inconsistent solutions in the decision making processes. The preference judgment is supposed to be assessed by means of fuzzy preference relations or linguistic terms instead of exact numerical values to deal with vague information.

With regard to such elicitation techniques, we can frequently find that most researchers have focused on dealing with qualitative preference relations but they involve numerical values to provide them.

However, the decision making process has much complexity and uncertainty. In order to avoid the expert making appraisals with difficulty or doing subjective judgments, this study develops an incomplete qualitative preference method under the belief function framework. This theory is chosen since It is considered as a useful theory for representing and managing uncertain knowledge [10]. It provides a convenient framework for dealing with incomplete and uncertain information, notably those given by experts. Using our approach, the decision maker has only to express his opinions qualitatively, based on knowledge and experience that he provides in response to a given question rather than direct quantitative information. Then, our model transforms these preference relations into numerical values using the belief function theory. By adopting our approach, we try to closely imitate the expert reasoning without adding any additional information.

The remainder of this paper is organized as follows. In section 2, we focus on preference relation approaches. Next, we present some useful definitions needed for belief function context. Section 4 reviews some existing qualitative belief function approaches and represents a comparative example

between AHP, fuzzy PreRa and qualitative belief function method. Then, in section 5, our suggested solution will be approached. Section 6 presents an example to illustrate our method. Finally, Section 7 ends this work.

II. Preference Relations

A. Multiplicative Preference Relation (AHP Method)

The AHP approach is a decision-making technique developed by Saaty [2], [3] to solve complex problems of choice and prioritization. The problem is structured hierarchically at different levels. The purpose of constructing this hierarchy is to evaluate the influence of the criteria on the alternatives to attain objectives. So, an AHP hierarchy has at least three levels: The highest level consists of a unique element that is the overall objective. Then, each level of the hierarchy contains criteria or sub-criteria that influence the decision. Alternative elements are put at the lowest level. Once the hierarchy is built, the decision maker starts the prioritization procedure. Elements of each level are paired (with respect to their upper level decision elements) and then compared. This method elicits preferences through pair-wise comparisons based on a nine-point scale [2], which translates the preferences of a decision maker into crisp numbers.

After filling all the pair-wise comparison matrices, the local priority weights are determined by using the eigenvalue method. The objective is then to find the weight of each criterion, or the score of each alternative by calculating the eigenvalue vector. With these values, the AHP method permits to compute a consistency ration to check if the matrix is consistent or not.

B. Fuzzy Preference Relations (Fuzzy PreRa)

Fuzzy PreRa method was proposed by Herrera-Viedma et al. [9] , which focus on avoiding inconsistent solutions in the decision-making processes.

The concept of this method is that if there are n alternatives $X = \{a_1, \ldots, a_n, n \geq 2\}$ then we can obtain the pair-wise preference relation data, from comparing and constructing a consistent reciprocal fuzzy preference relations $\{p_{12}, p_{23}, \ldots, p_{n-1n}\}$. This method follows the one of traditional AHP method characteristics, which is preference relation satisfied transitivity property.

Herrera-Viedma et al. [9] had proof that for a reciprocal additive fuzzy preference relation $P = (p_{ij})$, the following statements are equivalent:

$$p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \forall i < j < k \tag{1}$$

$$p_{i(i+1)} + p_{(i+1)(i+2)} + \ldots + p_{(j-1)j} + p_{ji} = \frac{j-i+1}{2} \forall i < j \tag{2}$$

A decision maker's preference on a set of alternatives X is denoted by a positive preference relation matrix, with membership function: $\mu(a_i, a_j) = p_{ij}$, denotes the preference degree or intensity of the alternative a_i over a_j. If $p_{ij} = 1/2$ implies that there is no difference between a_i and a_j, $p_{ij} > 1/2$ implies a_i preferred to a_j, $p_{ij} = 1$ indicates that a_i is absolutely preferred to a_j, $p_{ij} = 0$ indicates that a_j is absolutely preferred to a_i. P is assumed additive reciprocal, given by: $p_{ij} + p_{ji} = 1 \forall i, j \in \{1, \ldots, n\}$. More details can be found in [9].

III. Belief Function Theory

A. Basic Concepts

The TBM is a model to represent quantified belief functions [11]. Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^Θ the set of all the subsets of Θ [10].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba), called initially by Shafer, basic probability assignment [10]:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{3}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A. The events having positive bbm's are called focal elements. Let $\mathcal{F}(m) \subseteq 2^\Theta$ be the set of focal elements of the bba m.

Associated with m is the belief function is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0 \tag{4}$$

The degree of belief $bel(A)$ given to a subset A of the frame Θ is defined as the sum of all the basic belief masses given to subsets that support A without supporting its negation.

The plausibility function pl expresses the maximum amount of specific support that could be given to a proposition A in Θ. It measures the degree of belief committed to the propositions compatible with A. $pl(A)$ is then obtained by summing the bbm's given to the subsets B such that $B \cap A \neq \emptyset$ [10]:

$$pl(A) = \sum_{B \cap A \neq \emptyset} m(B), \quad \forall A \subseteq \Theta \tag{5}$$

B. Uncertainty Measures

Different uncertainty measures have been defined, such as the composite measures [12]:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 (\frac{|A|}{m(A)}) \tag{6}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

IV. Qualitative Belief Function method

The problem of eliciting qualitatively expert opinions and generating basic belief assignments have been addressed by many researchers [13] [14] [15].

A. Wong and Lingras' Method.

Wong and Lingras [15] proposed a method for generating quantitative belief functions from qualitative preference assessments. So, given a pair of propositions, experts may express which of the propositions is more likely to be true. Thus, they defined two binary relations preference $\succ$ and indifference $\sim$ defined such as:

$$a \succ b \text{ is equivalent to } bel(a) > bel(b) \tag{7}$$

$$a \sim b \text{ is equivalent to } bel(a) = bel(b) \tag{8}$$

where $a, b \in 2^{\Theta}$.

This approach is based on two steps. The first one consists in considering that all the propositions that appear in the preference relations are potential focal elements. However, some propositions are eliminated according to the following condition: if $a \sim b$ for some $a \subset b$, then a is not a focal element.

After that, the basic belief assignment is generated using the two presented Equations 7 and 8. This formulation has multiple belief functions that are consistent with the input qualitative information, and so their procedure only generates one of them.

B. Ben Yaghlane et al.'s Method.

Ben Yaghlane et al. proposed a method for generating optimized belief functions from qualitative preferences [13]. So giving two alternatives, an expert can usually express which of the propositions is more likely to be true, thus they used two binary preference relations: the preference and the indifference relations. The objective of this method is then to convert these preferences into constraints of an optimization problem whose resolution, according to some uncertainty measures (UM), allows the generation of the least informative or the most uncertain belief functions defined as follows:

$$a \succ b \Rightarrow bel(a) - bel(b) \geq \varepsilon \tag{9}$$

$$a \sim b \Rightarrow |bel(a) - bel(b)| \leq \varepsilon \tag{10}$$

where ε is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions A and B. Note that ε is a constant specified by the expert before beginning the optimization process.

Ben Yaghlane et al. proposed a method that requires that propositions are represented in terms of focal elements, and they assume that Θ (where Θ is the frame of discernment) should always be considered as a potential focal element. Then, a mono-objective technique was used to solve such constrained optimization problem:

$$\begin{gathered} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \\ bel(a) - bel(b) \leq \varepsilon \\ bel(a) - bel(b) \geq -\varepsilon \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1, m(a) \geq 0, \forall a \subseteq \Theta; m(\emptyset) = 0 \end{gathered} \tag{11}$$

where the first, second and third constraints are derived from Eqs 9 and 10, representing the quantitative constraints corresponding to the qualitative preference relations.

Furthermore, the proposed method addresses the problem of inconsistency. In fact, if the preference relations are consistent, then the optimization problem is feasible. Otherwise no solutions will be found.

C. A Comparative Example

In this section, we briefly describe the decision problem and the methods used to derive the priorities to be assigned to each criterion. We apply the same example used in [16] [17]. Thus, we consider a "Software selection problem": a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system. There are six important factors used to measure the performance of MAS products based on their technical capabilities and their ability to fulfill managerial expectations. It includes: development interfaces (DI), graphic support (GS), multi-media data support (MS), data file support (DS), cost effectiveness (CE) and vendor support (VS).

1) AHP method.: Lai et al. [16] used the AHP method to solve the selection problem of MASs. Decision makers judge the importance of one criterion over another can be made subjectively and converted to a numerical value using a scale of 1-9 where 1 denotes equal importance and 9 denotes the highest degree of favoritism. The preference relation matrix for pair-wise comparison of criteria is shown in Table I. Thus, the relative ranking of criteria is presented in the last column of Table I.

Table I
PREFERENCE RELATION MATRIX FOR PAIR-WISE COMPARISON OF CRITERIA (AHP)

	DI	GS	MS	DS	CE	VS	Priority	order
DI	1	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	5	7	0.1208	4
GS	3	1	$\frac{1}{3}$	$\frac{1}{4}$	5	6	0.1606	3
MS	4	3	1	2	7	8	0.3596	1
DS	3	4	$\frac{1}{2}$	1	6	8	0.2786	2
CE	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{6}$	1	5	0.0548	5
VS	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{5}$	1	0.0256	6

2) Fuzzy PreRa.: In this section, we use Fuzzy PreRa method re-computation with the six evaluation criteria. So, after transforming the obtaining data presented in Table I and using the Fuzzy PreRa approach, we get the following result (Table II). Thus, the relative ranking of criteria is introduced in the last column of Table II.

Table II
PREFERENCE RELATION MATRIX FOR PAIR-WISE COMPARISON OF CRITERIA (FUZZY PRERA)

	DI	GS	MS	DS	CE	VS	Priority	Order
DI	0.5	0.25	0	0.1577	0.5655	0.9317	0.1469	4
GS	0.75	0.5	0.25	0.4077	0.8155	1.1817	0.1968	3
MS	1	0.75	0.5	0.6577	1.0655	1.4317	0.2466	1
DS	0.8423	0.5923	0.3423	0.5	0.9077	1.2740	0.2452	2
CE	0.4345	0.1845	−0.0655	0.0923	0.5	0.8662	0.1338	5
VS	0.0683	−0.1817	−0.4317	−0.2740	0.1338	0.5	0.0608	6

3) Qualitative Belief Function Method.: In this section, we re-examine the numerical example investigated by Lai et al. [16] using a qualitative belief function method. We propose to use Ben Yaghlane et al. approach. First, we propose to transform the obtained matrix in Table I into preference relation matrix without quantifying the degree of preferences (see Table III). By adopting this approach, the expert is not obliged to fill all the pair-wise comparisons; he is able to express his preferences freely.

Table III
PREFERENCES RELATION MATRIX

	DI	GS	MS	DS	CE	VS
DI	-	-	-	-	$\succ$	$\succ$
GS	$\succ$	-	-	-	$\succ$	$\succ$
MS	$\succ$	$\succ$	-	$\succ$	$\succ$	$\succ$
DS	$\succ$	$\succ$	-	-	$\succ$	$\succ$
CE	-	-	-	-	-	$\succ$
VS	-	-	-	-	-	-

Once the preferences relations are defined, the corresponding basic belief assignment (bba) should be constructed. We suggest to transform our problem into a constrained optimization model in order to choose the optimal solution. We assume that $\varepsilon = 0.01$ and the uncertainty measures is H since it has a unique maximum as defined in Equation 6.

The results obtained in Table IV show that the qualitative belief function method coincides with the AHP and the Fuzzy PreRa approaches. We can find whatever the decision maker gives any pair-wise comparison types, the six criteria show the same ranking. By adopting a qualitative belief function method, expert is not obliged to transform qualitative preferences into quantitative information. Therefore, the evaluating process can be more simple and more flexible.

Table IV
THE OBTAINED BBA USING BEN YAGHLANE ET AL. METHOD

Criteria	DI	GS	MS	DS	CE	VS	Θ
bba	0.079	0.089	0.109	0.099	0.069	0.059	0.495
order	4	3	1	2	5	6	

However, in real life, a decision maker could have many difficulties when expressing his opinions. He may develop an incomplete preference ordering in which some assessments cannot be provided. Therefore, in next section, we introduce our proposed approach dealing with incomplete and incomparable preferences.

V. BELIEF PREFERENCE RELATION UNDER DECISION MAKING PROCESS

As presented above, representing efficiently the expert preferences is a crucial task in elaborating the necessary data for a considered problem. Therefore, we propose a realistic model that is able to efficiently imitate the expert reasoning and to construct belief functions when dealing with incomplete and incomparable preference relations.

A. Incompleteness in the Belief Function Theory

By adopting our approach, we try to differentiate incomplete preferences from incomparable ones. This situation is illustrated by complete ignorance, missing information, lack of knowledge or an ongoing preference elicitation process. Incompleteness represents then simply an absence of knowledge about the relationship between these pairs of alternatives. In other words, a partial order allows some relations between pairs of alternatives to be unknown.

Example. Given three alternatives $\Theta = \{a, b, c\}$, an incomplete order can be for example: $(a \succ c, b \sim c)$ or $(c \succ a, a \succ b)$, where some relations between pairs of alternatives are unknown.

B. Incomparability in the Belief Function Theory

A missing value in a linguistic preference relation is not always equivalent to a lack of preference of one alternative over another. A missing value can also be the result of the incapacity of an expert to compare one alternative over another because they are too different. To model this situation, we first consider how to represent the incomparability relation. In fact, our problem here is that incomparability is expressed entirely in terms of negations: if $a?b$ then $\neg(a \succ b) \wedge \neg(b \succ a)$.

By definition, a couple of alternatives (a, b) belongs to the incomparability relation if and only if the expert is unable to compare a and b. Furthermore, it is hard to see what kind of behavior could correspond to this relation. If neither a nor b is chosen, then the expert may not be able to tell which alternative is better, since not $a \succ b$, not $b \succ a$ and not $a \sim b$. In other terms, we apply incomparability when the preference profiles of two alternatives are severely conflicting.

The question now is how to formalize this situation in the belief function framework. In order to build this new

preference relation, we may accept that there exist positive reasons which support the relation $\neg(a \succ b)$ and also there exist sufficient negative information to establish the relation $(a \succ b)$. These two assumptions can properly model the contradictory information. Besides, we can surely establish that "a is preferred to b" as there are not sufficient reasons supporting the opposite and there are sufficient information against it, while we can also surely establish that "b is preferred to a" for the same reasons. Therefore, a and b are in conflicting position. On the other hand, and based on the belief function framework and as defined by Boujelben et al. [18], the incomparability situation appears between two alternatives when their evaluations given by basic belief assessments differ significantly.

Consider two alternatives a and b, as proved in Wong et al. [15], the belief function exists since the preference relation $\succ$ satisfies the following axioms:

1) Asymmetry: $a \succ b \Rightarrow \neg(b \succ a)$
2) Negative Transitivity: $\neg(a \succ b)$ and $\neg(b \succ c) \Rightarrow \neg(a \succ c)$
3) Dominance: For all $a, b \in 2^{\Theta}, a \supseteq b \Rightarrow a \succ b$ or $a \sim b$
4) Partial monotonicity: For all $a, b, c \in 2^{\Theta}$, if $a \supset b$ and $a \cap c = \emptyset \Rightarrow (a \cup c) \succ (b \cup c)$

So, Wong et al. have justified the existing of the following relation: $a \succ b \Leftrightarrow bel(a) > bel(b)$

In other words, Wong et al. have proved that it may exist functions other than the belief functions, which are also compatible with a preference relation such that for every $a, b \in 2^{\Theta}$: $a \succ b \Leftrightarrow f(a) > f(b)$.

If and only if the relation $\succ$ satisfies the previous axioms.

Similarly to this idea, we can prove that the plausibility function also exists since the preference relation $\succ$ satisfies the previous axioms. Besides, we can conclude that it exists a plausibility function pl: $2^{\Theta} \rightarrow [0, 1]$ such as:

$$a \succ b \Leftrightarrow pl(a) > pl(b) \tag{12}$$

To summarize, we can get the following relation:

$$a \succ b \Leftrightarrow bel(a) > bel(b) \text{ and } pl(a) > pl(b) \tag{13}$$

As we have defined previously, the incomparability situation appears between two alternatives when their preference profiles are severely conflicting. That is when their evaluations given by basic belief assessments differ significantly. We can then intuitively conclude from Equation 13 that, if a is incomparable with b, then:

$$a?b \Leftrightarrow bel(a) \geq bel(b) \text{ and } pl(a) \leq pl(b) \tag{14}$$

The first part of Equation 14 supports the assumption "a is preferred to b" however the second one supporting the opposite affirmation. Also, the second part of the Equation supports the assumption "b is preferred to a" and the first part affirms the opposite assumption.

Consequently, our purpose is then to prove the existing of the previous Equation 14 in order to correctly represent the bba relative to the incomparability relation.

Proof: According to the definition of the plausibility function, we have:

$$pl(a) = bel(\Theta) - bel(\bar{a}).$$

We start from the second part of the Equation 14, our assumption is: $pl(a) \leq pl(b)$
$\Leftrightarrow bel(\Theta) - bel(\bar{a}) \leq bel(\Theta) - bel(\bar{b})$
$\Leftrightarrow$ - $bel(\bar{a}) \leq -bel(\bar{b})$
$\Leftrightarrow bel(\bar{a}) \geq bel(\bar{b})$
Using the Equation 7, we can therefore conclude that: $\bar{a} \succ \bar{b}$*, which means that* $\neg(a \succ b)$*. This contradicts with the assumption "if* $pl(a) \leq pl(b)$ *then* $a \succ b$*". Hence, if we have* $pl(a) \leq pl(b)$*, then* $b \succ a$*. However, from the first part of the assumption, we have:* $a \succ b$*.* ■

As a conclusion, such representation of incomparability (Equation 13), enables us to correctly express the conflicting information produced by the alternative a and the alternative b. In fact, the first part of the Equation 14 "$bel(a) \geq bel(b)$" implies that a is preferred to b. Then, the plausibility function is used since it expresses the maximum amount of specific support that could be given to a proposition a. However, when we define the second part of the Equation 14, we propose to assume that $pl(a) \leq pl(b)$ which means that b is preferred to a. This contradicts the first assumption, and can properly express the conflicting information's produced by a and b.

C. Computational Procedure

Now and after modeling the incompleteness and the incomparability preferences, we propose to extend Ben Yaghlane et al. method [13]. We transform these preferences relations into constraints as presented in section 3.4, we get:

$$\begin{gathered} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \\ bel(a) - bel(b) \leq \varepsilon \\ bel(a) - bel(b) \geq -\varepsilon \\ bel(a) \geq bel(b) \\ pl(a) \leq pl(b) \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1; m(a) \geq 0; \forall a \subseteq \Theta; m(\emptyset) = 0 \end{gathered} \tag{15}$$

where the first, second and third constraints of the model are derived from the preference and indifference relations. The fourth and fifth constraints correspond to the incomparability relation. ε is a constant specified by the expert before beginning the optimization process.

VI. Example

Let us consider a problem of eliciting the weight of the candidate criteria. The problem involves four criteria: $\Theta =$

$\{a, b, c, d\}$. After eliciting the expert opinions, the decision maker has validate the following comparison matrix (see Table V).

Table V
PREFERENCES RELATION MATRIX

	a	b	c	d
a	-	$\succ$	-	$\succ$
b	-	-	$\succ$	$\sim$
c	-	-	-	?
d	-	-	-	-

Next, these obtained relations are transformed into optimization problem according to our proposed method (Table VI). We assume that $\varepsilon = 0.01$ and the uncertainty measures is H since it has a unique maximum as defined in Equation 6.

Table VI
THE OBTAINED BBA USING OUR PROPOSED MODEL

	a	b	c	d	a, b, d	a, b, c	Θ
m	0.084	0.074	0.064	0.064	0.214	0.214	0.285
bel	0.084	0.074	0.64	0.064	0.436	0.436	1
pl	0.797	0.787	0.563	0.563	0.935	0.935	1

By applying our presented solution, it is easy to see that our method aggregates all the elicited data. Here, in the present example, all the incomparabilities are detected. We obtain for example $c?d$. Thus, a quantitative information is constructed from incomplete and even incomparable criteria.

VII. CONCLUSION

In this study, a qualitative belief function method is used to elicitate expert preferences in an uncertain environment. Indeed, a new model for constructing quantitative information from elicited expert opinions has been defined, that takes into account the incomplete and even the incomparable alternatives or criteria. The originality of our model is its ability to generate quantitative information from qualitative preferences only.

Comparing with AHP and Fuzzy PreRa approaches, the qualitative belief function method provide a greater flexibility for eliciting expert opinions. Then, an extension of Ben Yaghlane et al. model has been proposed to deal with imperfect assessment. An interesting future work is to combine our proposed model with other MCDM methods.

REFERENCES

[1] M. Zeleny, *Multiple Criteria Decision Making*. McGraw-Hill Book Company, 1982.

[2] T. Saaty, "A scaling method for priorities in hierarchical structures," *Journal of Mathematical Psychology*, vol. 15, pp. 234–281, 1977.

[3] ——, *The Analytic Hierarchy Process*. McGraw-Hill, New-York, 1980.

[4] P. V. Laarhoven and W. Pedrycz, "A fuzzy extension of Saaty's priority theory," *Fuzzy Sets and Systems*, vol. 11, pp. 199–227, 1983.

[5] F. A. Lootsma, *Fuzzy Logic for planning and Decision-Making*. Kluwer Academic Publishers, 1997.

[6] B. Schoner and W. C. Wedley, "Ambiguous criteria weights in AHP: consequences and solutions," *Decision Sciences*, vol. 20, pp. 462–475, 1989.

[7] M. Beynon, B. Curry, and P. Morgan, "The Dempster-Shafer theory of evidence: An alternative approach to multicriteria decision modelling," *OMEGA*, vol. 28(1), pp. 37–50, 2000.

[8] A. Ennaceur, Z. Elouedi, and E. Lefevre, "Handling partial preferences in the belief AHP method: Application to life cycle assessment," *Proceedings of the International Conference of the Italian Association for Artificial Intelligence, LNAI 6934, Springer-Verlag*, pp. 396–401, 2011.

[9] E. Herrera-Viedma, F. Herrera, F. Chiclana, and M. Luque, "Some issues on consistency of fuzzy preference relations," *European Journal of Operational Research*, vol. 154, pp. 98–109, 2004.

[10] G. Shafer, *A Mathematical Theory of Evidence*. Princeton University Press, 1976.

[11] P. Smets and R. Kennes, "The Transferable Belief Model," *Artificial Intelligence*, vol. 66, pp. 191–234, 1994.

[12] N. Pal, J. Bezdek, and R. Hemasinha, "Uncertainty measures for evidential reasoning I: A review," *International Journal of Approximate Reasoning*, vol. 7, pp. 165–183, 1992.

[13] A. Ben Yaghlane, T. Denoeux, and K. Mellouli, "Constructing belief functions from expert opinions," *Proceedings of the 2nd International Conference on Information and Communication Technologies: from Theory to Applications (ICTTA06), Damascus, Syria*, pp. 75–89, 2006.

[14] N. Bryson and A. Mobolurin, "A process for generating quantitative belief functions," *European Journal of Operational Research*, vol. 115, pp. 624–633, 1999.

[15] S. Wong and P. Lingras, "Representation of qualitative user preference by quantitative belief functions," *IEEE Transactions on Knowledge and Data Engineering*, vol. 6, pp. 72–78, 1994.

[16] V. Lai, R. Trueblood, and B. Wong, "Software selection: a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system," *Information and Management*, vol. 36, pp. 221–232, 1999.

[17] T. Wang, Y. Chen, and Y. Chiang, "Incomplete linguistic preference relations to evaluate multimedia authoring system," *WSEAS TRANSACTIONS on MATHEMATICS*, vol. 7, pp. 214–223, 2008.

[18] M. A. Boujelben, Y. D. Smet, A. Frikha, and H. Chabchoub, "A ranking model in uncertain, imprecise and multi-experts contexts: The application of evidence theory," *International Journal of Approximate Reasoning*, vol. 52, pp. 1171–1194, 2011.